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A Distributed Control Strategy for Parallel DC-DC Converters

Mahdieh S. Sadabadi

***Abstract*— This letter addresses the problem of voltage regulation and balanced current sharing in a parallel connection of heterogeneous DC-DC converters sharing a common ZIP (constant impedance, constant current, and constant power) load. To this end, a distributed dynamic control approach is developed. The proposed control approach does not rely on the load profile and the number of active converters. The paper describes theoretical aspects in rigorous Lyapunov-based stability analysis, load-independent characteristic, scalability, and plug-and-play feature of the control design, and verifies the performance of the proposed control mechanism via simulation case studies in MATLAB/Simscape Electrical environment.**

***Index Terms*— Distributed control, parallel DC-DC converters, stability analysis, constant power load.**

I. Introduction

DUE to a limited output power capacity of a single DC-DC converter, the parallel interconnection of DC-DC converters is drawing continually increasing attention. A parallel-converter system offers improved efficiency, high reliability, and ease of maintenance [1], [2]. Additionally, a parallel collection of DC-DC converters may reduce the size and weight of the required power units [1]. With more widespread use of parallel converter systems in high-performance low-voltage/high-current applications, the appropriate design of control strategies that ensure stability and reliability plays an important role.

A. Literature Review on Parallel DC-DC Converters

A main challenge in the parallel interconnection of DC-DC converters is to ensure voltage stability and balanced current sharing amongst all the active converters. An inappropriate current sharing causes overloading and overheating of the constituent converters, leading to the reduced system reliability and/or the failure of the overall system [3]. Several current sharing approaches have been proposed in the literature. The current sharing strategies can be classified into droop (e.g. [4], [5]) and active-sharing categories (e.g. [1], [3], [6]–[13]). The droop methods are based on a virtual resistance added to the output characteristic of the individual converters. In these approaches, the output voltage of each converter droops by increasing the load current. Although droop-based approaches are simple and have a decentralized structure, they might lead to poor voltage regulation [7]. Active current-sharing schemes are widely used control approaches in the parallel-connected converter systems. These approaches generally rely on a dual-loop control structure, composed of an outer voltage control loop and an inner current control loop, where the dynamics of the control loops are decoupled based on a frequency consideration basis [2]. However, as shown in [14], the frequency separation and different bandwidth of each loops deteriorate the transient performance and might affect the closed-loop stability. To deal with this issue, a geometrical decomposition-based approach has been recently proposed in [14]. In this approach, the dynamics of the load voltage regulation and current sharing are decoupled with no frequency separation assumptions. The results of [14] have been presented in a port-controlled-Hamiltonian framework in [15]. In these techniques, the voltage regulation and current sharing are achieved via a centralized control framework, where the controller of each converter has access to the sum of all the converter currents. This centralized communication protocol is crucial for stability and proper operation of such control schemes. Although these approaches could tackle the problem of frequency separation in the active current-sharing control strategies, their centralized control architecture limits their reliability and scalability. Furthermore, due to the dependency of these approaches on the number of existing converters and the physical parameters of the converters, they are susceptible to parametric uncertainties and cannot provide a plug-and-play framework.

Distributed averaging-based control approaches for DC systems have been developed in [16]–[19]. Although these approaches do not rely on the frequency separation arguments for the current sharing and voltage regulation, they are based on several assumptions and significant simplifications which are not necessary valid in the real-world applications or hard to satisfy. These include considering lossless inductors and linear loads (e.g. [16]), a simplified model of the individual converters as ideal voltage sources or first-order dynamics (e.g. [18], [19]), and specific voltage/control state initialization (e.g. [17], [19]).

B. Contributions

Motivated by the challenges stated above, in this letter, a distributed dynamic control approach for the parallel interconnection of heterogeneous DC-DC buck converters is proposed. By means of the proposed control scheme, the voltage regulation and current sharing are simultaneously achieved with

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M. S. Sadabadi is with the Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield, United Kingdom (e-mail: m.sadabadi@sheffield.ac.uk).

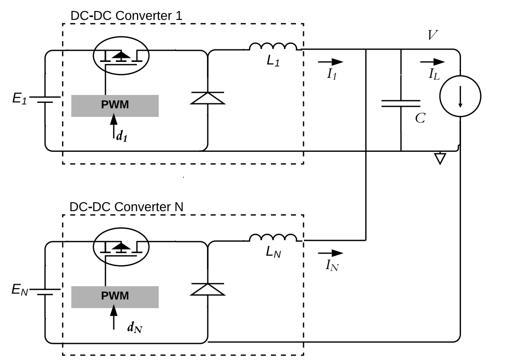

Fig. 1. Configuration of a parallel connection of N DC-DC converters.

no frequency separation assumptions. We provide theoretical foundations and concise stability certificates of the control design that are grounded on the Lyapunov stability methods and graph theory. We also propose a new framework based on switched system theory to characterize the plug-and-play functionality of the DC-DC converters. By virtue of this framework, the control mechanism allows the plug-and-play operation of the DC-DC converters as long as the communication topology of the controllers forms an undirected and a connected graph. The performance of the proposed control scheme is verified using simulation case studies carried out in the MATLAB/Simscape Electrical. The obtained results confirm the effectiveness of the proposed distributed control approach in load voltage regulation and current sharing.

The contributions of this letter lie in the following aspects: (i) Compared with the recent works in [14], [15], the proposed control mechanism has a distributed structure and does not rely on load characteristics, the number of active converters, and the output capacitance. It also guarantees the robust stability against the load variations and uncertainties in the converter parameters. (ii) Unlike the proposed current-sharing schemes in [1], [3], [6]–[12], the proposed control strategy in this letter simultaneously achieves balanced current sharing and voltage regulation without any frequency separation assumptions and/or decoupling between voltage regulation and current distribution dynamics. (iii) Furthermore, unlike the proposed distributed averaging control approaches in [16], the proposed control approach in this letter is robust against the parasitic resistance of the inductors in the DC-DC converters. (iv) Moreover, the proposed control strategy in this letter adopts a nonlinear ZIP (constant impedance, constant current, and constant power) load, which is not considered in [3], [7], [14]–[16], and provides a rigorous robust local stability certificate for the parallel DC-DC converters.

