On quantum reading, quantum illumination, and other notions

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In this brief review, we start by clarifying the crucial differences between three different protocols of quantum channel discrimination. In some recent literature, there has been confusion between the protocols of quantum illumination [1, 2], quantum reading [3], and a scheme of communication within a discrete-variable quantum computer [4]. While all these protocols are based on the model of quantum channel discrimination, they have completely different applications and features, which is the reason why they have different names and should not be naively confused. We also discuss the notion of quantum reading capacity [5] of an ensemble of quantum channels, clarifying how this is easily extended to an adaptive formulation and discussing the mathematical conditions under which it can be connected to the different notion of dense coding capacity [6] of a quantum channel.

I. INTRODUCTION

The protocols of quantum illumination [1, 2], quantum reading [3], and the communication of quantum computation [4] can all be represented as schemes of quantum channel discrimination (QCD). Recall that QCD is a very general problem where an ensemble of quantum channels is prepared in a black box. One is allowed to probe the input of the box by preparing a suitable quantum state, and to detect the output of the box by applying a suitable quantum measurement. The aim is understand which channel from the ensemble is present in the box.

Despite being interpreted as schemes of QCD, all protocols mentioned above have different aims and features, which is the reason why they should not be naively confused one with the other. In particular, the scheme of Ref. [4] is about communication between registers of a quantum computer, clearly not "quantum reading" of a classical/digital memory, nor "quantum illumination" of a remote target. With these two protocols, it only shares the basic connection with QCD.

Quantum illumination uses QCD to model a problem of target detection in bright noise environments, while quantum reading uses QCD to model a digital memory and the associated storage/readout of classical information. Most importantly, both these protocols show 'quantum advantage' in their respective tasks over classical strategies. For this reason, they are 'quantum' protocols. We will re-iterate this point in the manuscript.

We briefly review these protocols in Secs. II-IV and provide clarifications in Sec. IV A. Then, in Sec. V we consider the notion of quantum reading capacity [5] and its immediate and implicit extension to adaptive protocols. In Sec. VI, we discuss the (adaptive) notion of dense coding capacity [6] and describe the precise mathematical conditions under which this capacity can be connected to a quantum reading capacity. Sec. VII is for conclusions.

II. QUANTUM ILLUMINATION

Quantum illumination [1, 2] (see also [7–24]) is the use of input quantum resources (such as entanglement) and output quantum measurements to enhance the detection of a remote low-reflectivity object in a bright thermal-noise environment. It can be represented as a QCD problem, because the presence or absence of the target can be associated with the binary discrimination of two channels, one including a partial reflection from the target and the other one being a completely thermalizing channel (replacing the input with the state of the environment). Here one can show that, despite initial entanglement is lost in the sender-receiver path, the benefits of quantum illumination still survive in the forms of output correlations. These allow one to enhance the sensitivity of detecting the presence of the target-object with respect to the use of classical sources of light (in particular separable states in the DV version of the protocol [1], and mixtures of coherent states in the CV version [2]). It is called "quantum" illumination because it proves a quantum advantage with respect to classical strategies under the same conditions (e.g., the same mean number of input photons).

III. QUANTUM READING

Quantum reading [3] (see also [25–41]) is the use of input quantum resources (such as entanglement) and output quantum measurements to enhance the retrieval of classical information stored in the cells of an optical memory. In its basic binary formulation, it can be represented as a QCD problem where the discrimination is between two quantum channels associated with two different reflectivities of a memory cell (used to encode a bit of information). We therefore have two bosonic Gaussian channels, generally characterized by different values of loss (and thermal noise), that encode a classical bit.

Contrary to quantum illumination, the scheme is in the very near range, typically works with high reflectivities, and also allows one to use codewords to encode information in blocks of many cells (so that quantum reading capacities can be defined). Most importantly, the use of quantum resources (e.g. entanglement) allows one to enhance the data readout in terms of bits per cell with respect to the use of classical strategies (in particular

the use of coherent states or their mixtures). It is called "quantum" reading because it proves a quantum advantage with respect to classical strategies using the same amount of energy (mean number of photons).

The two schemes of quantum illumination and quantum reading have a specific peculiarity (quantum enhancement) that gives them the "quantum name" while, at the same time, it is clear that they are both associated with QCD. Further discussions can be found in Section V.H "Gaussian channel discrimination and applications" of the Gaussian information review [42] and also in the recent review on photonic quantum sensing [43].

