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Nursing Homes' Competition and Distributional Implications when the Market is Two-Sided

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Abstract

We investigate the effect of competition in the nursing homes sector with a two-sided market approach. Using a Hotelling model, our key findings are that i) the two-sidedness of the market leads to higher wages for nurses; ii) this is then passed to residents in the form of higher prices; iii) nursing homes profits are instead unaffected. In contrast, when nurses wages are regulated, the two-sidedness of the market implies a transfer between residents and nursing homes. When residents price are regulated, it implies a transfer between nurses and nursing homes. These key results are generally robust to institutional settings which employ pay-for-performance schemes, the presence of altruistic motives of nursing homes and the heterogeneity in residents reservation utility.

Keywords: nursing homes; competition; two-sided markets; distribution.

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1 Introduction

Long term care expenditure is expected to rise driven by an ageing population. Projections suggest that it might more than double by 2060 in several high-income countries (OECD, 2011). Nursing homes represent a significant proportion of long term care expenditure (around 0.5% and 1.5% of GDP in most countries; OECD, 2017) and this is likely to remain the case in the future despite governments policies that encourage informal care. Governments also increasingly encourage residents' choice: by developing quality ratings and spreading this information widely, they can encourage providers to compete. Quality is a key concern in the nursing homes sector and this is mainly driven by the care that nurses provide within the home. In the US for instance, nursing homes have been historically understaffed and the low levels of quality have motivated the introduction of minimum nurse staffing ratios in nursing homes.

This study investigates competition among nursing homes when providers compete both for residents and for nurses (who provide care to residents), and nurses are altruistic and care about the level of quality provided. The nursing homes industry is not a textbook example of two-sided markets (Rochet and Tirole, 2003, 2006) because quality is a prominent feature of this sector. However, we claim that the nursing home is two-sided precisely because the quality delivered to residents depends on nurses' workload, that in turn, depends on the staff ratio of nursing homes. This situation generates network externalities between both sides, residents and nurses, that make the business model of nursing homes two-sided. Our main objective is to explore the distributional implications of the competition across the three key actors involved (residents, nurses and nursing homes) that arise from the two-sidedness of the market.[1]

Nursing homes compete for residents on quality and possibly on price. Differently from the hospital sector, where prices are regulated in most OECD countries (except for the US outside of Medicare and Medicaid), nursing homes are free to set prices in most OECD countries within a competitive market.[2] Nursing homes also compete on quality, which is an important aspect

[1] We focus on the distributional implications of competition between nursing homes in this two-sided setting in a model where quality is decided by nurses, rather than on how competition affects quality itself (that has been the focus of previous literature). In Section 6, however, we provide an extension that allows for market expansion effects (under uncovered, rather than covered, market). In this case, competition generated by the two-sideness of the market also affects quality.

[2] To take into account the diversity of institutional contexts, we will study the equilibrium in nursing homes' market, first without regulation, and next with regulations, respectively on salary paid to nurses and price charged to residents.

of residents experience.

One way to influence the quality of care is by attracting a larger number of nurses. More nurses can improve quality of care through a better matching between residents and nurses (residents more likely to get along with the nurse) and a relaxed time constraint for nurses: for a given number of residents, more nurses implies that each nurse can spend more time with each individual resident which allows them to provide better care. The empirical evidence also supports that nurses staffing levels affect quality of care, as measured by deficiencies related to quality of care and quality of life (Lin, 2014), and incidence of pressure sores and urinary tract infections (Konetzka, Stearns and Park, 2008). The effect can be quantitatively large. Increasing nurses staffing by one standard deviation increases quality by more than 16% (Lin, 2014).

The importance of nurses staffing levels in affecting the demand for nursing homes is exemplified in the Medicare web portal *Nursing Home Compare* which provides case-mix adjusted staffing measures that prospective residents can use to choose the nursing home.[3] The empirical evidence also supports that demand responsiveness to quality can be enhanced by publicly reported quality information, which includes nurses staffing ratios in addition to clinical indicators (Werner *et al.*, 2012). In summary, nursing homes have to compete not only on price (when it is permitted) but also on salary to attract nurses. In our model, nurses are altruistic (or intrinsically motivated) and decide the amount of quality provided. The marginal benefit from quality, which arises from the altruistic component, is traded-off against the marginal cost of providing quality, which decreases in the residents-staff ratio of nursing homes. Thus, everything else equal, a higher salary paid by nursing homes attracts more nurses, which reduces the residents-staff ratio and therefore enhances the quality provided by nurses. In turn, higher quality increases the demand for residents (Zhao, 2016).[4]

The nursing homes' market is then two-sided because: i) a higher number of nurses can affect demand for residents because it implies higher quality (relaxed time constraints for nurses and better matching between residents and nurses), and ii) a higher number of residents affects nurses

[3] https://www.medicare.gov/nursinghomecompare/Data/About.html

[4] The review by Toode *et al.* (2011) highlights that one key factor affecting nurses' supply are working conditions. In our set-up, the altruism parameter is exogenous but the amount of quality decided by nurses depends on the residents-staff ratio that captures working conditions.

labour supply by affecting nurses working conditions (nurses working under higher pressure with a larger volume of residents). Both effects create different types of network externalities between nurses and residents: a traditional positive network externality from nurses to residents as it is standard in two-sided models (*e.g.* see Rochet and Tirole, 2003), plus a common network externality as in Bardey *et al.* (2012, 1014) that captures the quality supplied by nursing homes and that is valued (possibly with different intensities) by both sides.

The *distributional* consequences that arise in the presence of two-sidedness of the market across residents, nurses and nursing homes, as a result of the common network externality are as follows. The two-sidedness of the market leads to more intense competition for nurses since their number impacts positively the quality supplied by their nursing homes, which contributes to higher wages offered to nurses. In other words, nurses benefit from the two-sidedness of the market. Such increases in wage are however passed to the residents in the form of higher prices, so that residents are worse off. Nursing-home profits are instead unaffected since the increase in nurses wages is offset by the increase in residents price. By offering a higher wage a nursing home increases nurses' utility directly but also indirectly by reducing the residents-nurse ratio which is valued by nurses (because of lower workload) and residents (because it implies a higher quality). These effects depend critically on the assumption that the marginal cost of providing quality for nurses depends on the resident-nurse ratio, and on the nurses being altruistic and caring about the level of quality provided.[5]

The two-sidedness of the market matters also if either residents prices or nurses wages are *regulated*, but has different distributional implications. When *nurses' wages* are regulated, the two-sidedness of the market still increases residents' prices, which makes residents worse off, if the regulated wages are not too high. In turn, this implies a transfer between residents and nursing homes (rather than between residents and nurses, as in the main model). More precisely, when the regulated wage is higher than its equilibrium value (without regulation), residents' price is

[5] In most of the analysis nurses quality does not respond directly to the two-sidedness parameter in the *symmetric equilibrium.* Our horizontal differentiation framework therefore allows us to isolate the transfer effects across the different key actors (nurses, residents and nursing homes). However, the results are qualitatively similar in two extensions where quality is not fixed even in equilibrium, as quality can respond to policy interventions. This is the case in Section 4.2 where we investigate pay for performance with incentive schemes for staff, with a higher fee implying higher quality since pay for performance schemes increase the intensity of competition in quality, and in Section 6 where a segment of demand responds to quality and price, and therefore affects quality in equilibrium through the residents-staff ratio.

lower, and both effects work at the expense of nursing homes' profits.

Differently, when *residents' price* are regulated, the two-sidedness of the market affects nurses' wages and nursing homes' profits, and it implies a transfer between nurses and nursing homes. A higher regulated price lowers residents' utility, while it increases nurses' wage with an ambiguous effect on nursing homes' profits. A fuller discussion dealing with the distributional implications of both regulations is given in the conclusion.

Pay for performance (P4P) schemes are increasingly used to incentivise quality of nursing homes (Miller and Singer Babiarz, 2014). We show that the key insights in terms of the effect of the two-sidedness of the market on residents prices and nurses wages also hold, and can be even strengthened, when P4P schemes are used to incentivise quality. Moreover, we distinguish two types of P4P schemes. In the first scenario, a regulator uses a P4P scheme to reward nursing homes according to their performance on quality indicators. In the second scenario, nursing homes are allowed themselves to introduce P4P within their organization and reward nurses with higher quality. In both cases, we still find that the two-sidedness of the market makes the residents worse off. In the former case, quality remains unchanged (since nurses are not directly incentivised by the scheme) but P4P amplifies the competition effects arising from the two-sidedness of the market which in turn increases nurses wages even more, which are again passed to residents in the form of higher prices. In the latter, we show that the P4P scheme is such that the quality fee paid to nurses is equal to residents' valuation of quality. Nurses are better off as a result since the fee more than compensates for their increase in effort. Residents are worse off since they are charged a higher price which does not compensate for the higher quality. Nursing homes pass the higher costs to the residents so that profits remain unchanged.

As an extension, we allow for heterogeneity in residents reservation utility due to differences in the degree of autonomy and dependency. Some residents may not be willing to go to a nursing home if the price is too high relative to living by themselves or with an informal carer. We show that if the uncovered market is sufficiently small, it is still the case that the two-sidedness of the market increases nurses' wage and residents' price but now it also reduces demand, which in turn increases quality. The welfare effects are generally similar to the ones identified under the covered market, but can be reinforced or weakened through the additional effects on quality and demand, and this is also the case for price and wage regulation.

Finally, when we allow nursing homes to have altruistic motives towards their residents in addition to profits. If these concerns relate mostly to quality as opposed to price, then the results due to the common network externality are further amplified leading to further increases in nurses' salary at the expense of the residents.

Although our model is designed to capture features of the nursing homes' sector, within the context of primary care our model may also capture some features of emerging online platforms that bring patients and doctor together. Patients and doctors join these platforms which are then used usually to arrange medical appointments. Patients that use online platforms may value positively the number of doctors available to increase the probability that a better matching occurs. Instead, doctor time and availability depends on doctors/patient ratio which in turn affects the quality supplied to patients. Depending on the context, doctors can charge a fee to patients directly and the platform charge a fixed fee to both sides to use the platform.[6]

In many countries however, primary care providers are paid by capitation with a fixed price for each additional patient registered in the practice. In some countries, like in England, primary care is organised in large primary care practices. The GP practice has an owner (generally a GP) who recruits other health professionals (other salaried GPs, nurses and midwives). Therefore, the model is likely to apply to primary care practices where there is a clear distinction between the owner of the practice (equivalent to nursing homes in our model) and the employees working for the practice (the nurses in our model) treating patients (taking care of residents in our model). In those settings, only the results in Section 3.2 with price regulation would apply. The primary care market may be two-sided if: i) a larger number of staff (doctors, specialised nurses working in the same primary care practice) affects demand for residents because it implies a wider range of services and broadly higher quality (including more time spent with patients and better matching between staff and patient based on their health conditions and staff specialisation and expertise), and ii) a higher number of individuals registered with the practice affects staff labour supply by affecting working conditions (staff having to cope with high workload by extending working hours, or working more intensively). Therefore both types of network externalities are

[6] In France for example the platform Doctolib provides services to patients and doctors. The service is free for patients and doctors pay a fee for each patient visit. Doctors can also buy additional services to the platform. See Doctolib.fr. Moreover, in France doctors can charge extra billings, *i.e.* an additional fee on top of the regulated price reimbursed by the funder (the so-called "Secteur 2") introducing an element of price competition.

still present.

Our model is related to two strands of literature: (i) the one that investigates quality and price competition within an horizontal differentiation framework (Hotelling/Salop models) within the health sector,[7] and (ii) the literature on two-sided markets that has dramatically grown during the last decade after the seminal articles of Rochet and Tirole (2003, 2006) but which has mainly been applied to sectors like banking or retail markets that are distinct from the nursing homes one. Nevertheless, there is a smaller literature which combines both elements. Bardey and Rochet (2010) analyse the competition between PPO and HMO which both compete for policyholders one side and to affiliate health care providers on the other side. We borrow from this study that consumers value having a larger pool of providers (doctors, nurses) but in order to work in an horizontal differentiation set-up we assume that all consumers value them in the same way.[8] Pezzino and Pignataro (2007) make a similar assumption that consumers value having access to more doctors in a context of hospital competition under regulated price. By contrast, we assume that the quality is not a decision variable from nursing homes but rather is endogenously decided by their nurses. Such a decision depends on their workload, which in turn depends on the resident/nurse ratio. As quality enters positively in nurses and customers utility function, this assumption can be viewed as a micro foundation of the common network externalities framework introduced by Bardey *et al.* (2012, 2014). We develop our two-sided analysis in a Hotelling framework, following Armstrong's (2006) two-sided model.

Bardey *et al.* (2014) focus on the general properties of the quality function that depends on the numbers of individuals from both sides, characterising the common network externality. When such quality can be represented by a homogeneous function, they show how the homogeneity degree[9] affects platforms' profits and price. In particular, they highlight that a zero homogeneity degree of the quality function does not affect platforms' profits but instead only transfer rents from one side to the other. In this study, we also focus on the distributional implications that arise from the two-sidedness (of the nursing homes market) in a richer envi-

[7] See for example Ma and Burgess (1993), Gravelle (1999), and Brekke, Siciliani and Straume (2012).

[8] In other words, the adverse selection effect pointed out by these authors is assumed away here. Boilley (2012) extends this article and analyses the case where PPO and HMO compete for the same health care providers whereas Bardey and Rochet (2010) make a local monopoly assumption on this side.

[9] When the number of affiliated on both sides is multiplied by the same factor, the quality is scaled by another constant that can be higher, equal or lower.

ronment where nurses quality is endogenised, and residents externalities explicitly modelled. In this respect, our analysis is closer to Bardey *et al.* (2012), who study different payment schemes (capitation, salary and fee-for-service) and their respective distributional properties. Differently, we focus on a fixed remuneration (wage) that matches nurses' remuneration, but we shed light on different regulatory regimes, both on nurses' side and on residents' side. We also consider a wider set of pay-for-performance schemes that correspond to what is observed in nursing homes industry. Finally, we study the welfare implications of this nursing homes' two-sided market structure when residents' side is uncovered.