Notation: The notation used in this paper is standard. In particular, $\mathbf{1}_n$ is an $n\times 1$ vector of ones and $\mathbf{I}_n$ is an $n\times n$ Identity matrix. The symbols X^T and $X=[x_{i,j}]$ denote the transpose of matrix X and a matrix with entries $x_{i,j}$. Throughout the paper, $\mathrm{col}(x_i) = \begin{bmatrix} x_1^T & x_2^T & \dots & x_n^T \end{bmatrix}^T$ and $[a] = \mathrm{diag}(a_1, a_2, \dots, a_n)$. For a symmetric matrix X, the positive definite and positive semidefinite operators are respectively shown by $X \succ 0$ and $X \succeq 0$. We define the sets $\mathbb{R}_+ := \{x \in \mathbb{R} \mid x > 0\}$ and $\mathbb{R}_{\geq 0} := \{x \in \mathbb{R} \mid x \geq 0\}$.

II. Parallel DC-DC Converters

Fig. 1 shows the general configuration of a parallel connection of N DC-DC buck converters sharing a common load. The load is considered as a nonlinear ZIP (constant impedance, constant current, and constant power) load. In this figure, $V(t)$, $I_L(t)$, $I_i(t)$, $d_i(t)$, and E_i respectively the load voltage, the load current, the converter current, the duty cycle of the converter, and the voltage of the input side of the converter i. The parameters L_i and C are the inductance of the converter i and the output capacitance, respectively.

A. Model of Parallel DC-DC Buck Converters

Based on an average model, the dynamics of each DC-DC converter can be described as follows [20]:

$$\begin{aligned} C\dot{V}(t) &= \sum_{i=1}^{N} I_i(t) - I_L(t), \\ L_i \dot{I}_i(t) &= -V(t) - r_i I_i(t) + u_i(t), \\ I_L(t) &= \frac{V(t)}{R} + I_L^* + \frac{P_L}{V(t)}, \end{aligned} \tag{1}$$

for $i=1,\dots,N$, where $u_i(t) = E_i d_i(t)$, $0 \leq d_i \leq 1$, $r_i \in \mathbb{R}_+$, $R \in \mathbb{R}_+$, $I_L^* \in \mathbb{R}_{\geq 0}$, and $P_L \in \mathbb{R}_{\geq 0}$ are the control input, the duty cycle of the DC-DC converter, the internal resistance of L_i, the load resistance, the constant current, and the constant power demand of the common load, respectively.

B. Control Objectives

The main objective is to design a control algorithm for the parallel-connected DC-DC converters such that 1) the steady-state value of the load voltage $V(t)$ is regulated at a given reference value V^* for the unknown load profile (*Voltage Regulation*), and 2) the total current demand is proportionally distributed amongst the converters at the steady state (*Current Sharing*). The objectives can be mathematically formulated as

$$\begin{aligned} &\lim_{t\to\infty} V(t) = \bar{V} = V^*, \\ &\lim_{t\to\infty} I_i(t) = \bar{I}_i = i^*, \quad i=1,\dots,N \end{aligned} \tag{2}$$

where $i^* \in \mathbb{R}$ is a constant, $\bar{V}$ and $\bar{I}_i$ respectively are the steady-state value of $V(t)$ and the converter current $I_i(t)$.

III. Proposed Control Strategy

This section presents a load-independent distributed dynamic control mechanism for the voltage regulation and balanced current sharing problem in the parallel-connected DC-DC buck converters.

A. Distributed Control Approach

In order to guarantee both voltage regulation and current sharing problems in (2), the following control law is proposed for each converter i, $i=1\dots,N$:

$$\begin{aligned} u_i(t) =& k_{1,i}V + k_{2,i}I_i + k_{3,i}w_i + (1-k_{1,i})\alpha_i(v_i - I_i), \\ T_{w_i}\dot{w}_i(t) =& -V(t) + V^* + \alpha_i(v_i(t) - I_i(t)), \\ T_{v_i}\dot{v}_i(t) =& -\alpha_i(v_i - I_i) - K_P\sum_{j=1}^{N}\eta_{i,j}(v_i - v_j) - K_I\sum_{j=1}^{N}\gamma_{i,j}\left(\theta_i - \theta_j\right), \\ T_\theta\dot{\theta}_i(t) =& \sum_{j=1}^{N}\gamma_{i,j}\left(v_i(t) - v_j(t)\right), \end{aligned} \tag{3}$$

where $T_{w_i} \in \mathbb{R}_+$, $T_{v_i} \in \mathbb{R}_+$, $T_\theta \in \mathbb{R}_+$, $K_I \in \mathbb{R}_+$, $\eta_{ij} \in \mathbb{R}_{\geq 0}$, $\gamma_{ij} \in \mathbb{R}_{\geq 0}$, $K_P \in \mathbb{R}_+$, $\alpha_i \in \mathbb{R}_+$, and $(k_{1,i}, k_{2,i}, k_{3,i})$ are the design parameters of the distributed control protocol that can be used to tune the closed-loop performance. In the following, we show that the proposed control scheme in (3) guarantees the voltage regulation and balanced current sharing amongst the DC-DC converters. The parameters $\eta_{ij} = \eta_{ji}$ and $\gamma_{ij} = \gamma_{ji}$ determine the communication topology amongst different converters.

