# Controlling the Lithium Intercalation Voltage in the Li(Mn<sub>1-x</sub>Ni<sub>x</sub>)<sub>2</sub>O<sub>4</sub> Spinel *via* Tuning of the Ni Concentration: a Density Functional Theory Study

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### ABSTRACT

 $LiMn_2O_4$  spinel is a promising cathode material for secondary lithium-ion batteries. Despite showing a high average voltage of lithium intercalation, the material is structurally unstable, undergoing lowering of the crystal symmetry due to Jahn-Teller distortion of the six-fold  $Mn^{3+}$  cations. Although Ni has been proposed as a suitable substitutional dopant to improve the structural stability of  $LiMn_2O_4$ , and enhance the average lithium intercalation voltage, the thermodynamics of the Ni incorporation and its effect on the electrochemical properties of this spinel material are not yet known. In this work, we have employed density functional theory calculations with a Hubbard Hamiltonian (DFT+*U*) to investigate the thermodynamics of cation mixing in the  $Li(Mn_{1-x}Ni_x)_2O_4$  solid solution. Our results suggest  $LiMn_{1.5}Ni_{0.5}O_4$  is the most stable composition from room temperature up to at least 1000 K, in agreement with experiments. We also found that the configurational entropy is much lower than the maximum entropy at 1000 K, indicating that higher temperatures are required to reach a fully disordered solid solution. A maximum average lithium intercalation voltage of 4.8 eV was calculated for the  $LiMn_{1.5}Ni_{0.5}O_4$  composition, which is very close to the experimental value. The temperature was found to have a negligible effect on the Li intercalation voltage of the most stable composition. The findings reported here support the application of  $LiMn_{1.5}Ni_{0.5}O_4$  as a suitable cathode material for lithium-ion batteries, with a highly stable voltage of intercalation under a wide range of temperatures.

#### KEYWORDS

Spinel, equilibrium concentration, mixing thermodynamics, solid-state chemistry and lithium voltage of intercalation.

#### 1. Introduction

The spinel-structured lithium manganese oxide LiMn<sub>2</sub>O<sub>4</sub> (LMO), which can be easily and reversibly de-lithiated, is an environmentally acceptable compound with a low fabrication cost.<sup>1</sup> This material has attracted considerable attention over the last few decades due to its application as a cathode of rechargeable lithium-ion batteries.<sup>2</sup> However, the commercial exploitation of this spinel requires improvement in the relatively low structural stability of its charged form<sup>3</sup> and an increase in the average lithium intercalation voltage. The crystal structure of LMO is severely degraded after a few operational cycles of lithiation and de-lithiation due to the strong Jahn-Teller (JT) distortions of the octahedrally coordinated high-spin Mn<sup>3+</sup> cations, especially below the Verwey-like temperature  $(T_V)$  of 283.5 K.<sup>4</sup> The uneven occupation of the Mn<sup>3+</sup>  $\hat{e_{g^2}}$  state and the interaction with the oxygen p orbitals in LMO causes a tetragonal elongation of this cation in the direction of the  $d_{z^2}$  orbital which lowers its energy. However, the JT effect vanishes in the de-lithiated MnO2, as all the Mn cations become oxidized to the highly stable 4+ state with a half-filled electronic  $t_{2g}$  level.

In the normal LMO spinel, the monovalent Li cations fill the Wyckoff 8*a* tetrahedral holes, while the divalent O atoms form a

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face-centred cubic (*fcc*) arrangement. Moreover, the Mn ions are embedded in the 16*d* octahedral positions and hold an effective 3.5+ oxidation state. The 3*d* itinerant electrons move freely between the Mn<sup>3+</sup> and Mn<sup>4+</sup> cations, which renders these cations equivalent.

