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ARTICLE

Barter markets, indivisibilities, and Markovian core

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Abstract

We study a general barter market where every agent is endowed with several heterogeneous indivisible items and wishes to exchange. There is no medium of exchange like money. Agents have general preferences over their interested bundles of items and may acquire several items. We propose a practical and sensible solution called a Markovian core, generalizing the classical notion of the core. A Markovian core allocation is individually rational, Pareto-efficient, and stable against any coalition deviation by comparison with their current assignments instead of their initial endowments and is shown to be a natural outcome of a decentralized market process.

KEYWORDS

barter market, efficiency, indivisibility, Markovian core, stability

JEL CLASSIFICATION

C71, C78, D01, D30

1 | INTRODUCTION

This article aims to investigate a general barter market with multiple heterogeneous indivisible commodities and establish an innovative, practical, and reasonable solution to this class of markets. In recent years, barter markets have attracted attention far beyond their traditional boundary of economics; see, for example, Gewertz (1978), Pearson (2003), Argumedo and Pimbert (2010), and Ye et al. (2018). These markets are undoubtedly the oldest form of markets through which people exchange their commodities and services without using money or any medium of exchange.

Nowadays, they are still widely used and spreading into various new territories such as online peer-to-peer trade and human organ exchange. A great advantage of these markets is that they can avoid using money. Bartering gains popularity when money is tight, and bartering is necessary when money is not allowed. For instance, according to Mickey Meece's report in the *New York Times* (November 12, 2008), barter exchanges had double-digit increases in membership in 2008 due to the financial crisis. Countries that are short of international currencies (see, e.g., Bank, 1985) or want to avoid using international currencies often swap their commodities and services. It is also fairly common to see that developed nations exchange their industrial products for natural resources from developing nations. Financial companies frequently swap their assets. Howell and Chmielewski (2009) reported that more than 70,000 businesses make cashless transactions annually throughout the USA. A new development of the barter trade is organ exchange. In almost all countries, human organs are not permitted to sell or to buy but can be donated or mutually exchanged.

Marshall first studied barter exchange in the second half of the 19th century and later devoted one chapter to the theory of barter exchange in Marshall (1952). In response to Marshall's idea of barter exchange, Edgeworth (1881, 1925) formulated a market process for a simple barter economy with two perfectly divisible goods. Uzawa (1962) extended Edgeworth's process to a general economy with multiple perfectly divisible goods and proved that his continuous price adjustment process could reach equilibrium under certain conditions. Mukherji (1974) further examined the Edgeworth–Uzawa process under Uzawa's conditions and found that the path of price vectors generated by the process has a unique limit, which is an equilibrium of the economy. These authors defined an equilibrium of the barter market as the limit of a continuous price adjustment process. This is in marked contrast to Walrasian equilibrium and core that are defined on the primitives of the economy.

The barter market of Shapley and Scarf (1974) stands out as a celebrated model in the fields of microeconomics and cooperative game theory. The top trading cycle (TTC) procedure described in their paper has found important applications in mechanism design, two-sided matching, kidney exchange, and school choice, etc. In this market, there are finitely many traders or agents each of whom owns initially an indivisible item, for example, a house. Every agent has preferences over all the houses but has no use for more than one item. There is no money or other medium of exchange. Agents swap their houses with each other to obtain their favorite possible houses. Shapley and Scarf (1974) proved that this market is a balanced nontransferable utility (NTU) game and therefore has a nonempty core by a theorem of Scarf (1967), and both core allocation and a competitive equilibrium can be found by the TTC procedure. Since then, many remarkable properties of this model and the procedure have been discovered.

Roth and Postlewaite (1977) demonstrated that if every agent has strict preferences, this market has a unique competitive equilibrium coinciding with the unique strict core allocation. Roth (1982) proved that the TTC procedure can induce every agent to behave honestly. Wako (1984) illustrated that the strict core can be a proper subset of the set of competitive equilibrium allocations. Qin (1993, 1994) took the possibility of randomization of coalitions into account and proposed the solution of the inner core as a refinement of the core. Ma (1994) proved that under strict preferences, a procedure is individually rational, Pareto-efficient, and strategy-proof if and only if it is the TTC procedure. Konishi et al. (2001) studied a generalization of the Shapley–Scarf model by allowing each agent to initially own, say, one house and one car. They demonstrated that many distinctive features of the Shapley–Scarf economy cannot carry over to this extended model. But they found that if there are only three agents and all agents have additively separable strict preferences, then the core of this market is not empty. Inoue (2008) examined an extension of the Shapley–Scarf model and established that if the aggregate upper contour set of all agents is discretely convex, then the core is nonempty and that especially if the upper contour set of every agent is M-convex the core is not empty.

