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Twin nucleation and variant selection in Mg alloy: an integrated crystal plasticity modelling and experimental approach

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Abstract

Extension twin nucleation and variant selection in magnesium alloy WE43 is investigated in experimentally characterised and deformed microstructures replicated in crystal plasticity models. Stored (dislocation) energy density is found to identify the experimentally observed locations of twins which are not otherwise explained by global Schmid factors or local resolved shear stress criteria. A critical stored energy of 0.015 Jm^{-2} was determined below which twin nucleation does not occur. The stored energy density explains the locations of the observed twins and the absence of twins in parent grains anticipated to be favourable for twin nucleation. Twin variant selection has been shown to be driven by minimising locally stored shear energy density, while the geometric compatibility and strain compatibility factors only aid in partial prediction. All experimentally observed variants were correctly determined.

Keywords: HCP; Twin nucleation; Rare-earth alloy; Stored energy density; Variant selection; Crystal plasticity.

1. Introduction

1 There is increasing interest in light metals in automobile industries and good strength to weight
2 ratio makes Magnesium (Mg) alloy one of the candidate materials [Joost and Krajewski
3 (2017)]. However, Magnesium and its alloys are limited by their poor formability and ductility
4 due to the hexagonal close-packed (HCP) crystallographic anisotropy. Mg alloys commonly
5 exhibit strong ‘basal’ texture with the basal poles aligned with the sheet normal direction after
6 rolling [Styczynski et al. (2004), Mackenzie and Pekguleryuz (2008)]. Such texture results in
7 modest work hardening and non-uniform elongation during deformation. It has been reported
8 that the addition of rare earth (RE) elements, e.g. Cerium (Ce), Neodymium (Nd) and Yttrium
9 (Y), can modify the conventional rolling texture [Bohlen et al. (2007), Hantzsche et al. (2010),
10 Hadorn et al. (2013), Imandoust et al. (2017)]. Typical Mg-RE alloys exhibit a weak basal
11 texture with the basal pole tilted towards the normal direction and transverse direction after
12 rolling and/or annealing [Al-Samman and Li (2011), Guan et al. (2017a)]. The formability of
13 Mg alloys can thus be improved by achieving a RE-texture which reduces the anisotropy.

14 At low temperatures, basal slip in Mg is activated at lower shear stress than prismatic or
15 pyramidal systems, making $\langle a \rangle$ -basal slip the preferred slip mode [Partridge (1967)]. Due to
16 the limited number of easy slip systems in Mg alloys, deformation twinning that can
17 accommodate strain along the crystal c-direction provides an alternative mode of plastic
18 deformation. Twinning occurs by the formation of a nucleus which then propagates along the
19 twin direction on the corresponding twin plane, where it continuously shears and reorients the
20 parent grain [Christian and Mahajan (1995)]. It is therefore essential to understand the twinning
21 mechanism to enable control over the microstructure and texture evolution.

22 The commonly observed twin mode in Mg alloys is the $\{10\bar{1}2\}[\bar{1}011]$ extension twin
23 [Partridge (1967), El Kadiri et al. (2015)]. Due to the symmetry of the HCP crystal structure,
24 there are six possible extension twin variants in Mg. Extensive work has been carried out to

25 establish the mechanistic basis of twin nucleation and variant selection. The most widely
26 reported criterion is based on global Schmid factor that assumes a stress state resulting from
27 remote loading conditions. Jonas et al. (2011) have shown that only half of activated twins have
28 the highest global Schmid factors (e.g. >0.3). Similarly, statistical analysis of twin nucleation
29 and corresponding microstructural features in Mg [Beyerlein et al. (2010)] and Zr [Capolungo
30 et al. (2009)] show that 47%~60% of twins have the highest global Schmid factors on the
31 activated variant, while the 21%~27% have the second highest. It is suggested that the local
32 stress variation is responsible for the activation of twin variants with smaller global Schmid
33 factors, although the local resolved shear stress has not been investigated quantitatively (see
34 also [Livescu et al. (2019)]. In many modelling approaches, critical resolved shear stress
35 (CRSS) is typically used to describe twin formation [Proust et al. (2009), Wang et al. (2013),
36 Wu et al. (2015)]. A probabilistic twin nucleation model has been proposed by Beyerlein and
37 Tomé (2010) which assumes a Poisson distribution of threshold shear stress to predict twin
38 nucleation. This model was incorporated within crystal plasticity-phase field modelling [Liu
39 et al. (2018)] to study the spatial twin formation process. However, experimental observations
40 suggest that global Schmid factors calculated from macroscopic loading cannot fully explain
41 the observed twins [Molodov et al. (2016), McClelland et al. (2015), Sevillano (2008), Livescu
42 et al. (2019)]. This indicates that the twins are not always driven by stress alone. Gradients in
43 plasticity established within parent grains can be mitigated by twin nucleation to achieve strain
44 accommodation [McClelland et al. (2015)]. In the cases where twin nucleation is not stress
45 driven, low/negative Schmid factors (non-Schmid) have been observed [Molodov et al.
46 (2016)]. The negative Schmid factors may also result in negative local resolved shear stress
47 associated with the observed active twin variant in the microstructure.

48 The non-Schmid twin variant selection is typically explained using the geometric compatibility
49 factor and the shear strain accommodations. Both direct and indirect slip transmission may

50 occur at grain boundaries. Various models including the N factor [Livingston and Chalmers
51 (1957)], the geometric compatibility factor (m') [Luster and Morris (1995)], and the residual
52 burgers vector method (e.g. [Lim and Raj (1985)]) have been utilised to investigate slip transfer.
53 Of these three factors, the geometric compatibility factor (m') has been extensively used as it
54 shows better predictions [Bieler et al. (2014), Guo et al. (2015)], and is based on the degree of
55 coplanarity of incoming and outgoing slip systems across the grain boundary. It was later
56 extended to investigate twin variant selection and twin-assisted-twinning in HCP materials
57 [Wang et al. (2010), Xin et al. (2015), Guo et al. (2014), Nervo et al. (2016), Kumar et al.
58 (2016)]. In addition, the selection of non-Schmid variants by the parent grain has also been
59 investigated by studying the shear strain accommodations that are required in the neighbouring
60 grain to permit twin variant formation [Jonas et al. (2011)]. It has been shown that the twin
61 variant which requires least strain accommodation by prismatic slip and the highest by basal
62 slip is selected by the parent grain.

63 Energy based criteria have also been employed to study twin formation. For instance, Wang
64 et al. (2012) proposed a deformation energy criterion for twin variant selection. The correct
65 prediction of variant selection increased from 50% using a global Schmid factor criterion to
66 80% using the energy criterion. The crystal plasticity finite element (CPFE) modelling work
67 by Cheng et al. [Cheng and Ghosh (2015), Cheng and Ghosh (2017)] also uses an energetic
68 criterion, where twin nucleation occurs when the initial energy of $\langle c + a \rangle$ dislocations
69 generated in the parent grain exceeds the energy of $\langle c + a \rangle$ dislocation dissociation event.
70 Similarly, the energetic twin nucleation model introduced by Capolungo and Beyerlein (2008)
71 is based on the dissociation of $\langle a \rangle$ type dislocations. From these studies it is evident that the
72 energy arising from slip to maintain the local dislocation structure may lead to twin nucleation.
73 In this context the local stored energy density, which is based on evolution of dislocation

74 structure, and which has been shown to be a driver for fatigue crack nucleation in materials
75 [Wan et al. (2014), Chen et al. (2018)], may also be a useful approach.

76 Twin nucleation and variant selection are complex phenomena requiring knowledge of
77 macroscopic loading, material properties, microstructure and the local stress and stored energy
78 variations. In this paper, we utilise a systematic methodology which integrates a physically
79 based crystal plasticity model and experiments aiming to establish new understanding of both
80 twin nucleation and variant selection in WE43 alloy. In the next section, the experimental
81 procedure and crystal plasticity approach are briefly outlined. Results are then presented,
82 assessing twin nucleation and variant selection from the experimental observations, together
83 with material property determination and comparison of the results with the CPFPE analysis of
84 twin nucleation. Finally, we investigate twin variant selection utilising both the experimental
85 observations and CPFPE calculations based on local stored energy. This is followed by
86 discussion and conclusions.

87 **2. Methodology**

88 **2.1 Experimental procedure**

89 Mg alloy WE43 was received as an extruded bar supplied by Magnesium Elektron with the
90 composition and texture shown in Guan et al. (2017b). Compression samples with dimensions
91 of 4×4×8 mm (ASTM E9-09) were cut to calibrate the constitutive model and to investigate
92 twin nucleation. The compression was performed along the transverse direction (TD) until ~5%
93 strain in a Zwick/Roell™ 100 kN machine at room temperature at a strain rate of 0.1 s⁻¹.

94 Electron backscattered diffraction (EBSD) was performed on the sample before and after
95 deformation in ZEISS Sigma 300™ SEM equipped with Bruker high resolution EBSD detector
96 with voltage of 20 kV and ~13 mm working distance. The sample was mechanically polished
97 until 4000 grit silicon carbide papers followed by fine polishing using 50 nm colloidal silica
98 suspension for about 30 minutes. The sample was then Ar ion polished with a PECS-II system

99 under dual beam condition using 4.0 keV beam energy with rotational speed of 4 rpm for about
 100 40 minutes, followed by 20 minutes fine polishing using 2.0 keV and a speed of 2 rpm. A large
 101 area of $831 \times 505 \mu\text{m}^2$ was scanned with step size of $3 \mu\text{m}$ to obtain the undeformed
 102 microstructure. After deformation, a sub-region of interest of area $159 \times 122 \mu\text{m}^2$ from this large
 103 area was further scanned for high resolution electron backscattered diffraction (HR-EBSD)
 104 analysis with a step size of $0.3 \mu\text{m}$. The Kikuchi patterns with binning of 2×2 were saved for
 105 offline digital image cross-correlation.

106 **2.2. Constitutive framework**

107 The strain gradient and rate dependent crystal plasticity finite element formulation developed
 108 in Dunne et al. (2007) is utilized to study twin nucleation. A brief description of the formulation
 109 is presented here. The total deformation gradient \mathbf{F} is decomposed into elastic (\mathbf{F}^e) and plastic
 110 deformation gradient tensors (\mathbf{F}^p) as

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p \quad (1)$$

111 The plastic velocity gradient considers the contributions from all slip systems and is expressed
 112 as

$$\mathbf{L}^p = \sum_i \dot{\gamma}^i \mathbf{n}^i \otimes \mathbf{s}^i \quad (2)$$

113 where \mathbf{n}^i and \mathbf{s}^i are the plane normal and direction of slip system i respectively. The slip rate
 114 considering both forward and backward thermally activated dislocations escape is given by

$$\dot{\gamma}^i = \rho_m b^i v_D \exp\left(-\frac{\Delta F}{kT}\right) \sinh\left(\frac{(\tau^i - \tau_c^i) \Delta V^i}{kT}\right) \quad (3)$$

115 in which ρ_m is the mobile dislocation density, b^i the Burgers vector magnitude, v_D the Debye
 116 frequency, k the Boltzmann constant, T the temperature, τ^i and τ_c^i are the resolved shear stress

117 (RSS) and the corresponding critical resolved shear stress (CRSS) for slip system i . The strain
 118 rate sensitivity is determined by an activation energy ΔF and a corresponding activation volume
 119 ΔV^i for slip system i . The hardening law is given by the evolution of the CRSS based on the
 120 development of dislocation density as

$$\tau_c^i = \tau_{c0}^i + \alpha G b^i \sqrt{\rho_{SSD} + \sum_{i=1}^n \rho_{GND}^i} \quad (4)$$

121 where τ_{c0}^i is the initial slip resistance on the slip system, α the hardening coefficient and G the
 122 shear modulus. ρ_{SSD} and ρ_{GND} are the density of statistically stored (SSD) and geometrically
 123 necessary dislocations (GND) respectively. The SSD density is evolved as a function of plastic
 124 strain rate \dot{p} at a material point as

$$\rho_{SSD} = \int_0^t \gamma' \dot{p} dt \quad (5)$$

125 where γ' determines the rate of density evolution. The distribution of GND density is calculated
 126 from strain gradients accommodating lattice curvatures. Further details of the GND calculation
 127 can be found in Dunne et al. (2012). In summary, the Nye tensor is related to the GND density
 128 on an individual slip system by

$$\text{curl}(\mathbf{F}_p) = \sum_{i=1}^n \mathbf{\Lambda}^i \rho_{gnd}^i \quad (1)$$

129 where $\mathbf{\Lambda}^i$ is the second order tensor that holds the slip system geometry information (details in
 130 Dunne et al. (2012)) and ρ_{GND}^i the GND density on slip system i .