Gal-Or et al. (2019) develop a two-sided set up that contains horizontal and vertical differentiation to analyse competition in equity crowfunding markets. In their model platforms compete in quality that works as an investment variable. In our model the quality variable is determined by a common network externality, which introduces one component of the two-sidedness of the market and is not directly chosen by the firm but by the employees working in the firm. Instead, the firm makes salary decisions to attract workers. The institutional context is also very different. They model equity crowfunding platforms who invest in quality to attract investors and startup and charge fees to both. Our focus is on nursing homes where fees are charged to residents and only passed to the employees through salaries.

The rest of the study is organised as follows. Section 2 provides the main model. Section 3 investigates price and wage regulation and compares the solution with and without regulation. Section 4 extends the main model to pay for performance. Section 5 extends the model to allow heterogeneity in residents' reservation utility making the market uncovered. Section 6 allows for altruistic concerns of nursing homes, in addition to profits. Section 7 concludes.

2 The model

We use a Hotelling set up with a market characterized by two nursing homes (providers) $i = \{1, 2\}$ that are located at the endpoints of the unit line $Y = [0, 1]$. Residents (consumers) are uniformly distributed on Y with a total mass normalised to 1. The utility of a resident who chooses nursing home i and travels a distance y is:

$$U_i(y) = \theta q_i - p_i - t_r y + 2\beta N_i \int_0^{\frac{1}{2N_i}} (v - z)\, dz, \tag{1}$$

where q_i is the quality of care provided to residents by nursing home i, p_i is the price charged by the nursing home to their residents, t_r is the marginal disutility of distance (for example related to distance to family and friends), θ is the marginal benefit of quality, and N_i is the number of nurses employed.

We assume that a higher number of nurses increases residents' utility and satisfaction as a result of higher chance of a good match between the resident and the nurse. This is captured by the last term in (1). Analytically, v gives the highest (gross) benefit to residents from staying in a nursing home which is reduced by an amount z if the resident is not matched with her ideal nurse. Therefore, z captures the cost of matching, which for analytical simplicity, we assume to be uniformly distributed between zero and one. This degree of satisfaction cannot be known *ex ante* as it depends on an *ex post* interaction between the nurse and the resident. When residents choose their nursing home, they will take into account that nursing homes with more nurses are associated in expected terms with a better match and therefore a higher utility. β is a preference parameter related to the marginal benefit of residents from a better nurse-resident match.[10] Solving for the integral in (1) residents' utility can be written as:

$$U_i(y) = \theta q_i - p_i - t_r y + \beta \left(v - \frac{1}{4N_i} \right). \tag{2}$$

Assuming that quality is high enough to ensure that the market is covered, the demand functions of nursing homes 1 and 2 are respectively:

$$\begin{aligned} D_1 &= \frac{1}{2} + \frac{\theta}{2t_r}(q_1 - q_2) + \frac{1}{2t_r}(p_2 - p_1) - \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{N_2} \right), && (3) \\ D_2 &= 1 - D_1. && (4) \end{aligned}$$

These demand functions suggest that nursing homes with higher quality, lower prices, and a higher number of nurses attract a larger number of residents.

[10] See Gal-Or (1997) for a similar assumption applied to healthcare markets.

We now turn to the market for nurses. Nurses are also uniformly distributed on Y with a total mass normalised to 1. The utility of a nurse who works for nursing home i and travels distance y is:

$$V_i(y) = w_i + \alpha q_i - \frac{1}{2} \frac{c}{(k-\gamma) + \gamma \frac{N_i}{D_i}} q_i^2 - t_N y, \tag{5}$$

where w_i is the fixed wage paid to the nurse by nursing home i, and t_N is the transportation costs for nurses, which reflects the desire to work close to home. We assume that nurses are altruistic and care about the residents, as it is often argued that nursing is a vocational job. The assumption that nurses are motivated or altruistic has been recognized for long time within the health economics literature,[11] and more recently in the literature on motivated agents in the broader public sector.[12] Altruism is captured by the positive parameter α. We assume that residents value quality weakly more than nurses, $\theta \geq \alpha$. Providing quality is however costly to the nurse. The positive parameter c (k) is (inversely) related to the marginal disutility of providing quality.

We also critically allow for congestion effects through the positive parameter γ. We make the intuitive assumption that for nurses it is more costly to provide quality when the residents-nurses ratio is high, everything else equal. Nurses will have to work harder to offer the same attention to their residents if the number of residents per nurse increases. We enter γ in the cost function so that the marginal cost of quality is unchanged in equilibrium. Therefore, comparative statics with respect to γ is equivalent to the strength of the two-sidedness of the market in a *Common Network Externality* framework.[13]

The timing of the game is the following:

1. Nursing homes set the price p_i and the salary w_i.

2. Residents and nurses decide whether to be affiliated to nursing homes.

[11] See, *e.g.*, Ellis and McGuire (1986), Chalkley and Malcolmson (1998), Eggleston (2005), Heyes (2005), Jack (2005), Kaarbøe and Siciliani (2011).

[12] See, for example, Francois (2000), Murdock (2002), Glazer (2004), Besley and Ghatak (2005), and Francois and Vlassopoulos (2008) for a literature review.

[13] The standard models of two-sided markets focus on positive inter-group network externalities, where the utility of agents from one side depends positively on the number of agents affiliated to the platform of the other side. Differently, the network externalities at play in our model correspond to the Common Network Externality introduced in Bardey *et al.* (2012, 2014). This situation occurs when both sides value the quality provided and the quality depends positively on the number of providers (nurses) and negatively on the number of consumers (residents).

3. Nurses choose the level of quality delivered to residents that maximise their utility.

As usual, we proceed by backward induction. Maximising nurse's utility working for nursing home i with respect to quality yields:

$$q_i^* = \frac{\alpha}{c}\left(k - \gamma + \gamma\frac{N_i}{D_i}\right). \tag{6}$$

The level of quality decided by nurses increases in their altruism and in the nurse-resident ratio of the nursing home since a higher ratio increases the marginal disutility of providing quality.

After substitution, the indirect utility function of the nurse working for nursing home i is:

$$V_i = w_i + \frac{\alpha^2}{2c}\left(k - \gamma + \gamma\frac{N_i}{D_i}\right) - t_N y, \tag{7}$$

which is decreasing in the residents-nurses ratio. *Ceteris paribus*, it is more pleasant to work in nursing home i if the nurse has fewer residents to take care of.

The supply functions of nurses in nursing homes 1 and 2 are given by:

$$N_1 = \frac{1}{2} + \frac{1}{2t_N}(w_1 - w_2) + \frac{\gamma\alpha^2}{4ct_N}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right), \tag{8}$$

$$N_2 = 1 - N_1. \tag{9}$$

Everything else equal, the nursing home that pays a higher wage will attract more nurses. Moreover, nurses are more willing to work for nursing homes that have low resident-nurse ratios, which could be interpreted broadly as better working conditions. After substituting for quality, we can also re-write the demand functions for residents as:

$$D_1 = \frac{1}{2} + \frac{1}{2t_r}(p_2 - p_1) - \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{N_2}\right) + \frac{\gamma\alpha\theta}{2t_r c}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right), \tag{10}$$

and $D_2 = 1 - D_1$. It is worth to emphasise the two-sided nature of the market by noticing that residents' demand and nurses' supply are inter-related. This arises because the number of nurses affects positively residents' demand through better matching (third term in (10)) and through higher quality, due to lower congestion (fourth term in (10)). In turn, a higher number of residents affects nurses labour supply through higher workload and therefore worse working conditions (third term in (8)).

The comparative static of residents' demand and nurses' supply with respect to residents' price and nurses' wage is

$$
\begin{aligned}
\frac{dD_1}{dp_1} &= -\frac{1}{8\Delta}\left(4ct_N - \gamma\alpha^2\left(\frac{1}{D_1} + \frac{1}{1-D_1}\right)\right), && (11)\\
\frac{dN_1}{dp_1} &= \frac{\gamma\alpha^2}{8\Delta}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) > 0, && (12)\\
\frac{dD_1}{dw_1} &= \frac{\beta c}{16\Delta}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right) + \frac{\gamma\alpha\theta}{4\Delta}\left(\frac{1}{D_1} + \frac{1}{1-D_1}\right) > 0, && (13)\\
\frac{dN_1}{dw_1} &= \frac{ct_r}{2\Delta} + \frac{\gamma\alpha\theta}{4\Delta}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) > 0, && (14)
\end{aligned}
$$

where Δ is defined in Appendix 8.1 and is positive if the problem is well behaved. This highlights again the two-sidedness of nursing home competition. First, an increase in nurses' wage by nursing home 1 increases nurses' supply. This in turn increases residents' demand since residents value more nurses, through a better matching process. These two effects are respectively captured by dN_1/dw_1 and dD_1/dw_1 when setting $\gamma = 0$. There are however two additional effects at work: a higher number of nurses also reduces the resident-nurse ratio. This reduces the cost for nurses to provide quality, which further increases nurses' willingness to work for nursing home 1, and ultimately leads to an increase in quality, which is valued by residents.

An increase in prices charged to residents by nursing home 1 reduces residents' demand and therefore also reduces the resident-nurse ratio. The latter makes nurses more willing to work for nursing home 1 because of better working conditions, and also has a feedback effect on demand: although facing a higher price residents value the reduced resident-nurse ratio. We assume that the latter is a second order effect, so that an increase in price always decreases demand. This seems the most plausible and realistic scenario. At the symmetric equilibrium this implies $ct_N > \gamma\alpha^2$, which as shown below is required for the nursing home maximisation problem to be well behaved.

2.1 Best reply functions of nursing homes

The profit function of nursing home $i, j = \{1, 2\}$ is:

$$\pi_i = (p_i - g) D_i (p_i, p_j, w_i, w_j) - w_i N_i (p_i, p_j, w_i, w_j), \tag{15}$$

with $j \neq i$ and where g denotes the marginal cost of having a resident in a nursing home.

The first order conditions with respect to residents' price and nurses' wage are:

$$\frac{\partial \pi_i}{\partial p_i} = (p_i - g) \frac{\partial D_i}{\partial p_i} + D_i - w_i \frac{\partial N_i}{\partial p_i} = 0, \tag{16}$$

$$\frac{\partial \pi_i}{\partial w_i} = (p_i - g) \frac{\partial D_i}{\partial w_i} - N_i - w_i \frac{\partial N_i}{\partial w_i} = 0. \tag{17}$$

The first two terms of the optimality condition for residents' price p_i are in line with the traditional monopolistic pricing rule. An increase in price raises revenues on all infra-marginal resident but also reduces demand. Moreover, this optimality condition (16) contains a new additional term, which is negative. When setting the price, nursing homes have to take into account that a higher price will reduce the resident-nurse ratio which will attract a larger number of nurses that will translate into higher nurses expenditure. This last effect therefore tends to reduce the price.

The first order condition for nurses' wage w_i is such that it trades off the benefits from a larger residents' demand generated by an increase in quality (better matching and lower resident-nurse ratio) with the cost of higher wage for nurses. The two-sidedness of this optimality condition comes from the fact that residents' demand increases in the wage paid to nurses.[14]

Within a nursing home, prices charged to residents and wages paid to nurses tend to be *complement*:

$$\frac{\partial^2 \pi_i}{\partial p_i \partial w_i} = \frac{\partial D_i}{\partial w_i} - \frac{\partial N_i}{\partial p_i}. \tag{18}$$

The two-sidedness of the nursing homes' market makes that a higher wage paid to nurses increases the number of nurses working for a nursing home, which in turn increases quality and

[14] The second-order conditions are given by: $\frac{\partial^2 \pi_i}{\partial p_i^2} = 2 \frac{\partial D_i}{\partial p_i} < 0$, $\frac{\partial^2 \pi_i}{\partial w_i^2} = -2 \frac{\partial N_i}{\partial w_i} < 0$, and $\frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial^2 \pi_i}{\partial w_i^2} > \left(\frac{\partial^2 \pi_i}{\partial p_i \partial w_i} \right)^2$ where $\frac{\partial^2 \pi_i}{\partial p_i \partial w_i} = -\frac{\partial N_i}{\partial p_i} + \frac{\partial D_i}{\partial w_i}$.

decrease demand sensitivity, therefore allowing nursing homes to charge a higher price. There is however another effect going in the opposite direction. A higher wage makes a marginal increase in price more costly since there are more nurses willing to work for the nursing home when prices are higher due to the lower workload. The latter is generally smaller, so that residents' price and nurses' wage are generally complements. A sufficient condition to ensure that this is the case is that $\theta \geq \alpha$ and $D_i \leq 0.75$ (see Appendix 8.2). We assume this is the case below.

The next expressions summarise the strategic complementarity or substitutability between nursing homes' decision variables, namely residents' prices and nurses' wages:

$$\frac{dp_i}{dp_j} = -\frac{1}{\Lambda}\left(\underset{+}{\frac{\partial D_i}{\partial p_j}}\underset{-}{\frac{\partial^2 \pi_i}{\partial w_i^2}} + \underset{+}{\frac{\partial^2 \pi_i}{\partial p_i \partial w_i}}\underset{-}{\frac{\partial N_i}{\partial p_j}}\right), \tag{19}$$

$$\frac{dw_i}{dw_j} = \frac{1}{\Lambda}\left(\underset{-}{\frac{\partial N_i}{\partial w_j}}\underset{-}{\frac{\partial^2 \pi_i}{\partial p_i^2}} + \underset{+}{\frac{\partial^2 \pi_i}{\partial p_i \partial w_i}}\underset{-}{\frac{\partial D_i}{\partial w_j}}\right), \tag{20}$$

where $\Lambda > 0$ by the second order conditions (see Appendix 8.2), $\partial D_i/\partial p_j = -\partial D_i/\partial p_i > 0$, $\partial N_i/\partial p_j = -\partial N_i/\partial p_i < 0$, $\partial N_i/\partial w_j = -\partial N_i/\partial w_i < 0$ and $\partial D_i/\partial w_j = -\partial D_i/\partial w_i < 0$. It suggests that prices are strategic complements. Nurses wages are also strategic complements if the complementarity between residents prices and nurses wages (captured by $\partial^2 \pi_i/\partial p_i \partial w_i$) or the residents' demand responsiveness to competitor's nurses' wage is not too high.