In stark contrast to the markets with perfectly divisible goods which always have a nonempty core in general environments (see Scarf, 1967; Arrow & Hahn, 1971) and to the markets with indivisible goods and quasi-linear utilities in money which admit competitive equilibria under several regular conditions (see Kelso & Crawford, 1982; Bikhchandani & Mamer, 1997; Ma, 1998; Gul & Stachetti, 1999; Sun & Yang, 2006, 2014; Baldwin & Klemperer; 2019), the core of markets with indivisibility which are more general than the Shapley–Scarf market can easily become empty. The difficulty is caused both by the absence of money and by the presence of indivisibility which is an extreme form of nonconvexity. For instance, Shapley and Scarf (1974, pp. 32–34) themselves showed the nonexistence of core in an apparently natural market with three agents having symmetric holdings in a tract of nice houses. In their example, agent j owns houses j, j', and j'' for $j = 1, 2, 3$. Moulin (1995) pointed out that the core generally does not exist in the market where each agent owns a car and a house and views cars and houses as complements. Konishi et al. (2001) demonstrated that the core can be empty even in the class of economies with several identical items and agents consuming multiple units. In such economies, there is no complementarity among the items. Inoue (2008, pp. 102–103) gave a different type of example with an empty core.

The notion of the core cannot be used as a solution to many markets where agents initially possess more than one item and may acquire several items and for which the core is generally not guaranteed to be nonempty. It is, therefore, necessary to find an appropriate alternative solution to such markets. On the one hand, as the core is one of the most important solution concepts in the context of competition and cooperation, it will be important for any proposed solution to maintain some basic properties of the core such as individual rationality, Pareto-efficiency, and stability. On the other hand, any proposed solution has to be general enough to handle a variety of situations. To achieve this, the conditions imposed on the core, such as full rationality, will have to be appropriately relaxed. We aim to establish such a solution for a general barter market.

In this paper, we study a general barter market in which each agent is initially endowed with several inherently indivisible items and wishes to exchange with other agents. There is no medium of exchange like money. Agents have general preferences over their interested bundles of items and may acquire several items. Unfortunately, the core as a widely recognized solution cannot be applied to this general market because it can be empty and void. However, in practice, barter trade does take place in volume and on a large scale. So there is a gap between the existing theory and reality.

To bridge the gap and have a better understanding of the nature of the market, we attempt to explore a more practical approach by taking human behavioral aspects and cognitive factors such as myopia, impatience, and bounded rationality into consideration and propose a constructive and sensible solution called a Markovian core. This new solution not only meets those criteria mentioned above but also fits the context of barter markets well, because barter trade happens often when money is short or not allowed. In such (sometimes desperate) situations, agents usually want to get an immediate improvement of their current position and may not have enough time or even try to think about how to achieve the best from all possible potentially beneficial exchanges. This can lead them to act myopically and hastily and therefore not necessarily optimally or fully rationally. Briefly speaking, our solution can be described as follows. Every Markovian core allocation is individually rational with respect to their initial endowments, Pareto-efficient, and stable against any coalition deviation by comparison with their current assignments instead of their initial endowments. We prove that the market has always a nonempty strict Markovian core. A strict

Markovian core allocation will be shown to be a natural outcome generated by a decentralized market process. This result offers a plausible explanation for the very existence of a variety of barter markets. Furthermore, we demonstrate that a natural random decentralized process converges almost surely to a strict Markovian core allocation.

In a nutshell, our paper introduces a large class of general barter markets with indivisible goods in which the widely used solution-core fails to exist. We propose a new, practical, and reasonable solution called the Markovian core and establish its existence and shed light on the existence of many real-world barter markets. Our arguments are simple and intuitive.

