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A mobility decomposition of absolute measures of panel distributional change*

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Abstract

This paper proposes a decomposition of a general class of absolute measures of panel welfare change into growth, dispersion and exchange mobility components, which is useful for both intergenerational and intragenerational mobility assessments. We show that this decomposition is the only one within a broad set of possibilities that satisfies a key dispersion-sensitivity property and that prioritising improvements among those initially poorest is identical to a favourable view of exchange mobility.

Keywords: Egalitarian growth, mobility decomposition, panel distributional change.

JEL Codes: I3.

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1 Introduction

Social scientists have long been interested in describing and ethically judging distributional change in well-being indicators across two periods.[1] This interest was further aroused by contributions to the measurement of 'pro-poor' growth, characterised by requiring faster growth among those initially poorest (e.g. Ravallion and Chen, 2003). Meanwhile in the mobility measurement literature, Van Kerm (2004) showed how to decompose overall distributional change in a well-being indicator into three components: *growth in average*, *dispersion*, and *exchange mobility*. The dispersion component captures anonymous changes in the shape of the distribution affecting inequality levels; whereas, the exchange mobility component embeds change due to non-anonymous re-rankings.[2] These decompositions can be applied to either longitudinal data following the same units (e.g. people in intragenerational assessments) or retrospective data (e.g. families, when studying intergenerational mobility). Several tools for this type of assessment exist in the literature.[3]

Unlike most existing studies dwelling on income growth rates, this paper focuses on *absolute change* owing to the growing interest in non-monetary social metrics.[4] Furthermore, we focus on a general class of decomposable absolute indices of pro-poor panel well-being change that provides an intuitive measure of absolute change in a well-being indicator as the weighted average of changes in individual achievements where the weights are chosen in order to prioritise the well-being improvements of those most disadvantaged in the initial period (e.g. the parents' generation).[5] We propose a novel decomposition of the general class of measures into components of *growth in average*, *dispersion* and *exchange mobility* in the same spirit of the decomposition pioneered by Van Kerm (2004). We show: (i) that this decomposition is the only one satisfying a key dispersion-sensitivity property within a broad set of possible decompositions, and (ii) that prioritising improvements among those initially poorer is identical to a favourable view of exchange mobility.

The rest of the paper proceeds as follows. Section 2 presents the general class of measures of absolute change. Section 3 shows how only one type of decomposition satisfies a key dispersion-sensitive property. Section 4 establishes the link between prioritising improvements among those poorer and exchange mobility. Section 5 offers some concluding remarks.

2 A class of decomposable absolute measures of panel change

Suppose a hypothetical society consists of a fixed set of n (≥ 2) individuals and we are interested in assessing social change in an indicator across two time periods. Let $x_i \in \mathbb{R}$ be the *achievement* of individual i in the first period and vector $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$ be the distribution of all achievements in the first period.[6] Without loss of generality, we assume that achievements are ordered in $\mathbf{x}$, such that $x_1 \geq x_2 \geq \cdots \geq x_n$. Similarly,

[1] See e.g. Cowell (1985), Fields and Ok (1999), Demuynck and Van de Gaer (2012), Jenkins and Van Kerm (2016), Palmisano and Van de Gaer (2016) and Bossert and Dutta (2019).

[2] Structural mobility is normally associated with the combination of *growth in average* and *dispersion* components (Van Kerm, 2004).

[3] See Fields and Ok (1999), Van Kerm (2004), Demuynck and Van de Gaer (2012), Dhongde and Silber (2016), Jenkins and Van Kerm (2016) and references therein.

[4] See e.g. Sustainable Development Goals: https://www.un.org/sustainabledevelopment/sustainable-development-goals/.

[5] Palmisano and Van de Gaer (2016) provide an axiomatic characterisation of the class of absolute measures of non-anonymous changes; whereas, Bossert and Dutta (2019) consider anonymous changes. Both propose rank-dependent weights drawn from the class of generalised-Gini social evaluation functions.