Substitutional doping of LMO is an option to avoid the JT-active 3+ oxidation state in the six-fold Mn cations. Replacing the appropriate proportion of this cation by transition metal atoms<sup>5</sup> with a preferred stable oxidation state below 3+ has proven so far to be the best strategy to ensure that manganese is exclusively in the Mn<sup>4+</sup> form in LMO. For example, 25 %<sup>6</sup> doping by the highly stable Ni<sup>2+</sup>, with the  $e_g$  state half filled, promotes the oxidation of all Mn ions to the JT-inactive 4+ oxidation state. In the discharged spinel, the dopant oxidizes to the very stable low-spin Ni<sup>4+</sup> with a full  $t_{2g}$  level, whereas the rest of the cations remain as Mn<sup>4+</sup>.

In order to gain detailed atomic-level insight into the doping behaviour of the LMO material, we present here a computational study of the  $\text{Li}(\text{Mn}_{1-x}\text{Ni}_x)_2\text{O}_4$  solid solution. We have generated the complete configurational space and the inequivalent configurational subspace for each spinel composition and report the mixing thermodynamics and average voltage of lithium intercalation for all the Ni concentrations and its dependence on the temperature for the most energetically stable



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composition. We have also determined the cation ordering based on the configurational entropy. The approach presented here shows that moderate Ni doping of the  $\text{LiMn}_2\text{O}_4$  leads to a substantial change in the average voltage of lithium intercalation, suggesting an attractive route to tuning the cathode properties of this spinel.

#### 2. Computational Methods

We have carried out spin-polarized density functional theory (DFT) calculations within the Vienna Ab-initio Simulation Package (VASP)<sup>7,8</sup> to determine the structures and energies of  $Li(Mn_{1-x}Ni_x)_2O_4$ . We have employed the Perdew, Burke, and Ernzerhof functional corrected for solids (PBEsol) within the generalized gradient approximation (GGA),<sup>9</sup> where we have included the long-range dispersion corrections via the semiempirical method of Grimme with the Becke and Johnson damping [D3-(BJ)].<sup>10,11</sup> The cut-off for the kinetic energy of the plane wave basis was set at 730 eV, and the effective Hubbard parameters were  $U_{\rm eff}$  = 4.0 eV for Mn and 5.5 eV <sup>12</sup> for Ni.<sup>13</sup> A Γ-centred Monkhorst-Pack k-point mesh with a uniform spacing of *ca*. 0.16 Å<sup>-1</sup>, equivalent to a  $5 \times 5 \times 5$  grid, was adopted for the Brillouin-zone integrations. To improve the convergence of the electronic partial occupancies during geometry optimizations, we employed the Gaussian and the Methfessel-Paxton order one methods<sup>14</sup> for the spinel phases and lithium metal, respectively, with a smearing width of 0.05 eV . A final static calculation was performed after geometry optimizations using the tetrahedron method with Blöchl corrections to obtain highly accurate energies, as well as the electronic and magnetic properties.<sup>15</sup> The projector augmented wave (PAW) method was used to describe the atomic frozen cores and their interaction with the valence electrons, which were defined as 1s<sup>2</sup>2s<sup>1</sup> for Li, 3p<sup>6</sup>3d<sup>5</sup>4s<sup>2</sup> for Mn,  $3p^6 \ 3d^8 \ 4s^2$  for Ni and  $2s^2 2p^4$  for O. During our simulations, we allowed the relaxation of both the cell shape and internal atomic positions of all structures to their ground state, using the conjugate-gradient method, until the Hellmann-Feynman forces on all atoms dropped below 0.02 eVÅ<sup>-1</sup>.

The fully lithiated and de-lithiated forms of the nickelmanganese oxide spinel were modelled starting from the conventional cubic unit cell, while the body-centred cubic (*bcc*) lithium was simulated using the asymmetric unit cell. The initial electronic magnetic moments were oriented parallel within the octahedral sub-lattice and were allowed to relax during the simulations. For simplicity, we have used the normal cation arrangement for the spinel phases, where Li occupies the tetrahedral positions, whereas Mn and Ni are confined to the octahedral sites, ignoring the partial inversion degree of x = 0.30reported experimentally and computationally.<sup>16</sup>

We have used the site occupancy disorder (SOD) code to generate the symmetry-adapted ensemble of configurations within the disordered Li( $Mn_{1-x}Ni_x$ )<sub>2</sub>O<sub>4</sub> solid solution and to carry out the thermodynamic analysis.<sup>17</sup> Within this approach,