2.2 Symmetric equilibrium

The symmetric equilibrium is summarised in the following proposition:[15]

Proposition 1 *Suppose that $t_N - \gamma\alpha(\alpha + 2\theta)/c < \beta \leq t_N + t_r$ to ensure equilibrium existence. At a symmetric equilibrium, residents' prices and nurses' wages are such that:*

$$p^* = g + t_r + \frac{\gamma\alpha(\alpha + 2\theta)}{c},$$
$$w^* = \beta + \frac{\gamma\alpha(\alpha + 2\theta)}{c} - t_N,$$

[15] Since in our setting providers are symmetric, we focus on the symmetric equilibrium. Formally ruling out asymmetric equilibria involves deriving the stability conditions based on (4x4) matrix of the second and cross-partial derivatives of providers' profit functions in the (p_1, p_2, w_1, w_2) space. However, these are rather involved leading to no additional insights.

and nursing homes' profits are:

$$\pi^* = \frac{1}{2}\left(t_r + t_N - \beta\right).$$

See Appendix 8.3 for proof. The price mark-up charged by nursing homes on residents' side depends on the transportation cost (t_r), which is in line with the standard Hotelling model. Lower transportation costs imply more competition and a more responsive demand function to price, which in turn reduce the price. Similarly, the wage paid to nurses is negatively related to nurses transportation cost (t_N). Lower transportation costs for nurses imply that more nurses are more willing to switch provider for a given increase in wage, which implies that nursing homes will compete more aggressively to attract nurses by offering a higher wage. The wage paid to nurses is positively related to the marginal residents evaluation of having more nurses through better matching process. In summary, more aggressive competition for nurses (a lower t_N) implies higher nurses' salary and lower nursing homes' profits (and this holds also for the limiting case $t_N \to 0$).

Both residents' prices and nurses' wages have an additional term, which is due to the two-sided nature of the market. This is a key result of the model which depends on the simultaneous presence of the common network externality parameter γ and the presence of altruistic providers α. The parameter γ plays a critical role in the analysis, and reflects the extent to which it is more costly for nurses to provide quality when the residents-nurses ratio is high. If γ is zero, the marginal cost of providing quality is independent of resident-nurse ratio, and the solution reduces to $p^* = g + t_r$, $w^* = \beta - t_N$. If $\alpha = 0$, then the nurse has no motivation to provide quality, and therefore quality is zero (or at the minimum enforceable level) regardless of the marginal cost of quality. It is therefore the combination of altruism and the common network externality parameter that introduces the two-sidedness of the market. When γ is positive and the marginal cost of quality depends on the resident-nurse ratio, nursing homes will compete more aggressively on wages: by offering a higher wage a nursing home can attempt to attract more nurses, which also has the potential additional effect of reducing the resident-nurse ratio which is valued directly by nurses (because of lower workload) and residents (because it implies a higher quality). Residents however are charged a higher price as a result, and this higher price is entirely transferred from residents to nurses, such that nursing homes' profit is ultimately independent of any quality considerations. This arises because both sides, *i.e.* residents like nurses, value quality. This quality component enters into the class of *common* network externalities studied in Bardey *et al.* (2014) because it depends on the nurse/resident ratio, which affects both residents' demand and nurses' supply (see (10) and (8)).[16]

The results in relation to the common network externality γ are different from those related to the other *inter-group* network externality β, that is related to the valuation the residents place from a better matching between nurses and residents. Here, a higher evaluation for nurses β triggers more competition for nurses which again translates into higher wages, as for the other type of externality, but critically it does not affect residents prices, in contrast to the other type of externality, and instead lowers profits. This arises because this form of inter-group externality affects demand but does not affect nurses supply (see again (10) and (8)). Notice that our results on the common network externality do not rely on the presence of the inter-group externality (when $\beta = 0$).

Perhaps counter-intuitively, higher altruism leads to higher nurses' wage. From a contract theory perspective we could have expected that nursing homes would take advantage of nurses' vocations to pay them a lower wage, as pointed out in Heyes (2005). Higher altruism amplifies the mechanisms introduced by the two-sidedness of the market (captured by γ) leading to an even more aggressive competition for nurses.

Finally, a note on equilibrium existence. We require profits to be weakly positive and nurses' wage to be strictly positive. The first requires residents' evaluation of quality to be not too high ($t_r + t_N > \beta$). Otherwise, competition leads to ruinous competition where nursing homes make negative profits in equilibrium. The second requires that residents' valuation is not too low that it leads to negative wages ($\alpha\gamma\left(\alpha + 2\theta\right)/c + \beta > t_N$). If the inter-group network externality is absent, then the profit condition is always satisfied, while we require the common network externality to be sufficiently important relative to nurses transportation costs.

[16] More generally, if common network externalities (CNE hereafter) are represented by an homogeneous function, Bardey *et al.* (2014) show that the rents obtained by the providers, here the nursing homes, at the symmetric (and covered market) equilibrium depend on the homogeneity degree of such CNE. As the ratio is a 0-degree homogeneous function, nursing homes profits are independent of this quality component.

2.3 Residents' and nurses' utility and welfare

In the symmetric equilibrium, quality provided is equal to $q^* = \alpha k/c$, and does not depend on the two-sidedness of the market parameter γ. Residents' utility at the symmetric equilibrium is:

$$U_i^*(y) = \theta q^* - p^* + \beta\left(v - \frac{1}{2}\right) - t_r y = \frac{\alpha\left(\theta k - \gamma\left(2\theta + \alpha\right)\right)}{c} - g - t_r + \beta\left(v - \frac{1}{2}\right) - t_r y, \quad (21)$$

which unambiguously decreases in γ. If the market is two-sided, quality does not vary, neither it changes the number of nurses in the market, but it increases residents price and therefore reduces residents' utility.

The nurses' utility is given by:

$$V_i^*(y) = \beta - t_N + \frac{\alpha}{2c}\left(\alpha k + 2\gamma\left(\alpha + 2\theta\right)\right) - t_N y. \quad (22)$$

Nurses utility instead increases in γ. If the market is two-sided, their workload is unaffected in equilibrium but they receive a higher wage.

Finally, the total welfare is given by the sum of residents and nurses utility and nursing homes profits:

$$\begin{aligned} W^* &= U^* + V^* + \pi^* = 2\left[\int_0^{1/2} U^*(y)dy + \int_0^{1/2} V^*(y)^* dy + \pi^*\right] && (23)\\ &= \left(\alpha + 2\theta\right)\frac{\alpha k}{2c} - \frac{t_N}{4} - \frac{t_r}{4} - g + \beta\left(v - \frac{1}{2}\right). && (24)\end{aligned}$$

The two-sidedness of the market γ implies a transfer from residents to nurses, and does not affect profits, so that the social welfare remains unchanged. Instead, an increase in the diversity value component (β), which relates to residents potential benefits from better matching with the nurses, implies a transfer from nursing homes to nurses. Higher β affects the welfare function because it increases the utility to residents that is not compensated by an increase in price.

3 Price or wage regulation

Institutional arrangements can vary both across and within a country, so that price or wages can be regulated. Price regulation is common in the U.S. nursing home market where Medicaid covers the majority of nursing homes days. Nurses' wages can also be determined by regulation, negotiation or unions, rather than by the market. For instance, Sojourner *et al.* (2010, 2012) point out that in U.S., unions cover about 10% of nursing home workers (25% in the middle Atlantic region). Thus, in this section, we compare the results of the main model with other institutional settings. First, we investigate the scenario when nurses wages are regulated and residents prices are endogenously determined. We show that the two-sidedness of the market still affects residents' prices despite nurses wages being regulated. Second, we investigate the scenario when prices are fixed or regulated and nurses wage are endogenous. Again, the two-sidedness of the market affects nurses wages despite prices being regulated.[17]

3.1 Symmetric equilibrium under regulated nurses wages

Suppose that nurses' wage is regulated at the value $\overline{w}$. Using (16) we obtain:

Proposition 2 *At a symmetric equilibrium with a regulated wage $\overline{w}$, the price charged to residents is:*

$$\bar{p} = g + t_r + \frac{\gamma\alpha}{ct_N - \gamma\alpha^2}\left(\alpha\left(\beta - \bar{w}\right) + 2\theta t_N\right). \qquad (25)$$

We require $ct_N - \gamma\alpha^2 > 0$ for the second order condition of the price to hold.[18] If the market is not two-sided, we obtain the traditional mark-up pricing: $\bar{p} = g + t_r$. Instead, if the market is two-sided, compared to the familiar benchmark, the price charged to residents at equilibrium depends on the regulated wage. As for Proposition 1, this result depends on the presence of both nurses' altruism and the common network externality parameter. An increase in regulated nurses' wage reduces residents' price. This arises because a higher price tends to attract nurses at the margin through a more favorable resident-nurse ratio and therefore tends to exacerbate

[17] As in section 2, the quality provided in the symmetric equilibrium is equal to $q^* = \alpha k/c$, and this holds also under price and wage regulation. Moreover, equilibrium quality does not depend on the two-sidedness parameters. In Section 6, we show that this is not the case anymore if the residents market is uncovered.

[18] The Second Order Condition is $\frac{\partial^2 \pi_1}{\partial p_1^2} = 2\frac{\partial D_1}{\partial p_1} < 0$, and $\frac{dD_1}{dp_1} = -\frac{1}{2\Delta}\left(ct_N - \frac{\gamma\alpha^2}{4D_1(1-D_1)}\right)$, which evaluated at the symmetric equilibrium gives: $\frac{dD_1}{dp_1} = -\frac{1}{2\Delta}\left(ct_N - \gamma\alpha^2\right) < 0$.

wage expenditure for nurses. This latter effect, analogous to Section 2, is larger (in absolute terms) the higher is the regulated wage. This leads to the (possibly) counter-intuitive result, that the price charged to residents decreases with the regulated nurses' wage (when intuitively we may expect the higher nurses wage to be passed on to the resident through higher prices). As a result, residents are always better off when nurses' wage increase. Whether the price under regulated wages is higher or lower compared to the unregulated market depends on the level of the regulated wage. Re-writing (25) as:

$$\bar{p} = p^* + \frac{\gamma\alpha^2}{ct_N - \gamma\alpha^2}\left(w^* - \overline{w}\right), \tag{26}$$

suggests that the price under regulated wages will be lower whenever the regulated wage is higher than the wage when nursing homes compete for nurses. The effect of the two-sidedness parameter on residents' price is, differently from Section 2, ambiguous:

$$\frac{\partial\bar{p}}{\partial\gamma} = \frac{\alpha ct_N\left(\alpha\left(\beta - \overline{w}\right) + 2\theta t_N\right)}{\left(ct_N - \gamma\alpha^2\right)^2}. \tag{27}$$

On one hand, it increases the price charged to customers because a higher price tends to attract nurses at the margin through a more favorable resident-nurse ratio. On the other hand, it exacerbates the wage expenditure for nurses. The first effect dominates when the regulated wage is not too high ($\overline{w} < \overline{w}_\gamma \equiv \beta + 2\theta t_N/\alpha$), and the two-sidedness works in the same direction as in Section 2, so that the price increase is not transferred to nurses wage anymore.

Equilibrium existence requires weakly positive profits:

$$\overline{\pi} = \frac{1}{2}\left(\bar{p} - g - \bar{w}\right) = \frac{1}{2}\left(t_r + \gamma\alpha\frac{\alpha\left(\beta - \bar{w}\right) + 2\theta t_N}{ct_N - \gamma\alpha^2} - \bar{w}\right) \geq 0, \tag{28}$$

which implies that nurses' (regulated) wage cannot be too high: $\bar{w} \leq \bar{w}_\pi \equiv t_r + 2\theta/c + \gamma\alpha^2\left(\beta - t_r\right)/ct_N$. Differently from Section 2, the two-sidedness of the market affects profits, through its effect on prices. Similarly, residents' utility is only affected through changes in prices. Recall that in the symmetric equilibrium, quality provided by the nurses is equal to $q^* = \alpha k/c$, and does not depend on the two-sidedness of the market parameter γ. Since nurses' wages are regulated, nurses' utility in equilibrium is not affected by the two-sidedness parameter.

Total welfare is also not affected. The two-sidedness of the market implies a transfer between residents and the nursing home, rather than between the residents and the nurses when wages are endogenously determined (as in Section 2).

Table 1 compares the results with the unregulated market, and provides comparative statics. Under nurses' *wage* regulation, if the regulated wage is higher than under the unregulated market, then residents and nurses are better off but nursing homes are worse off. Therefore, wage regulation can improve residents and nurses utility at the expense of nursing homes' profits. As expected, lower residents transportation costs imply a more responsive demand, which reduce residents' price. Similarly to Section 2, higher altruism increases residents' prices when the markets is two-sided if the regulated wage is not too high. Differently from Section 2, nurses' transportation costs now affect residents price when the market is two-sided, and a more responsive nurses' supply function implies an increase in residents' price if the wage is not too high.