[6] $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{R}_{++}$ denote the set of real numbers and its non-negative and strictly positive counterparts, respectively.

vector $\mathbf{y} = (y_1, \ldots, y_n) \in \mathbb{R}^n$ is the distribution of all n achievements in the second period. For any $\mathbf{a} \in \mathbb{R}^n$, we define the mean operator as $\mu(\mathbf{a}) = \frac{1}{n}\sum_{i=1}^{n} a_i$. Thus, for any $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$, $\mu(\mathbf{a}-\mathbf{b}) = \mu(\mathbf{a}) - \mu(\mathbf{b})$.

We restrict our attention to the class of additively decomposable non-anonymous measures of absolute distributional change to track changes in social evaluations of well-being:

$$G(\mathbf{y}, \mathbf{x}; \mathbf{w}) = \sum_{i=1}^{n} w_i (y_i - x_i), \tag{1}$$

where the weights, $\mathbf{w} = (w_1, \ldots, w_n) \in \mathbb{R}^n_{++}$, depend on the achievement ranks in the initial period such that: $w_i < w_j$ if and only if $x_i > x_j$ for any pair $\{x_i, x_j\}$ in $\mathbf{x}$. We also conveniently normalise the weights so that they sum to unity, i.e. $\sum_{i=1}^{n} w_i = 1$, because welfare comparisons based on Equation 1 remain ordinally equivalent when general weights are normalised. Such additive class of measures with normalised weights have been used for analysing both anonymous (Bossert and Dutta, 2019) and non-anonymous welfare changes (Palmisano and Van de Gaer, 2016). In fact, Palmisano and Van de Gaer (2016) axiomatically characterised the class of absolute measures of panel change with $w_i = [i^\delta - (i-1)^\delta]/n^\delta$ for $\delta \geq 1$ such that $\sum_{i=1}^{n} w_i = \frac{1}{n^\delta}\sum_{i=1}^{n}[i^\delta - (i-1)^\delta] = 1$. Given that achievements in the first period (i.e. $\mathbf{x}$) are ranked in descending order, the measures in G prioritise changes in achievements of those with lower initial values.

Following the terminology of Palmisano and Van de Gaer (2016), each measure in G can be conveniently decomposed into an *average change* component, $\mu(\mathbf{y}-\mathbf{x})$, and a *progressivity* component , $I(\mathbf{y}, \mathbf{x}; \mathbf{w})$, as:

$$G(\mathbf{y}, \mathbf{x}; \mathbf{w}) = I(\mathbf{y}, \mathbf{x}; \mathbf{w}) + \mu(\mathbf{y}-\mathbf{x}), \tag{2}$$

where

$$I(\mathbf{y}, \mathbf{x}; \mathbf{w}) = \sum_{i=1}^{n} w_i [(y_i - x_i) - \mu(\mathbf{y}-\mathbf{x})]. \tag{3}$$

The progressivity component I measures the change that rewards pro-poor improvements or larger changes among those with lower achievements in the first period. Clearly, if $y_i - x_i = \mu(\mathbf{y}-\mathbf{x})$ for all $i = 1, \ldots, n$, then $I(\mathbf{y}, \mathbf{x}; \mathbf{w}) = 0$ and $G(\mathbf{y}, \mathbf{x}; \mathbf{w}) = \mu(\mathbf{y}-\mathbf{x})$ for all $\mathbf{w}$. Otherwise, if $\mu(\mathbf{x}) \geq 0$ and $y_i - x_i$ is lower in magnitude among those with higher initial achievements, then $I(\mathbf{y}, \mathbf{x}; \mathbf{w}) > 0$ for all $\mathbf{w}$, signalling a pro-poor improvement.

3 Dispersion-sensitive social mobility

The measures of absolute panel change described in section 2 can be adapted to social mobility analysis by further decomposing the progressivity component into a *dispersion* and an *exchange mobility* component (i.e. mobility through pure re-rankings). Traditionally, these mobility elements are isolated using counterfactual distributions (e.g. see Van Kerm, 2004). In most decompositions, alternative counterfactual distributions are deployed, and in potentially different sequences.