131 The microstructure-sensitive stored energy density methodology presented by Wan et al.
 132 (2014), argued that 5% of energy due to plastic deformation is stored as dislocation structures
 133 and 95% is dissipated as heat. They derived the accumulated stored energy density G_{SE} at each
 134 microstructural point as

$$G_{SE} = \int \frac{\zeta |\boldsymbol{\sigma} : d\boldsymbol{\varepsilon}^p|}{\sqrt{\rho_{SSD} + \sum_{i=1}^n \rho_{GND}^i}} \quad (7)$$

135 in which ζ represents the fraction of plastic energy stored locally, i.e. $\zeta = 0.05$. The stored
136 energy criterion has been extensively used to study crack nucleation and growth, and therefore
137 to predict the fatigue life of materials [Chen et al. (2018), Wilson et al. (2019)]. In the present
138 work, the accumulation of stored plastic energy in the material by the creation and evolution
139 of dislocation structures is argued to be a key factor for twin nucleation. In the context of
140 deformation twinning, the accumulation and dissociation of dislocations at the grain boundaries
141 lead to the formation of twin nuclei [Jeong et al. (2018)]. Hence local stored energy density
142 considered also by other authors [Capolungo and Beyerlein (2008), Cheng and Ghosh (2015),
143 Cheng and Ghosh (2017)] is investigated, but here the particular form of energy in eqn. (7) is
144 that stored by dislocation structure within an area determined by the dislocation density. The
145 underpinning dislocation mechanistic basis is described in Zheng et al. (2019) and it is noted
146 that the energy is stored predominantly by geometrically necessary dislocations.

147 **3. Results**

148 **3.1 Deformed microstructure and experimental twin variant identification**

149 Fig. 1(a) shows the microstructure before deformation with average grain size of 80 μm , where
150 the highlighted dashed square indicates the section of microstructure considered for CPFE
151 analysis and solid square indicates the region of interest (ROI) for HR-EBSD analysis. The
152 deformed microstructure in the ROI (Fig. 1(b)), along with the HCP unit cells to indicate the
153 grain orientations with respect to loading direction (TD), shows the presence of deformation
154 twins with low area fractions. The primary observation is that twinning is not activated in the
155 grain with most favourable crystallographic orientation for twinning (Grain P), but instead it is
156 active in surrounding grains with least favourable crystallographic orientations. These grains

157 are henceforth referred to as grains A, B, C and D as shown in Fig. 1(b). In order to investigate
 158 twin nucleation, it is necessary to locate twin tips in these grains. Among grains A-D, the twin

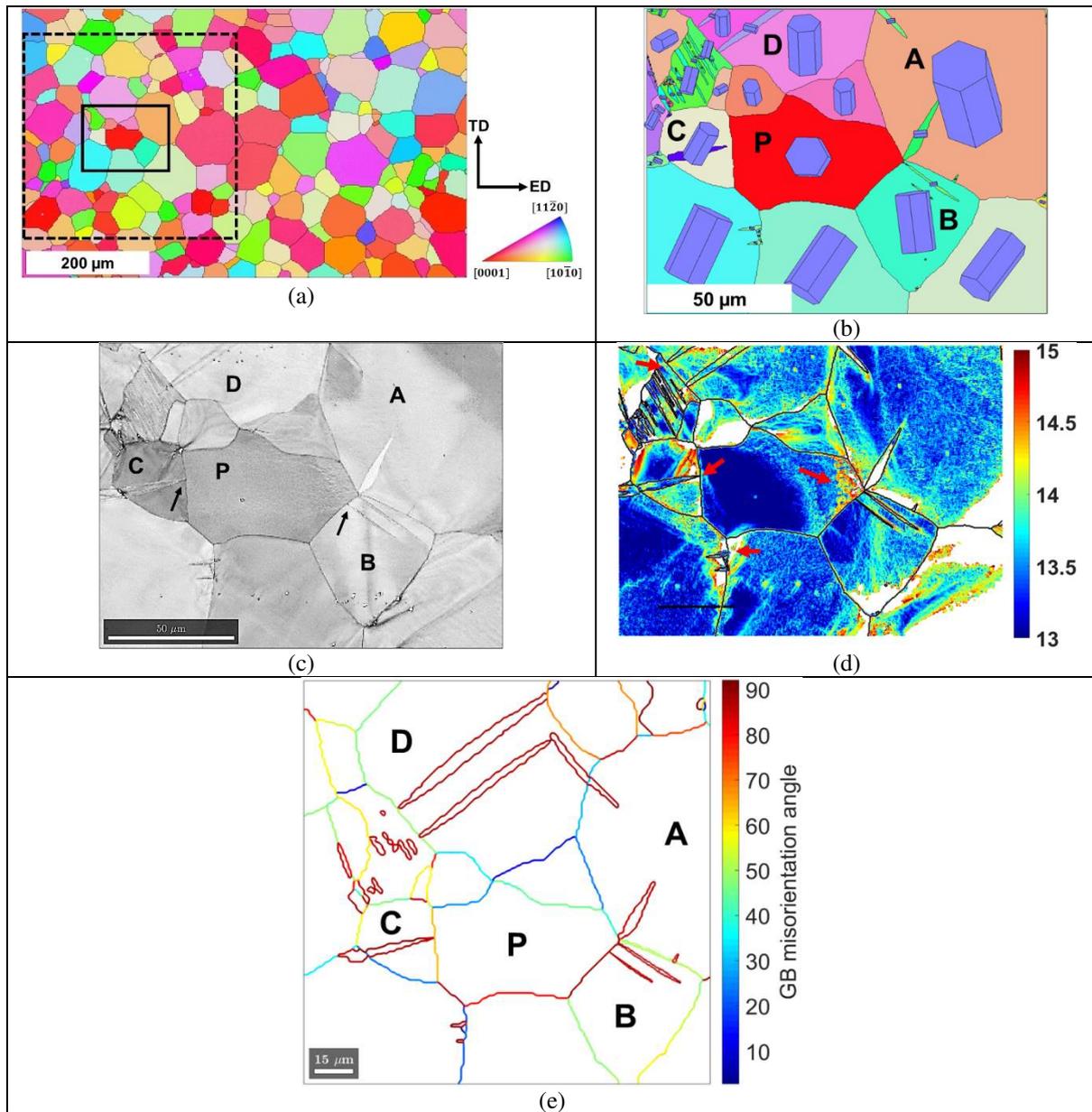


Figure 1: Uniaxial compression results showing (a) Undeformed microstructure, where the dashed square indicates the region of interest (ROI) for CPFE simulations and the solid line square indicates ROI for HR-EBSD analysis, (b) the ROI for HR-EBSD analysis after deformation with unit-cell orientations, (c) image quality map of the ROI and (d) GND density distribution in the ROI, where the red arrows show high GND density at twin tips (e) Grain boundaries (GB) of grains A-D and P showing misorientation angle

159

160 tips in grain C (right-hand tip) and B (left-hand tip) appear uncertain (see Fig. 1(b)). Therefore

161 the image quality map (Fig. 1(c)) of the EBSD data is investigated, which confirms that the

162

163 tips of these twins in grain C and B are indeed at the respective parent grain boundaries
164 (indicated using black arrows in Fig. 1(c)). Fig. 1(d) shows the distribution of GND density in
165 the microstructure shown in Fig. 1(b). The GND distribution is heterogeneous within the grains
166 and in particular at the vicinity of deformation twins, with high density at the twin tips
167 (indicated using red arrows) compared to their respective twin boundaries. Fig. 1(d) also shows
168 heterogenous distribution of GND density within the twin of grain C. It is shown in several
169 studies that nucleation of twins at the high angle grain boundaries is typically assisted by local
170 dislocations (slip-assisted), where the local high stress fluctuations drive twin nucleation and
171 variant selection [Beyerlein et al. (2010), Beyerlein et al. (2011), Khosravani et al. (2015)].
172 Fig. 1(e) shows the grain boundary misorientation angles in the region of interest comprising
173 grains A-D and P. One of the twins in grain A, which shares a boundary with grain D, appears
174 to have nucleated at the grain boundary with a misorientation angle of $\sim 35^\circ$. This
175 misorientation angle and the presence of an adjoining twin in neighbouring grain D indicates
176 that these twins may have nucleated through twin-assisted-twinning, which is consistent with
177 independent studies [Khosravani et al. (2015)]. In contrast, the other twin in grain A that shares
178 the boundary with grain B and P, appears to have nucleated close to the triple junction. Twins
179 in grains B-D have nucleated at grain boundaries with higher misorientation angles ($>40^\circ$),
180 indicating that these are slip-assisted nucleation events.

181 There are six variants of extension twins (variants of $\{10\bar{1}2\}\langle 10\bar{1}1\rangle$) as shown in Table 1,
182 therefore the next step is the identification of variant type of the twins in grains A-D. There are
183 several methods to identify the twin variants such as by trace and orientation analysis [Jiang
184 et al. (2008), Pei et al. (2012)]. In the current study the method of orientations is employed,
185 where the crystallographic orientations of all six twin variants corresponding to parent grain
186 orientation are calculated using the procedure outlined in [Niewczas (2010)]. Then the

Variant type	Crystallography
1	$(10\bar{1}2)[\bar{1}011]$
2	$(01\bar{1}2)[0\bar{1}11]$
3	$(\bar{1}102)[1\bar{1}01]$
4	$(\bar{1}012)[10\bar{1}1]$
5	$(0\bar{1}12)[01\bar{1}1]$
6	$(1\bar{1}02)[\bar{1}101]$

187

188 experimentally measured twin orientations are compared with these theoretical ones to identify
 189 the active variants in the microstructure. Fig. 2 shows the basal pole figures consisting of
 190 numerical and experimental twin orientations corresponding to grains A-D. Following the
 191 procedure briefed earlier, the active variants are identified as variant 1 in grains A, B and D,