IV. CAPACITY OF QUANTUM COMPUTATION

The scheme of Ref. [4] is about the communication capacity of quantum computation. Clearly, it is not about target detection or optical storage, but rather communication between registers of a discrete-variable quantum computer. In this scheme, there is a "memory" register (M) where the sender encodes a classical variable i in N pure quantum states $|i\rangle_M \langle i|$ with some probability p_i. Then, the receiver has a computation register (C) prepared in some initial state ρ_C^0. The initial state of the two registers is therefore the tensor-product

$$\sum_i p_i |i\rangle_M \langle i| \otimes \rho_C^0. \tag{1}$$

The two registers are then fed into a quantum computer, which applies the unitary $\hat{U}_i$ onto register C conditionally on the value i of register M. Here $\hat{U}_i$ represents a series of quantum gates which describes some quantum algorithm. For instance, i may be an integer, and the computational output $\rho_C^i = \hat{U}_i \rho_C^0 \hat{U}_i^\dagger$ may be its factorization according to Shor's algorithm.

In general, for the input state as in Eq. (1), the quantum computer provides the output

$$\sum_i p_i |i\rangle_M \langle i| \otimes \rho_C^i. \tag{2}$$

The receiver measures register C so as to discriminate between the possible output states ρ_C^i, or equivalently between the possible unitary operations $\hat{U}_i$. The optimal information accessible to the receiver is the Holevo bound

$$I(C : i) = S(C) - S(C|i), \tag{3}$$

where $S(C)$ is the von Neumann entropy of the reduced state of C, and $S(C|i)$ is the corresponding conditional von Neumann entropy. This is clearly maximized when p_i is uniform and the states ρ_C^i are pure and orthogonal, so that it takes the maximum value $I(C : i) = \log_2 N$.

By construction, it is clear that $I(C : i)$ represents the capacity of the quantum computation $\{\hat{U}_i\}$ because it tells you how good the quantum computer is in providing distinguishable output states (solutions) for different inputs. When the maximum $\log_2 N$ is achieved, it means that the quantum computation is perfect over the entire input alphabet of N letters.

A. Clarifications on protocols

Apart from being interpreted as a possible protocol of QCD, the scheme of Ref. [4] is clearly different from both quantum illumination and quantum reading.

- First of all, Ref. [4] is a communication scheme, where sender's input alphabet is decoded by a receiver. More specifically, it is *spatial* communication between two registers which is mediated by a quantum computation. It is a two-register description where the unitaries are $\hat{U}_i$ are not stored in the computational register M but rather applied in the dynamical process of the quantum computer (they are in fact control-unitaries). In this regard it is clearly different from the static scenario where a classical variable is physically and stably stored into a black box by an ensemble of channels (to describe presence/absence of a target, or the different reflectivities of a memory cell). This means that Ref. [4] is not about readout from *storage*.

- The input-output process is based on a single signal system processed by a unitary. Today, we know that unitary discrimination can be perfectly solved in a finite number of uses [44]. It is therefore different from what happens in the more general discrimination of quantum channels, where perfect discrimination is not guaranteed (at finite energies) and the optimal states may require the use of idler systems, which are not sent through the box but directly to the output measurement in order to assist the entire process.

- The signal-idler structure, which is missing in Ref. [4], is one of the main features for both quantum illumination and quantum reading. The use of input entanglement and, more generally, quantum correlations, is the main working mechanism of these two protocols under completely general conditions of decoherence. As a matter of fact, as already said above, the "quantum" name in "illumination" and "reading" exactly comes from the comparison of a quantum resource at the input (entanglement) with respect to the use of classical input states (separable states, mixtures of coherent states).

- The communication scheme of Ref. [4] is only for a discrete-variable Hilbert space. The most important setting for both quantum illumination and quantum reading is *bosonic*. Quantum illumination provides the possible working mechanism of a lidar (in the optical case) and a radar (in the microwave case). The main setting for quantum reading is also

bosonic and at the optical frequencies, which is the relevant physical regime in optical storage.

V. QUANTUM READING CAPACITY

In this section and the next one, we discuss the two different notions of quantum reading capacity and dense coding capacity. First we discuss how quantum reading capacity of an ensemble of quantum channels is implicitly and immediately extended to an adaptive formulation. Then, we clarify the mathematical conditions under which the (adaptive) dense coding capacity of a quantum channel can be reduced to the (adaptive) quantum reading capacity of a corresponding ensemble of encoding channels.