In fact we can consider two counterfactual distributions by ranking the two periods' actual distributions ($\mathbf{x}$ and $\mathbf{y}$) in descending order independently. The first counterfactual distribution is $\dot{\mathbf{y}} = \mathbf{y}P$ such that $\dot{y}_1 \geq \dot{y}_2 \geq \cdots \geq \dot{y}_n$, where P is a permutation matrix.[7] The second couterfactual distribution is $\dot{\mathbf{x}} = \mathbf{x}Q$, where Q is also a permutation matrix, such that $\dot{\mathbf{x}}$ contains all achievements of $\mathbf{x}$ but the achievements in $\dot{\mathbf{x}}$ are ranked according to the achievements in the second period distribution $\mathbf{y}$.

[7]A permutation matrix is a square matrix such that every one of its rows and columns contains one element equal to one and the rest of the elements equal to zero.

In order to clarify the relationship between the four achievement distributions $(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{y}, \dot{\mathbf{y}})$, consider two distributions $\mathbf{x} = (120, 100, 88, 67)$ and $\mathbf{y} = (115, 130, 70, 80)$. Then, $\dot{\mathbf{y}} = (130, 115, 80, 70)$, where the achievements are ordered according to those in $\mathbf{x}$; and $\dot{\mathbf{x}} = (100, 120, 67, 88)$, where the achievements are ranked according to those in $\mathbf{y}$. Clearly, $\mu(\mathbf{y}) = \mu(\dot{\mathbf{y}})$ and $\mu(\mathbf{x}) = \mu(\dot{\mathbf{x}})$.

The first path through which the progressivity component can be decomposed is the fictional sequence leading from $\mathbf{x}$ to $\dot{\mathbf{x}}$ (i.e. exchange mobility) and then from $\dot{\mathbf{x}}$ to $\mathbf{y}$ (i.e. dispersion mobility); whereas a second path is the fictional sequence leading from $\mathbf{x}$ to $\dot{\mathbf{y}}$ (i.e. dispersion mobility) and then from $\dot{\mathbf{y}}$ to $\mathbf{y}$ (i.e. exchange mobility). That is:

Path 1: $I(\mathbf{y}, \mathbf{x}; \mathbf{w}) = I(\mathbf{y}, \dot{\mathbf{x}}; \mathbf{w}) + I(\dot{\mathbf{x}}, \mathbf{x}; \mathbf{w})$.
Path 2: $I(\mathbf{y}, \mathbf{x}; \mathbf{w}) = I(\mathbf{y}, \dot{\mathbf{y}}; \mathbf{w}) + I(\dot{\mathbf{y}}, \mathbf{x}; \mathbf{w})$.

These two decompositions follow from the following facts: (1) the difference $y_i - x_i$ can easily be decomposed into $[(y_i - \dot{x}_i) + (\dot{x}_i - x_i)]$ as well as $[(y_i - \dot{y}_i) + (\dot{y}_i - x_i)]$; (2) we already know that $\mu(\mathbf{y}) = \mu(\dot{\mathbf{y}})$ and $\mu(\mathbf{x}) = \mu(\dot{\mathbf{x}})$. Plugging this information into Equation 3 yields both mobility decompositions respectively. Additionally, we can obtain another path through a convex combination of Paths 1 and 2:[8]

Path 3: $I(\mathbf{y}, \mathbf{x}; \mathbf{w}) = \alpha I(\mathbf{y}, \dot{\mathbf{y}}; \mathbf{w}) + (1-\alpha) I(\dot{\mathbf{x}}, \mathbf{x}; \mathbf{w}) + (1-\alpha) I(\mathbf{y}, \dot{\mathbf{x}}; \mathbf{w}) + \alpha I(\dot{\mathbf{y}}, \mathbf{x}; \mathbf{w})$ for some $\alpha \in (0,1)$.