The rest is organized as follows. Section 2 presents the model. Section 3 establishes the main results, and Section 4 concludes.

2 | MODEL

Consider a general barter market in which there are m agents and n different types of indivisible commodities. Let $M = \{1, 2, \ldots, m\}$ denote the set of agents. Each agent $j \in M$ is initially endowed with a nonzero bundle $\omega^j \in \mathrm{Z}^n_+$ of indivisible items and has a general preference relation $\succeq_j$ on a family of possible consumption bundles $F^j \subseteq \mathrm{Z}^n_+$ containing his initial endowment ω^j, where Z^n_+ represents the space of indivisible commodities, that is, the collection of all nonnegative n-dimensional integer vectors. Both F^j and $\succeq_j$ depend on the agent and may vary from one agent to another. The preference relation $\succeq_j$ is assumed to be complete and transitive. There is no medium of exchange like money in the economy. As the imposed condition is mild and minimal, the model is very general, substantially extending that of Shapley and Scarf (1974) in which every agent $j \in M$ is endowed with one item $e(j)$—the j-th unit vector in IR^m and has the consumption set $F^j = \{e(h) \mid h \in M\}$. We use $E = (\succeq_j, \omega^j, F^j \mid j \in M)$ to represent our current general market. All agents try to swap their items in order to improve their welfare.

Given the market, an allocation is a redistribution $X = (x^i \mid i \in M)$ of all items among all agents such that $\sum_{j\in M} x^j = \sum_{j\in M} \omega^j$ and $x^j \in F^j$ for every $j \in M$. At allocation X, agent j receives bundle x^j.

This market can be naturally formulated as an NTU cooperative game. The conventional solution to this kind of game is the notion of the core (see Scarf, 1967). It is defined to be a set of redistributions of all items among all agents that cannot be profitably blocked by any coalition of agents by comparing with their initial endowments. A coalition is a nonempty subset of the set M.

Definition 1. An allocation $(x^j \mid j \in M)$ is a *core allocation* if there do not exist a coalition S and an allocation $(x^j \mid j \in S)$ with $\sum_{j\in S} x^j = \sum_{j\in S} \omega^j$ such that $x^j \succ_j \omega^j$ and $x^j \in F^j$ for all $j \in S$. An allocation $(x^j \mid j \in M)$ is a *strict core allocation* if there do not exist a coalition S and an allocation $(x^j \mid j \in S)$ with $\sum_{j\in S} x^j = \sum_{j\in S} \omega^j$ such that $x^j \succeq_j \omega^j$ and $x^j \in F^j$ for all $j \in S$ with at least one strict inequality.

Shapley and Scarf (1974) considered a market in which each agent initially owns one item like one house, has preferences over all items, but has no use for more than one item. They showed that this market has a nonempty core.

Because the core is not guaranteed to be nonempty, it cannot be used as a solution to many markets where agents initially possess more than one item and may acquire several items, and it is therefore necessary and useful to find an appropriate alternative solution to such markets. It will be important for such a solution to possess at least individual rationality, Pareto-efficiency, and stability. The Markovian core to be introduced has these desirable properties. From the classical notion of the core (Definition 1), we understand that every agent has to be totally rational, infinitely patient, and fully capable of cognitive thinking and computing and to have a whole and clear picture of the entire market and that every member in every possible blocking coalition always compares with his initial endowment. Unlike this classical solution, the proposed solution has to relax some of these assumptions by taking both incomplete information, human myopic behavior, impatience, and bounded rationality into account. We may imagine an economic milieu in which people initially own several items and wish to exchange with each other. At the beginning, they may not know everyone except their neighbors and friends but they gradually get to know each other as time goes on. They may be myopic or not patient enough but just want to grab every opportunity to improve their current position. In this process, agents haggle with each other and exchange with each other as long as improvement can be made. Agents act rationally at the moment they exchange but not necessarily optimally or fully rationally if the whole process is taken into account. When trade happens, ownership will automatically change hands. Owing to possible exchanges of ownerships, the traditional core definition must be adapted to this fact. Trade terminates until none have incentive to trade any further. The final state of the market process may well be seen as a natural solution for stability. There is a tradition of treating an equilibrium of an economy as a final state of certain market process. For instance, following Edgeworth (1881), Uzawa (1962) directly defined the equilibrium of a barter economy with perfectly divisible goods to be the final state of his continuous price adjustment process. In the current paper, we will not only introduce a new solution concept in a similar way as Walrasian equilibrium and core are defined on the primitives of the market but also show that this new solution will be a natural outcome of a market process.