SOD produces the complete configurational space for each Ni concentration in a  $1 \times 1 \times 1$  supercell of the conventional cubic unit cell of spinel, followed by a reduction to the sub-space of the symmetrically inequivalent configurations. All configurations were constructed *via* atomic substitutions in LiMn<sub>2</sub>O<sub>4</sub> by using the group of symmetry operators of the space group  $Fd\overline{3}m$  (No. 227) of the parent structure. Two configurations are considered equivalent if they can be interconverted following an isometric transformation. Figure 1a represents the extreme LiMn<sub>2</sub>O<sub>4</sub> spinel composition with lattice constant a = 8.34 Å, whereas Fig. 1b shows the extreme LiNi<sub>2</sub>O<sub>4</sub> spinel composition with lattice constant a = 8.03 Å. In the conventional unit cell of the spinel structure, the O atoms are arranged in a face-centred cubic lattice occupying the 32*e* sites, with the tetrahedral (8*a*) cation sites containing Li atoms and the octahedral cation (16d) sites filled by the Mn cations. In the configurational space considered for each Ni concentration, the Ni takes up its position in the octahedral 16d sites, as shown in Fig. 1c with lattice constant a = 8.22 Å.

#### 3. Results and Discussion

Table 1 displays the prohibitively large total number of configurations (N) for each Ni concentration in the complete configurational space. However, the total number of configurations can be reduced by more than two orders of magnitude to the symmetrically inequivalent configurations (M) in the reduced configurational space. We have chosen to work only with those cell compositions containing an even number of Ni atoms to reduce the number of simulations further.

We have used statistical mechanics to estimate the thermodynamic properties from the sub-space of the symmetrically inequivalent configurations. We have assumed a Boltzmannlike distribution for the estimation of the occurrence probability  $\widetilde{P}_m$  at a temperature *T* of each inequivalent configuration *m* of energy  $E_m$  according to:

$$\widetilde{P}_m = \frac{\Omega_m}{Z} \exp(-E_m/k_B T), \qquad (1)$$

**Table 1** The total number of configurations (*N*) and the number of symmetrically inequivalent configurations (*M*) for each spinel composition in the  $1 \times 1 \times 1$  supercell of the cubic unit cell of Li(Mn<sub>1-x</sub>Ni<sub>x</sub>)<sub>2</sub>O<sub>4</sub>.

| Cell composition                                 | X      | Ν    | М  |  |
|--|--------|------|----|--|
| $Li_8Mn_{16}O_{32}$                              | 0.0000 | 1    | 1  |  |
| $Li_8Mn_{14}Ni_2O_{32}$                          | 0.1250 | 120  | 3  |  |
| $Li_8Mn_{12}Ni_4O_{32}$                          | 0.2500 | 1820 | 22 |  |
| $Li_8Mn_{10}Ni_6O_{32}$                          | 0.3750 | 8008 | 65 |  |
| $Li_8Mn_6Ni_{10}O_{32}$                          | 0.6250 | 8008 | 65 |  |
| $Li_8Mn_4Ni_{12}O_{32}$                          | 0.7500 | 1820 | 22 |  |
| $Li_8Mn_2Ni_{14}O_{32}$                          | 0.8750 | 120  | 3  |  |
| Li <sub>8</sub> Ni <sub>16</sub> O <sub>32</sub> | 1.0000 | 1    | 1  |  |
|  |        |      |    |  |



Figure 1 Schematic representation of the conventional unit cells (a)  $LiMn_2O_4$  (b)  $LiNi_2O_4$  and (c)  $LiMn_{1.5}Ni_{0.5}O_4$ . Lattice geometry and ionic positions for these structures are provided in the Supplementary information.

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where  $\Omega_m$  represents the degeneracy or number of times that a given configuration is repeated in the complete space of all configurations, *m* can take integer values from 1 to *M*,  $k_B$  is the Boltzmann constant and the configurational partition function  $Z = \sum_{m=1}^{M} \Omega_m \exp(-E_m/k_B T)$  ensures that the sum of the proba-

bilities of the complete configurational space is equal to 1.