Using vector notation, the overall parallel-connected DC-DC converters in (1) with the proposed control strategy in (3) can be described as follows:

$$\begin{aligned}
C\dot{V}(t) &= \mathbf{1}_N^T I(t) - \frac{1}{R}V(t) - I_L^* - \frac{P_L}{V(t)}, \\
[L]\dot{I}(t) &= [k_1 - 1]\,\mathbf{1}_N V(t) + ([k_2] - [1-k_1]\,[\alpha] - [r])\,I(t) \\
&\quad + [k_3]\,w(t) + [1-k_1]\,[\alpha]\,v(t), \\
[T_w]\,\dot{w}(t) &= -\mathbf{1}_N V(t) + \mathbf{1}_N V^* + [\alpha]\,(v(t) - I(t)), \\
[T_v]\,\dot{v}(t) &= -[\alpha]\,(v(t) - I(t)) - K_P \mathscr{L}_{\mathscr{P}} v(t) - K_I \mathscr{L}_{\mathscr{I}} \theta(t), \\
T_\theta \dot{\theta}(t) &= \mathscr{L}_{\mathscr{I}} v(t),
\end{aligned} \tag{4}$$

where $[k_1 - 1] = [k_1] - \mathbf{I}_N$, $u(t) = \mathrm{col}(u_i(t))$, $w(t) = \mathrm{col}(w_i(t))$, $v(t) = \mathrm{col}(v_i(t))$, $\theta(t) = \mathrm{col}(\theta_i(t))$, and $I(t) = \mathrm{col}(I_i(t))$ for $i = 1, \ldots, N$. Matrix $\mathscr{L}_{\mathscr{P}} \in \mathbb{R}^{N\times N}$ is the proportional Laplacian matrix with an adjacency matrix $\mathscr{B}_{\mathscr{P}} = [\eta_{i,j}]$ and $\mathscr{L}_{\mathscr{I}} \in \mathbb{R}^{N\times N}$ is the integral Laplacian matrix with an adjacency matrix $\mathscr{B}_{\mathscr{I}} = [\gamma_{i,j}]$.

Assumption 1: It is assumed that the communication graphs associated with $\mathscr{L}_{\mathscr{P}}$ and $\mathscr{L}_{\mathscr{I}}$ are connected and undirected.

Lemma 1: Let $\mathscr{L}_{\mathscr{I}}$ be a Laplacian matrix associated with a connected undirected graph. For any $T_\theta \in \mathbb{R}_+$, the trajectory $\theta(t)$ in (4) starting from any initial condition $\theta(0) = \mathrm{col}(\theta_i(0))$ satisfies the following condition:

$$\sum_{i=1}^{N} \theta_i(t) = \sum_{i=1}^{N} \theta_i(0), \quad \forall t \geq 0. \tag{5}$$

Proof: The row vector $\mathbf{1}_N^T$ is the left eigenvector of $\mathscr{L}_{\mathscr{I}}$ associated with a zero eigenvalue [21]. Hence, pre-multiplying both sides of $T_\theta \dot{\theta}(t) = \mathscr{L}_{\mathscr{I}} v(t)$ in (4) by $\mathbf{1}_N^T$ leads to $\mathbf{1}_N^T \dot{\theta}(t) = 0$. As a result, $\mathbf{1}_N^T \theta(t) = \mathbf{1}_N^T \theta(0)$ $\forall t \geq 0$. ■

B. Steady-State Performance and Existence of Equilibria

Lemma 2: Consider the dynamics of the overall parallel-connected converters in (4) with a nonsingular matrix $[k_3]$[1]. Under Assumption 1, the following statements hold:

1) The proposed distributed control approach in (3) simultaneously achieves voltage regulation and balanced current sharing objectives in (2).
2) The steady-state solutions $(\bar{V}, \bar{I}, \bar{v}, \bar{\theta}, \bar{w})$ are obtained as follows:

$$\begin{aligned}
&\bar{V} = V^*, \quad \bar{I} = \frac{1}{N}\mathbf{1}_N\left(\frac{1}{R}V^* + I_L^* + \frac{P_L}{V^*}\right), \\
&\bar{v} = \bar{I}, \quad \bar{\theta} = \frac{1}{N}\mathbf{1}_N\left(\mathbf{1}_N^T \theta(0)\right), \\
&\bar{w} = [k_3]^{-1}\left([1-k_1]\mathbf{1}_N V^* + ([r] - [k_2])\,\bar{I}\right).
\end{aligned} \tag{6}$$

[1]In Theorem 1, it is shown that $k_{3,i} \neq 0$, $i = 1, \ldots, N$.

Proof: Consider the closed-loop system in (4) with the distributed dynamic control strategy given in (3). In the steady state, one obtains that

$$\mathscr{L}_{\mathscr{I}} \bar{v} = 0, \tag{7a}$$
$$-[\alpha]\,(\bar{v} - \bar{I}) - K_P \mathscr{L}_{\mathscr{P}} \bar{v} - K_I \mathscr{L}_{\mathscr{I}} \bar{\theta} = 0, \tag{7b}$$
$$-\mathbf{1}_N \bar{V} + \mathbf{1}_N V^* + [\alpha]\,(\bar{v} - \bar{I}) = 0, \tag{7c}$$
$$\mathbf{1}_N^T \bar{I} - \frac{1}{R}\bar{V} - I_L^* - \frac{P_L}{\bar{V}} = 0, \tag{7d}$$
$$-\mathbf{1}_N \bar{V} - [r]\,\bar{I} + \bar{u} = 0, \tag{7e}$$
$$-\bar{u} + [k_1]\,\mathbf{1}_N \bar{V} + [k_2]\,\bar{I} + [k_3]\,\bar{w} - [k_1 - 1]\,[\alpha]\,(\bar{v} - \bar{I}) = 0, \tag{7f}$$

where $(\bar{V}, \bar{I}, \bar{u}, \bar{\theta}, \bar{v}, \bar{w})$ are the steady-state values of the load voltage $V(t)$, the converter current $I(t)$, the control input $u(t)$, and controller states $(\theta(t), v(t), w(t))$, respectively.

From (7a) one obtains $\bar{v} = \mathbf{1}_N v^*$, where $v^* \in \mathbb{R}$ is a constant. By left multiplying both sides of (7b) by $\mathbf{1}_N^T$ and replacing $\bar{v}$ with $\mathbf{1}_N v^*$, the following equation is obtained.

$$-\mathbf{1}_N^T [\alpha]\,(\bar{v} - \bar{I}) - \mathbf{1}_N^T K_P \mathscr{L}_{\mathscr{P}} \mathbf{1}_N v^* - K_I \mathbf{1}_N^T \mathscr{L}_{\mathscr{I}} \bar{\theta} = 0. \tag{8}$$

Since $[\alpha] \succ 0$, $\mathscr{L}_{\mathscr{P}} \mathbf{1}_N = 0$, and $\mathbf{1}_N^T \mathscr{L}_{\mathscr{I}} = 0$, we can conclude that $\mathbf{1}_N^T [\alpha]\,\bar{v} = \mathbf{1}_N^T [\alpha]\,\bar{I}$. Left multiplying (7c) by $\frac{1}{N}\mathbf{1}_N^T$ and considering $\mathbf{1}_N^T [\alpha]\,(\bar{v} - \bar{I}) = 0$ yield that $\bar{V} = V^*$ (*Voltage Regulation*).