Definition 2. An allocation $(x^j \mid j \in M)$ is *strongly M-blocked by a coalition S* if there exists $(y^j \mid j \in S)$ such that $\sum_{j\in S} y^j = \sum_{j\in S} x^j$ and $y^j \succ_j x^j$ and $y^j \in F^j$ for all $j \in S$. An allocation $(x^j \mid j \in M)$ is *M-blocked by a coalition S* if there exists $(y^j \mid j \in S)$ such that $\sum_{j\in S} y^j = \sum_{j\in S} x^j$ and $y^j \succeq_j x^j$ and $y^j \in F^j$ for all $j \in S$ with at least one strict inequality.

In the definition, S will be called *an M*-blocking coalition of X. We use the M-blocking coalition in order to differentiate it from the classical notion of blocking coalition: S is *a blocking coalition of an allocation* $(x^j \mid j \in M)$ if there exists $(y^j \mid j \in S)$ such that $\sum_{j\in S} y^j = \sum_{j\in S} \omega^j$ and $y^j \succeq_j x^j$ and $y^j \in F^j$ for all $j \in S$ with at least one strict inequality.

An allocation $(x^j \mid j \in M)$ is *individually rational* if, for every agent $j \in M$, x^j is at least as good as his initial endowment ω^j.

Definition 3. An allocation is a *Markovian core allocation* if it is individually rational and cannot be strongly M-blocked by any coalition. An allocation is a *strict Markovian core allocation* if it is individually rational and cannot be M-blocked by any coalition.

By definition, a Markovian core allocation is individually rational with respect to the initial endowment of every agent. This property is shared with the classical notion of core. The current definition of M-blocking coalitions with at least two members is similar to that in the classical core (Definition 1) but differs from the latter in one major aspect that the current definition requires agents in each blocking coalition to compare the proposed assignments (y^j) with their current assignments (x^j) instead of their initial endowments (ω^j). Our Markovian core allocation is Pareto-efficient and immune from any possible coalition deviation by comparing with every coalition member's current assignment and captures some behavioral aspects and limited ability of human decision-making. That we use the term of Markovian core is to try to reflect behind the model a process of decision-making in which agents make their decision rationally but not necessarily optimally at every time based solely on their present states.

3 | EXISTENCE RESULTS

An allocation $(x^j \mid j \in M)$ is *a Pareto-improvement* of an allocation $(y^j \mid j \in M)$ if $x^j \succeq_j y^j$ for all $j \in M$ with at least one strict inequality. An allocation $(x^j \mid j \in M)$ is *a strong Pareto-improvement* of an allocation $(y^j \mid j \in M)$ if $x^j \succ_j y^j$ for all $j \in M$. An allocation is *strongly Pareto-efficient* if it has no Pareto-improvement. An allocation is *Pareto-efficient* if it has no strong Pareto-improvement. Our first theorem shows that the market has a nonempty strict Markovian core. It will be proved constructively as a natural outcome of a successive Pareto-improvement market process. Our arguments for the following two results are simple and intuitive.

Theorem 1. *The market $E = (\succeq_j, \omega^j, F^j \mid j \in M)$ has a nonempty strict Markovian core.*

Proof. We prove this by constructing a finite number of successive Pareto-improvements. Imagine that the market opens at day 0 with the initial market state $X^0 = (\omega^j \mid j \in M)$. We also denote it by $X^0 = (x^{0,j} \mid j \in M)$. If X^0 is not a strict Markovian core allocation, then there must exist an M-blocking coalition S^1 with $(y^j \mid j \in S^1)$ against X^0. Then we $(x^j \mid j \in M)$ obtain a new market state $X^1 = (x^{1,j} \mid j \in M)$ on day 1 with $x^{1,j} = y^j$ for all $j \in S^1$ and $x^{1,j} = \omega^j$ for all $j \in M \setminus S^1$. Clearly, X^1 is a new allocation at which none is worse than his initial state and at least one is strictly better off. So X^1 is a Pareto-improvement of X^0. Again, if X^1 is not a strict Markovian core allocation, there must exist an M-blocking coalition S^2 with $(y^j \mid j \in S^2)$ against X^1. We have a new allocation X^2 on day 2 being a Pareto-improvement of X^1. We repeat this process until there is no M-blocking coalition anymore. This is a monotonic process and must stop in finite time as the number of allocations is finite. Therefore, the market must have a nonempty strict Markovian core. □