The Helmholtz free energy of mixing  $\Delta F_{mix}$  of the solid solution was obtained from:

 $\Delta F_{\rm mix} = F[\text{Li}(\text{Mn}_{1-x}\text{Ni}_{x})_2\text{O}_4 - (1-x)F[\text{Li}\text{Mn}_2\text{O}_4] - xF[\text{Li}\text{Ni}_2\text{O}_4]$ (2)

where the configurational free energies  $F = -k_B T \ln Z$  were calculated directly from the partition function.

The equilibrium geometries and energies of all configurations in the reduced configurational space of the  $\text{Li}(\text{Mn}_{1-x}\text{Ni}_x)_2\text{O}_4$  solid solution were obtained from the DFT calculations. Our results indicate that only one configuration is more stable than the others for the entire Ni concentration range, suggesting a large degree of order within the LiMn\_2O\_4-LiNi\_2O\_4 system.

Figure 2 shows the free energy of mixing for the  $Li(Mn_{1-x})$  $Ni_x)_2O_4$  system. Both curves have negative values of  $\Delta F_{mix'}$ which indicates that the compositions are miscible and can form stable solid solutions. At both 300 K and 1000 K, the same energetically stable structure, at the intermediate Ni composition of x = 0.25, has the lowest free energy of mixing. The low free energy of mixing is potentially owing to magnetic interactions, which play a crucial role in the thermodynamics of mixing of these solid solutions, especially at different temperatures. The most stable ordered structure at x = 0.25 (-0.93634 eV/cell and -0.92571 eV/cell at 300 and 1000 K, respectively) not only has the lowest energy but it is also weighted most highly, with a probability of occurrence above 99 % at 300 K and 85 % at 1000 K, as compared to the other 21 configurations. Experimentally, LiMn<sub>1.5</sub>Ni<sub>0.5</sub>O<sub>4</sub> has been suggested as an attractive and promising material because of its acceptable stability, good cyclic property and relatively high capacity.<sup>18</sup> Li(Mn<sub>1-r</sub>Ni<sub>r</sub>)<sub>2</sub>O<sub>4</sub> has therefore been the subject of a variety of studies that have reported on synthesis methods, thermal stability, effects of ordered and disordered local structure, cation ordering, particle size, and changes in composition.<sup>19-23</sup>

In Fig. 3, we have plotted the variation of the configurational entropy of the solid solution for  $\text{Li}(\text{Mn}_{1-x}\text{Ni}_x)_2\text{O}_4$  with  $0 \le x \le 1$  as calculated in the  $1 \times 1 \times 1$  simulation cell at different temperatures.

The compositions x = 0 and x = 1 correspond to either Mn or Ni occupancy in all the 16 equivalent octahedral positions in the simulation cell. The configurational entropy is therefore



**Figure 2** Free energies of mixing ( $\Delta F$ ) for the Li(Mn<sub>1-x</sub>Ni<sub>x</sub>)<sub>2</sub>O<sub>4</sub> solid solution at 300 and 1000 K.



**Figure 3** Configurational entropy as a function of temperature for the  $1 \times 1 \times 1$  supercell of Li(Mn<sub>1-x</sub>Ni<sub>x</sub>)<sub>2</sub>O<sub>4</sub>.

independent of the temperature. However, for the remaining concentrations of Ni, and in particular for x = 0.375, 0.625 and 0.750, we see that the configurational entropies increase with temperature. At high temperatures, the configurational entropy increasingly tends towards its maximum value, as there are a large number of possible inequivalent configurations for these intermediate Ni concentrations. Interestingly, this configurational entropy behaviour suggests a large degree of site occupancy disorder for the Ni and Mn ions at high temperatures. Experimental investigations of LiMn<sub>1.5</sub>Ni<sub>0.5</sub>O<sub>4</sub> samples, prepared by annealing at 973.15 K,<sup>24</sup> have shown long-range Ni and Mn ordering, which agrees with our results.