Now, these alternative choices of counterfactual distributions generally yield different values for both mobility elements. The exchange mobility components, $I(\dot{\mathbf{x}}, \mathbf{x}; \mathbf{w})$ and $I(\mathbf{y}, \dot{\mathbf{y}}; \mathbf{w})$, are expected to capture pro-poor improvements merely due to re-rankings; whereas, the dispersion mobility components, $I(\mathbf{y}, \dot{\mathbf{x}}; \mathbf{w})$ and $I(\dot{\mathbf{y}}, \mathbf{x}; \mathbf{w})$, are expected to capture distributional change due to anonymous changes in dispersion of achievements. Thus, the dispersion components are expected to satisfy the distributional property that we refer to as *dispersion-sensitivity*, which requires that if a distribution $\mathbf{a} \in \mathbb{R}^n_+$ is obtained from another distribution $\mathbf{b} \in \mathbb{R}^n_+$ through a rank-preserving progressive transfer from a better-off person to a worse-off person, then the dispersion mobility component should increase signalling an improvement:

Dispersion-sensitivity: For any $\mathbf{b}, \mathbf{a} \in \mathbb{R}^n$ and some $\theta > 0$, $I(\mathbf{b}, \mathbf{a}; \mathbf{w}) > 0$ whenever $a_i < b_i = a_i + \theta \leq b_j = a_j - \theta < a_j$ for at least one pair $\{i, j\}$ and $b_k = a_k$ for all $k \neq \{i, j\}$.

Proposition 1 shows that the dispersion component $I(\dot{\mathbf{y}}, \mathbf{x}; \mathbf{w})$ satisfies the dispersion-sensitivity property, unlike the dispersion component $I(\mathbf{y}, \dot{\mathbf{x}}; \mathbf{w})$. Intuitively, the main reason for this result is that the weight assigned to each element $\dot{y}_i - x_i$ is inversely proportional to its rank, which is not the case for $y_i - \dot{x}_i$.

Proposition 1 The dispersion mobility component $I(\dot{\mathbf{y}}, \mathbf{x}; \mathbf{w})$ satisfies the dispersion-sensitivity property, whereas the dispersion mobility component $I(\mathbf{y}, \dot{\mathbf{x}}; \mathbf{w})$ violates it.

Proof. First, suppose, $\dot{\mathbf{y}} \in \mathbb{R}^n$ is obtained from $\mathbf{x} \in \mathbb{R}^n$ by a rank-preserving progressive transfer involving a pair $\{i, j\}$ such that for some $\theta > 0$: $x_j < \dot{y}_j = x_j + \theta \leq \dot{y}_i = x_i - \theta < x_i$ and $\dot{y}_k = x_k$ for all $k \neq \{i, j\}$. Using Equation 3, we obtain $I(\dot{\mathbf{y}}, \mathbf{x}, \mathbf{w}) = \theta[w_j - w_i]$. Now, $I(\dot{\mathbf{y}}, \mathbf{x}, \mathbf{w}) > 0$ because $w_i < w_j$ as the achievements in $\mathbf{x}$ are ranked in decreasing order. Thus, $I(\dot{\mathbf{y}}, \mathbf{x}, \mathbf{w}) > 0$ satisfies the dispersion-sensitivity property. Next, suppose, $\mathbf{y} \in \mathbb{R}^n$ is obtained from $\dot{\mathbf{x}} \in \mathbb{R}^n$ using a rank-preserving progressive transfer involving a pair $\{i, j\}$ such that for some $\theta > 0$: $\dot{x}_i < y_i = \dot{x}_i + \theta \leq y_j = \dot{x}_j - \theta < \dot{x}_j$ and $y_k = \dot{x}_k$ for all $k \neq \{i, j\}$. Then, again by Equation 3, $I(\mathbf{y}, \dot{\mathbf{x}}; \mathbf{w}) = \theta[w_i - w_j]$. Now that the achievements in $\dot{\mathbf{x}}$ are not ranked in decreasing order, it is possible that $w_i \gtreqless w_j$ and so $I(\mathbf{y}, \dot{\mathbf{x}}; \mathbf{w}) \gtreqless 0$. Hence, $I(\mathbf{y}, \dot{\mathbf{x}}; \mathbf{w})$ does not satisfy the dispersion-sensitivity property. ■

[8] In fact, Path 3 is equivalent to a Shapley decomposition when $\alpha = 0.5$.