By replacing $\bar{V}$ with V^* in (7c), we obtain that $\bar{I} = \bar{v}$; therefore, $\bar{I} = \mathbf{1}_N v^*$ (*Balanced Current Sharing*), where $v^* = \frac{1}{N}\left(\frac{V^*}{R} + I_L^* + \frac{P_L}{V^*}\right)$. Moreover, replacing $\bar{I} = \bar{v}$ and $\bar{v} = \mathbf{1}_N v^*$ in (7b) yields that $\mathscr{L}_{\mathscr{I}} \bar{\theta} = 0$ and hence $\bar{\theta} = \mathbf{1}_N \theta^*$, where $\theta^* \in \mathbb{R}$. Considering $\mathbf{1}_N^T \bar{\theta} = \mathbf{1}_N^T \theta(0)$ (see Lemma 1) imposes that $\theta^* = \frac{1}{N}\mathbf{1}_N^T \theta(0)$. From (7e) and (7f), it follows that

$$\bar{w} = [k_3]^{-1}\left([1-k_1]\mathbf{1}_N \bar{V} + ([r] - [k_2])\,\bar{I}\right). \tag{9}$$

■

IV. Stability Analysis

In this section, we discuss the stability analysis of the dynamical closed-loop system in (4) with the proposed distributed control mechanism in (3). By change of the variables as $e_V(t) = V(t) - \bar{V}$, $e_I(t) = I(t) - \bar{I}$, $e_v(t) = v(t) - \bar{v}$, $e_\theta(t) = \theta(t) - \bar{\theta}$, and $e_w(t) = w(t) - \bar{w}$, where $e_V(t) \in \mathbb{R}$, $e_I(t) \in \mathbb{R}^{N\times 1}$, $e_v(t) \in \mathbb{R}^{N\times 1}$, $e_w(t) \in \mathbb{R}^{N\times 1}$, and $e_\theta(t) \in \mathbb{R}^{N\times 1}$, the equilibria of the closed-loop system are shifted to origin with their corresponding dynamics as:

$$\begin{aligned}
C\dot{e}_V(t) &= \mathbf{1}_N^T e_I(t) - \frac{1}{R}e_V(t) + \frac{P_L}{(e_V(t) + \bar{V})\bar{V}}e_V(t), \\
[L]\,\dot{e}_I(t) &= [k_1 - 1]\,\mathbf{1}_N e_V(t) + ([k_2] - [1-k_1]\,[\alpha] - [r])\,e_I(t) \\
&\quad + [k_3]\,e_w(t) + [1-k_1]\,[\alpha]\,e_v(t), \\
[T_v]\,\dot{e}_v(t) &= -[\alpha]\,(e_v(t) - e_I(t)) - K_P \mathscr{L}_{\mathscr{P}} e_v(t) - K_I \mathscr{L}_{\mathscr{I}} e_\theta(t), \\
T_\theta \dot{e}_\theta(t) &= \mathscr{L}_{\mathscr{I}} e_v(t), \\
[T_w]\,\dot{e}_w(t) &= -\mathbf{1}_N e_V(t) + [\alpha]\,(e_v(t) - e_I(t)).
\end{aligned} \tag{10}$$

In the following theorem, we show that the overall parallel-converter system in (4) is locally stable.

Theorem 1: Let $\mathscr{L}_{\mathscr{P}}$ and $\mathscr{L}_{\mathscr{I}}$ be Laplacian matrices associated with connected and undirected communication graphs. If

$(T_\theta, K_P, K_I) \in \mathbb{R}_+$, $[\alpha] \succ 0$, $[T_v] \succ 0$, $[T_w] \succ 0$, and $(k_{1,i}, k_{2,i}, k_{3,i})$ belongs to the set

$$\chi^1_{[i]} = \left\{ \begin{array}{c} k_{1,i} < 1, \quad k_{2,i} < r_i, \\ 0 < \frac{k_{3,i}}{T_{w_i}} < \frac{1}{L_i} (1 - k_{1,i})(r_i - k_{2,i}) \end{array} \right\}. \tag{11}$$

and (R, P_L, V^*) belongs to the following set

$$\chi^2 = \left\{ (R, P_L, V^*) : \quad \frac{1}{R} - \frac{P_L}{V^{*2}} > 0 \right\}. \tag{12}$$

then, the origin in (10) is locally asymptotically stable. Moreover, the origin in (10) is globally asymptotically stable if $P_L = 0$ (Z and ZI load).

Proof: We should show that the conditions given in $\chi^1_{[i]}$ and χ^2, guarantee the local asymptotic stability of the origin of the closed-loop system in (10). To this end, the following quadratic-type Lyapunov function $\mathbf{V}$ is proposed:

$$\begin{aligned} \mathbf{V} =& \frac{1}{2} C e_V^2(t) + \frac{1}{2} e_v^T(t) [T_v] \; e_v(t) + \frac{K_I}{2} e_\theta^T(t) T_\theta e_\theta(t) \\ &+ \frac{1}{2} \sum_{i=1}^{N} [e_{I_i}(t) \;\; e_{w_i}(t)] P_i [e_{I_i}(t) \;\; e_{w_i}(t)]^T , \end{aligned} \tag{13}$$

where $P_i \succ 0$ is defined as follows:

$$P_i = \left[\begin{array}{cc} L_i \rho_i & -\frac{L_i}{T_{w_i}} \rho_i \nu_i \\ -\frac{L_i}{T_{w_i}} \rho_i \nu_i & \nu_i \left(1 + \frac{L_i}{T_{w_i}^2} \rho_i \nu_i \right) \end{array} \right], \tag{14}$$

where $\rho_i > 0$ and $\nu_i > 0$ are determined based on $(k_{1,i}, k_{2,i}, k_{3,i}, T_{w_i})$ in $\chi^1_{[i]}$ in (11) as follows:

$$\rho_i = \frac{(r_i - k_{2,i})}{(r_i - k_{2,i})(1 - k_{1,i}) - \frac{L_i}{T_{w_i}} k_{3,i}}, \;\; \nu_i = T_{w_i} \frac{k_{3,i}}{(r_i - k_{2,i})}. \tag{15}$$