The above theorem shows that a strict Markovian core allocation exists in a very general barter market. It might lead some casual reader into thinking that the strict core can be very similar to the strict Markovian core. In fact, it is not. To see this point, we will use the following example of Shapley and Scarf (1974, section 5) to show that when agents are indifferent between some items, the strict core can be empty.

Example 1. There are three traders. Trader 1 initially owns house h_1, trader 2 house h_2, and trader 3 house h_3. Their preferences over the houses are given by

$$\succeq_1 = h_2, [h_1, h_3]$$
$$\succeq_2 = [h_1, h_3], h_2$$
$$\succeq_3 = h_2, [h_1, h_3].$$

This means that traders 1 and 3 prefer house h_2 to both houses h_1 and h_3 but are indifferent between the latter two houses, while trader 2 are indifferent between houses h_1 and h_3 but prefers both to house h_2.

In this market, there are four core allocations: $\pi^1 = (h_1, h_3, h_2)$, $\pi^2 = (h_2, h_1, h_3)$, $\pi^3 = (h_2, h_3, h_1)$, and $\pi^4 = (h_3, h_1, h_2)$. It is easy to see that π^1 is blocked by coalition $\{1, 2\}$, π^2 by $\{2, 3\}$, π^3 by $\{2, 3\}$, and π^4 by $\{1, 2\}$. Consequently, there is no strict core allocation in the market.

The proof of Theorem 1 motivates us to consider a natural market process. A sequence $(X^s \mid s = 0, 1, \dots, t^*)$ of allocations in the market $E = (\succeq_j, \omega^j, F^j \mid j \in M)$ is said to be *a sequence of successive Pareto-improvement market states* if $X^s = (x^{s,j} \mid j \in M)$ for every $s = 0, 1, 2, \dots, t^*$ with $X^0 = (\omega^j \mid j \in M)$ and X^s is a Pareto-improvement of X^{s-1} for $s = 1, 2, \dots, t^*$ and there is no Pareto-improvement of X^{t^*}. X^0 is called *the initial market state* and X^{t^*} *the final market state*. Because there are only a finite number of allocations in the market, this sequence must be finite. Any market process which generates such a sequence is called a *successive Pareto-improvement market process*. Clearly, any such market process must terminate in a finite number of iterations. Then the following result follows immediately.

Remark. The final market state generated by any successive Pareto-improvement market process for the market $E = (\succeq_j, \omega^j, F^j \mid j \in M)$ is a strict Markovian core allocation.

This shows that the strict Markovian core allocation is a practical and sensible solution as it is an outcome generated by a natural decentralized successive Pareto-improvement market process. It should be noted that the basic idea of barter process is due to Edgeworth (1881) albeit for a simple economy with two perfectly divisible goods. For the models of Edgeworth (1881), Uzawa (1962), and Mukherji (1974) with perfectly divisible goods, prices are continuously adjusted to balance demand and supply of each good. In contrast, for our current model with indivisible goods, the market process has to adjust quantities of each good among traders involved. More precisely, in our market process as long as the market has not reached an equilibrium namely, a strict Markovian core allocation, an M-blocking coalition will emerge and members in the coalition will adjust their currently holding bundles among themselves so that at least one member in the coalition will get better off and none in the coalition becomes worse off. Members outside the coalition maintain their status quo. This adjustment results in a Pareto-improvement.

Now we consider a very practical and realistic situation where agents may not have a clear and complete knowledge of the whole market but are somehow well-informed in a sense that as long as there will be opportunities for some coalition of agents to improve themselves, this coalition can grasp such opportunities with a positive probability. To be precise, the market opens at day 0 with every agent coming with his initial endowment. Trade takes place everyday $t = 0, 1, \ldots,$ as long as agents can find opportunities to exchange and improve themselves. As agents do not have a complete knowledge of the market, we can only impose a mild condition upon the economy that on each day t, every blocking coalition should occur with a positive probability.