Table 1. Wage regulation $\bar{w} > w^*$

Comparison with unregulated market			
ΔU	ΔV	$\Delta\pi$	ΔW
> 0	> 0	< 0	$= 0$
Comparative statics			
$\frac{\partial \bar{p}}{\partial t_r} = 1,\quad \frac{\partial \bar{p}}{\partial t_N} = -\gamma\alpha^2 \frac{2\theta\alpha\gamma + c(\beta - \bar{w})}{(ct_N - \gamma\alpha^2)^2},\quad \frac{\partial \bar{p}}{\partial \alpha} = 2\gamma c t_N \frac{\theta t_N (1+\alpha^2) + \alpha(\beta - \bar{w})}{(ct_N - \gamma\alpha^2)^2}.$			

3.2 Symmetric equilibrium under regulated prices

Suppose now that prices are regulated with $p_1 = p_2 = \widehat{p}$ but nursing homes compete for nurses. Using (17) we obtain:

Proposition 3 *At a symmetric equilibrium with a regulated price $\widehat{p}$, nurses' wage $\widehat{w}$ is:*

$$\widehat{w} = \frac{(\widehat{p} - g)(2\alpha\theta\gamma + \beta c) + \alpha^2\gamma(t_r - \beta)}{ct_r + 2\alpha\theta\gamma} - t_N. \tag{29}$$

We assume that the regulated price is always sufficiently high to rule out corner solutions, *i.e.* $\widehat{p} - g \geq A/(c\beta + 2\alpha\theta\gamma)$, where $A := ct_r t_N + \alpha\gamma(\alpha(\beta - t_r) + 2\theta t_N)$. The optimal nurses' wage increases with the price mark up. The higher the mark up, the stronger is the incentive for nursing homes to attract nurses to induce higher quality, higher demand and revenues. The incentive to increase nurses wage is reinforced when this translates into a larger marginal increase in residents' demand (through better matching process, and higher quality due to more favorable resident-nurse ratio). Whether the wage under regulated prices is higher or lower compared to the scenario where nursing homes compete on residents' price depends on the level of the regulated price:

$$\widehat{w} = w^{*} + \frac{c\left(\beta + 2\gamma\alpha\theta\right)}{ct_{r} + 2\gamma\alpha\theta}\left(\widehat{p} - p^{*}\right). \tag{30}$$

If the regulated price is below the price when nursing homes compete in an unregulated market, then nurses wages will also be lower.[19] The effect of the two-sidedness parameter on nurses wages price is, differently from Section 2, ambiguous:

$$\frac{\partial \widehat{w}}{\partial \gamma} = \frac{\alpha c\left(t_{r} - \beta\right)\left(2\theta\left(\widehat{p} - g\right) + \alpha t_{r}\right)}{\left(ct_{r} + 2\gamma\theta\alpha\right)^{2}}. \tag{31}$$

This effect is positive only if residents transportation costs are high compared to their valuation of the benefits from a better match between nurses and residents. Since residents prices are regulated, residents' utility in equilibrium is not affected by the two-sidedness parameter. Recall again that in the symmetric equilibrium, quality provided by the nurses is equal to $q^{*} = \alpha k/c$, and does not depend on the two-sidedness of the market parameter γ. The two-sidedness of the market therefore implies a transfer between the nurses and the nursing home, rather than between the nurses and the residents when both prices and wages are endogenously determined (as in Section 2).

Equilibrium profit is given by:

$$\widehat{\pi} = \frac{1}{2}\left(\widehat{p} - g - \widehat{w}\right) = \frac{1}{2}\left(t_{N} + \left(t_{r} - \beta\right)\frac{\left(\widehat{p} - g\right)c - \gamma\alpha^{2}}{ct_{r} + 2\alpha\theta\gamma}\right), \tag{32}$$

which is increasing in the regulated price only if the direct effect on profits is not offset by an increase in nurses' wage. To ensure at least zero profit the following condition has to hold

[19] Even though we keep the number of nurses fixed, our result that nurses salary is increasing in the regulated price is broadly in line with Hackmann (2019) that suggests that higher regulated price by Medicaid has an impact on nurses supply (see also Lin, 2014).

$(\widehat{p} - g) \geq A/c\,(\beta - t_r)$.[20]

Table 2 compares the results with the unregulated market, and provides comparative statics. A reduction in the price charged to residents *reduces* nurses wage, and therefore regulation has opposite effects on nurses and residents' utility, while the effect on nursing homes' profits is indeterminate and is determined by whether the price effects dominates on the wage effect. Lower nurses transportation costs imply a more responsive supply function to wages, which reduce nurses' wage. Differently from Section 2, residents transportation costs affect nurses' wages: lower residents transportation costs (more competition for residents) have an indeterminate effect on nurses' wages. Similarly to Section 2, altruism affects nurses' wages but the effect is indeterminate. It is positive only if and only if residents' transportation costs are high compared to the benefits valuation from matching residents with nurses.

Table 2. Price regulation $\bar{p} < p^*$

Comparison with unregulated market			
ΔU	ΔV	$\Delta \pi$	ΔW
> 0	< 0	$\gtrless 0$	$= 0$
Comparative statics			

$$\frac{\partial \widehat{w}}{\partial t_N} = -1, \quad \frac{\partial \widehat{w}}{\partial t_r} = \frac{\alpha^2 \gamma (c\beta + 2\alpha\theta\gamma) - c(\widehat{p} - g)}{(ct_r + 2\alpha\theta\gamma)^2}, \quad \frac{\partial \widehat{w}}{\partial \alpha} = \frac{2\gamma(t_r - \beta)[\theta c(\widehat{p} - g) + \alpha(2ct_r + \alpha\theta\gamma)]}{(ct_r + 2\alpha\theta\gamma)^2}.$$

4 Pay for performance

P4P schemes which reward quality are increasingly popular within the health and long-term care settings. However, the empirical evidence in relation to its effectiveness is mixed (Miller and Singer Babiarz, 2014).[21] In this section, we extend the main model in Section 2 by introducing P4P in two plausible scenarios. First, we consider a P4P scheme financed by a public funder (*e.g.* a local or central government) under which nursing homes receive financial incentives that depend on the level of quality provided. Second, we assume that nursing homes can remunerate nurses not only through the fixed wage but also through a pay-for-performance scheme.

[20] This condition is always satisfied if $\beta < t_r$. But if $\beta > t_r$, then this condition is more stringent than the one required for a weakly positive nurses salary. Therefore, to ensure that both profits and salaries are weakly positive the following has to hold: $\widehat{p} - g \geq \max\{A/c\,(\beta - t_r)\,, A/(c\beta + 2\alpha\theta\gamma)\}$.

[21] For instance, Werner *et al.* (2013) did not find that nursing homes significantly improved quality following the introduction of P4P. The authors argue that current P4P programs may fail to achieve quality improvements because the incentives were paid to the nursing homes, rather than to their individual staff members.

4.1 Pay for performance at the nursing home level

We assume that nursing homes receive a financial incentive τ, which is paid by a public funder, for each unit quality provided to residents. The profit function of nursing home 1 is:

$$\pi_1 = (p_1 - g) D_1 (p_1, p_2, w_1, w_2) - w_1 N_1 (p_1, p_2, w_1, w_2) + \tau q_1. \tag{33}$$

In Appendix 8.4, we obtain the following proposition.

Proposition 4 *In the presence of a P4P scheme on quality, we have:*

$$\begin{aligned} w' &= \beta - t_N + \frac{\gamma(\alpha + 2(\theta + \tau))\alpha}{c}, \;\; p' = g + t_r + \frac{\gamma(\alpha + 2(\theta + \tau))\alpha}{c}, \\ q' &= \frac{\alpha k}{c}, \quad \pi' = \frac{1}{2}\left(t_r + t_N - \beta + \tau\frac{\alpha k}{c}\right). \end{aligned}$$

When P4P is paid to nursing homes by a public funder, we find that P4P does not affect quality (in line with Werner *et al.,* 2013). Nevertheless, P4P has distributional implications. Nursing homes' profit increases by the amount of the financial incentives, but the P4P scheme exacerbates the positive (negative) externality that nurses (residents) generate on quality. Thus, nurses also benefit from a wage increase while residents face a higher price.[22]

4.2 Pay for performance for staff

Consider now that the P4P scheme is paid to nurses by nursing homes, as a result of nursing homes using P4P as an internal management tool to increase quality. Each nursing home can set a fee f_i for each unit of quality provided. The indirect utility of a nurse who works for nursing home i evaluated at quality $q_i^* = (\alpha + f_i)\left(k - \gamma + \gamma\frac{N_i}{D_i}\right)/c$ is:

$$V_i(y) = w_i + \frac{1}{2c}(\alpha + f_i)^2 \left(k - \gamma + \gamma\frac{N_i}{D_i}\right) - t_N y. \tag{34}$$

In Appendix 8.4 we obtain the following proposition.

[22] Analytically, the effect of an increase in the P4P fee τ on prices and wages is equivalent to an increase in the marginal valuation of quality θ, but has a different effect on profits, which increase with the fee but are not affected by residents' valuation of quality.

Proposition 5 *In the symmetric equilibrium when nursing homes also compete in P4P we have:*

$$
\begin{aligned}
f^{**} &= \theta,\ q^{**} = \frac{(\alpha + f^{**})\,k}{c}, \\
p^{**} &= g + t_r + \frac{\left[\gamma\left(\alpha + 2\theta\right) + kf^{**}\right](\alpha + f^{**})}{c}, \\
w^{**} &= \beta - t_N + \frac{\gamma\left(\alpha + f^{**}\right)(\alpha + 2\theta)}{c}, \\
\pi^{**} &= \frac{1}{2}\left(t_r + t_N - \beta\right).
\end{aligned}
$$

The key result is that nursing homes set a quality fee which is equal to the residents' quality valuation. The quality fee does not depend on the nurses' altruism so that the level of quality reflects the sum of residents and nurses valuation, $(\alpha + \theta)$. Nursing homes' profit is not affected by the introduction of the fee. Nurses overall payment, $w^{**} + f^{**}q^{**}$, is increased with P4P. This arises because the fee is paid on top of the wage, and because P4P stimulates competition for nurses, which further increases their fixed wage. However, the increase in nurses' payment is passed to the residents through an increase in price so that nurses' profits remain unchanged.

It is straightforward to show that $p^{**} > p^{*}$, $w^{**} > w^{*}$ and $\pi^{**} = \pi^{*}$, with $p^{**} - p^{*} = f^{**}q^{**} + w^{**} - w^{*}$. Nurses always gain from the introduction of the P4P scheme: $V^{**} - V^{*} = \theta\left(\gamma\left(2\theta + \alpha\right) + \frac{k}{2}\left(2\alpha + \theta\right)\right)/c > 0$. Whether residents gain or lose is in principle indeterminate: they gain from higher quality but pay a higher price:

$$
U^{**} - U^{*} = \theta\left(q^{**} - q^{*}\right) - \left(p^{**} - p^{*}\right) = -\frac{\theta}{c}\left[k\alpha + \gamma\left(\alpha + 2\theta\right)\right] < 0. \tag{35}
$$

The price effect however dominates, and residents have lower utility with P4P. The increase in nurses utility is however higher than the reduction in residents' utility, and welfare is increased: $W^{**} - W^{*} = \Delta V + \Delta U = \theta^2 k/2c > 0$. The following proposition summarises.

Proposition 6 *When the pay for performance scheme is paid by nursing homes to nurses, the quality fee is equal to residents' valuation of quality. The scheme induces an increase in quality and nurses' wage which is passed on to residents' prices so that profits are unchanged. Residents are worse off as a result of higher prices and despite the higher quality. Nurses are better off and total welfare is increased.*

See Appendix 8.4 for proof. The proposition highlights that quality and welfare enhancements may come at the cost of higher prices and lower residents utility.

4.3 Comparison

Table 3 compares the results obtained under the unregulated market with the two P4P schemes explored in this section. The introduction of pay-for-perfomance schemes increases quality only when it is paid directly to nurses, otherwise quality is unchanged and P4P only has distributional implications. Perhaps surprisingly, when P4P are paid to nurses then residents are worse off under P4P because the increase in quality is more than offset by an increase in price. This arises because nurses succeed to extract residents surplus through two channels. On the one hand, they receive an additional fee for this quality increase which is just equal to residents valuation of quality. On the other hand, it also stimulates competition between nursing homes to attract nurses that contributes to increase their wage. Both effects contribute to increase residents price.

Table 3. Introduction of pay for performance

	ΔU	ΔV	$\Delta \pi$	ΔW
$\tau > 0$, introduction of P4P at nursing homes level	< 0	> 0	> 0	$= 0$
$f^* > 0$, introduction of P4P at nurses level	< 0	> 0	$= 0$	> 0

5 Uncovered residents' market

In this section, we investigate the scenario when the reservation utility of some residents is not high enough to guarantee a covered market. We assume that there are two types of residents $m \in \{H, L\}$ who differ in their gross valuation of treatment in proportion λ and $1-\lambda$, respectively. Their utility function is:

$$U_i^m(y) = \begin{cases} S + \theta q_i - p_i - t_r y + \beta \left(v - \frac{1}{4N_i} \right) & \text{if } m = H, \\ s + \theta q_i - p_i - t_r y + \beta \left(v - \frac{1}{4N_i} \right) & \text{if } m = L, \end{cases} \tag{36}$$

where $S > s$. This parameter could be related to the degree of autonomy (*e.g.* degree of independence in their daily activities) so that some potential residents ($m = L$) are not willing to go to a nursing home if the price is too high or the quality too low relative to living by themselves or with an informal carer. The nurses' supply function remains unchanged. First, we characterise the symmetric equilibrium in such an environment. We focus on equilibria where the H-segment is fully covered, and the L-segment is only partially covered. Second, we compare such equilibrium to the scenarios where either nurses' wages or residents' prices are regulated. To keep the computations simpler we set $\beta = 0$. Total demand for nursing home i is:

$$D_i = \lambda \left[\frac{1}{2} + \frac{1}{2t_r}(p_j - p_i) + \frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\left(\frac{N_i}{D_i} - \frac{N_j}{D_j}\right)\right] + \frac{1-\lambda}{t_r}\left[s + \frac{\alpha\theta}{c}\left(k - \gamma + \gamma\frac{N_i}{D_i}\right) - p_i\right], \qquad \forall i, j = \{1, 2\}. \tag{37}$$

The following proposition is obtained (Appendix 8.5).

Proposition 7 *At the symmetric equilibrium ($D_1^* = D_2^* = D^*$ and $N_1^* = N_2^* = 1/2$), residents' price and nurses' wage are:*

$$\begin{aligned} p^* &= g + t_r \frac{2D^*}{(2-\lambda)} + \frac{\alpha\gamma\left(\alpha + 4\theta D^*\right)}{4c\left(D^*\right)^2}, \\ w^* &= \frac{\gamma\alpha}{c}\left(\frac{\alpha}{2D^*} + \frac{2\theta}{2-\lambda}\right) - t_N, \end{aligned}$$

where equilibrium demand is: $D^* = \frac{\lambda}{2} + \frac{1-\lambda}{t_r}\left[s + \frac{\alpha\theta}{c}\left(k - \gamma + \frac{\gamma}{2D^*}\right) - p^*\right]$.