Note that $trace(P_i) > 0$ and $det(P_i) > 0$ ($P_i \succ 0$). Moreover, $\mathbf{V} \geq 0$ for all $(e_V, e_I, e_v, e_w, e_\theta)$ and $\mathbf{V} = 0$ if and only if $e_V = 0$, $e_I = 0$, $e_v = 0$, $e_w = 0$, and $e_\theta = 0$. The time derivative of $\mathbf{V}$ along the closed-loop trajectories of (10) is obtained as:

$$\begin{aligned} \dot{\mathbf{V}} =& -\left(\frac{1}{R} - \frac{P_L}{(e_V(t) + \bar{V})\bar{V}} \right) e_V^2(t) + \mathbf{1}_N^T e_I(t) e_V(t) \\ &- \frac{1}{2} \left(e_v^T(t) [\alpha] (e_v(t) - e_I(t)) + (e_v(t) - e_I(t))^T [\alpha] e_v(t) \right) \\ &- \frac{K_P}{2} e_v^T(t) \left(\mathscr{L}_\mathscr{P} + \mathscr{L}_\mathscr{P}^T \right) e_v(t) - \frac{K_I}{2} e_v^T(t) \mathscr{L}_\mathscr{I} e_\theta(t) \\ &- \frac{K_I}{2} e_\theta^T(t) \mathscr{L}_\mathscr{I}^T e_v(t) + \frac{K_I}{2} \left(e_\theta^T(t) \mathscr{L}_\mathscr{I} e_v(t) + e_v^T(t) \mathscr{L}_\mathscr{I}^T e_\theta(t) \right) \\ &+ \frac{1}{2} \sum_{i=1}^{N} \left[e_{I_i}(t) \;\; e_{w_i}(t) \right] Q_i \left[e_{I_i}(t) \;\; e_{w_i}(t) \right]^T \\ &+ \frac{1}{2} \sum_{i=1}^{N} \left(e_V(t) B_i^T P_i \left[e_{I_i}(t) \;\; e_{w_i}(t) \right]^T + \left[e_{I_i}(t) \;\; e_{w_i}(t) \right] P_i B_i e_V(t) \right) \\ &- \frac{1}{2} \sum_{i=1}^{N} \alpha_i \left[e_{I_i}(t) \;\; e_{w_i}(t) \right] P_i B_i (e_{v_i}(t) - e_{I_i}(t)) \\ &- \frac{1}{2} \sum_{i=1}^{N} \alpha_i (e_{v_i}(t) - e_{I_i}(t)) B_i^T P_i \left[e_{I_i}(t) \;\; e_{w_i}(t) \right]^T . \end{aligned} \tag{16}$$

where $Q_i = A_i^T P_i + P_i A_i$,

$$A_i = \left[\begin{array}{cc} \frac{-r_i + k_{2,i}}{L_i} & \frac{k_{3,i}}{L_i} \\ 0 & 0 \end{array} \right], \quad B_i = \left[\begin{array}{c} \frac{-1 + k_{1,i}}{L_i} \\ -\frac{1}{T_{w_i}} \end{array} \right]. \tag{17}$$

By direct calculations and taking into account (14)-(15), it follows that

$$\rho_i \left((k_{1,i} - 1) + \frac{L_i}{T_{w_i}^2} \nu_i \right) = -1 \tag{18}$$

Therefore, it can be shown that Q_i and $P_i B_i$ can be written as

$$Q_i = -2\rho_i \left[\begin{array}{cc} (r_i - k_{2,i}) & -\frac{(r_i - k_{2,i})}{T_{w_i}} \nu_i \\ -\frac{(r_i - k_{2,i})}{T_{w_i}} \nu_i & \frac{\nu_i^2}{T_{w_i}^2} (r_i - k_{2,i}) \end{array} \right], \; P_i B_i = \left[\begin{array}{c} -1 \\ 0 \end{array} \right]. \tag{19}$$

Since $Q_i \in \mathbb{R}^{2 \times 2}$, $det(Q_i) = 0$, and $trace(Q_i) < 0$ in the set $\chi^1_{[i]}$ ($k_{2,i} < r_i$), we can conclude that $Q_i \preceq 0$. Hence, taking into account (19) and invoking the properties of the Laplacian matrix as $\mathscr{L}_\mathscr{I}^T = \mathscr{L}_\mathscr{I}$ and $\mathscr{L}_\mathscr{P}^T = \mathscr{L}_\mathscr{P}$, $\dot{\mathbf{V}}$ can be rewritten as follows:

$$\begin{aligned} \dot{\mathbf{V}} =& -\left(\frac{1}{R} - \frac{P_L}{(e_V(t) + \bar{V})\bar{V}} \right) e_V^2(t) + \mathbf{1}_N^T e_I(t) e_V(t) \\ &- \frac{1}{2} \left(e_v^T(t) [\alpha] (e_v(t) - e_I(t)) + (e_v(t) - e_I(t))^T [\alpha] e_v(t) \right) \\ &- K_P e_v^T(t) \mathscr{L}_\mathscr{P} e_v(t) + \frac{1}{2} \sum_{i=1}^{N} \left[e_{I_i}(t) \;\; e_{w_i}(t) \right] Q_i \left[e_{I_i}(t) \;\; e_{w_i}(t) \right]^T \\ &- \sum_{i=1}^{N} e_V(t) e_{I_i}(t) + \sum_{i=1}^{N} \alpha_i e_{I_i}(t) (e_{v_i}(t) - e_{I_i}(t)). \end{aligned} \tag{20}$$

Therefore,

$$\begin{aligned} \dot{\mathbf{V}} =& -\left(\frac{1}{R} - \frac{P_L}{(e_V(t) + \bar{V})\bar{V}} \right) e_V^2(t) - K_P e_v^T(t) \mathscr{L}_\mathscr{P} e_v(t) \\ &- (e_v(t) - e_I(t))^T [\alpha] (e_v(t) - e_I(t)) \\ &+ \frac{1}{2} \sum_{i=1}^{N} [e_{I_i}(t) \;\; e_{w_i}(t)] Q_i [e_{I_i}(t) \;\; e_{w_i}(t)]^T . \end{aligned} \tag{21}$$