In 2011, the notion of quantum reading capacity was introduced [5]. One considers a memory where each cell encodes a classical label x by means of a quantum channel $\mathcal{E}_x$ chosen with some marginal probability p_x. In the basic readout strategy, each cell can be identically and independently probed by a quantum state ρ which records the classical information x into the output state ρ_x. See also Fig. 1(a). More generally, the encoder can use classical codewords, so that a message $x_1, \ldots, x_n$ is written in a block of cells. Given a marginal ensemble of outputs $\{\rho_x\}$, a reader can retrieve at most the Holevo information $\chi(\{\rho_x\})$. What the encoder can do is to optimize over the marginal probability p_x, while the decoder can optimize over the choice of the input state ρ. Thus, for a memory cell described by the marginal channel ensemble $\{\mathcal{E}_x\}$, we can write the following (one-shot) quantum reading capacity

$$C_1(\{\mathcal{E}_x\}) = \max_{\rho, p_x} \chi(\{\rho_x\}). \qquad (4)$$

Because quantum reading is ultimately a problem of QCD, its formulation automatically extends to an adaptive version using the tools of Ref. [45]. In fact, following the adaptive formulation of QCD [45], one can immediately extend the protocol in Fig. 1(a) where the optimization over the input state is replaced by an optimization over interleaved quantum operations (QOs). Therefore, the combination of Refs. [5] and [45] directly provides the most general (adaptive) definition of quantum reading, together with the associated notion of adaptive quantum reading capacity $C(\{\mathcal{E}_x\})$.

The next ingredient from Ref. [45] is the notion of joint teleportation-covariance. An ensemble of channels $\{\mathcal{E}_x\}$ is *jointly tele-covariant* if, for any Pauli unitary U_k, we may write

$$\mathcal{E}_x(U_k \rho U_k^\dagger) = V_k \mathcal{E}_x(\rho) V_k^\dagger, \qquad (5)$$

for generally-different unitaries V_k. Recall that this is a stronger condition than the simple teleportation covariance [46–48]. In fact, we need to assure that the output unitary V_k does not depend on the classical label x, so that the property of Eq. (5) holds jointly for
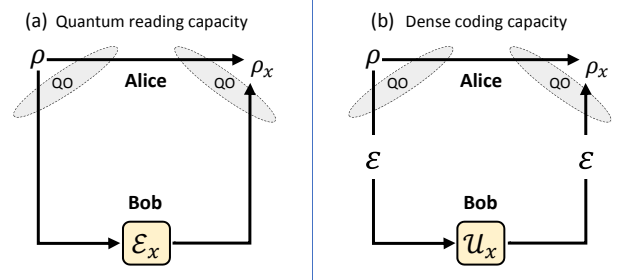

FIG. 1: Setting of the quantum reading capacity (a) compared to the setting of the dense coding capacity (b). Quantum operations (QOs) are added in the adaptive versions of the protocols.

any $\mathcal{E}_x$ in the ensemble. For instance, this condition is true for ensembles of erasure channels or ensembles of Holevo-Werner channels [49–53]. In the specific case where $V_k = U_k$ holds for any k, we say that $\{\mathcal{E}_x\}$ is *jointly Pauli-covariant*. For instance this is the case for ensembles of Pauli channels (such as depolarizing or dephasing channels). However, this stronger condition is not true for erasure channels, Holevo-Werner channels and others.

Because of Eq. (5), we can then use the teleportation protocol as a universal operation to simulate all the channels $\mathcal{E}_x$ in the ensemble by using their Choi matrices $\rho_{\mathcal{E}_x}$ as program states. As a result, for a memory cell made by jointly tele-covariant channels, we can use channel simulation and collapse all the QOs in the protocol, following exactly the procedure of Ref. [45]. It is immediate to see that this leads to the bound

$$C(\{\mathcal{E}_x\}) \leq \max_{p_x} \chi(\{\rho_{\mathcal{E}_x}\}). \qquad (6)$$

Since a possible strategy is to use maximally-entangled states to read each cell, we have that the bound is also achievable and, therefore,

$$C(\{\mathcal{E}_x\}) = \max_{p_x} \chi(\{\rho_{\mathcal{E}_x}\}). \qquad (7)$$

VI. DENSE CODING CAPACITY

Let us now clarify the relations between quantum reading capacity and dense coding capacity defined in Ref. [6]. As we can see from Fig. 1(a), the quantum reading capacity is defined over an ensemble of quantum channels $\{\mathcal{E}_x\}$, read by assuming an identity channel before and after the memory cells. This is a different situation from the dense coding capacity which is defined over a single quantum channel $\mathcal{E}$ used back and forth by Alice and Bob, and where the classical information is encoded by applying Pauli unitaries $\mathcal{U}_x() := U_x() U_x^\dagger$.