If $\lambda = 1$, we recover the solution in proposition 1, highlighting that any difference in equilibrium under uncovered market is due to differences in demand. Using Cramer's rule (Appendix 8.5), we obtain:

$$\begin{aligned} \frac{dD^*}{d\gamma} &= -\frac{1-\lambda}{t_r}\frac{\alpha}{2c\Upsilon D^*}\left[\theta\left(1 + 2D^*\right) + \frac{\alpha}{2D^*}\right], \\ \frac{dp^*}{d\gamma} &= \frac{\alpha}{2c\Upsilon D^*}\left[\frac{\left(\alpha + 4\theta D^*\right)}{2\left(D^*\right)}\left(1 + \frac{1-\lambda}{t_r}\frac{\alpha\theta\gamma}{2c\left(D^*\right)^2}\right)\right. \\ &\quad \left. + \frac{1-\lambda}{t_r}\theta\left(1 - 2D^*\right)\left(\frac{2t_r}{2-\lambda} - \frac{\alpha\gamma\left(\alpha + 2\theta D^*\right)}{2c\left(D^*\right)^3}\right)\right], \end{aligned}$$

with $\Upsilon = 1 + \frac{1-\lambda}{t_r}\left(\frac{\gamma\alpha\theta}{2c(D^*)^2} + \frac{2t_r}{2-\lambda} - \frac{\alpha\gamma(\alpha+2\theta D^*)}{2c(D^*)^3}\right)$. An increase in the two-sidedness of the market increases price if the uncovered market is sufficiently small. In turn, this reduces equilibrium demand (as long as $\Upsilon > 0$, which again is satisfied when the uncovered market is sufficiently small). The effect of the two-sidedness parameter on nurses' wage is given by:

$$\frac{dw^*}{d\gamma} = \frac{\alpha}{c}\left(\frac{\alpha}{2D^*} + \frac{2\theta}{(2-\lambda)} - \frac{\alpha\gamma}{2\left(D^*\right)^2}\frac{dD^*}{d\gamma}\right). \tag{38}$$

In this case, the effect of the two-sidedness parameter on wages is positive and reinforced if demand is reduced. Overall, the results in Section 2 are robust to the inclusion of an uncovered market as long as this market is relatively small, which is in line with the empirical evidence suggesting that the demand for nursing homes is generally inelastic (Grabowski and Gruber, 2007; Mommaerts, 2018) and that it is only residents with lower dependence who are likely to respond to changes in market conditions.

5.1 Residents' and nurses' utility and welfare

Residents' utility at the symmetric equilibrium is:

$$U^*(y) = \begin{cases} S + \theta q^* - p^* - t_r y & \text{if } m = H, \\ s + \theta q^* - p^* - t_r y & \text{if } m = L, \end{cases} \tag{39}$$

where

$$\theta q^* - p^* = \frac{\alpha}{c}\left[\theta k - \gamma\left(\theta\left(\frac{1}{2D^*} + 1\right) + \frac{\alpha}{4(D^*)^2}\right)\right] - g - t_r\frac{2D^*}{(2-\lambda)}, \tag{40}$$

and $q^* = (\alpha/c)\left(k - \gamma + \gamma/\left(2D^*\right)\right)$. We have already established that when the uncovered market is small, an increase in the two-sidedness parameter increases price. It also increases quality, given that $\frac{dq^*}{d\gamma} = \frac{\alpha}{k}(\frac{1}{2D^*} - 1) - \frac{\alpha}{k}\gamma\frac{1}{2(D^*)^2}\frac{dD^*}{d\gamma}$. This is because with $D^* < 1/2$ an increase in γ reduces the marginal cost of providing quality, and therefore increases quality, and this is further reinforced by the reduction in demand which further reduces the marginal cost of quality. The effect of the two-sidedness parameter on residents' utility is therefore in principle indeterminate. However, (40) suggests that the price effects tends to dominate for a given demand, and this price effect can be reinforced or weakened by a reduction in demand (proof omitted).

Nurses' utility is given by:

$$V^*(y) = \frac{\alpha}{2c}\left[\alpha k + \gamma\left(\alpha\left(\frac{3}{2D^*} - 1\right) + \frac{4\theta}{2-\lambda}\right)\right] - t_N - t_N y. \tag{41}$$

For a given demand, nurses' utility increases with the two-sidedness of the market, and this effect is reinforced by the reduction in equilibrium demand.

Nursing homes' profits are:

$$\pi^* = \frac{1}{2}\left(t_r \frac{4D^2}{2-\lambda} + t_N\right) + \frac{\theta\alpha\gamma}{c}\frac{1-\lambda}{2-\lambda}. \tag{42}$$

This suggests that the two-sidedness parameter increases profits driven by higher prices but it is partially offset by a reduction in demand.

Finally, overall welfare is given by (Appendix 8.5):

$$W^* = 2\left[\lambda \int_0^{1/2} (S + \theta q^* - p^* - t_r y)\, dy + (1-\lambda) \int_0^{D^{*L}} (s + \theta q^* - p^* - t_r y)\, dy + \int_0^{1/2} V^*(y) dy + \pi^*\right],$$

which can be re-written as:

$$W^* = \lambda S + (1-\lambda) 2D^L s - t_r \left(\frac{\lambda}{4} + (1-\lambda)\left(D^L\right)^3\right) + \left(\frac{\alpha}{2} + 2D^*\theta\right) q^* - \frac{t_N}{4} - 2gD^*. \tag{43}$$

If $\lambda = 1$, then $q^* = (\alpha k)/c$ and welfare is, as previously obtained, equal to: $W^* = S - t_r(\lambda/4) - t_N/4 - g + (\alpha + 2\theta)(\alpha k/2c)$. As expected, the two-sidedness of the market affects welfare only through changes in quality and demand, which instead were fixed with covered markets. The effect of the two-sidedness of the market on welfare is:

$$\frac{\partial W^*}{\partial \gamma} = \left(\frac{\alpha}{2} + 2D^*\theta\right)\frac{\partial q^*}{\partial \gamma} + \frac{\partial W^*}{\partial D^L}\frac{\partial D^{*L}}{\partial \gamma},$$

where $\frac{\partial D^{*L}}{\partial \gamma} = \frac{\partial D^{*L}}{\partial \gamma}\frac{1}{2(1-\lambda)}$ and $\frac{\partial W^*}{\partial D^L} = 2(1-\lambda)\left(s + \theta q^* - t_r\left(D^{*L}\right)^2 - g\right)$. We have shown above that an increase in the two-sidedness of the market reduces demand due to the price increase but increases quality. The effect on welfare is therefore indeterminate (with further substitution of $\partial q^*/\partial\gamma$, $\partial W^*/\partial D^L$ and $\partial D^{*L}/\partial\gamma$ giving no additional insights).

5.2 Price or wage regulation with uncovered residents' market

When nurses' wage is regulated, nursing homes maximise profits with respect to price.

Proposition 8 *At the symmetric equilibrium when residents' market is uncovered and the wage* *paid to nurses is regulated ($\overline{w}$), the price charged to residents is given by:*

$$\overline{p} = g + \frac{2\overline{D}t_r}{(2-\lambda)} + \frac{\alpha\gamma}{2\left(2ct_N - \frac{\gamma\alpha^2}{\overline{D}}\right)} \left\{ \frac{2\theta t_N c(2-\lambda) - \frac{\alpha^2\gamma\theta(1-\lambda)}{\overline{D}}}{\overline{D}(2-\lambda)c} - \frac{\alpha}{2\overline{D}^2}\overline{w} \right\},$$

with $\overline{D} = \frac{\lambda}{2} + \frac{1-\lambda}{t_r}\left[s + \frac{\alpha\theta}{c}\left(k - \gamma + \frac{\gamma}{2\overline{D}}\right) - \overline{p}\right]$.

As in the benchmark case, an increase in the regulated nurses' wage reduces residents' price. It is still the case, as in Section 3.1, that the two-sidedness parameter has an ambiguous effect on price, and it also has an ambiguous effect on demand. In terms of nurses' utility, the two-sidedness parameter reduces the marginal cost of quality and, by the envelope theorem, increases nurses utility as long as the two-sidedness parameter reduces demand (easing their workload) or the demand effect is sufficiently small. Residents' utility is affected through changes in price, which is indeterminate, and now also in quality, which tends to increase if demand effects are small. The effect on profits is determined by the effect on prices and demand.

Turning to price regulation, nursing homes only compete through nurses' wage.

Proposition 9 *When the price charged to residents is regulated ($\widehat{p}$), nurses' wage is given by:*

$$\widehat{w} = \frac{(\widehat{p} - g)\,2\alpha\theta\gamma + \alpha^2\gamma\left(t_r + \frac{\theta\alpha\gamma(1-\lambda)}{4c\widehat{D}^2}\right)}{2\widehat{D}ct_r + \frac{\theta\alpha\gamma(2-\lambda)}{2\widehat{D}}} - t_N,$$

with $\widehat{D} = \frac{\lambda}{2} + \frac{1-\lambda}{t_r}\left[s + \frac{\alpha\theta}{c}\left(k - \gamma + \frac{\gamma}{2\widehat{D}}\right) - \widehat{p}\right]$.

As in Section 3.2, nurses' wage increases with the regulated price $\widehat{p}$. If the uncovered market segment is sufficiently small, and given our assumption of $\beta = 0$, in line with Section 3.2, the two-sidedness of the market increases nurses salary. Combined with the reduction in the marginal cost of quality, nurses have higher utility. Given that prices are regulated, residents are better off as a result of the higher quality. Profits are generally reduced as a result of higher nurses wage, but are also affected by changes in demand, which tends to increase due to higher quality.

6 Altruistic motives

In this section, we extend the main model in Section 2 under covered market, and allow the nursing homes objective function to include altruistic concerns about residents. For example, some nursing homes may be (partially) publicly owned as a result of public-private procurement contracts. We adopt the following general objective function:

$$\widetilde{\pi}_i = \pi_i + \delta \widetilde{U}_i, \tag{44}$$

where δ is the weight given to residents, relative to profits, and could be interpreted as an additional form of regulation (in addition to price and wage regulation) that is indirectly imposed via (partial) public ownership,[23] and

$$\widetilde{U}_i = \theta q_i + \beta \left(v - \frac{1}{4N_i} \right) - \sigma p_i, \tag{45}$$

is the utility component of the average resident, excluding transportation costs.[24] Therefore, in addition to profits, nursing homes take into account the effect of quality and price on the utility of the average resident. It could be argued that nursing homes altruistic component mostly relates to quality, which is a key aspect of the care they provide, and less about price. To allow for this, we include the parameter $\sigma \in [0, 1]$ which is the weight that the nursing home gives to price. With $\sigma = 0$ the altruistic component of nursing homes only includes quality. In Appendix 8.6, the following proposition is obtained.

Proposition 10 *When nursing homes have altruistic concerns towards residents' utility, the symmetric equilibrium is given by:*

$$\begin{aligned} p^A &= g + t_r + \frac{\gamma\alpha\left(\alpha + 2\theta\right)}{c} + \delta \left(\frac{2\theta\alpha\gamma}{c} \left(1 - 2\sigma\right) - 2\sigma t_r \right), \\ w^A &= \beta - t_N + \frac{\gamma\alpha\left(\alpha + 2\theta\right)}{c} + \delta \left(\frac{2\theta\alpha\gamma}{c} + \beta \right) \left(1 - 2\sigma\right), \\ \pi^A &= \frac{1}{2} \left\{ t_r + t_N - \beta - \delta \left[2ct_r + (1 - 2\sigma)\beta \right] \right\}. \end{aligned}$$

[23] We thank an anonymous reviewer for suggesting this extension.

[24] We exclude transportation costs because we conjecture that nurses homes care about the utility components over which they have an impact, namely quality and price, while distances are exogenous to the nursing homes.

Compared to our benchmark case, there are three additional mechanisms at work. First, the common network externality component, which implies a transfer from residents to nurses, is increased (decreased) by $\delta[2\theta\alpha\gamma(1-2\sigma)]/c$ if $\sigma < 1/2$ ($\sigma > 1/2$). The result is intuitive. When nursing homes care mostly about quality, *i.e.* $\sigma < 1/2$, the effect of the common network externality is amplified and further increases the nurses' salary at the expense of residents' welfare. However, the opposite arises if nursing homes give a significant weight to prices, because the effect of the network externality on prices, which is positive and tends to reduce residents' utility, is internalised by the nursing home.

Second, altruistic concerns also affects the relation between the parameter β with price and wages. In the benchmark case, a higher valuation of nurses, through better matching, intensifies competition for nurses, which in turn increases nurses salary. With altruistic concerns, this effect is intensified if the nursing home gives a high weight to quality ($\sigma < 1/2$), but is instead weakened if the nursing home gives sufficient weight to price as well ($\sigma > 1/2$). As in the benchmark model, the residents equilibrium price is still independent of β.

Third, in the benchmark model the price increases in the market power t_r. If the altruistic component takes prices into account, then nursing homes exercise less market power and charge residents a lower price, and this price unambiguously decreases with the altruistic component on price.

In summary, when $\delta > 0$, we obtain the following insights. If the altruistic component gives a low weight to price, $\sigma < 1/2$, residents are better off only if the third mechanism outweights the first one, while if $\sigma > 1/2$ residents are always better off as a result of lower prices. Therefore, residents are not necessarily better off when nursing homes have altruistic concerns in a two-sided market. Nurses are instead better (worse) off when the altruistic component gives a low (high) weight to price as a result of more intense quality competition. Profits for nursing homes tend to reduce as a result of lower price under the third mechanism, but may increase as a result of the possible increase in nurses' salary under the second mechanism.

7 Conclusion

This study has investigated the market for nursing homes using a *two-sided market* approach. Our key assumptions, which are in line with the empirical evidence (see Introduction) are that i) a higher number of nurses can affect demand for residents because it potentially implies higher quality (through better matching and relaxed time constraints), and ii) a higher number residents affects nurses labour supply by affecting nurses' working conditions (nurses working under higher pressure with a larger volume of residents). It is the combination of these two assumptions, together with nurses' altruistic motives, which makes the market two-sided.