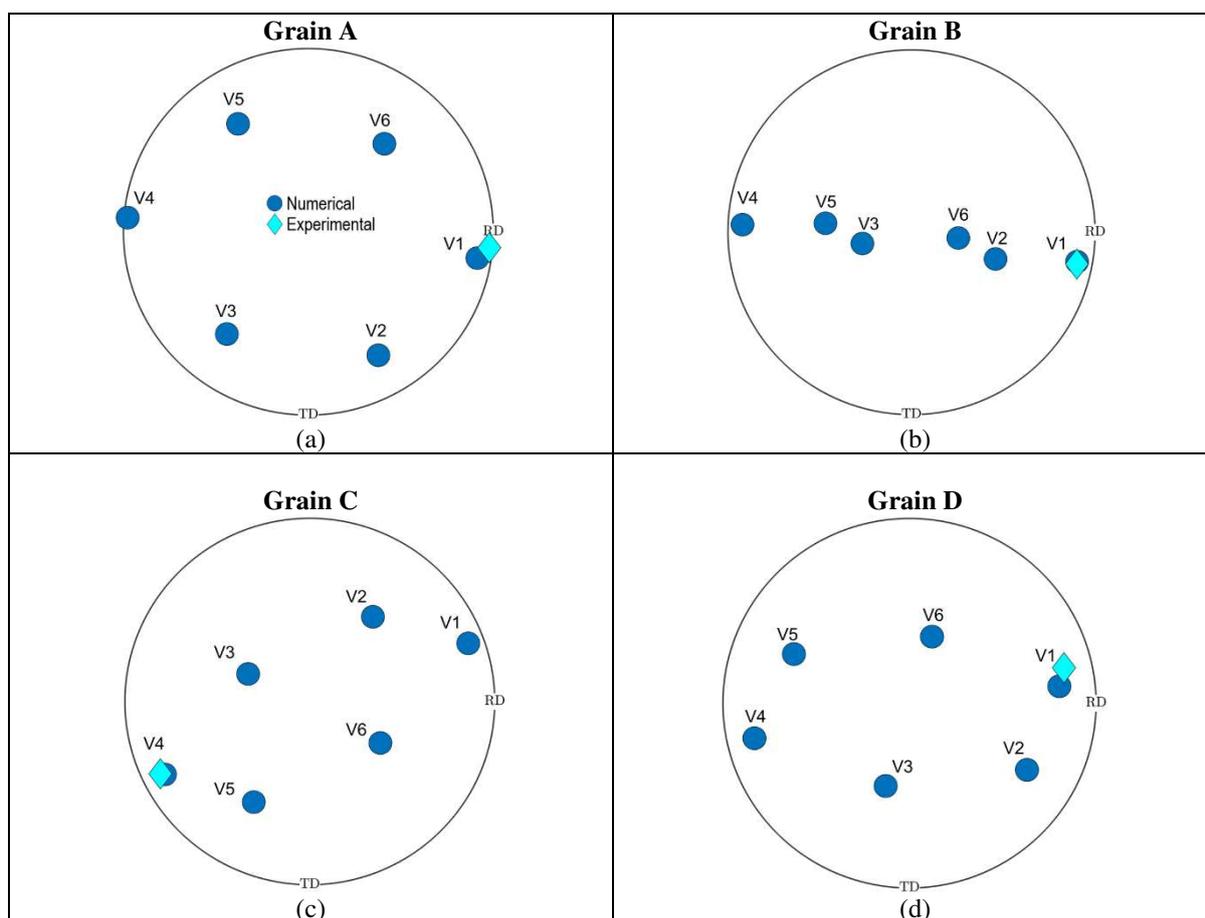


Figure 2: Discrete basal pole figures showing the numerical (blue dots) and experimental (diamonds) crystallographic orientations of twins in (a) Grain A, (b) Grain B, (c) Grain C and (d) Grain D. V# stands for variant (V) of type #.

192

193

194 while it is variant 4 in grain C. For the sake of completeness global Schmid factors, which rely
 195 on macroscopic loading direction, are also calculated to identify the type of twin variant active
 196 in the microstructure. Table 2 shows the Bunge Euler angles and global Schmid factors of all
 197 six variants of grains A-D, where the variants in bold are the active twin variants in the
 198 microstructure. From the table, except for grain C the global Schmid factors of active twin

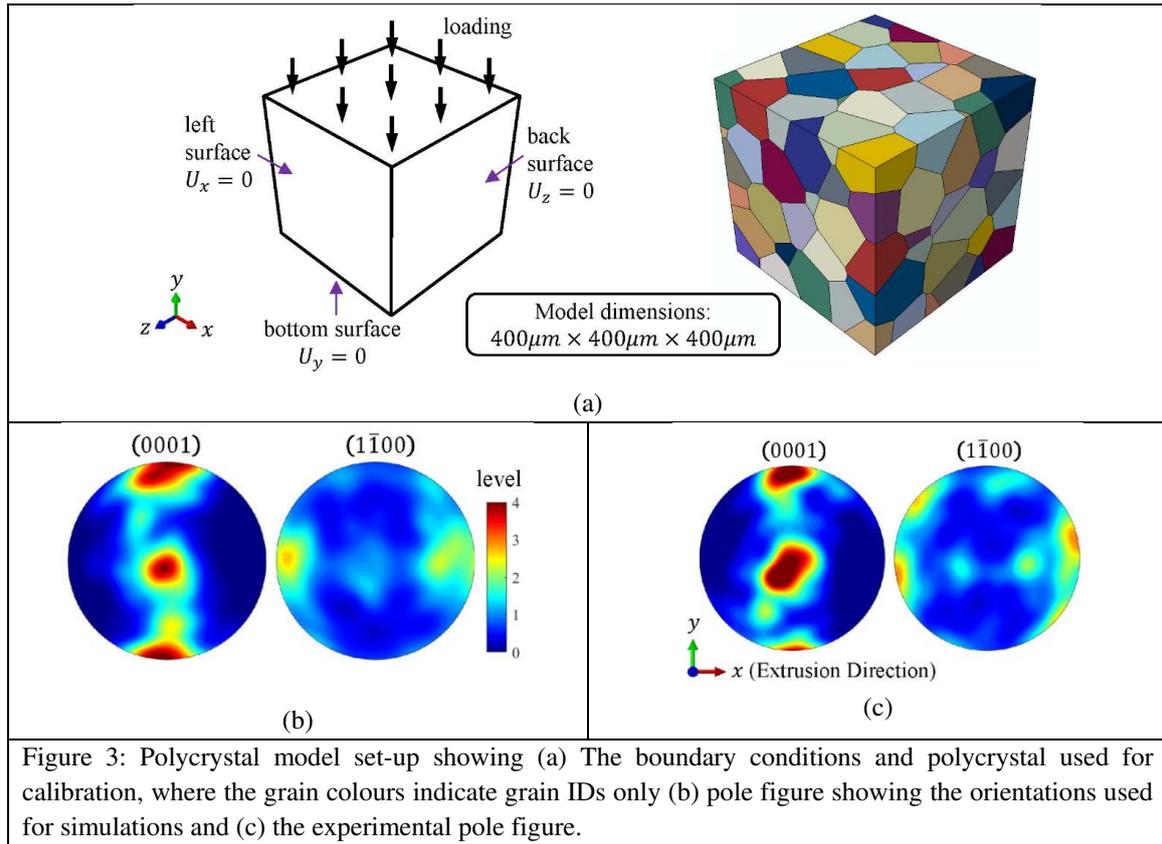
Table 2: Global Schmid factors of twins, where bold indicates active twin variants

Grain ID	Euler angles			Global Schmid factors					
				Variant 1	Variant 2	Variant 3	Variant 4	Variant 5	Variant 6
A	194	36	171	-0.15	0.16	0.07	-0.16	0.09	0.02
B	187	86	171	-0.48	-0.47	-0.49	-0.49	-0.49	-0.49
C	150	161	193	0.17	0.37	-0.01	0.20	0.41	0.00
D	183	56	158	-0.34	-0.25	-0.16	-0.32	-0.29	-0.22

199 variants are all negative. Deformation twinning is a 3D unidirectional defect as opposed to slip.
 200 This implies that the global Schmid factors of active twin variants should always be positive.
 201 Therefore, the global Schmid factors fail to explain the nucleation and selection of twin variants
 202 in grains A, B and D. In the case of grain C non-Schmid twin variant selection is followed,
 203 where the variant corresponding to the third rank is selected. The other alternative explanation
 204 for the nucleation of these twins is the activation by fluctuations of local (favourable) stress
 205 states, which is now explored further.
 206

207 3.2 Model material property determination

208 In order to determine the slip rule (eq. 3) properties for subsequent modelling, a three-
 209 dimensional polycrystalline CPFE model comprising 125 grains as shown in Fig. 3(a) was
 210 used. The grain morphologies were generated using 3D Voronoi tessellation software
 211 VGRAIN [Zhang et al. (2011)] with the average grain size of 80 μ m to be representative of the
 212 experimental microstructure, which are then discretised using C3D20R finite elements. The



213

214 crystal orientations were assigned from the EBSD map of the undeformed sample. The overall
 215 texture represented in the model and recorded from the experiment are shown in Fig. 3(b) and
 216 (c). Only 12 of the total 30 available slip systems $\{0001\} \langle 11\bar{2}0 \rangle$, 3 prismatic slip systems
 217 $\{10\bar{1}0\} \langle 11\bar{2}0 \rangle$, 6 pyramidal slip systems $\{\bar{1}\bar{1}22\} \langle 11\bar{2}3 \rangle$ - are considered in the current
 218 study. The elastic stiffness tensor is defined as $C_{11}=58.0$, $C_{12}=25.0$, $C_{13}=20.8$, $C_{33}=61.2$,
 219 $C_{44}=16.6$ (GPa) [Tromans (2011)]. The material properties following the calibration process
 220 are listed in Table 3. The rate sensitivity parameters close to those experimentally measured by
 221 Bhattacharyya et al. (2016) are used in this study, where the activation volume of basal and
 222 prismatic slip is $100 b^3$ and that of pyramidal slip is $6.25 b^3$. The Debye frequency of Mg is
 223 calculated to be $\sim 1.2 \times 10^{13} \text{ s}^{-1}$ [Kwak et al. (2016)].

224

225

Table 3: Single crystal parameters for WE43 at room temperature

Mode	τ_{c0} (MPa)	ΔV (b ³)	ΔF (J)	ρ_m (μm^{-2})	V_D (s ⁻¹)	γ' (μm^{-2})	α
Basal	14	100					
Prismatic	72.8	100	7.4×10^{-20}	1	1.2×10^{13}	1000	0.75
Pyramidal II order	126	6.25					

226

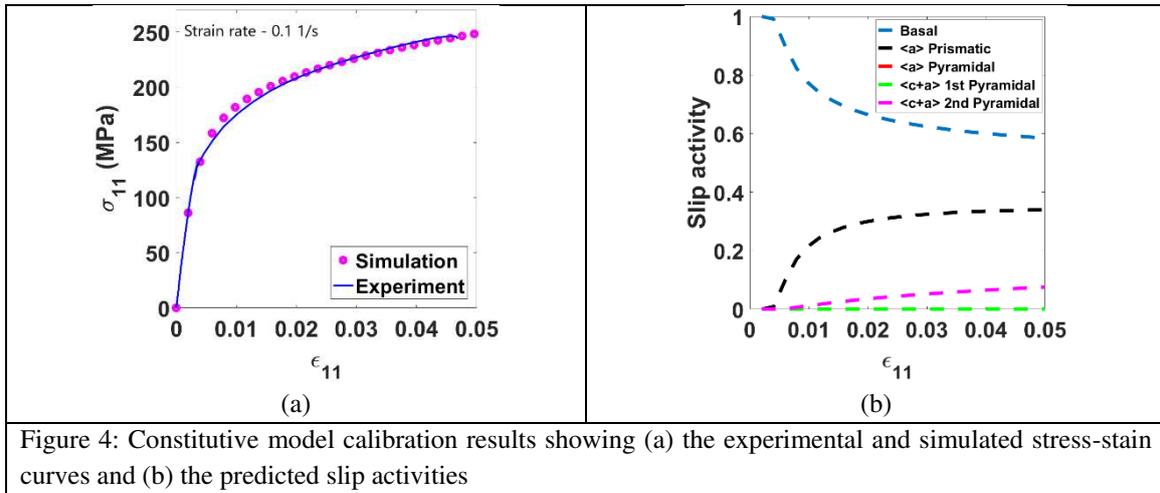


Figure 4: Constitutive model calibration results showing (a) the experimental and simulated stress-stain curves and (b) the predicted slip activities