A direct and convenient consequence of proposition 1 is that *Path 2 is the only sensible mobility decomposition path, involving the exclusive use of $\dot{\mathbf{y}}$ as the counterfactual distribution for mobility decomposition*. The dispersion mobility component $I(\dot{\mathbf{y}},\mathbf{x};\mathbf{w})$ measures change in absolute inequality in the absence of re-rankings, with $I(\dot{\mathbf{y}},\mathbf{x};\mathbf{w}) > 0$ signalling a reduction in absolute inequality. In fact, for the class of indices proposed by Palmisano and Van de Gaer (2016), $I(\dot{\mathbf{y}},\mathbf{x};\mathbf{w})$ is the *difference* between indices from a generalised-Gini class respecting the absolute Lorenz partial ordering and characterised by (1) non-positive values; (2) increasing (i.e. becoming less negative) in the event of rank-preserving progressive transfers; and (3) reaching the value of 0 when distributions are egalitarian.[9]

4 Prioritisation of those initially poorer and exchange mobility

We know from proposition 1 that the dispersion component satisfies the dispersion-sensitivity property only in Path 2. How does the exchange component in Path 2, namely $I(\mathbf{y},\dot{\mathbf{y}};\mathbf{w})$, behave? We already know that measures in G prioritise the improvements of those initially poorer because, by definition, $w_i > w_j$ if and only if $x_i < x_j$. We now show in proposition 2 that this restriction on the set of admissible weights is also both necessary and sufficient to ensure that the exchange mobility component is either positive or null (in the absence of re-rankings). That is, $I(\mathbf{y},\dot{\mathbf{y}};\mathbf{w})$ measures pro-poor improvements due to re-rankings. In other words, exchange mobility is always deemed welfare-enhancing with the panel change measures in G (by contrast, dispersion mobility is only welfare-enhancing if it reflects inequality reduction).

Proposition 2 For any $\mathbf{y} \in \mathbb{R}^n$ and $\dot{\mathbf{y}} = \mathbf{y}P$, where P is a permutation matrix, such that $\dot{y}_1 \geq \dot{y}_2 \geq \cdots \geq \dot{y}_n$, and $\dot{\mathbf{y}} \neq \mathbf{y}$, $I(\mathbf{y},\dot{\mathbf{y}};\mathbf{w}) > 0$ if and only if $w_i > w_j$ for every pair $\{i, j\}$ such that $i > j$.

Proof. From Equation 3, we know that:

$$I(\mathbf{y},\dot{\mathbf{y}};\mathbf{w}) = \sum_{i=1}^{n} w_i[(y_i - \dot{y}_i) - \mu(\mathbf{y} - \dot{\mathbf{y}})]. \tag{4}$$

Whenever P is an identity matrix, $\dot{\mathbf{y}} = \mathbf{y}$. Then, clearly $I(\mathbf{y},\dot{\mathbf{y}};\mathbf{w}) = 0$.

Now, let $\dot{\mathbf{y}} \neq \mathbf{y}$. Summing Equation 4 by parts with Abel's formula (Guenther and Lee, 1988), we obtain:

$$I(\mathbf{y},\dot{\mathbf{y}};\mathbf{w}) = w_n \sum_{i=1}^{n} (y_i - \dot{y}_i) + \sum_{i=1}^{n-1} (w_{i+1} - w_i) \left(\sum_{k=i+1}^{n} (y_k - \dot{y}_k) \right). \tag{5}$$

Since $\mu(\dot{\mathbf{y}}) = \mu(\mathbf{y})$, the first term of the right-hand side in Equation 5 is equal to zero; therefore:

$$I(\mathbf{y},\dot{\mathbf{y}};\mathbf{w}) = \sum_{i=1}^{n-1} (w_{i+1} - w_i) \left(\sum_{k=i+1}^{n} (y_k - \dot{y}_k) \right). \tag{6}$$