Our main result is that the two-sidedness in the market has *distributional* implications as it leads to more intense competition for nurses and to higher wages, so that nurses are better off. Such increases in wage are then passed to the residents in the form of higher price, which makes the residents worse off. Nursing homes' profits are instead generally unaffected since the increase in nurses' wages is exactly offset by the increase in price. By offering a higher wage a nursing home increases nurses' utility directly but also indirectly by reducing the residents-nurse ratio which is valued by nurses (because of lower workload) and residents (because it translates into a higher quality). These incentive effects depend critically on how the resident-nurse ratio affects nurses' utility and therefore the quality they provide. When the resident-nurse ratio does not affect directly nurses' utility, both nurses' wages and residents' prices tend to be lower. When the residents' market is uncovered, the results are qualitatively similar but the two-sidedness of the market also reduces demand, which in turn implies a higher quality, and this is what drives any additional welfare effect.

The two-sidedness of the market matters and has marked different distributional implications if either residents' price or nurses' wage is *regulated*. When *nurses' wages* are regulated, the two-sidedness of the market implies a transfer between residents and nursing homes. When *residents' price* is regulated, it instead implies a transfer between nurses and nursing homes.

Our results have therefore implications for the regulation of nursing homes sector. Suppose that an unregulated market, where prices and wages are endogenous, leads to residents' prices which are considered excessive by the regulator (*e.g.* an anti-trust authority, health ministry, local or federal authorities). Two regulatory interventions are possible. We show, counter-intuitively, that an increase in the regulated nurses wage implies a *reduction* in residents price. Then, regulating the nurses wages at a level which is higher than the wage in the unregulated market, will also reduce residents price. Both nurses and residents are better off as a result, while nursing homes' profits will correspondingly reduce. An alternative way to reduce residents' price is for the regulator to introduce price regulation.

The introduction of a regulated price, which is below the price in an unregulated market, will make residents better off, but in this scenario will instead compress nurses' wage, and might also reduce nurses' utility. This form of price regulation therefore generates a transfer from nurses (and potentially nursing homes) to residents. In summary, if a regulator is concerned about excessive prices paid by residents, both forms of regulations (either directly on prices or, indirectly, through nurses wages) will favour residents.

Moreover, we show that policy interventions which facilitate the introduction of pay for performance schemes (*e.g.* by developing reliable quality metrics) at the nursing homes level, rather than the staff level, do not affect the quality provided, which seems in line with mixed empirical evidence in relation to its effectiveness (Miller and Singer Babiarz, 2014). In contrast, P4P which are paid by nursing homes to nurses have the intended effect of improving quality. They also increase the scope for competition for nurses, which translates into larger wages for nurses and higher nurses utility. Our analysis however highlights an adverse effect for residents. Despite benefiting from higher quality, residents are worse off since the higher price that they are charged does not compensate for the higher quality, while nursing homes profits are unchanged. Again, the two-sidedness of the market does not favour residents. Overall, our analysis highlights that policy evaluations that empirically test the effect of the introduction of P4P schemes in nursing homes markets should not only focus on quality outcomes but also on its effect on nursing homes' profits and residents' prices, and more broadly its distributional consequences.

Our model assumes that nursing homes maximise profits. As an extension, we also investigate the effect of allowing for altruistic concerns towards the residents in the nursing home payoff function. Future work could investigate the effect of the two-sidedness of the market in a mixed market where public providers compete with private providers.

In the specific context of the development of online platforms in primary care, these platforms direct patients to their affiliated doctors in exchange of a fixed fee to doctors for being affiliated to the platform. Instead, they do not charge the patients. If such platforms start to charge positive prices to patients, our results tend to show that patients are *a priori* better off when the fee paid to doctors is regulated (rather than when patients' prices are regulated). Future work could incorporate additional features of these online platforms.

Price regulation for primary care, in form of capitation payments, is common across high-income countries. For countries where primary care is organised in large practices, as in England, price regulation implies that patients will not be affected by the two-sidedness of the market in such institutional settings, but will instead imply a transfer between the owner of the GP practice and its employees (salaried doctors, nurses and other health professionals working in the practice). Our model was designed to capture features of the nursing homes market. Future work could incorporate features that further characterise the primary care sector (*e.g.* gatekeeping, interface with hospital sector, alternative payment arrangements *etc.*).

Our model contains features that also apply to the education sector. Schools and universities can be viewed as platforms that compete for students on one side and for teachers or professors on the other side. It is well documented that the quality of education depends on the pupil/teacher ratio, which through the benefits of higher quality affects both students and teachers. Angrist and Lavey (1999) find that lower pupil/teacher ratio are associated with higher test scores for the children, and that a reduction of class size improves teachers working conditions by lightening their workload and easing classroom management (Buckingham, 2003).[25] Moreover, a higher availability of teachers or professors improves the likelihood of matching between teacher and pupils (e.g. getting along) or professor and student (e.g. match of research interest and sub-discipline). Our main model however applies only to private education systems where providers compete on both price and quality. Instead, price regulation is common in the education sector across OECD countries, especially for primary and secondary schools in which case the results in Section 3.2 will apply. In many instances, the private education sector is mostly composed by non-for-profit institutions, and the results in Section 6 are also likely to hold.

More broadly, our analysis has relevance for the public sector where altruistic concerns or intrinsic motivation are important, and employees are affected by the workload through the ratio of staff and volume of provision of public services. Outside of the public sector, the model

[25] Card and Krueger (1992) also find that a smaller class size tends to increase average future earnings.

still holds each time that the quality of good or services provided by intermediaries depends positively on the number of providers and negatively on the number of customers, and such quality is commonly valued by customers and providers, albeit by different intensities. We show that differences in institutional features or market structure may justify different forms of regulation.

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8 Appendix

8.1 Demand function analysis

The demand functions system is given by:

$$\begin{aligned}
D_1 &= \frac{1}{2} + \frac{\theta}{2t_r}(q_1 - q_2) + \frac{1}{2t_r}(p_2 - p_1) - \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{N_2}\right), \\
D_2 &= 1 - D_1, \\
N_1 &= \frac{1}{2} + \frac{1}{2t_N}(w_1 - w_2) + \frac{\alpha^2\gamma}{4ct_N}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right), \\
N_2 &= 1 - N_1.
\end{aligned} \tag{46}$$

Consider the following functions:

$$\begin{aligned}
\Gamma_D &= D_1 - \frac{1}{2} - \frac{\alpha\theta\gamma}{2ct_r}\left(\frac{N_1}{D_1} - \frac{1-N_1}{1-D_1}\right) - \frac{1}{2t_r}(p_2 - p_1) + \frac{\beta}{8t_r}\left(\frac{1}{N_1} - \frac{1}{1-N_1}\right), \\
\Gamma_N &= N_1 - \frac{1}{2} - \frac{1}{2t_N}(w_1 - w_2) - \frac{\alpha^2\gamma}{4ct_N}\left(\frac{N_1}{D_1} - \frac{1-N_1}{1-D_1}\right).
\end{aligned} \tag{47}$$

Totally differentiating, we obtain:

$$\begin{bmatrix} 1 + \frac{\alpha\theta\gamma}{2ct_r}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) & -\frac{\alpha\theta\gamma}{2ct_r}\left(\frac{1}{D_1} + \frac{1}{1-D_1}\right) - \frac{\beta}{8t_r}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right) \\ \frac{\alpha^2\gamma}{4ct_N}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) & 1 - \frac{\alpha^2\gamma}{4ct_N}\left(\frac{1}{D_1} + \frac{1}{1-D_1}\right) \end{bmatrix} \begin{bmatrix} dD_1 \\ dN_1 \end{bmatrix} = -\begin{bmatrix} \frac{1}{2t_r} \\ 0 \end{bmatrix} dp_1. \tag{48}$$

Applying the Cramer's rule we obtain:

$$\frac{dD_1}{dp_1} = -\frac{1}{8\Delta}\left(4ct_N - \alpha^2\gamma\left(\frac{1}{D_1} + \frac{1}{1-D_1}\right)\right), \tag{49}$$

where

$$\begin{aligned}
\Delta &: = ct_r t_N \det = ct_r t_N + \alpha\gamma\left(\frac{\alpha\beta}{32}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right)\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right) - \frac{\alpha t_r}{4}\left(\frac{1}{D_1} + \frac{1}{1-D_1}\right)\right. \\
&\qquad \left. + \frac{\theta t_N}{2}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right)\right),
\end{aligned} \tag{50}$$

which is positive under minimal regularity conditions. Similarly, we have:

$$\frac{dN_1}{dp_1} = \frac{\alpha^2\gamma}{8\Delta}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right). \tag{51}$$

Differentiating with respect to w_1 and applying Cramer's rule gives:

$$\frac{dD_1}{dw_1} = \frac{\alpha\theta\gamma}{4\Delta}\left(\frac{1}{D_1} + \frac{1}{1-D_1}\right) + \frac{\beta c}{16\Delta}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right), \tag{52}$$

$$\frac{dN_1}{dw_1} = \frac{ct_r}{2\Delta} + \frac{\alpha\theta\gamma}{4\Delta}\left(\frac{N_1}{D_1^2} + \frac{1-N_1}{(1-D_1)^2}\right). \tag{53}$$

8.2 Profit complementarity and substitutability in price and wage

The effect of an increase in wage on the marginal profitability of price is:

$$\frac{\partial^2\pi_1}{\partial p_1 \partial w_1} = \frac{\gamma\alpha}{4\Delta}\frac{2\theta D_1(1-D_1) - \alpha\left[(1-N_1)D_1^2 + N_1(1-D_1)^2\right]}{2D_1^2(1-D_1)^2} + \frac{\beta c}{16\Delta}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right). \tag{54}$$

Suppose that we set altruism at the highest possible value, $\alpha = \theta$, then:

$$\frac{\partial^2\pi_1}{\partial p_1 \partial w_1} = \frac{\gamma\alpha\theta}{4\Delta}\frac{(2D_1-1)N_1 + D_1(2-3D_1)}{2D_1^2(1-D_1)^2} + \frac{\beta c}{16\Delta}\left(\frac{1}{N_1^2} + \frac{1}{(1-N_1)^2}\right), \tag{55}$$

which is always positive when $D_1 \leq 0.75$.

Differentiating the first-order conditions with respect to p_2 gives:

$$\begin{bmatrix} \frac{\partial^2\pi_1}{\partial p_1^2} & \frac{\partial^2\pi_1}{\partial p_1\partial w_1} \\ \frac{\partial^2\pi_1}{\partial w_1\partial p_1} & \frac{\partial^2\pi_1}{\partial w_1^2} \end{bmatrix} \begin{bmatrix} dp_1 \\ dw_1 \end{bmatrix} = -\begin{bmatrix} \frac{\partial D_1}{\partial p_2} \\ -\frac{\partial N_1}{\partial p_2} \end{bmatrix} dp_2. \tag{56}$$

Applying Cramer's rule yields:

$$\frac{dp_1}{dp_2} = \frac{1}{\Lambda}\begin{vmatrix} -\frac{\partial D_1}{\partial p_2} & \frac{\partial^2\pi_1}{\partial p_1\partial w_1} \\ \frac{\partial N_1}{\partial p_2} & \frac{\partial^2\pi_1}{\partial w_1^2} \end{vmatrix} = -\frac{1}{\Lambda}\left(\frac{\partial D_1}{\partial p_2}\frac{\partial^2\pi_1}{\partial w_1^2} + \frac{\partial^2\pi_1}{\partial p_1\partial w_1}\frac{\partial N_1}{\partial p_2}\right) > 0, \tag{57}$$

where

$$\Lambda := \frac{\partial^2\pi_1}{\partial p_1^2}\frac{\partial^2\pi_1}{\partial w_1^2} - \left(\frac{\partial^2\pi_1}{\partial p_1\partial w_1}\right)^2 > 0. \tag{58}$$

Differentiating the first-order conditions with respect to w_2 gives:

$$\begin{bmatrix} \frac{\partial^2 \pi_1}{\partial p_1^2} & \frac{\partial^2 \pi_1}{\partial p_1 \partial w_1} \\ \frac{\partial^2 \pi_1}{\partial w_1 \partial p_1} & \frac{\partial^2 \pi_1}{\partial w_1^2} \end{bmatrix} \begin{bmatrix} dp_1 \\ dw_1 \end{bmatrix} = - \begin{bmatrix} \frac{\partial D_1}{\partial w_2} \\ -\frac{\partial N_1}{\partial w_2} \end{bmatrix} dw_2. \tag{59}$$

Applying Cramer's rule yields:

$$\frac{dw_1}{dw_2} = \frac{1}{\Lambda} \begin{vmatrix} \frac{\partial^2 \pi_1}{\partial p_1^2} & -\frac{\partial D_1}{\partial w_2} \\ \frac{\partial^2 \pi_1}{\partial w_1 \partial p_1} & \frac{\partial N_1}{\partial w_2} \end{vmatrix} = \frac{1}{\Lambda} \left(\frac{\partial N_1}{\partial w_2} \frac{\partial^2 \pi_1}{\partial p_1^2} + \frac{\partial^2 \pi_1}{\partial p_1 \partial w_1} \frac{\partial D_1}{\partial w_2} \right). \tag{60}$$

8.3 Symmetric equilibrium

The first-order conditions evaluated at the symmetric equilibrium are:

$$\frac{\partial \pi_1}{\partial p_1} = (p_1 - g) \frac{\partial D_1}{\partial p_1} + \frac{1}{2} - w_1 \frac{\partial N_1}{\partial p_1} = 0, \tag{61}$$

$$\frac{\partial \pi_1}{\partial w_1} = (p_1 - g) \frac{\partial D_1}{\partial w_1} - \frac{1}{2} - w_1 \frac{\partial N_1}{\partial w_1} = 0. \tag{62}$$

where

$$\frac{dD_1}{dp_1} = -\frac{ct_N - \gamma\alpha^2}{2A}, \ \frac{dN_1}{dp_1} = \frac{\gamma\alpha^2}{2A}, \tag{63}$$

$$\frac{dN_1}{dw_1} = \frac{ct_r + 2\alpha\theta\gamma}{2A}, \frac{dD_1}{dw_1} = \frac{\beta c + 2\alpha\theta\gamma}{2A}, \tag{64}$$

with

$$A =: ct_r t_N + \alpha\gamma \left(\alpha \left(\beta - t_r \right) + 2\theta t_N \right). \tag{65}$$