It is illustrated that $[\alpha] \succ 0$, $\mathscr{L}_\mathscr{P} \succeq 0$, $Q_i \preceq 0$. To show that $\dot{\mathbf{V}} \leq 0$, one needs to show that $\frac{1}{R} - \frac{P_L}{(e_V(t) + \bar{V})\bar{V}} > 0$. Since $(R, P_L, V^*) \in \chi^2$, there exists an open interval of $e_V(t)$ containing the origin such that $\frac{1}{R} - \frac{P_L}{(e_V(t) + \bar{V})\bar{V}} > 0$. Furthermore, it is always possible to find a (sufficiently small) compact level set of the Lyapunov function $\mathbf{V}$ that is included in a neighbourhood in which $\frac{1}{R} - \frac{P_L}{(e_V(t) + \bar{V})\bar{V}} > 0$ holds. Hence, $\dot{\mathbf{V}} \leq 0$ implies that such a compact set is forward invariant, hence the LaSalle's invariance principle [22] can be applied. To this end, we should show that the the largest invariant set in $\mathscr{Z} = \left\{ (e_V, e_I, e_v, e_w, e_\theta) : \; \dot{\mathbf{V}}(e_V, e_I, e_v, e_w, e_\theta) = 0 \right\}$ is origin. Based on (21), the set $\mathscr{Z}$ is characterized as follows:

$$\mathscr{Z} = \left\{ \begin{array}{l} \left(\frac{1}{R} - \frac{P_L}{(e_V(t) + \bar{V})\bar{V}} \right) e_V^2 = 0, \\ (e_v - e_I)^T [\alpha] (e_v - e_I) = 0, \; e_v^T \mathscr{L}_\mathscr{P} e_v = 0, \\ \left[\begin{array}{c} e_{I_i}(t) \\ e_{w_i}(t) \end{array} \right]^T Q_i \left[\begin{array}{c} e_{I_i}(t) \\ e_{w_i}(t) \end{array} \right] = 0, \; i = 1, \ldots, N \end{array} \right\}. \tag{22}$$

From the above set, it follows that $e_V = 0$, $e_v = e_I$, and $e_v = \mathbf{1}_N e_v^*$, where $e_v^* \in \mathbb{R}$. From the system dynamics in (10), one obtains that $\mathbf{1}_N^T e_I = 0$; therefore, $\mathbf{1}_N^T e_v = 0$, $e_v^* = 0$, $e_v = e_I = 0$, $e_w = 0$, and $\mathscr{L}_\mathscr{I} e_\theta = 0$. Hence, $e_\theta = \mathbf{1}_N e_\theta^*$, where $e_\theta^* \in \mathbb{R}$. According to Lemma 1, $\mathbf{1}_N^T e_\theta = \mathbf{1}_N^T \theta - \mathbf{1}_N^T \bar{\theta} = 0$; hence, $e_\theta^* = 0$ and $e_\theta = 0$. As a result, the largest invariant set of $\mathscr{Z}$ is origin.

TABLE I

PARAMETERS OF THE SYSTEM UNDER STUDY.

Converter 1	$L_1 = 1.3mH$	$r_1 = 0.1\Omega$	$E_1 = 24V$
Converter 2	$L_2 = 1.2mH$	$r_2 = 0.1\Omega$	$E_2 = 24V$
Converter 3	$L_3 = 1.6mH$	$r_3 = 0.1\Omega$	$E_3 = 24V$
Converter 4	$L_4 = 1.4mH$	$r_4 = 0.1\Omega$	$E_4 = 24V$
Load parameters	$R = 1\Omega$	$I_L^* = 5A$	$P_L = 120W$
Load capacitance	$C = 40\mu F$		
Switching frequency	$f_{sw} = 40kHz$	Sampling frequency	$f_s = 20kHz$

Hence, the origin in (10) is locally asymptotically stable. By virtue of the stability certificate and the conditions given in (11) and (12), the solutions to (4) asymptotically converge to the steady-state values $(\bar{V}, \bar{I}, \bar{v}, \bar{\theta}, \bar{w})$ in (6), achieving the voltage regulation and balanced current sharing objectives. If $P_L = 0$, the condition in (12) is always feasible; therefore, the origin in (10) is globally asymptotically stable for Z/ZI loads. ■

Remark 1: (Robustness Feature of the Proposed Control Strategy) If the parameters of the ZIP load are uncertain and belong to the intervals $\underline{R} \leq R \leq \bar{R}$, $\underline{I}_L^* \leq I_L^* \leq \bar{I}_L^*$, and $\underline{P}_L \leq P_L \leq \bar{P}_L$, then it can be shown that the robust local stability of the parallel connection of the converters is guaranteed if $(\bar{R}, \bar{P}_L, V^*) \in \chi^2$. Moreover, the voltage regulation and current sharing are achieved for all values of $L_i \in \mathbb{R}_+$ and $C \in \mathbb{R}_+$.

Remark 2: (Plug-and-Play Operation of DC-DC Converters) The plug-and-play (PnP) operation of converters affects the dynamic topology of the system. The parallel-converter system with the dynamic topology can be modeled by a switched dynamical system with a piecewise constant switching function $\sigma(t) : [0, \infty) \to \{1, \dots, k\}$, where k is the total number of possible changes in the dynamic communication graph due to changes in the system dynamics. For simplicity of the presentation, it is assumed that $\mathscr{L} = \mathscr{L}_{\mathscr{P}} = \mathscr{L}_{\mathscr{I}}$. The finite set of all possible communication network topologies is denoted by $\Gamma = \{\mathscr{G}^1, \dots, \mathscr{G}^k\}$. It is assumed that the communication graphs $\{\mathscr{G}^1, \dots, \mathscr{G}^k\}$ are undirected and connected. Therefore, the Lyapunov function proposed in (13) can be considered as a common Lyapunov function for the switched dynamical behaviour of the parallel converter system, providing a guarantee of the stability under the PnP operation of the converters [23].