In the dense coding protocol, a bipartite input state ρ is prepared by Alice. One system is retained by her, while the other system is sent to Bob through the quantum channel $\mathcal{E}$. Bob applies a Pauli unitary $\mathcal{U}_x$ and sends

the system back through the same channel $\mathcal{E}$. Overall the transmitted system undergoes the composition of channels $\mathcal{E}_x := \mathcal{E} \circ \mathcal{U}_x \circ \mathcal{E}$, which generates the bipartite output state $\rho_x := \mathcal{I} \otimes \mathcal{E}_x(\rho)$. See Fig. 1(b).

By optimizing over the input state ρ and the probability p_x of the Pauli encoder $\mathcal{U}_x$, one can write the (one-shot) dense coding capacity of the channel $\mathcal{E}$ as

$$C_1(\mathcal{E}) = \max_{\rho, p_x} \chi(\{\rho_x\}), \qquad (8)$$

where $\{\rho_x\}$ is the ensemble of output states. Even though this formula has the same RHS as in Eq. (4), it is related to a completely different quantity, since it refers to the capacity defined for a single channel $\mathcal{E}$.

In the adaptive version of the protocol considered in Ref. [6], Alice has also the freedom to apply the most general QOs on the two systems, both during transmission and upon receipt of these systems. Considering these interleaved QOs $\{\Lambda_i\}$, we can define a corresponding (adaptive) dense coding capacity $C(\mathcal{E})$ for the channel $\mathcal{E}$. See Ref. [6] for more details.

Now it comes another crucial difference with respect to the quantum reading capacity. Instead of joint telecovariance, here we need the stronger condition of joint Pauli-covariance. In fact, because of the specific structure $\mathcal{E} \circ \mathcal{U}_x \circ \mathcal{E}$ of dense coding, we need to be sure that channel $\mathcal{E}$ is covariant with respect to the Pauli operators U_k. In this way, we can swap U_k through the entire chain $\mathcal{E}_x := \mathcal{E} \circ \mathcal{U}_x \circ \mathcal{E}$, i.e., we can write $\mathcal{E}_x(U_k \rho U_k^\dagger) = U_k \mathcal{E}_x(\rho) U_k^\dagger$. If this is possible, then we can simulate $\mathcal{E}_x$ with their Choi matrices and write

$$C(\mathcal{E}) = \max_{p_x} \chi(\{\rho_{\mathcal{E}_x}\}). \qquad (9)$$

In general, this procedure cannot be done if the channel $\mathcal{E}$ is tele-covariant but not Pauli-covariant. Given a Pauli unitary at the input of this type of channel $\mathcal{E}$, the output unitary is non-Pauli and could not commute (or anticommute) with Bob's Pauli encoders, which means that we cannot exploit the basic property that allows us to write Eq. (9). It is clear that the formula in Eq. (9) is proven under different (stronger) conditions than those valid for the quantum reading capacity of Eq. (7). The lack of

these crucial properties in the derivation of Eq. (7) means that such equation is not sufficient to imply Eq. (9), which is therefore a new result proven in Ref. [6].

To further discuss this point, assume that we compress the scheme in Fig. 1(b) into the scheme of Fig. 1(a), so that we interpret the dense coding capacity of $\mathcal{E}$ as the quantum reading capacity of the ensemble $\{\mathcal{E}_x\}$, where $\mathcal{E}_x := \mathcal{E} \circ \mathcal{U}_x \circ \mathcal{E}$. Then we must be careful not to lose the property of Pauli covariance. In other words, Eq. (7) can be written for dense coding if $\mathcal{E}_x$ are jointly Pauli-covariant, which means that the channel $\mathcal{E}$ in the decomposition $\mathcal{E}_x := \mathcal{E} \circ \mathcal{U}_x \circ \mathcal{E}$ must be Pauli-covariant. From this point of view, a contribution of the present work is to show and fully clarify the strong conditions under which the (adaptive) dense coding capacity of a channel can be reduced to the (adaptive) quantum reading capacity of an ensemble of encoding channels.

VII. CONCLUSION

In this brief review paper we have discussed various protocols based on the model of quantum channel discrimination (a primitive notion in many areas of quantum information theory). In particular, we have discussed and compared quantum illumination [1, 2], quantum reading [3], and the communication capacity of quantum computation [4]. These are schemes with different aims and features, not be naively confused one with the other. Then, we have discussed the notions of quantum reading capacity [5] (of an ensemble of quantum channels) and dense coding capacity [6] (of a quantum channel), clarifying their crucial differences and the mathematical conditions under which they can be connected. Specifically, this connection is possible under the condition of joint Pauli-covariance, not to be confused with joint teleportation covariance or other weaker conditions.

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