We obtain:

$$p_1^* - g = \frac{w_1^* \frac{\partial N_1}{\partial p_1} - \frac{1}{2}}{\frac{\partial D_1}{\partial p_1}}, \ w_1^* = \frac{(p_1^* - g) \frac{\partial D_1}{\partial w_1} - \frac{1}{2}}{\frac{\partial N_1}{\partial w_1}}. \tag{66}$$

Substituting for the price into the wage equation, we have:

$$w_1^* = \frac{\frac{\partial D_1}{\partial p_1} + \frac{\partial D_1}{\partial w_1}}{2 \left(\frac{\partial N_1}{\partial p_1} \frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1} \frac{\partial D_1}{\partial p_1} \right)} = \beta - t_N + \frac{\gamma\alpha \left(\alpha + 2\theta \right)}{c}. \tag{67}$$

Substituting the equilibrium wage into the price equation, we obtain:

$$p_1^* - g = \frac{w_1\gamma\alpha^2 - A}{-\left(ct_N - \gamma\alpha^2\right)} = t_r + \frac{\alpha\gamma\left(2\theta+\alpha\right)}{c}. \tag{68}$$

At the symmetric equilibrium, nursing homes' profit are:

$$\Pi_1 = \frac{1}{2}\left(t_r + \frac{\alpha\gamma\left(2\theta+\alpha\right)}{c} + t_N - \frac{\alpha\gamma\left(2\theta+\alpha\right)+\beta c}{c}\right) = \frac{1}{2}\left(t_r + t_N - \beta\right). \tag{69}$$

8.4 Pay for performance

Pay for performance at the nursing home level. The first order conditions for residents' price and nurses' wage are:

$$\frac{\partial \pi_1}{\partial p_1} = \left(p_1 - g\right)\frac{\partial D_1}{\partial p_1} + \frac{1}{2} - w_1\frac{\partial N_1}{\partial p_1} + \tau\frac{\gamma\alpha}{cD_1^2}\left(\frac{\partial N_1}{\partial p_1}D_1 - \frac{\partial D_1}{\partial p_1}N_1\right) = 0, \tag{70}$$

$$\frac{\partial \pi_1}{\partial w_1} = \left(p_1 - g\right)\frac{\partial D_1}{\partial w_1} - \frac{1}{2} - w_1\frac{\partial N_1}{\partial w_1} + \tau\frac{\gamma\alpha}{cD_1^2}\left(\frac{\partial N_1}{\partial w_1}D_1 - \frac{\partial D_1}{\partial w_1}N_1\right) = 0. \tag{71}$$

The first order condition for price is:

$$\left(p_1 - g\right) = \frac{\frac{1}{2} - w_1\frac{\partial N_1}{\partial p_1} + \tau\frac{\gamma\alpha}{cD_1^2}\left(\frac{\partial N_1}{\partial p_1}D_1 - \frac{\partial D_1}{\partial p_1}N_1\right)}{\frac{\partial D_1}{\partial p_1}}. \tag{72}$$

Substituting in the first order condition for wage, we obtain:

$$\frac{1}{2} - w_1\frac{\partial N_1}{\partial p_1} + \tau\frac{\gamma\alpha}{c}\left[\frac{\frac{\partial N_1}{\partial p_1}D_1 - \frac{\partial D_1}{\partial p_1}N_1}{D_1^2}\right]\frac{\frac{\partial D_1}{\partial w_1}}{\frac{\partial D_1}{\partial p_1}} - \frac{1}{2} - w_1\frac{\partial N_1}{\partial w_1} + \tau\frac{\gamma\alpha}{c}\left[\frac{\frac{\partial N_1}{\partial w_1}D_1 - \frac{\partial D_1}{\partial w_1}N_1}{D_1^2}\right] = 0.$$

Thus, we have:

$$\begin{aligned} w' &= \frac{\frac{\partial D_1}{\partial p_1} + \frac{\partial D_1}{\partial w_1}}{2\left(\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}\right)} + \frac{2\gamma\tau\alpha}{c}, \\ p' &= \frac{\frac{\partial N_1}{\partial p_1} + \frac{\partial N_1}{\partial w_1}}{2\left(\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}\right)} + \frac{2\gamma\tau\alpha}{c}. \end{aligned} \tag{73}$$

Pay for performance at the staff level. The resident demand function is the same as in (3) and nurse supply function is :

$$
\begin{aligned}
N_1 &= \frac{1}{2} + \frac{1}{2t_N}(w_1 - w_2) + \frac{\gamma\alpha^2}{4ct_N}\left(\frac{N_1}{D_1} - \frac{N_2}{D_2}\right) \\
&\quad + \frac{1}{4ct_N}\left((f_1)^2\left(k - \gamma + \gamma\frac{N_1}{D_1}\right) - (f_2)^2\left(k - \gamma + \gamma\frac{N_2}{D_2}\right)\right),
\end{aligned} \tag{74}
$$

with $D_2 = 1 - D_1$, $N_2 = 1 - N_1$. At the symmetric equilibrium we obtain:

$$
\begin{aligned}
\frac{dD_1}{dp_1} &= \frac{-1}{2ct_r t_N \det}\left[ct_N - (\alpha + f)^2\gamma\right], \\
\frac{dN_1}{dp_1} &= \frac{1}{2ct_r t_N \det}\left((\alpha + f)^2\gamma\right), \\
\frac{dD_1}{dw_1} &= \frac{1}{2ct_r t_N \det}\left[2(\alpha + f)\theta\gamma + \beta c\right], \\
\frac{dN_1}{dw_1} &= \frac{1}{2ct_r t_N \det}\left[ct_r + 2(\alpha + f)\theta\gamma\right], \\
\frac{dD_1}{df_1} &= \frac{k}{2ct_r t_N \det}\left[\theta t_N + (\alpha + f)\left(\theta\frac{\gamma}{c}(\alpha + f) + \beta\right)\right], \\
\frac{dN_1}{df_1} &= \frac{(\alpha + f)k}{2ct_r t_N \det}\left[t_r + \theta(\alpha + f)\frac{\gamma}{c}\right].
\end{aligned} \tag{75}
$$

Since the quality provided by nurses is received by each resident, each nursing home i has to pay $f_i q_i D_i$. Then, nursing home i profit function is:

$$
\pi_1 = (p_1 - g - f_1 q_1) D_1(p_1, p_2, w_1, w_2, f_1, f_2) - w_1 N_1(p_1, p_2, w_1, w_2, f_1, f_2). \tag{76}
$$

Substituting quality in nursing home profit yields:

$$
\begin{aligned}
\pi_1 &= \left(p_1 - g - \frac{f_1}{c}(\alpha + f_1)\left(k - \gamma + \gamma\frac{N_1}{D_1}\right)\right) D_1 - w_1 N_1, \\
&= \left(p_1 - g - \frac{f_1(\alpha + f_1)}{c}(k - \gamma)\right) D_1 - \left(w_1 + \frac{f_1(\alpha + f_1)\gamma}{c}\right) N_1.
\end{aligned} \tag{77}
$$

The first-order conditions with respect to p_1, w_1 and f_1 are respectively:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial p_1} &= \left(p_1 - g - \frac{f_1(\alpha+f_1)}{c}(k-\gamma)\right)\frac{\partial D_1}{\partial p_1} + D_1 - \left(w_1 + \frac{(\alpha+f_1)f_1\gamma}{c}\right)\frac{\partial N_1}{\partial p_1} = 0, \quad (78)\\
\frac{\partial \pi_1}{\partial w_1} &= \left(p_1 - g - \frac{f_1(\alpha+f_1)}{c}(k-\gamma)\right)\frac{\partial D_1}{\partial w_1} - N_1 - \left(w_1 + \frac{(\alpha+f_1)f_1\gamma}{c}\right)\frac{\partial N_1}{\partial w_1} = 0,\\
\frac{\partial \pi_1}{\partial f_1} &= \left(p_1 - g - \frac{f_1(\alpha+f_1)(k-\gamma)}{c}\right)\frac{\partial D_1}{\partial f_1} - \left(w_1 + \frac{(\alpha+f_1)f_1\gamma}{c}\right)\frac{\partial N_1}{\partial f_1}\\
&\quad - \frac{(2f_1+\alpha)}{c}\left(\gamma N_1 + (k-\gamma)D_1\right) = 0.
\end{aligned}$$

Differently from Section 2, nursing home 1 here also chooses the fee f_1 to maximize its profit. It is such that the marginal marginal revenue, *i.e.* the mark-up $p_1 - g - (k-\gamma)f_1(\alpha+f_1)/c$ multiplied by the marginal increase in demand $(\partial D_1/\partial f_1)$, is equal to the marginal cost due to the increase in the fee, given by $(2f_1+\alpha)(\gamma N_1 + (k-\gamma)D_1)/c$, and the marginal cost of increasing the number of nurses, given by $(w_1 + (\alpha+f_1)f_1\gamma/c)(\partial N_1/\partial f_1)$.

8.5 Uncovered residents' market

For the H-segment the market is covered and the demand functions are, with $\beta = 0$,:

$$\begin{aligned}
D_i^H &= \frac{1}{2} + \frac{1}{2t_r}(p_j - p_i) + \frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\left(\frac{N_i}{D_i} - \frac{N_j}{D_j}\right), \quad (79)\\
D_j^H &= 1 - D_i^H, \qquad \forall i,j = \{1,2\}.
\end{aligned}$$

For the L-segment, the market is uncovered and the demand functions are:

$$D_i^L = \frac{1}{t_r}\left[s + \frac{\alpha\theta}{c}\left(k - \gamma + \gamma\frac{N_i}{D_i}\right) - p_i\right], \qquad \forall i = \{1,2\}. \quad (80)$$

The residents' demand and nurses' supply functions are implicitly given by:

$$\begin{aligned}
\Gamma_D &= D_1 - \frac{\lambda}{2} - \frac{(1-\lambda)\alpha\theta(k-\gamma)}{t_r c} + \left(\frac{\lambda}{2t_r} + \frac{1-\lambda}{t_r}\right)p_1 - \frac{\lambda}{2t_r}p_2 \quad (81)\\
&\quad -\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r} + \frac{1-\lambda}{t_r}\right)\frac{N_1}{D_1} + \lambda\frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\frac{1-N_1}{D_2} = 0,\\
\Gamma_N &= N_1 - \frac{1}{2} - \frac{1}{2t_N}(w_1 - w_2) - \frac{\alpha^2\gamma}{4ct_N}\left(\frac{N_1}{D_1} - \frac{1-N_1}{D_2}\right) = 0. \quad (82)
\end{aligned}$$

Differentiation of this system with respect to p_1 and w_1 yields respectively:

$$\left(\begin{array}{cc} 1+\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{N_1}{(D_1)^2} & -\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{1}{D_1}-\lambda\frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\frac{1}{D_2} \\ \frac{\alpha^2\gamma}{4ct_N}\frac{N_1}{(D_1)^2} & 1-\frac{\alpha^2\gamma}{4ct_N}\left(\frac{1}{D_1}+\frac{1}{D_2}\right) \end{array}\right)\left[\begin{array}{c} dD_1 \\ dN_1 \end{array}\right]=-\left[\begin{array}{c} \left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right) \\ 0 \end{array}\right]dp_1,$$

$$\left(\begin{array}{cc} 1+\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{N_1}{(D_1)^2} & -\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{1}{D_1}-\lambda\frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\frac{1}{D_2} \\ \frac{\alpha^2\gamma}{4ct_N}\frac{N_1}{(D_1)^2} & 1-\frac{\alpha^2\gamma}{4ct_N}\left(\frac{1}{D_1}+\frac{1}{D_2}\right) \end{array}\right)\left[\begin{array}{c} dD_1 \\ dN_1 \end{array}\right]=-\left[\begin{array}{c} 0 \\ -\frac{1}{2t_N} \end{array}\right]dw_1.$$

Applying the Cramer's rule, we obtain:

$$\begin{aligned}
\frac{dD_1}{dp_1} &= -\frac{1}{\widetilde{\det}}\frac{(2-\lambda)}{8t_rct_N}\left[\left(4ct_N-\alpha^2\gamma\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\right)\right], \qquad (83)\\
\frac{dN_1}{dp_1} &= \frac{1}{\widetilde{\det}}\frac{(2-\lambda)\,\alpha^2\gamma}{8t_rct_N}\frac{N_1}{D_1^2},\\
\frac{dD_1}{dw_1} &= \frac{1}{\widetilde{\det}}\frac{\theta\alpha\gamma}{4ct_rt_N}\left(\frac{2-\lambda}{D_1}+\frac{\lambda}{D_2}\right),\\
\frac{dN_1}{dw_1} &= \frac{1}{\widetilde{\det}}\frac{1}{4ct_rt_N}\left[2ct_r+\theta\alpha\gamma\,(2-\lambda)\,\frac{N_1}{D_1^2}\right].
\end{aligned}$$

The determinant is given by:

$$\begin{aligned}
\widetilde{\det} &= \left[1+\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{N_1}{D_1^2}\right]\times\left[1-\frac{\alpha^2\gamma}{4ct_N}\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\right] \qquad (84)\\
&\quad+\frac{\alpha^2\gamma}{4ct_N}\frac{N_1}{D_1^2}\left[\frac{\theta\alpha\gamma}{c}\left(\frac{\lambda}{2t_r}+\frac{1-\lambda}{t_r}\right)\frac{1}{D_1}+\lambda\frac{\theta}{2t_r}\frac{\alpha\gamma}{c}\frac{1}{D_2}\right]\\
&= \frac{1}{8c^2t_rt_N}\left[8c^2t_rt_N-2ct_r\alpha^2\gamma\left(\frac{1}{D_1}+\frac{1}{D_2}\right)\right.\\
&\qquad\left.+\theta\alpha\gamma\,(2-\lambda)\,\frac{N_1}{(D_1)^2}4ct_N-2\theta\alpha\gamma\,(1-\lambda)\,\frac{N_1}{D_1^2}\alpha^2\gamma\frac{1}{D_2}\right].
\end{aligned}$$