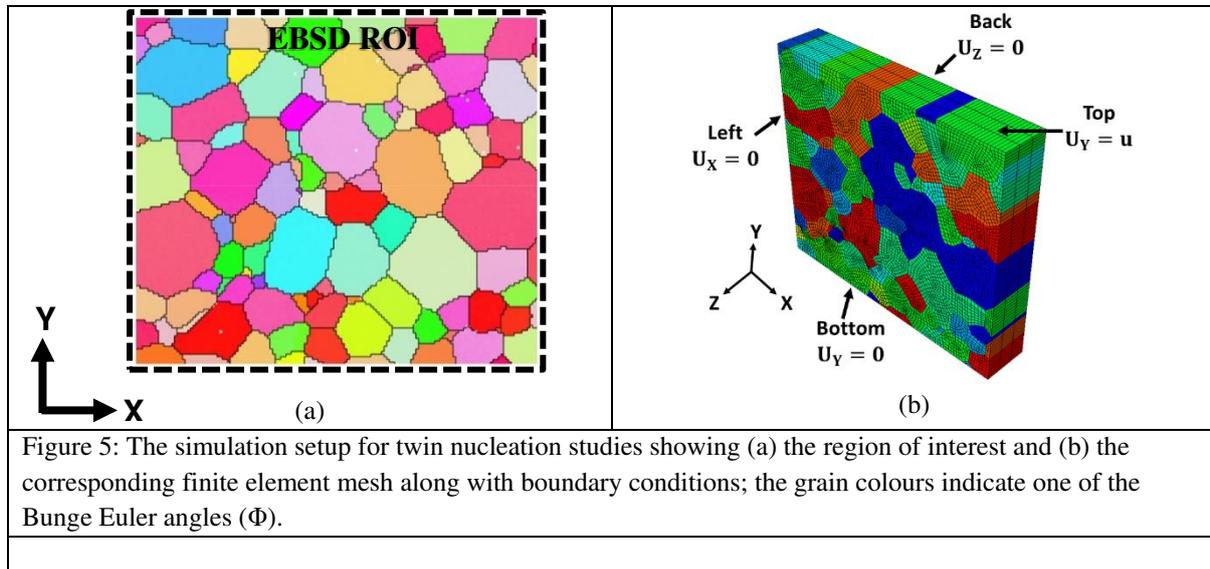
227

228 Fig. 4(a) shows the experimentally measured macroscopic true stress-strain response and the
 229 corresponding CPFE calculated response that shows good agreement with experiments. The
 230 slip activities during deformation are shown in Fig. 4(b). During compression, the basal slip
 231 system is activated first and is dominant at the onset of plastic deformation. Its activity
 232 decreases gradually with an increase in the contribution of other slip systems. Similar trends
 233 have been reported in independent studies (e.g. Bhattacharyya et al. (2016)).

234 3.3 Crystal plasticity analysis of twin nucleation

235 As discussed in section 3.1, global Schmid factors fail to explain the observed activation of
 236 twins. Therefore, the section of microstructure (Fig. 1(a)) including the grains of interest is
 237 explicitly replicated in the CPFE geometric model to study the local stress fields that may have

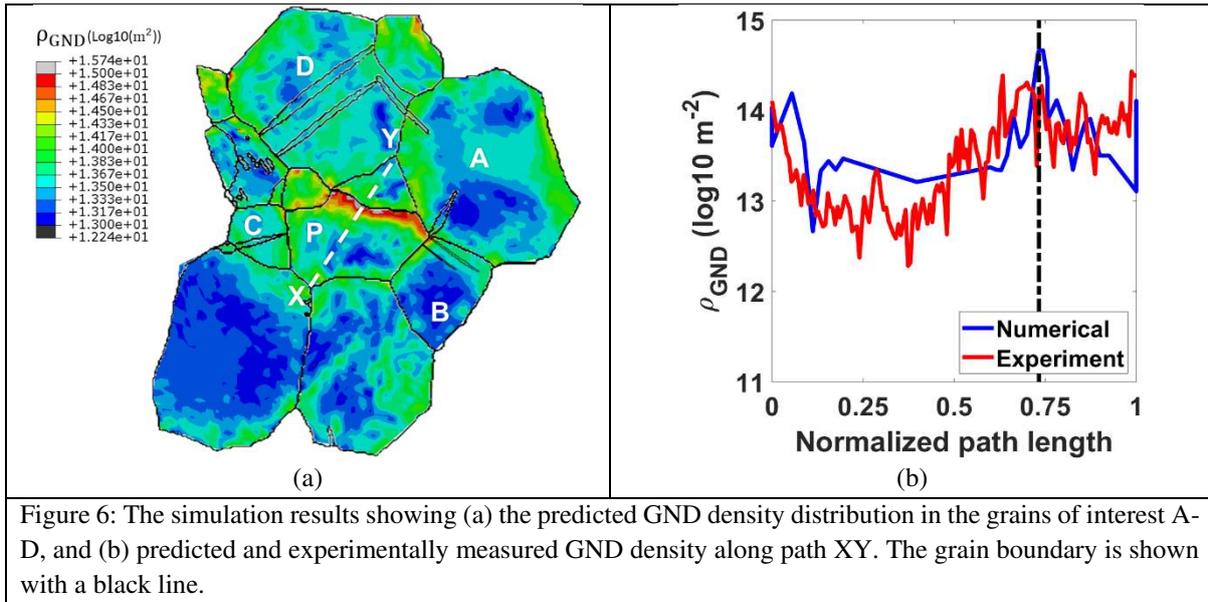
238 facilitated twin nucleation (prior to their incipient formation). Fig. 5(a) shows the
 239 microstructure and Fig. 5(b) the corresponding finite element mesh, which has an area of



240

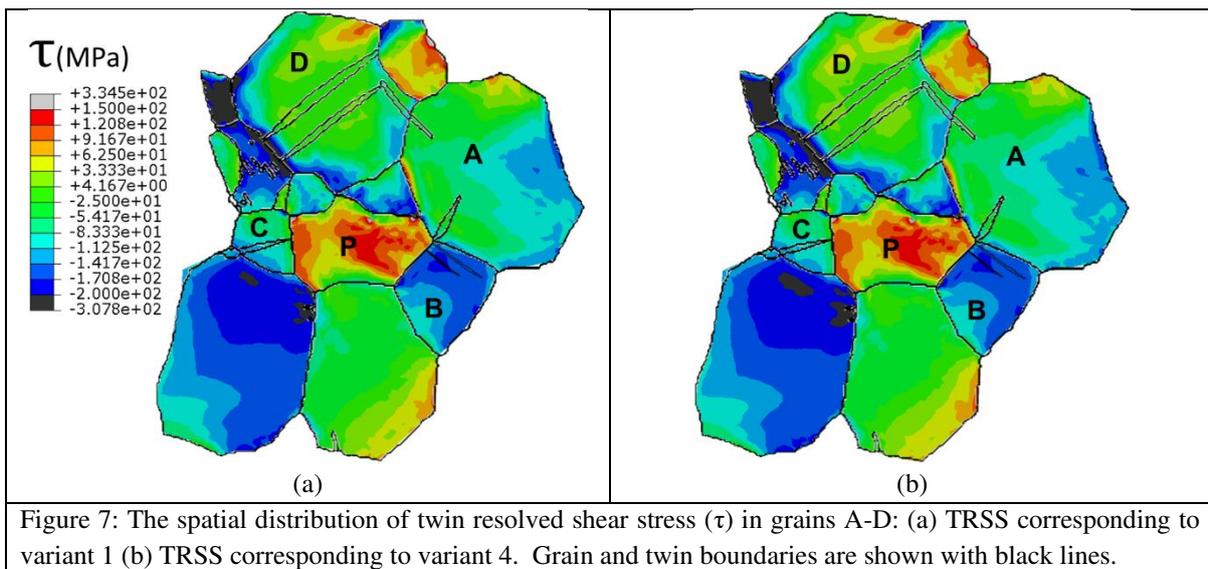
241 $446 \times 397 \mu\text{m}^2$ and thickness $80 \mu\text{m}$. Note that we proactively replicate the experimental
 242 microstructure prior to the nucleation of twins, which are not, therefore, explicitly represented
 243 in the geometric model, since this establishes the stress states leading to twin nucleation. A
 244 limitation of the current approach is that the 2-D microstructure is extruded in the Z-direction
 245 to obtain a pseudo 3-D model, which does not contain sub-structure information. Many studies
 246 have shown that the sub-surface microstructure influences the spatial distribution of
 247 deformation fields [Zeghadi et al. (2007), Zhang et al. (2018)]. However, Zhang et al. (2018)
 248 have also shown that the presence of the sub-surface mostly only influences the magnitudes of
 249 GND density, relative lattice rotations and strain fields but does not alter the observed trends
 250 on the free surface. The model is loaded along direction Y (corresponding to TD) until 5%
 251 strain at the strain rate of 0.1 s^{-1} . Once this strain is reached, the model is unloaded.

252 Fig. 6(a) shows the calculated GND distribution within the grains of interest, where the CPFE
 253 geometric model is superimposed with the experimental grain boundaries to indicate twins and
 254 path XY indicates that along which GND density is measured. The distribution of GND density



255

256 along the path XY is shown in Fig. 6(b), in which the CPFE satisfactorily captures the
 257 experimental trends. Twin nucleation has in the literature been attributed to local stress states,
 258 where a twin nucleates when the (positive) RSS driven by local stress states reaches the CRSS
 259 of the particular twin type. Therefore, RSS is calculated here at the peak applied strain and
 260 examined for the active twin variants in grains A-D; that is, variants 1 and 4. Fig. 7(a) shows
 261 the spatial distribution of TRSS for variant 1, while Fig. 7(b) shows that for variant 4. The



262

263 RSS in grain P is predominantly positive as expected considering the favourable
264 crystallographic orientation, but this grain does not nucleate any twins. Another observation is
265 that the RSSs corresponding to variant 1 (Fig. 7(a)) and variant 4 (Fig. 7(b)) are largely negative
266 within grains A-D. This is also expected as the grain orientations (Table 2 and Fig. 1(b)) are
267 least favourable. However, these are the grains within which twins are observed to nucleate,
268 contrary to convention based on resolved shear stress for extension twins. This implies that the
269 nucleation of these twins is not driven by local stresses that lead to high resolved shear stresses.
270 Therefore, the concept of twin nucleation based on CRSS (stress driven) does not explain the
271 activation of these twins in the microstructure. Nor does it explain the absence of twin
272 formation in grain P.

273 Fig. 8 shows the spatial distribution of effective plastic strain and basal, prism and pyramidal
274 slip respectively developed in grains A-D. The plastic strain and corresponding slip fields are
275 heterogeneous with some relationship to the twin nucleation sites within the grains A, B and
276 D. In particular, for grain A, one of the twins appears to nucleate from a location of high basal
277 slip. For grain B, a high localised pyramidal slip and local basal slip is potentially related to
278 twin nucleation, where the former is seen to make a considerable contribution to the effective
279 plastic strain compared to other slip systems. Grain C shows a localised region of high basal
280 slip on the right-hand boundary region close to where the narrow twin section is observed
281 (suggesting nucleation end). For grain D, it's again basal slip which seems potentially to be
282 related to both twins nucleating within this grain.