Theorem 2. *Assume that the market $E = (\succeq_j, \omega^j, F^j \mid j \in M)$ starts at day 0 with every agent $j \in M$ endowed with ω^j and that on each day $t = 0, 1, \ldots,$ every M-blocking coalition happens with a positive probability. Then the market will converge almost surely to a strict Markovian core allocation.*

Proof. We prove this by constructing a finite random process of successive Pareto-improvements. Imagine that the market opens at day 0 with the initial market state $X^0 = (\omega^j \mid j \in M)$. We also denote it by $X^0 = (x^{0,j} \mid j \in M)$. If X^0 is not a strict Markovian core allocation, then there must exist an M-blocking coalition against X^0. By hypothesis, we may take S^1 as a realized M-blocking coalition with $(y^j \mid j \in S^1)$ against X^0 with a positive probability. Then we $(x^j \mid j \in M)$ obtain a new market state $X^1 = (x^{1,j} \mid j \in M)$ on day 1 with $x^{1,j} = y^j$ for all $j \in S^1$ and $x^{1,j} = \omega^j$ for all $j \in M \setminus S^1$. Clearly, X^1 is a new allocation at which none is worse than his initial state and at least one is strictly better off. So X^1 is a Pareto-improvement of X^0. Again, if X^1 is not a strict Markovian core allocation, there must exist a realized M-blocking coalition S^2 with $(y^j \mid j \in S^2)$ against X^1 with a positive probability. We have a new allocation X^2 on day 2 being a Pareto-improvement of X^1. We repeat this process until there is no M-blocking coalition anymore. This is a monotonic process and must converge almost surely to a strict Markovian core allocation in finite time. More precisely, because of the finite number of market states and the assumption of the theorem, there exists a positive ϵ such that every M-blocking coalition happens with probability greater than a fixed common $\epsilon > 0$. Hence the probability of the infinite sequence of no occurrence of M-blocking coalitions is equal to zero since $(1-\epsilon)^n$ converges to zero as n goes to infinity. So an M-blocking coalition occurs at a finite time instance t with probability one, and then after t this process repeats as many times as possible till we reach a strict Markovian core allocation with probability one. □

The interested reader may refer to Chen et al. (2016), Fujishige and Yang (2017), Kojima and Ünver (2008), and Roth and Vande Vate (1990) for different random market processes.

4 | CONCLUSION

In this paper, we have introduced a general barter market with many heterogeneous indivisible items. Every agent is initially endowed with multiple items and may acquire several items. There is no medium of exchange like money. Agents have general preferences over their interested bundles of items. This model reduces to the well-known model of Shapley and Scarf (1974) and has a nonempty core when every agent is initially endowed with one item and has no use for more than one item. However, it is well known that the core of such an economy is typically empty when every agent is allowed to acquire more than one item. We have proposed a practical and appealing solution to this general resource allocation problem called a Markovian core, generalizing the classical solution of the core. A Markovian core allocation is individually rational, Pareto-efficient, and immune from any possible coalition deviation by comparison with their current assignments instead of their initial endowments. This solution has relaxed the requirement of full rationality and infinite patience of the classical notion of the core by taking incomplete information and some human behavioral aspects and cognitive factors, such as myopia, impatience, and bounded rationality, into account. We have shown that the market has always a nonempty strict Markovian core. A strict Markovian core allocation is proved to be an outcome of a natural decentralized market process. This result provides a plausible explanation as to why barter markets widely exist in a variety of circumstances. Furthermore, we have proved that a random decentralized process converges almost surely to a strict Markovian core allocation.

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REFERENCES

Argumedo, A., & Pimbert, M. (2010). Bypassing globalization: Barter markets as a new indigenous economy in Peru. *Development*, *53*, 343–349.

Arrow, K. J., & Hahn, F. (1971). *General competitive analysis*. Holden-Day.

Baldwin, E., & Klemperer, P. (2019). Understanding preferences: "Demand types" and the existence of equilibrium with indivisibilities. *Econometrica*, *87*(3), 867–932.