Symmetric equilibrium. We define:

$$\widetilde{\Delta}:=\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1}-\frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}=\frac{1}{\widetilde{\det}}\frac{2-\lambda}{4t_rt_N}. \qquad (85)$$

By substitution, the equilibrium price and wage are:

$$p_1 - g = \frac{1}{\widetilde{\Delta}}\left(\frac{1}{2}\frac{\partial N_1}{\partial p_1} + D_1\frac{\partial N_1}{\partial w_1}\right) = \frac{2t_r D_1}{(2-\lambda)} + \frac{\alpha\gamma\left(\alpha + 4D_1\theta\right)}{4cD_1^2}, \tag{86}$$

$$w_1 = \frac{1}{\widetilde{\Delta}}\left(\frac{1}{2}\frac{\partial D_1}{\partial p_1} + D_1\frac{\partial D_1}{\partial w_1}\right) = \frac{\alpha\gamma}{c}\left(\frac{\alpha}{2D_1} + \frac{2\theta}{(2-\lambda)}\right) - t_N. \tag{87}$$

Comparative statics. The system is given by:

$$p^* - g - 2t_r\frac{D^*}{(2-\lambda)} - \frac{\alpha\gamma\left(\alpha + 4\theta D^*\right)}{4c\left(D^*\right)^2} = 0, \tag{88}$$

$$D^* - \frac{\lambda}{2} - \frac{1-\lambda}{t_r}\left[s + \frac{\alpha\theta}{c}\left(k - \gamma + \frac{\gamma}{2D^*}\right) - p^*\right] = 0.$$

Total differentiation yields:

$$\begin{pmatrix} 1 & \frac{-2t_r}{2-\lambda} + \frac{\alpha\gamma(\alpha+2\theta D^*)}{2c(D^*)^3} \\ \frac{1-\lambda}{t_r} & 1 + \frac{1-\lambda}{t_r}\frac{\alpha\theta}{c}\frac{\gamma}{2(D^*)^2} \end{pmatrix}\begin{bmatrix} dp^* \\ dD^* \end{bmatrix} = -\begin{bmatrix} -\frac{\alpha(\alpha+4\theta D^*)}{4c(D^*)^2} \\ -\frac{1-\lambda}{t_r}\frac{\alpha\theta}{cD^*}\left(1 - 2D^*\right) \end{bmatrix} d\gamma, \tag{89}$$

Applying the Cramer's rule gives:

$$\frac{dD^*}{d\gamma} = -\frac{1-\lambda}{t_r}\frac{\alpha}{2c\Upsilon D^*}\left[\theta\left(1 + 2D^*\right) + \frac{\alpha}{2D^*}\right], \tag{90}$$

$$\frac{dp^*}{d\gamma} = \frac{\alpha}{2c\Upsilon D^*}\left[\frac{\left(\alpha + 4\theta D^*\right)}{2\left(D^*\right)}\left(1 + \frac{1-\lambda}{t_r}\frac{\alpha\theta\gamma}{2c\left(D^*\right)^2}\right) + \frac{1-\lambda}{t_r}\theta\left(1 - 2D^*\right)\left(\frac{2t_r}{2-\lambda} - \frac{\alpha\gamma\left(\alpha + 2\theta D^*\right)}{2c\left(D^*\right)^3}\right)\right]$$

with

$$\Upsilon = 1 + \frac{1-\lambda}{t_r}\left(\frac{\gamma\alpha\theta}{2c\left(D^*\right)^2} + \frac{2t_r}{2-\lambda} - \frac{\alpha\gamma\left(\alpha + 2\theta D^*\right)}{2c\left(D^*\right)^3}\right), \tag{91}$$

being the determinant of the matrix above.

Welfare function. The welfare function is:

$$W^* = 2\left[\lambda\int_0^{1/2}\left(S + \theta q^* - p^* - t_r y\right)dy + (1-\lambda)\int_0^{D^{*L}}\left(s + \theta q^* - p^* - t_r y\right)dy + \int_0^{1/2}V^*(y)dy + \pi^*\right].$$

Solving the integral and substituting for π^*, this reduces to:

$$W^* = \lambda\left(S + \theta q^* - p^* - \frac{t_r}{4}\right) + (1-\lambda)2D^{*L}\left(s + \theta q^* - p^* - \frac{t_r}{2}\left(D^{*L}\right)^2\right) \tag{92}$$
$$+w^* + \frac{\alpha^2}{2c}\left(k - \gamma + \gamma\frac{1}{2D^*}\right) - \frac{t_N}{4} + 2\left(p^* - g\right)D^* - w^*.$$

Further substituting for w^*, the result is obtained:

$$W^* = \lambda S + (1-\lambda)2D^L s - t_r\left(\frac{\lambda}{4} + (1-\lambda)\left(D^L\right)^3\right) + \left(\frac{\alpha}{2} + 2D^*\theta\right)q^* - \frac{t_N}{4} - 2gD^*. \tag{93}$$

Wage regulation. Under wage regulation, only the first-order condition on price matters:

$$p_1 - g = \frac{1}{\frac{\partial D_1}{\partial p_1}}\left(\tilde{w}\frac{\partial N_1}{\partial p_1} - D_1\right), \tag{94}$$

which after substitution, at the symmetric equilibrium, gives:

$$\overline{p} - g = -\frac{\overline{w}\left(\frac{1}{\widetilde{\det}}\frac{(2-\lambda)\alpha^2\gamma}{8t_r ct_N}\frac{1}{2\overline{D}^2}\right) - \overline{D}}{\frac{1}{\widetilde{\det}}\frac{(2-\lambda)}{4t_r ct_N}\left(2ct_N - \frac{\alpha^2\gamma}{\overline{D}}\right)}, \tag{95}$$

or, after the inclusion of $\widetilde{\det}$,

$$\overline{p} - g = \frac{2t_r\overline{D}}{(2-\lambda)} + \frac{\theta\alpha\gamma\left[2ct_N(2-\lambda) - \frac{\alpha^2\gamma(1-\lambda)}{\overline{D}}\right]}{\overline{D}(2-\lambda)c\left(4ct_N - 2\frac{\alpha^2\gamma}{\overline{D}}\right)} - \frac{\frac{\alpha^2\gamma}{2\overline{D}^2}\overline{w}}{\left(4ct_N - 2\frac{\alpha^2\gamma}{\overline{D}}\right)}. \tag{96}$$

Price regulation. From the first-order condition with respect to wage, at the symmetric equilibrium, we have:

$$\widehat{w} = \frac{(\widehat{p} - g)\frac{2}{\det}\frac{\theta\alpha\gamma}{4ct_r t_N\widehat{D}} - \frac{1}{2}}{\frac{1}{4ct_r t_N}\frac{1}{\widetilde{\det}}\left(2ct_r + \frac{\theta\alpha\gamma(2-\lambda)}{2\widehat{D}^2}\right)}, \tag{97}$$

and after substitution of $\widetilde{\det}$, we obtain:

$$\widehat{w} = -t_N + \frac{2\theta\left(\widehat{p} - g\right)\alpha\gamma + t_r\alpha^2\gamma + \frac{\theta\alpha^3\gamma^2(1-\lambda)}{4c\widehat{D}^2}}{2\widehat{D}ct_r + \frac{\alpha\theta\gamma(2-\lambda)}{2\widehat{D}}}. \tag{98}$$

8.6 Equilibrium with altruistic motives

After substitution of nurses quality (6), we have:

$$\widetilde{U}_i = \frac{\theta\alpha}{c}\left(k - \gamma + \gamma\frac{N_i}{D_i}\right) + \beta\left(v - \frac{1}{4N_i}\right) - \sigma p_i. \tag{99}$$

The first order conditions with respect to residents' price and nurses' wage are:

$$\frac{\partial\widetilde{\pi}_i}{\partial p_i} = \frac{\partial \pi_i}{\partial p_i} + \delta\frac{\partial\widetilde{U}_i}{\partial p_i} = 0, \tag{100}$$

$$\frac{\partial\widetilde{\pi}_i}{\partial w_i} = \frac{\partial \pi_i}{\partial w_i} + \delta\frac{\partial\widetilde{U}_i}{\partial w_i} = 0. \tag{101}$$

From the first order conditions at the symmetric equilibrium we obtain:

$$\begin{aligned}(p_1 - g)\frac{\partial D_1}{\partial p_1} &= -\frac{1}{2} + w_1\frac{\partial N_1}{\partial p_1} - \delta\frac{\partial\widetilde{U}_i}{\partial p_1}, \\ (p_1 - g)\frac{\partial D_1}{\partial w_1} &= \frac{1}{2} + w_1\frac{\partial N_1}{\partial w_1} - \delta\frac{\partial\widetilde{U}_i}{\partial w_1},\end{aligned} \tag{102}$$

which, after substitution, gives:

$$\begin{aligned} w_1 &= \frac{\frac{1}{2}\left(\frac{\partial D_1}{\partial p_1} + \frac{\partial D_1}{\partial w_1}\right) + \delta\left(\frac{\partial\widetilde{U}_i}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial\widetilde{U}_i}{\partial w_1}\frac{\partial D_1}{\partial p_1}\right)}{\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}}, \\ p_1 - g &= \frac{\frac{1}{2}\left(\frac{\partial N_1}{\partial p_1} + \frac{\partial N_1}{\partial w_1}\right) + \delta\left(\frac{\partial\widetilde{U}_i(y)}{\partial p_1}\frac{\partial N_1}{\partial w_1} - \frac{\partial\widetilde{U}_i}{\partial w_1}\frac{\partial N_1}{\partial p_1}\right)}{\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}}.\end{aligned} \tag{103}$$

Let us define:

$$B \equiv \frac{\partial\widetilde{U}_i}{\partial p_1}\frac{\partial N_1}{\partial w_1} - \frac{\partial\widetilde{U}_i}{\partial w_1}\frac{\partial N_1}{\partial p_1},\ C \equiv \frac{\partial\widetilde{U}_i}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial\widetilde{U}_i}{\partial w_1}\frac{\partial D_1}{\partial p_1}, \tag{104}$$

so we can re-write nurses' salary and residents' price as:

$$w_1 = \frac{\frac{1}{2}\left(\frac{\partial D_1}{\partial p_1} + \frac{\partial D_1}{\partial w_1}\right) + \delta C}{\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}},\ p_1 - g = \frac{\frac{1}{2}\left(\frac{\partial N_1}{\partial p_1} + \frac{\partial N_1}{\partial w_1}\right) + \delta B}{\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}}. \tag{105}$$

Let us compute B and C. First, note that:

$$\begin{aligned}
\frac{\partial \widetilde{U}_i}{\partial p_1} &= \frac{\theta\alpha\gamma}{c}\frac{\frac{\partial N_1}{\partial p_1}D_1 - \frac{\partial D_1}{\partial p_1}N_1}{(D_1)^2} + \frac{\beta}{4(N_1)^2}\frac{\partial N_1}{\partial p_1} - \sigma = \frac{2\theta\alpha\gamma}{c}\left(\frac{\partial N_1}{\partial p_1} - \frac{\partial D_1}{\partial p_1}\right) + \beta\frac{\partial N_1}{\partial p_1} - \sigma, \\
\frac{\partial \widetilde{U}_i}{\partial w_1} &= \frac{2\theta\alpha\gamma}{c}\left(\frac{\partial N_1}{\partial w_1} - \frac{\partial D_1}{\partial w_1}\right) + \beta\frac{\partial N_1}{\partial w_1}. \qquad (106)
\end{aligned}$$

so we obtain:

$$\begin{aligned}
B &= \frac{2\theta\alpha\gamma}{c}\left(\frac{\partial D_1}{\partial w_1}\frac{\partial N_1}{\partial p_1} - \frac{\partial D_1}{\partial p_1}\frac{\partial N_1}{\partial w_1}\right) - \sigma\frac{\frac{\partial N_1}{\partial w_1}}{\frac{\partial D_1}{\partial w_1}\frac{\partial N_1}{\partial p_1} - \frac{\partial D_1}{\partial p_1}\frac{\partial N_1}{\partial w_1}}, \qquad (107) \\
C &= \left(\frac{2\theta\alpha\gamma}{c} + \beta\right)\left(\frac{\partial N_1}{\partial p_1}\frac{\partial D_1}{\partial w_1} - \frac{\partial N_1}{\partial w_1}\frac{\partial D_1}{\partial p_1}\right) - \sigma\frac{\partial D_1}{\partial w_1}.
\end{aligned}$$

Substituting into the price and wage equations, we obtain the equilibrium wage and price at the symmetric equilibrium:

$$\begin{aligned}
p^A &= g + t_r + \frac{\gamma\alpha(\alpha+2\theta)}{c} + \delta\left(\frac{2\theta\alpha\gamma}{c}(1-2\sigma) - 2\sigma t_r\right), \qquad (108) \\
w^A &= \beta - t_N + \frac{\gamma\alpha(\alpha+2\theta)}{c} + \delta\left(\frac{2\theta\alpha\gamma}{c} + \beta\right)(1-2\sigma).
\end{aligned}$$

The profit of the nursing home at the symmetric equilibrium is:

$$\begin{aligned}
\pi^A &= \frac{1}{2}\left\{t_r + \frac{\gamma\alpha(\alpha+2\theta)}{c} + \delta\left[\frac{2\theta\alpha\gamma}{c}(1-2\sigma) - 2\sigma c t_r\right]\right. \qquad (109) \\
&\quad \left. - \left[\beta - t_N + \frac{\gamma\alpha(\alpha+2\theta)}{c} + \delta\left(\frac{2\theta\alpha\gamma}{c} + \beta\right)(1-2\sigma)\right]\right\} \\
&= \frac{1}{2}\left\{t_r + t_N - \beta - \delta\left[2t_r + (1-2\sigma)\beta\right]\right\}.
\end{aligned}$$