283 In summary, therefore, the hotspots in slip/plasticity are observed to have some relationship
284 to observed twin nucleation occurrences in this microstructure, but it by no means precisely
285 and unambiguously locates the twin nucleation sites in all cases. The study of local quantities
286 within the microstructure from the CPFE compared with experimental observations therefore
287 seem to suggest that while local GND density, resolved shear stress and accumulated slip are

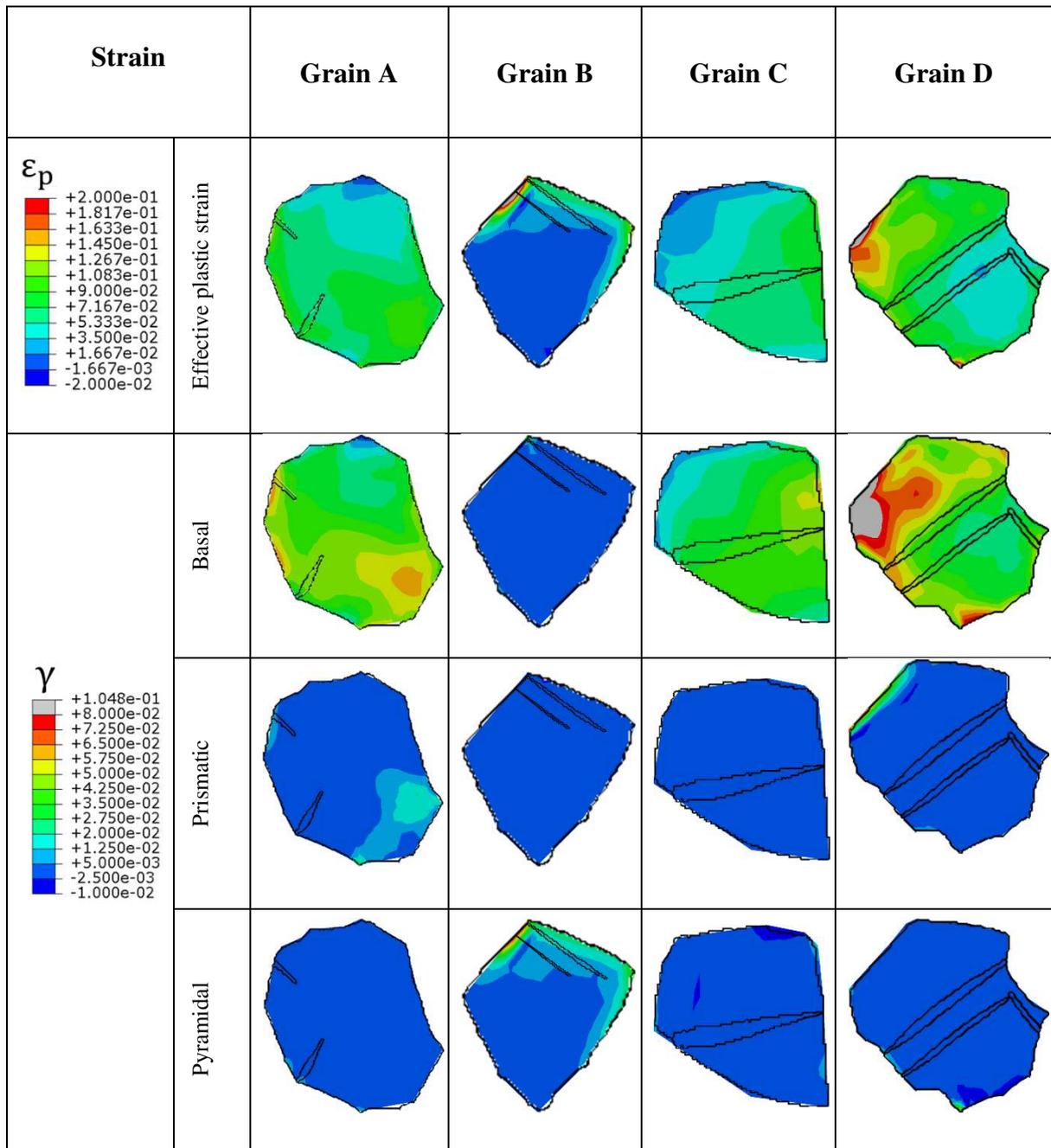


Figure 8: The spatial distribution of predicted effective plastic strain and the basal, prism and pyramidal slip fields in grains A-D as shown. Grain and twin boundaries are shown with black lines.

288

289 relevant and locally important to nucleating twins, they are not in their own right deterministic
 290 predictors of nucleation. We therefore turn to the energy approaches discussed in the
 291 introduction and in particular consider the dislocation-moderated stored energy density given
 292 in eqn. (7).

293 Fig. 9 (a)-(d) (i) shows the spatial distribution of stored energy density within grains A-D.
294 Dashed lines indicate paths along which stored energy density is extracted for line-graph plots
295 in (ii). Qualitatively, the stored energy density is observed to be consistently high within parent
296 grains at the twin nucleation sites. For instance, observe the high stored energies within grains
297 B and D in Fig. 9(b)(i) and Fig. 9(d)(i) at the vicinity of the twin nucleation sites.

298 Fig. 9 (a)-(d) (ii) shows the extracted stored energy densities measured along the paths shown
299 (Fig. 9(i)) in grains A-D respectively, in which the black vertical lines indicate the
300 experimentally observed twin nucleation sites. In grain A two twins are identified as AT1 and
301 AT2, where the nucleation of AT1 appears to be slip assisted while AT2 (dashed line in Fig. 9
302 (a)(ii)) seems to be related to the secondary twin in grain D (see Fig. 7)). The focus, therefore,
303 is on nucleation of AT1 but for the sake of completeness AT2 is also shown. From Fig. 9(a)(ii),
304 the experimental nucleation site of AT1 corresponds to the highest stored energy along path
305 P1-P2. Similarly for grain B in Fig. 9(b)(ii), the distinct peak in the stored energy corresponds
306 to twin nucleation site BT2 in Fig. 9(b)(i) and twin BT1 nucleates close-by at a location also
307 with high stored energy. While the latter is apparently not the highest energy density, the
308 nucleation of BT2 is likely to perturb the distribution of energy density, and were the spatially
309 resolved twin formation to be explicitly incorporated in the modelling, it is perfectly feasible
310 that the subsequent highest energy location would shift (from BT2) to BT1. This phenomenon
311 (redistribution of energy density) has been explicitly demonstrated in the context of secondary
312 crack nucleation in other studies [Chen et al. (2017)]. In the case of grain C, the location of
313 peak energy density corresponds exactly to the twin nucleation site. The stored energy,
314 however, appears quite uniform along this particular grain boundary R1-R2 but again, once
315 twin CT1 has been nucleated, the distribution of energy density and its magnitude is likely to
316 be very different, and sufficiently dissipated to reduce the energy driver for additional twin
317 nucleation anywhere else along this boundary.

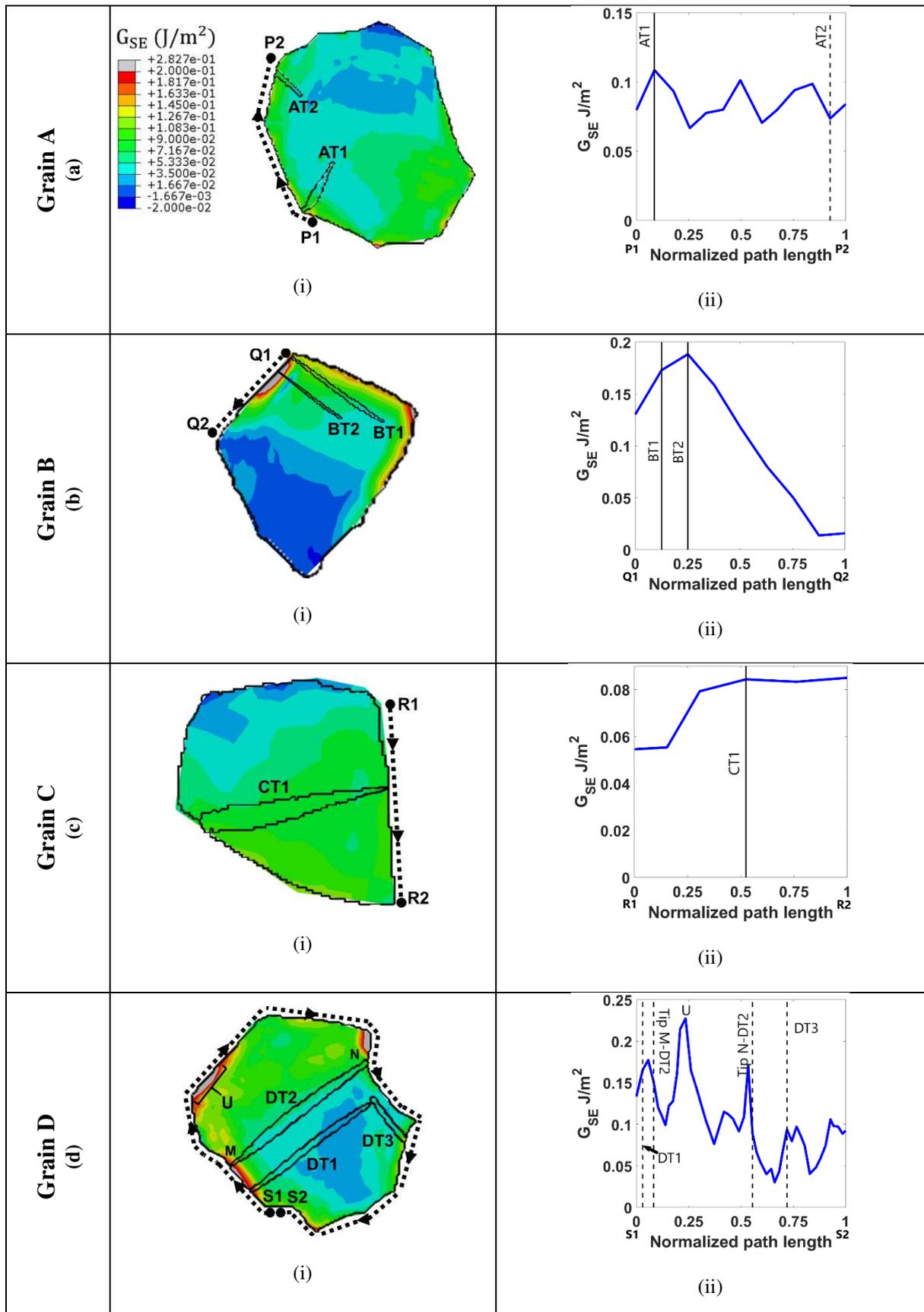


Figure 9: The calculated stored energy density (i) spatial distribution and (ii) along the paths (dashed lines) indicated in (i) for grains (a) A, (b) B, (c) C and (d) D. The solid black lines indicate the slip-assisted twin nucleation sites along respective paths.

318 Grain D nucleates three twins identified as DT1, DT2 and DT3. Twins DT1 and DT2 appear
319 to have nucleated at the grain boundary by slip assisted nucleation, while DT3 seems to be
320 secondary twin-assisted nucleation. The analysis in Fig. 9(d)(i) of the energy density at the left-
321 most tip of DT1 shows high stored energy compared to the right-hand tip. Therefore, DT1
322 seems to have nucleated at the grain boundary. Twin DT2 also seems to originate from the high
323 stored energy density location (M) common with DT1, but it is the case that a further region of
324 high energy density is observed close to, but not at, the right-hand tip of twin DT2 (N). Fig.
325 9(d)(ii) shows the distribution of stored energy density measured along the path S1-S2 (Fig.
326 9(d)(i)). The energy density at the vicinity of location U (Fig. 9(d)(i)) is noted to be the highest
327 along path S1-S2 and yet no twin is observed to nucleate at this location. In all the cases of
328 slip-driven twin nucleation observed and analysed in this study, location U in grain D is the
329 only example of a higher stored energy density being calculated in the absence of the
330 observation of a twin nucleation. This observation, and the nucleation of twins DT1 and DT2
331 at energy densities lower than that in region U, is explicitly investigated later in section 3.4.
332 All the results in Fig. 9 are shown for the peak applied strain (5%) such that any one value of
333 peak stored energy density should not be inferred as a *critical* value to drive twin nucleation.
334 Indeed, experimental observation suggests that in the alloy considered, twin nucleation initiates
335 early in the loading history at about 100 MPa [Guan et al. (2019)], corresponding to an applied
336 strain of about 0.002 (Fig. 4(a)). Hence, in order to extract out a definitive critical stored energy
337 density required to cause twin nucleation, it is necessary to have knowledge of the calculated
338 energy density at the point in the loading history when twin nucleation at a specific location is
339 observed.