V. SIMULATION RESULTS

In this section, the effectiveness of the proposed distributed dynamic control approach in (3) is illustrated on a parallel-converter system consisting of $N = 4$ DC-DC converters. The system parameters are given in Table. I. The simulation case studies are carried out in MATLAB/Simscape Electrical environment, where the realistic model of the PWM-based DC-DC converters is considered. It is assumed that the topology of the proposed distributed control strategy in (3) is based on the incidence matrices $\mathscr{B}_{\mathscr{P}} = [\eta_{i,j}]$ and $\mathscr{B}_{\mathscr{I}} = [\gamma_{i,j}]$, where

$$\eta_{i,j} = \gamma_{i,j} = \begin{cases} 1 & \text{if } \mid i-j \mid = 1 \text{ or } \mid i-j \mid = N-1, \\ 0 & \text{otherwise.} \end{cases} \quad (23)$$

The parameters of the controllers in (3) are chosen as $K_P = 10$, $K_I = 1$, $[\alpha] = 10 \times \mathbf{I}_4$, $T_\theta = 10^{-3}$, $[T_v] = 10^{-3} \times \mathbf{I}_4$, $[T_w] = 10^{-1} \times \mathbf{I}_4$, $[k_1] = 0.1 \times \mathbf{I}_4$, $[k_2] = -\mathbf{I}_4$, and $[k_3] = 30 \times \mathbf{I}_4$.

Voltage Tracking. In this case study, the performance of the proposed control strategy in (3) is assessed in terms of voltage regulation and balanced current sharing. To this end, it is assumed that the reference voltage V^* is stepped up from $12V$ to $18V$ at $t = 0.3s$. In order to show the independency of the current sharing and voltage regulation on the initial values of $\theta(t)$, $\theta(0)$ is randomly chosen. The dynamic responses the closed-loop parallel-converter system are depicted in Fig. 2.

Comparison with [14] and [16] for the Case of a Common Resistive Load. We compare our results (Approach I) with the distributed averaging control strategy in [16] (Approach II) and the centralized control approach in [14] (Approach III). The performance of all three control approaches is assessed in terms of voltage regulation and balanced current sharing. To this end, it is assumed that the reference voltage V^* is stepped up from $12V$ to $18V$ at $t = 1s$. The dynamic responses of all the control approaches are depicted in Fig. 3. Note that due to the voltage and current ripples caused by PWMs, it is not easy to observe the transient behaviour and compare the performance of the control approaches in voltage regulation and current sharing. Due to this reason, an ideal model of the DC-DC converters is employed in the second case study.

The results indicate that the performance of Approach I and Approach III is similar in terms of the settling time. However, Approach I relies on the less communication links among the converters. Moreover, the results show that Approach II is not robust against the internal resistances of the inductors. As it can be observed from Fig. 3 (part (a) and (c)), neglecting these resistances in the control design leads to steady-state errors in the load voltage and the current of the converters.

VI. CONCLUSION

In this letter, we address the problem of voltage regulation and balanced current sharing in the parallel interconnection of heterogeneous DC-DC converters with a common ZIP load. A distributed dynamic control approach is presented to simultaneously achieve voltage regulation and equal current distribution amongst the converters. A rigorous stability analysis based on the Lyapunov stability theory and the LaSalle's invariance principle is provided. The effectiveness of the results is illustrated both theoretically and numerically.

REFERENCES

[1] V. J. Thottuvelil and G. C. Verghese, "Analysis and control design of paralleled DC/DC converters with current sharing," *IEEE Trans. Power Electronics*, vol. 13, no. 4, pp. 635–644, Jul. 1998.

[2] S. Luo, Z. Ye, R. L. Lin, and F. C. Lee, "A classification and evaluation of paralleling methods for power supply modules," in *30th Annual IEEE Power Electronics Specialists Conference*, Charleston, SC, USA, USA, Jul. 1999, pp. 901–908.

[3] S. Moayedi, V. Nasirian, F. Lewis, and A. Davoudi, "Team-oriented load sharing in parallel DC–DC converters," *IEEE Trans. Industry Applications*, vol. 51, no. 1, pp. 479–490, Jan./Feb. 2015.

[4] J. W. Kim, H. S. Choi, and B. H. Cho, "A novel droop method for converter parallel operation," *IEEE Trans. Power Electronics*, vol. 17, no. 1, pp. 25–32, Jan. 2002.

[5] S. Anand and B. G. Fernandes, "Modified droop controller for paralleling of DC–DC converters in standalone DC system," *IET Power Electronics*, vol. 5, no. 6, pp. 782–789, Jul. 2012.

[6] Y. Huang and C. K. Tse, "Circuit theoretic classification of parallel connected DC–DC converters," *IEEE Trans. Circuits and Systems-I*, vol. 54, no. 5, pp. 1099–1108, May 2007.

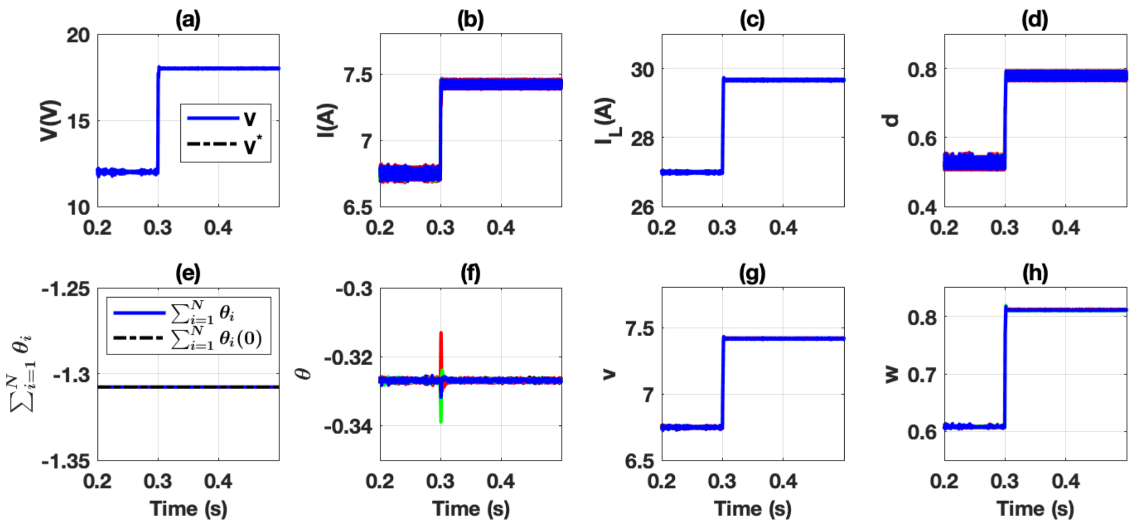

Fig. 2. Dynamic responses of the parallel converters due to changes in the load voltage reference at $t = 0.3s$: (a) load voltage, (b) converter currents, (c) load current, (d) duty cycles of the DC-DC converters, (e) $\sum_{i=1}^{N} \theta_i(t)$, (f) controller states $\theta(t)$, (g) controller states $v(t)$, and (h) controller states $w(t)$.