Bank, G. (1985). Constrained markets, "surplus" commodities and international barter. *Kyklos*, *38*, 249–267.

Bikhchandani, S., & Mamer, J. (1997). Competitive equilibrium in an exchange economy with indivisibilities. *Journal of Economic Theory*, *74*(2), 385–413.

Chen, B., Fujishige, S., & Yang, Z. (2016). Random decentralized market processes for stable job matchings with competitive salaries. *Journal of Economic Theory*, *165*, 25–36.

Edgeworth, F. (1881). *Mathematical psychics*, Kegan Paul.

Edgeworth, F. (1925). *Relating to political economy* (Vol. II). Macmillan.

Fujishige, S., & Yang, Z. (2017). On a spontaneous decentralized market process. *Journal of Mechanism and Institution Design*, *2*, 1–37.

Gewertz, D. (1978). Tit for tat: Barter markets in the Middle Sepik. *Anthropological Quarterly*, *51*, 36–44.

Gul, F., & Stacchetti, E. (1999). Walrasian equilibrium with gross substitutes. *Journal of Economic Theory*, *87*(1), 95–124.

Howell, J., & Chmielewski, T. (2009). Alpha Books.

Inoue, T. (2008). Indivisible commodities and the nonemptiness of the weak core. *Journal of Mathematical Economics*, 44, 96–111.

Kelso, A. S., & Crawford, V. P. (1982). Job matching, coalition formation, and gross substitutes. *Econometrica*, *50*(6), 1483–1504.

Kojima, F., & Ünver, M. (2008). Random paths to pairwise stability in many-to-many matching problems: A study on market equilibration. *International Journal of Game Theory*, *36*, 473–488.

Konishi, H., Quint, T., & Wako, J. (2001). On the Shapley–Scarf economy: The case of multiple types of indivisible goods. *Journal of Mathematical Economics*, *35*, 1–15.

Ma, J. (1994). Strategy-proofness and the strict core in a market with indivisibilities. *International Journal of Game Theory*, *23*, 75–83.

Ma, J. (1998). Competitive equilibrium with indivisibilities. *Journal of Economic Theory*, *82*(2), 458–468.

Marshall, A. (1952). *Principles of economics* (8th ed.). Macmillan.

Moulin, H. (1995). *Cooperative microeconomics*. Princeton University Press.

Mukherji, A. (1974). The Edgeworth–Uzawa barter stabilizes prices. *International Economic Review*, *15*, 236–241.

Pearson, R. (2003). Argentina's barter network: New currency for new times?. *Bulletin of Latin American Research*, *22*, 214–230.

Qin, C-Z. (1993). The inner core and the strictly inhibitive set. *Journal of Economic Theory*, *59*, 96–106.

Qin, C-Z. (1994). The inner core of an N-person game. *Games and Economic Behavior*, *6*, 431–444.

Roth, A. (1982). Incentive compatibility in a market with indivisibilities. *Economics Letter*, *9*, 127–132.

Roth, A., & Postlewaite, A. (1977). Weak versus strong domination in a market with indivisible goods. *Journal of Mathematical Economics*, *4*, 131–137.

Roth, A., & Vande Vate, J. H. (1990). Random paths to stability in two-sided matching. *Econometrica*, *58*, 1475–1480.

Scarf, H. (1967). The core of ann-person game. *Econometrica*, *35*, 50–69.

Shapley, L., & Scarf, H. (1974). On cores and indivisibilities. *Journal of Mathematical Economics*, *1*, 23–37.

Sun, N., & Yang, Z. (2006). Equilibria and indivisibilities: Gross substitutes and complements. *Econometrica*, *74*(5), 1385–1402.

Sun, N., & Yang, Z. (2014). An efficient and incentive compatible dynamic auction for multiple complements. *Journal of Political Economy*, *122* (2), 422–466.

Uzawa, H. (1962). On the stability of Edgeworth's barter process. *International Economic Review*, *3*, 218–232.

Wako, J. (1984). A note on the strong core of a market with indivisible goods. *Journal of Mathematical Economics*, *13*, 189–194.

Ye, S., Viswanathan, S., & Hann, I. H. (2018). The value of reciprocity in online barter markets: An empirical investigation. *MIS Quarterly*, *42*, 521–549..

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