340 The peak energy densities extracted for grains A to D at applied peak strain of 5% (from Fig.
341 9 (ii)) are of order 0.11, 0.2, 0.08, and 0.175 Jm⁻² respectively, such that a critical value must
342 therefore be 0.08 Jm⁻² (corresponding to grain C) or less. The full histories of the energy

343 densities at each of the key locations of twin nucleation can be extracted from the crystal
 344 plasticity analysis. On the basis that first twin nucleation was observed by Guan et al. (2019)
 345 at an applied stress of 100 MPa (corresponding to an applied strain of $\sim 0.2\%$), it becomes
 346 possible to estimate the *critical* value of stored energy for twin nucleation using the model
 347 results for grain C, and the critical stored energy is estimated to be 0.015 Jm^{-2} . The absence of
 348 twins in grain P is notable, since this grain is apparently well-orientated and anticipated to
 349 nucleate twins. However, the accumulation of grain-averaged stored energy density with
 350 applied strain in grains A-D and P is shown in Fig. 10. From this figure, the accumulation is
 351 the least in grain P compared to grains A-D. Hence while twin nucleation criteria based on
 352 global Schmid factor and resolved shear stress would indicate grain P should twin, the stored
 353 energy density does not, in agreement with experimental observations. In addition, it is noted
 354 that grains A-C are immediate neighbours of grain P (Fig. 1(b)), wherein grains A and C
 355 nucleate twins much earlier in the deformation history. Hence it is argued that the nucleation
 356 of twins in grains A-D, as a consequence of higher accumulation of stored energy, relaxes and
 357 redistributes the energy such that grain P never attains the *critical* energy density.

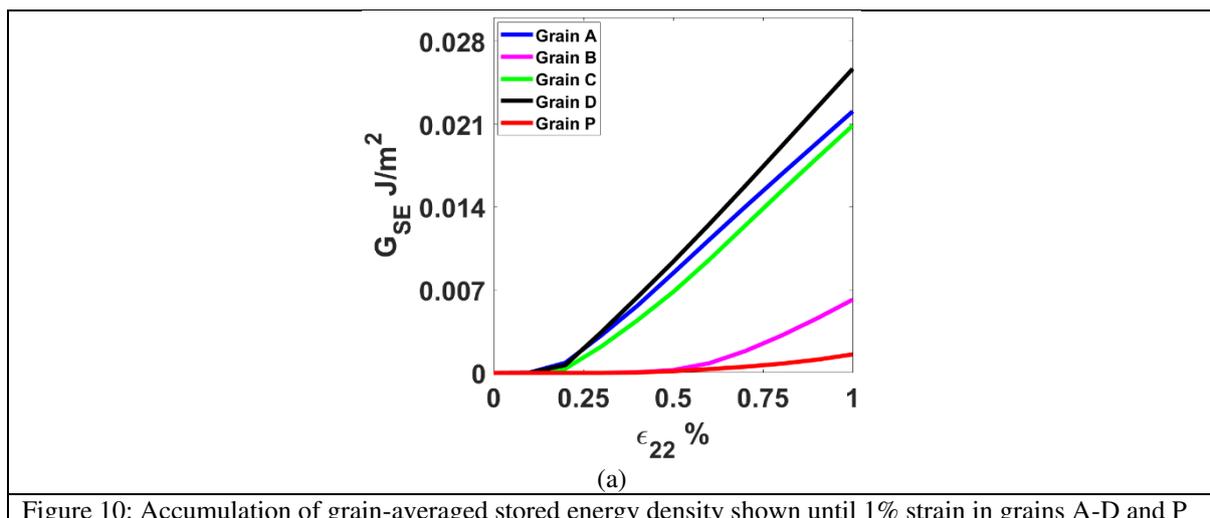


Figure 10: Accumulation of grain-averaged stored energy density shown until 1% strain in grains A-D and P

358

359

360

361 **3.4 Twin variant selection analysis**

362 The experimental observations in the current study indicate that neither the global Schmid
363 factor nor the twin RSS analyses explain the twin nucleation and variant selection (Table 2 and
364 Fig. 7) observations. Hence the geometric compatibility factor and shear strain
365 accommodations are firstly explored to study the twin variant selection observations in grains
366 A and B.

367 **3.4.1 Geometric compatibility factor and shear strain accommodation analysis**

368 Similar to slip transfer through interfaces, the geometric compatibility (m') between twin and
369 slip can be defined as [Luster and Morris (1995)]

$$370 \quad m' = \cos \varphi \times \cos \omega \quad (8)$$

371 where φ and ω are the angles between plane normal and shear direction of slip system in the
372 neighbouring grain and twin systems in parent grain respectively. High m' means the slip
373 system is well aligned with the twin variant and vice versa. The likelihood of strain transfer
374 across the grain boundary by the nucleation of a twin in the parent grain increases as the value
375 of m' approaches unity, while a low value (~ 0) of m' implies that the grain boundary is
376 impenetrable. However, in the present experimental study the parent grain crystallographic
377 orientations (A-D) are less favourable for twinning. Therefore, the value of m' is expected to
378 be low compared to other independent studies.

379 The selection of non-Schmid twin variants by the parent grain has also been explained in the
380 independent literature (e.g. [Jonas et al. (2011)]) on the basis of shear strain accommodation.
381 In that work, the twin variant selected within the parent grain is expected to be that which
382 requires least strain accommodation by prismatic slip and the highest by basal slip in the
383 neighbouring grain. However, these studies were performed with grains that are favourably

384 oriented for twinning, which generate twins at lower strains and largely activate basal slip. In
 385 the current study both the prismatic and basal slip collectively accommodate strain post-yield
 386 (Fig. 4(b)). Therefore, the total $\langle a \rangle$ type shear is considered containing contributions from both
 387 basal and prismatic slip. Thus, in this study, we firstly assess the criterion for variant selection
 388 within the parent grain which is based on maximum $\langle a \rangle$ type shear accommodation in the
 389 neighbouring grain in order to put the current work in the context of that in the open literature.
 390 In addition, it transpires that the shear accommodation hypothesis and m' do not work. Hence
 391 this is followed by the introduction of a new shear energy density criterion.

392 Following the procedure outlined in Jonas et al. (2011), the determination of accommodation
 393 strains involve defining a displacement gradient tensor \mathbf{S} in the twin reference frame, where
 394 the X, Y and Z directions correspond to the shear direction, the shear plane normal, and the
 395 cross product of X and Y directions respectively. All the components of \mathbf{S} except the $\partial u/\partial z =$
 396 s are zero, where s is the characteristic shear associated with a twin system (0.129 for extension
 397 twins). The tensor \mathbf{S} is given as

$$398 \quad \mathbf{S} = \begin{pmatrix} 0 & 0 & s \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (9)$$

399 This displacement gradient tensor is then rotated onto the crystallographic reference frame of
 400 the neighbouring grain to result in a new displacement gradient tensor e_{ij} , which contains all
 401 nine non-zero components. The non-zero shear components are given a physical interpretation
 402 in terms of the amount of shear strain required on a deformation mode in the neighbouring
 403 grain to accommodate the formation of a twin variant in the parent grain [Jonas et al. (2011)].
 404 Therefore, the main focus is on the shear components that correspond to shear on basal and
 405 prismatic slip systems. In particular the shear on the basal slip is determined as $(|e_{xz}| + |e_{yz}|)$,
 406 on prismatic slip is $(|e_{xy}| + |e_{yx}|)$ and finally the summation from shear on basal and prismatic
 407 is the total $\langle a \rangle$ type shear.

408 Table 4 and 5 show the shear strains accommodation and m' for all six possible twin variants
 409 for grains A and B respectively. The experimental results show that the twin variant 1 is
 410 selected by grain A (Fig. 2(a)). From Table 4, variants 1 and 6 have the maximum shear

411 Table 4. Shear accommodation and m' analysis of Grain A (bold indicates
 412 the observed active variant)

Twin variants	Shear accommodation			m'	
	Basal	Prismatic	$\langle a \rangle$ type	Basal	Prismatic
413 1	0.14	0.07	0.21	0.41	0.28
2	0.04	0.09	0.13	0.64	0.09
414 3	0.00	0.02	0.02	0.52	0.22
4	0.02	0.07	0.09	0.23	0.26
415 5	0.11	0.06	0.17	0.04	0.07
6	0.16	0.05	0.21	0.11	0.20

416 accommodation but among these two variants, the geometric compatibility factors for both
 417 basal and prismatic slip systems of variant 1 are greater than that of variant 6. Therefore, the
 418 preferred variant for grain A based on the two criteria is variant 1.

419 In grain B, the experimental active variant is also variant 1 (Fig. 2(b)). Among all the available
 420 six variants (Table 5), the 1st, 4th and 5th variants possess the same shear strain
 421 accommodation from the neighbouring grain, which means that these variants are equally
 422 probable.

423 Table 5. Shear accommodation and m' analysis of Grain B (bold indicates
 424 the observed active variant)

Twin variants	Shear accommodation			m'	
	Basal	Prismatic	$\langle a \rangle$ type	Basal	Prismatic
424 1	0.08	0.11	0.19	0.21	0.49
425 2	0.07	0.11	0.18	0.39	0.30
3	0.07	0.06	0.13	0.30	0.25
426 4	0.08	0.11	0.19	0.11	0.49
5	0.08	0.11	0.19	0.22	0.32
427 6	0.09	0.04	0.13	0.16	0.21

428 From the corresponding m' values, among variants 2, 4 and 5, variant 2 and 4 have the higher
429 compatibility offered by prismatic slip, while variant 5 has the highest for basal slip and the
430 least for prismatic slip. Therefore these two techniques lead to uncertainty in predicting the
431 active variant in grain B.

432 In summary, the shear strain accommodation and m' factors have been considered to
433 investigate the variant selection in parent grains A and B but transpiring only to be partially
434 correct. The values of m' depend on the 'alignment' of slip systems in the neighbouring grains
435 with the twin variant in the parent grain. The low values of m' , as reported in the current
436 investigation, have also been reported in independent studies [Wang et al. (2014), Liu et al.
437 (2016), Zhou et al. (2020)], which emphasize the importance of investigating the effect of local
438 deformation fields (stress, dislocation density etc.,) in twin variant selection. Zhou et al. (2020)
439 have shown that, in slip assisted twinning, it is necessary to consider the local slip activity
440 while predicting the twin variant selection using m' . Further, Wang et al. (2014) speculated
441 that the nucleation of twins (in one of their grains of interest) within the grains with negative
442 Schmid factor and low m' may occur due to high local stresses at the grain boundaries. Thus,
443 we return to local stored energy density considerations to explain the experimental observations
444 that may correctly identify the variant selection in all the cases.