Sufficiency: Suppose, $w_i > w_j$ for every pair $\{i, j\}$ such that $i > j$. Then, it must be the case that $w_{i+1} > w_i$ for all $i = 1, \ldots, n-1$. We now need to show that $\sum_{k=i+1}^{n}(y_k - \dot{y}_k) \geq 0$ for all $i = 1, \ldots, n-1$ and $\sum_{k=i+1}^{n}(y_k - \dot{y}_k) > 0$ for at least one i. This is equivalent to requiring that $\sum_{k=n-j+1}^{n}(y_k - \dot{y}_k) \geq 0$ for all

[9] Note that $I(\dot{\mathbf{y}},\mathbf{x};\mathbf{w}) = \sum_{i=1}^{n} w_i[\dot{y}_i - \mu(\dot{\mathbf{y}})] - \sum_{i=1}^{n} w_i[x_i - \mu(\mathbf{x})]$. For instance, when $w_i = \frac{2i-1}{n^2}$, $-I(\dot{\mathbf{y}},\mathbf{x};\mathbf{w})$ becomes the difference in the absolute Gini index.

$j = 1, \ldots, n-1$ and $\sum_{k=n-j+1}^{n}(y_k - \dot{y}_k) > 0$ for at least one j. Evidently, the requirement is met because $\dot{\mathbf{y}}$ contains the elements of $\mathbf{y}$ sorted in descending order.

Necessity: Let $w_{i+1} \leq w_i$. Since $\sum_{k=i+1}^{n}(y_k - \dot{y}_k) \geq 0$ for all $i = 1, \ldots, n-1$ and $\sum_{k=i+1}^{n}(y_k - \dot{y}_k) > 0$ for at least one i whenever $\dot{\mathbf{y}} = \mathbf{y}P$ (where P is a permutation matrix), then it must be the case that $I(\mathbf{y}, \dot{\mathbf{y}}; \mathbf{w}) \leq 0$ which contradicts the statement: $I(\mathbf{y}, \dot{\mathbf{y}}; \mathbf{w}) > 0$. ■

5 Discussion

An *anonymous* approach to absolute egalitarian progress (e.g. Bossert and Dutta, 2019) takes a Rawlsian perspective, in the sense of adopting a 'veil of ignorance' and placing higher weight on the absolute change of the lowest percentiles, regardless of whoever occupies them in either period. By contrast, the indices decomposed above measure different forms of pro-initially-poor growth, i.e. prioritising the absolute performance of those initially poorer. If we follow the latter's *non-anonymous* approach, we remain ultimately agnostic as to how these initially poorer individuals improved their condition more than their richer counterparts, as long as they did experience more improvement.

However, perhaps we should also care about the sources of non-anonymous absolute egalitarian progress. As the decomposition of $I(\mathbf{y}, \mathbf{x}; \mathbf{w})$ shows, there are two potential sources of pro-poor improvements in the non-anonymous approach (assuming also that $\mu(\mathbf{y} - \mathbf{x}) > 0$): absolute anonymous convergence stemming from the dispersion component and re-ranking swaps always favouring those initially poorer. These two components can, in principle, complement each other, but they can also occur independently of each other, and even counter each other's effects.

Meanwhile, in previous mobility assessments (relying on different measures of distributional change) the total decomposition is usually obtained through an aggregation rule over all possible decomposition sequences based on alternative counterfactual distributions (e.g. the Shapley-based canonical decomposition of Van Kerm, 2004). Conveniently, this methodological challenge is moot in our setting for two reasons. First, the isolation of the growth component from the progressivity counterpart is straightforward and unique (Palmisano and Van de Gaer, 2016). Second, we found that only one potential linear mobility decomposition of the progressivity component satisfies the desirable dispersion-sensitivity property related to its dispersion component. This is good news because, for a particular choice of weights, we can always quantify, directly without resorting to aggregation and averaging techniques, the extent to which total non-anonymous absolute egalitarian progress, i.e. what we called panel well-being change, was due to each of the two mobility sources (above and beyond the growth component), and inform our ethical judgement accordingly.

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