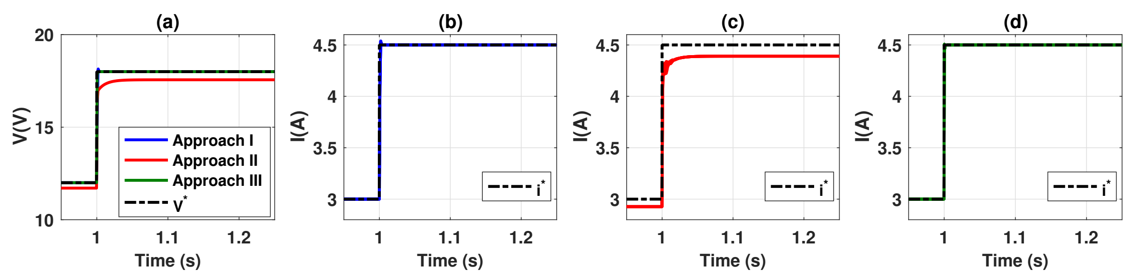

Fig. 3. Dynamic responses of the parallel converters via different control strategies: (a) load voltage via all three approaches, (b) converter currents via the proposed control strategy in (3) (Approach I), (c) converter currents via the distributed averaging control technique in [16] (Approach II), and (d) converter currents via the control approach in [14] (Approach III).

[7] H. Behjati, A. Davoudi, and F. Lewis, "Modular DC–DC converters on graphs: Cooperative control," *IEEE Trans. Power Electronics*, vol. 29, no. 12, pp. 6725–6741, Dec. 2014.

[8] S. K. Mazumder, M. Tahir, and K. Acharya, "Master–slave current-sharing control of a parallel DC–DC converter system over an RF communication interface," *IEEE Trans. Industrial Electronics*, vol. 55, no. 1, pp. 59–66, Jan. 2008.

[9] J. Shi, L. Zhou, and X. He, "Common-duty-ratio control of input-parallel output-parallel (IPOP) connected DC–DC converter modules with automatic sharing of currents," *IEEE Trans. Power Electronics*, vol. 27, no. 7, pp. 3277–3291, Jun. 2012.

[10] D. Sha, Z. Guo, and X. Liao, "Cross-feedback output-current-sharing control for input-series-output-parallel modular DC–DC converters," *IEEE Trans. Power Electronics*, vol. 25, no. 11, pp. 2762–2771, Nov. 2010.

[11] P. J. Grbovic, "Master/slave control of input-series- and output-parallel connected converters: Concept for low-cost high-voltage auxiliary power supplies," *IEEE Trans. Power Electronics*, vol. 24, no. 2, pp. 316–328, Feb. 2009.

[12] M. Li, C. K. Tse, H. H.-C. Iu, and X. Ma, "Unified equivalent modeling for stability analysis of parallel-connected DC/DC converters," *IEEE Trans. Circuits and Systems*, vol. 57, no. 11, pp. 898–902, Nov. 2010.

[13] R. Delpoux, J. F. Tregouet, J. Y. Gauthier, and C. Lacombe, "New framework for optimal current sharing of nonidentical parallel buck converters," *IEEE Trans. Control Systems Technology*, vol. 27, no. 3, pp. 1237–1243, May 2019.

[14] J. F. Tregouet and R. Delpoux, "New framework for parallel interconnection of buck converters: Application to optimal current-sharing with constraints and unknown load," *Control Engineering Practice*, vol. 87, pp. 59–75, 2019.

[15] J. Kreiss, J. F. Tregouet, D. Eberard, R. Delpoux, J. Y. Gauthier, and X. Lin-Shi, "Hamiltonian point of view on parallel interconnection of buck converters," *IEEE Trans. Control Systems Technology*, pp. 1–10, Jan. 2020.

[16] S. Trip, M. Cucuzzella, X. Cheng, and J. Scherpen, "Distributed averaging control for voltage regulation and current sharing in DC microgrids," *IEEE Control Systems Letters*, vol. 3, no. 1, pp. 174–179, Jan. 2019.

[17] M. Cucuzzella, S. Trip, C. D. Persis, X. Cheng, A. Ferrara, and A. van der Schaft, "A robust consensus algorithm for current sharing and voltage regulation in DC microgrids," *IEEE Trans. Control Systems Technology*, vol. 27, no. 4, pp. 1583–1595, Jul. 2019.

[18] J. Zhao and F. Dorfler, "Distributed control, load sharing, and dispatch in DC microgrids," in *American Control Conference (ACC)*, Chicago, IL, Jul. 2015, pp. 3304–3309.

[19] M. Tucci, L. Meng, J. M. Guerrero, and G. Ferrai-Trecate, "Stable current sharing and voltage balancing in DC microgrids: A consensus-based secondary control layer," *Automatica*, vol. 95, pp. 1–13, 2018.

[20] R. W. Erickson and D. Maksimovic, *Fundamentals of Power Electronics*. Norwell, MA, USA: Kluwer, 2001.

[21] F. Bullo, *Lectures on Network Systems*, 1st ed. Kindle Direct Publishing, 2020. [Online]. Available: http://motion.me.ucsb.edu/book-lns

[22] H. K. Khalil, *Nonlinear Systems*. New Jersey: Prentice Hall, 2006.

[23] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Automatic Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.