445 **3.4.2 Energy based investigation of variant selection**

446 In an earlier section, stored energy density was shown to identify correctly the locations of
447 experimentally observed twin nucleation sites, albeit with one location (region U) showing
448 high energies in the absence of observed twins. Fig. 11(a) shows a schematic representation of
449 one such location of high stored energy density (G_{SE}^{max}) in the parent grain along the grain
450 boundary that serves as a high energy and defect source for a twin embryo formation. In order
451 to assess variant selection, an appropriate energy density is also developed to account for the
452 energy stored in the parent grain due to formation of a given twin variant. Experimental data

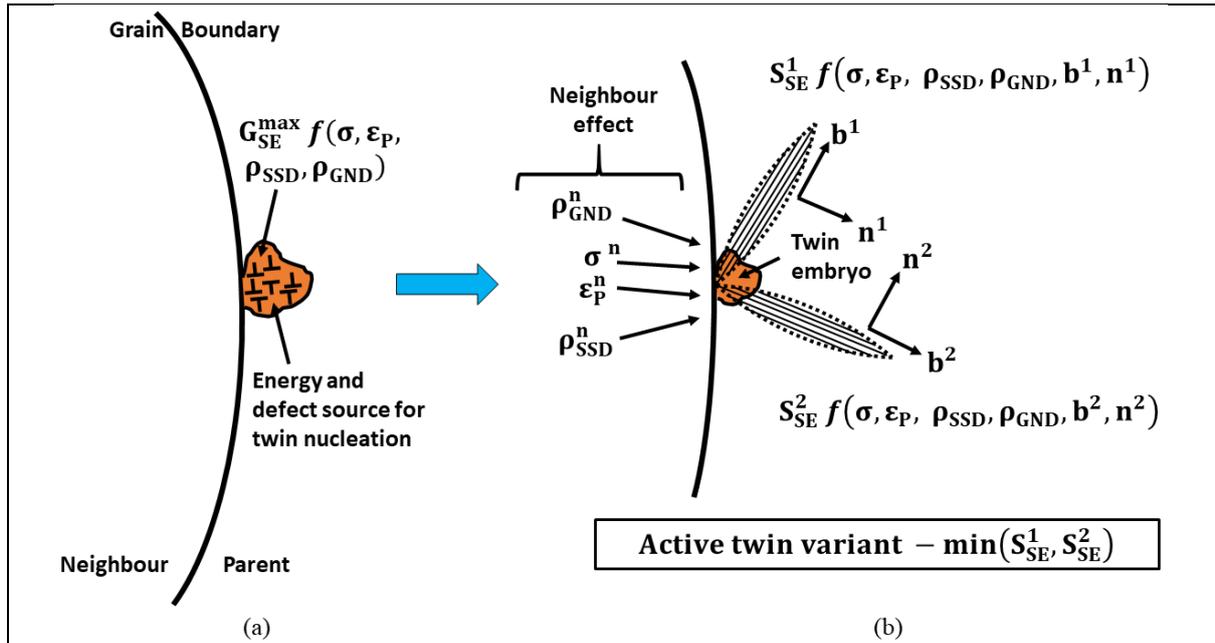


Figure 11: Schematic representation of twin nucleation and variant selection energy based criteria (a) location of high stored energy density (G_{SE}^{max}) along the grain boundary (b) twin variant selection criterion, operating at the location of G_{SE}^{max} , show only for 2 of 6 possible twin variants for the sake of clarity. S_{SE}^1 and S_{SE}^2 are the shear stored energy density of variants 1 and 2, and \mathbf{b} and \mathbf{n} are the twin shear direction and plane normal respectively.

453

454 reported in the literature show that the dislocation pile-up at the parent grain boundaries
 455 dissociate to form a twin nucleus, which further propagates and grows to form a twin.
 456 Therefore, the type of twin variant selected in the parent grain is decided at the incipient stage
 457 of nucleation at the parent grain boundaries. Beyerlein et al. (2011) have also observed a weak
 458 relationship between grain orientation and twin variant selection, and showed that the
 459 nucleation and variant selection are influenced strongly by the local high stress fluctuations at
 460 the grain boundaries. Hence an energy density term is established that is measured for all six
 461 twin variants at the parent grain boundaries to identify the twin variant.

462 It is shown in independent studies that a twin embryo forms by the dissociation of dislocations
 463 in the parent grain leading to formation of partials and twinning dislocations that shear the
 464 parent grain [Wang et al. (2009b)]. If φ^β is the total energy associated with the formation of
 465 an embryo of a twin variant β after the dissociation event, then this energy comprises that
 466 associated with the twin embryo (E_F^β) and work done (W_{ex}^β) by twinning dislocations

467 [Capolungo and Beyerlein (2008)]. This implies that the energy of the twin embryo (E_F^β) is
 468 stored in the material, while the work done is dissipated (lost). Therefore the total energy is
 469 given as [Nabarro (1952)]:

$$470 \quad \varphi^\beta = E_F^\beta + W_{\text{ex}}^\beta \quad (10)$$

471 On a continuum scale, for a given stress state ($\boldsymbol{\sigma}$) and plastic strain state ($\boldsymbol{\varepsilon}_p$) at a given
 472 microstructural location in the parent grain, the total energy associated with formation of a
 473 given twin variant post dislocation dissociation event can be estimated as:

$$474 \quad \varphi^\beta = (\boldsymbol{\sigma} : \mathbf{m}^\beta) (\boldsymbol{\varepsilon}_p : \mathbf{m}^\beta) = \tau^\beta \gamma_p^\beta \quad (11)$$

475 where the plastic strain ($\boldsymbol{\varepsilon}_p$) is given as

$$476 \quad \boldsymbol{\varepsilon}_p = \int_0^t \dot{\mathbf{d}}\boldsymbol{\varepsilon}_p dt \quad (12)$$

477 and \mathbf{m}^β is the symmetric part of the Schmid tensor defined as $\mathbf{m}^\beta = \frac{1}{2}((\mathbf{b}^\beta \otimes \mathbf{n}^\beta) +$
 478 $(\mathbf{n}^\beta \otimes \mathbf{b}^\beta))$, where \mathbf{n}^β and \mathbf{b}^β are twinning plane normal and direction respectively, τ^β the
 479 resolved shear stress and γ_p^β the resolved accumulated shear strain of twin variant β . When
 480 dislocation pile up in a parent grain dissociates to form multiple twinning dislocations, they
 481 tend to form several variants of the same twin type [Mendelson (1969)]. Therefore it is
 482 necessary to determine the energies associated with all six twin variants to understand the
 483 variant selection.

484 It is shown in molecular dynamics studies that a stable twin embryo consists of either twinning
 485 dislocations only or twinning and partial dislocations [Wang et al. (2009a)] that extend to a
 486 thickness of several crystallographic planes [Wang et al. (2009b)]. Therefore in the present
 487 formulation, a 3D representation of twin embryo is assumed that has a volume of ΔV_T , with an

488 area of A_T and length L_T , where the length L_T is taken to be the local dislocation mean free
 489 distance λ . Thus the volume is given as

$$490 \quad \Delta V_T = A_T L_T = \frac{A_T}{\sqrt{\rho_{SSD} + \sum_{i=1}^n \rho_{GND}^i}}$$

491 (13)

492 Then the shear energy density associated with the twin embryo that is stored in the system is
 493 given as

$$494 \quad S_{SE} = \frac{E_F^\beta}{A_T} = \frac{\varphi^\beta \Delta V_T}{A_T} = \frac{\tau^\beta \gamma_P^\beta}{\sqrt{\rho_{SSD} + \sum_{i=1}^n \rho_{GND}^i}}$$

495 (14) where the energy density associated with the twin embryo is called the shear stored energy,
 496 S_{SE} and equates to the stored twin embryo energy E_F^β . Further, for the formation of a stable twin
 497 embryo with an optimum distance between the twinning dislocations, the total energy
 498 associated with the formation of a twin variant (φ^β) should always be a minimum [Capolungo
 499 and Beyerlein (2008)], which implies that the energy of the twin embryo (E_F^β) should also be
 500 minimised for the chosen variant. Therefore in the current study, this energy S_{SE} ($= E_F^\beta/A_T$)
 501 is determined for all six twin variants at the parent grain boundaries where the total stored
 502 energy density is maximum in order to identify that twin variant which has minimum energy
 503 associated with its embryo formation. The schematic representation of one such twin embryo
 504 formation event is shown in Fig. 11(b), where for the sake of clarity only two of six twin
 505 variants are shown. At the location of peak G_{SE}^{\max} location, it remains possible for any of the
 506 possible six twin variants to form. The particular twin variant selected is argued to be that
 507 which minimises the shear stored energy density (S_{SE}) for its formation compared with any
 508 other variant.

509 Fig. 12 shows the shear stored energy density calculated for all the twin variants in grains A-
 510 D, where this energy is measured along the same paths as the stored energy (Fig. 9(i)). For
 511 grain B, the shear strain accommodation and m' compatibility analysis was shown not to result
 512 in the correct observed variant selection, and that in fact twin variant 1 developed in this grain.
 513 Examining grain B first, therefore, Fig. 12(b) shows that at the twin nucleation sites BT1 and
 514 BT2 (black lines) where the stored energy density is high, twin variant one leads to the least

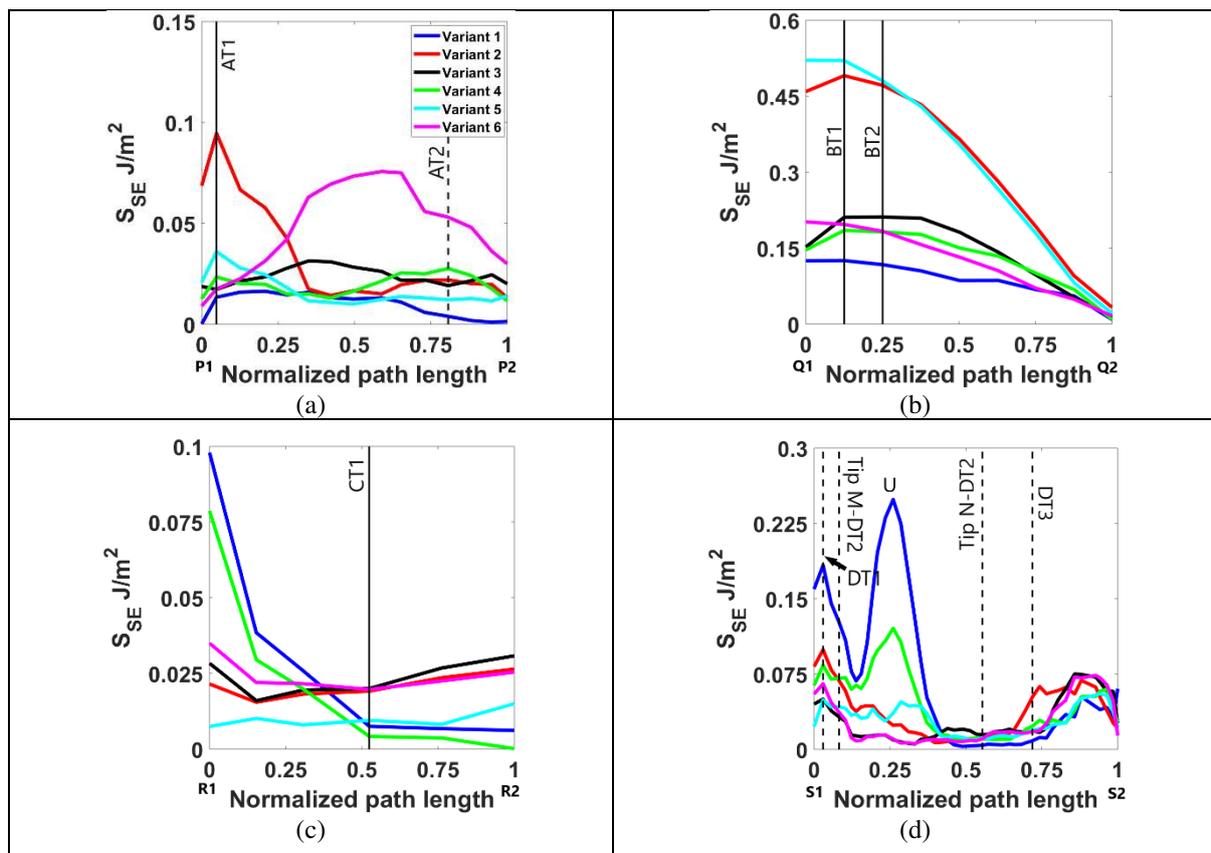


Figure 12: The shear stored energy quantified along the paths shown in Fig. 9(i) for grains (a) A, (b) B, (c) C and (d) D. The vertical lines indicate the experimentally observed twin nucleation sites, where the dashed lines indicate either the twin-assisted twin nucleation or ambiguous slip-assisted nucleation sites. The line colour indicates twin variant type shown in the legend.

515
 516 shear stored energy compared to the other variants (and notably variant 4 and 5) and hence is
 517 selected by the parent grain. Similarly for grain A, both the twins AT1 (slip assisted) and AT2
 518 (twin assisted) observed experimentally correspond to variant 1 and the predictions show least
 519 shear stored energy density for variant 1 at both the twin nucleation sites. This indicates that

520 the nucleation of twins assisted both by slip and twinning require least shear stored energy
521 density. Further, the shear stored energy criterion in this case is consistent with the shear strain
522 accommodation and m' analyses. For grain C, the shear stored energy correctly predicts the
523 experimentally observed active variant 4 in this grain. The distribution of stored energy density
524 in grain D led to ambiguity in predicting the twin nucleation site for twins DT1 and DT2 within
525 grain D (Fig. 9(d)(ii)). As mentioned earlier in the case of grain D, three twins DT1, DT2 and
526 DT3 of variant type 1 are experimentally observed. While the nucleation of DT3 is likely to be
527 twin-assisted, the nucleation of DT1 and DT2 are assisted by slip. DT1 has one of the twin tips
528 at the grain boundary and the other at the twin tip of DT3. From Fig. 9(d)(ii), high stored energy
529 indicates that DT1 may have nucleated at the grain boundary. However, Fig. 12(d) shows that
530 the shear stored energy for variant 1 at the grain boundary for DT1 is high. Therefore, it is
531 argued that the accumulation of high stored energy at the twin tip of DT3 due to its formation
532 may have nucleated DT1, which then propagated to the nearest grain boundary. It is shown in
533 independent studies that the formation of a twin results in high localization of stress [Kumar
534 et al. (2015)] and dislocation density [Guo et al. (2017)]. Similarly from Fig. 9(d)(ii), in the
535 case of DT2, high stored energy is observed at M and at the vicinity of N. However, the tip N
536 has the least shear stored energy for variant 1 compared to that of tip M. This implies that the
537 nucleation site of DT2 is indeed tip N instead of M (Fig. 12(d)). Further at tip N, the shear
538 stored energy of variant 1 is the least compared to the others. In addition the shear stored
539 energies for all the twin variant types at location U, which has the highest stored energy along
540 the path S1-S2 (Fig. 9(d)(i)), are high compared to those for both DT2 (tip N) and DT3. Hence
541 the absence of twin nucleation at region U for grain D results for two particular reasons. First,
542 the shear energy required for any twin variant in this region is very high compared with that at
543 observed twins DT2 and DT3. Second, the high stored energies and low shear energies
544 associated with twins DT2 and DT3 lead to early preferential nucleation of these twins, which

545 as a consequence dissipates energy, reducing the driving force for twin nucleation at region U.
546 Therefore, this detailed analysis implies that the stored energy density in conjunction with the
547 shear stored energy density appear to satisfactorily predict twin nucleation and variant
548 selection. A grain boundary location giving rise to high stored energy and low shear stored
549 energy is the most favourable for a twin nucleation site, where the twin variant selected is that
550 requiring the least shear energy.

551 **4. Discussion**

552 In the current study extruded Mg alloy WE43 (Fig. 1(a)) is compressed until ~5% strain to
553 nucleate extension twins with low/negative global Schmid factors. The CPFÉ calculations of
554 the resolved shear stresses determined by the local stress states are negative for all twin variants
555 (Fig. 7), which implies that these twins are not stress driven. The effective plastic strain shows
556 hotspots in the microstructure, but this distribution does not precisely predict the twin
557 nucleation sites. Therefore stored energy density, based on the evolution of local dislocation
558 structures, has been used to investigate twin nucleation. Fig. 9 shows that the locations of high
559 stored energy correspond to experimental twin nucleation sites in grains A-D. The stored
560 energy approach of twin nucleation is similar to other independent studies based on
561 heterogeneous twin nucleation theory [Capolungo and Beyerlein (2008), Beyerlein and Tomé
562 (2010), Ghazisaeidi and Curtin (2013)], according to which a twin nucleates from the existing
563 defects in the microstructure.

564 Independent experimental observations show that twins typically nucleate at low strain levels
565 for which the formation of dislocation structures (and correspondingly stored energy) is likely
566 to be limited, and for these cases, it is argued that the resolved shear stress (RSS) dominates
567 the total stored energy (i.e. not the shear energy density determining variant selection)
568 introduced above such that a simple RSS criterion is sufficient to explain twin nucleation (e.g.
569 [Liu et al. (2017), Paramatmuni and Kanjarla (2019)]). However, in contrast, nucleation of

570 twins with negative Schmid factors at lower strains that are insensitive to grain orientations
571 have also been observed [Livescu et al. (2019)]. While variant selection was not addressed in
572 [Livescu et al. (2019)], it was suggested that the nucleation of such twins may have been caused
573 by fluctuations in local stress states. Similarly, independent experimental studies have reported
574 such microstructure insensitive twins may nucleate to maintain the compatibility, orientation
575 and strain gradients in the microstructure [McClelland et al. (2015), Molodov et al. (2016)]. In
576 the present study, which examines twins of just this kind, it has been shown that such twins are
577 not in fact nucleated by local stress states, but rather, by the stored energy density.

578 Variant selection was then investigated. It is shown that shear strain accommodation and the
579 geometric compatibility factor explain the observed variant selection in grain A only.
580 Therefore, a new shear energy-based criterion has been introduced, to study variant selection.
581 It is accepted that high stress concentration at the grain boundaries supply the energy required
582 for the formation of twins. Further, the grain boundaries act as sources of defects (partials or
583 twinning dislocations) required for formation of twin nucleus. The shear energy based criterion
584 introduced in this study is based on local stress variations and is sensitive to such local
585 dislocation structures, which act as defect sources for twin embryo formation. The observed
586 twin variants selected within grains A-D correspond to those that require least predicted shear
587 energy density of formation. It is interesting to note that the twins that are nucleated by a twin
588 in a neighbouring grain (twin-assisted-twinning) also show least shear stored energy (grains A
589 and D). This indicates that the energy of formation for a twin embryo is minimum although it
590 is driven by the high stress concentration and defects supplied by twins in neighbouring grain.
591 In summary, it is shown that the stored energy in conjunction with the shear stored energy
592 locates twin nucleation site and further determines the twin variant type (Fig. 11).

593 Other energy-based variant selection approaches have also been developed. For instance, Wang
594 et al. (2012) used an analytical plastic strain energy based approach to predict the active twin

595 variants in the microstructure, where the strain energy depends largely on the strain in the twin
596 reference frame ($\epsilon_{33}^{\text{twin}}$) and the mean free path of the twin (\sqrt{L}), given as $\left(\epsilon_{33}^{\text{twin}}/\sqrt{L}\right)$. In their
597 studies, the variant with highest absolute value of $\left(\epsilon_{33}^{\text{twin}}/\sqrt{L}\right)$ was selected in the
598 microstructure. Alternatively, if the sign of $\left(\epsilon_{33}^{\text{twin}}/\sqrt{L}\right)$ is strictly considered, then the twin
599 variant with the least value of $\left(\epsilon_{33}^{\text{twin}}/\sqrt{L}\right)$ was chosen in their studies, which implies that the
600 twin variant with the least associated plastic strain energy was selected in the microstructure.
601 The shear stored energy approach is similar but more robust as it is based on the local stress
602 fluctuations and dislocation structures. Further, an advantage is its implementation in full field
603 approaches such as CPFE that includes the influence of neighbouring grains, evolving local
604 dislocation structures and the strain field.

605 The stored energy density has been investigated to understand the absence of twins in the
606 apparently most favourable grain P. Here, it was found that the rate of stored energy density
607 accumulation was lowest for grain P, such that the critical energy required for twin nucleation
608 was achieved earlier in other grains. The nucleation of twins in grains A-D in principle dissipate
609 energy, redistribute stress, and correspondingly local stored energy such that the critical value
610 required for twin nucleation does not develop in grain P. The *critical* stored energy for twin
611 nucleation in this material is estimated to be 0.015 Jm^{-2} . The independent molecular dynamics
612 and DFT studies have reported that the formation energy of twin boundaries decreases with
613 increasing twin thickness, where a stable 9 layers thick twin embryo has a formation energy of
614 $\sim 0.256 \text{ J/m}^{-2}$ (16 meV/A^{-2}) [Wang et al. (2009a)]. Assuming that this is the energy required for
615 the formation of a twin embryo, the energy reported in the current study is about an order of
616 magnitude less. This could be due to the length-scale and continuum approximation of the
617 current CPFE approach. Further due to the weak texture, majority of the applied strain is

618 accommodated by crystallographic slip in this material. The accumulation of these dislocations
619 at the grain boundaries leads to stress concentration and act also as defect sources necessary
620 for twin embryo formation. Therefore the energy and source barriers for the formation of twins
621 is minimum, which is argued here to result in low *critical* stored energy for twin nucleation
622 compared to MD studies [Wang et al. (2009a)]. Further, while the MD studies (e.g. [Wang
623 et al. (2009a)]) provide complete localized understanding of twin formation, they do not
624 consider the effect of existing defects, alloying elements and suffers from the limitations of
625 computational cell size (not enough neighbouring grains). In addition, it appears that the MD
626 studies in [Wang et al. (2009a)] are within the bulk of pure Mg away from the grain boundary.
627 This is crucial as the high stress concentrations at the grain boundary may aid in overcoming
628 the energy barriers and reduce the total energy associated with the twin embryo, which in turn
629 reduces the energy of formation of twins.

630 **5. Conclusions**

- 631 ● The experimental observations of compressed Mg alloy WE43 show the nucleation of
632 extension twins with low/negative global Schmid factors within unfavourable parent
633 grain orientations.
- 634 ● The finite element based crystal plasticity analysis shows that the local resolved shear
635 stresses for these twins are negative implying that they are not driven by local stress.
- 636 ● It is shown that the stored energy density offers mechanistic insight into twin nucleation
637 site, and acts as the driver of twin nucleation, indicating that these twins are driven
638 predominantly by local dislocation density.
- 639 ● Based on the investigation of accumulated stored energy density calculated at the point
640 in the loading history when twins are first observed, a *critical* value of stored energy
641 for twin nucleation is estimated to be 0.015 Jm^{-2} .

642 ● Twin variant selection is explained by the shear stored energy density, where the variant
643 selected is that which minimizes this energy density.

644 Therefore, the experimentally observed twin nucleation sites are identified by high stored
645 energy density (achieving the critical value first) and the variants selected by minimising the
646 stored shear energy which drives their formation.

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