Figure 4 shows schemes of the electronic configuration of the Mn(III, IV) and Ni(II, III, IV) ions when they are located in a high-spin octahedral field of anions. For the pure  $\text{LiMn}_2\text{O}_4$  spinel, the Mn<sub>B</sub> cations have the formal oxidation state 3+ and 4+, represented in Fig. 4a. For the Mn<sub>B</sub> cations in both oxidation states, the  $t_{2g\uparrow}^3$  level is highly stable since it is half-full.

However, the  $t_{2q\uparrow}^1$  orbital has an uneven occupation, which breaks the symmetry of the states and drives the Jahn-Teller distortion of the  $Mn_B^{3+}$  ion, which is ultimately responsible for the structural instability of LiMn<sub>2</sub>O<sub>4</sub>. The Jahn-Teller effect occurs when the non-centrosymmetric lobes of the  $d_{x^2-y^2}$  and  $d_{z^2}$  orbitals have a different occupation. Note that the empty  $e_g$ state of the  $Mn_B^{4+}$  cation does not affect the stability of LiMn<sub>2</sub>O<sub>4</sub>. Figure 4c illustrates the  $t^3_{2g\uparrow}t^2_{2g\downarrow}e^2_{2g\uparrow}$  and  $t^3_{2g\uparrow}t^1_{2g\downarrow}e^2_{2g\uparrow}$  electronic distribution of the Ni<sub>B</sub><sup>3+</sup> and Ni<sub>B</sub><sup>4+</sup> ions, respectively, in the extreme LiNi<sub>2</sub>O<sub>4</sub> composition. As in the lowest energy  $t_{2g}$  orbital of the  $Mn_B(III, IV)$  ions in  $LiMn_2O_4$ , the half-full  $e_{2g}$  state of  $Ni_B$ (III, IV) is very stable in LiNi<sub>2</sub>O<sub>4</sub>. Despite the  $t_{2g}$  orbitals of  $Ni_{R}$ (III, IV) being partially filled, the reduced stability and symmetry of these ions are not enough to produce a Jahn-Teller distortion. In the case of the  $d_{xy}$ ,  $d_{xz}$  and  $d_{yz}$  orbitals with unequal electron populations, their centrosymmetric lobes do not allow Jahn-Teller distortion. For the most stable solid solution with composition LiMn<sub>1.5</sub>Ni<sub>0.5</sub>O<sub>4</sub>, the highly stable electronic distribution of the Mn<sub>B</sub><sup>4+</sup> ion has already been discussed, which alongside the very stable  $t_{2g\uparrow}^3 t_{2g\downarrow}^3 e_{2g\uparrow}^2$  electronic configuration displayed by the Ni<sub>B</sub><sup>2+</sup> ion, with full  $t_{2g}$  and half-full  $e_{2g}$  levels, justifies the lowest energy in the free energy of mixing diagram represented in Fig. 2.

Next, we have calculated the average Li intercalation voltage for various Ni concentrations in  $Li(Mn_{1-x}Ni_x)_2O_4$  as:<sup>25,26</sup>

$$\overline{V} = -[E(\text{Li}(\text{Mn}_{1-x}\text{Ni}_{x})_{2}\text{O}_{4}) - E((\text{Mn}_{1-x}\text{Ni}_{x})_{2}\text{O}_{4}) - E(\text{Li})]$$
(3)

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**Figure 4** Scheme of the crystal field splitting for the Mn and Ni 3*d* electrons in high spin octahedral coordination environments (**a**)  $Mn_B^{3+}$  and  $Mn_B^{4+}$  in  $LiMn_2O_4$ , (**b**)  $Mn_B^{4+}$  and  $Ni_B^{2+}$  in  $LiMn_{1.5}Ni_{0.5}O_4$  and (**c**)  $Ni_B^{3+}$  and  $Ni_B^{4+}$  in  $LiNi_2O_4$ . The perturbation of the Mn and Ni 3*d* electrons in a spherical field is also represented.

The average voltage of an intercalated compound can be obtained from the Gibbs free energies, which at 0 K can be approximated by the lowest total energies (*E*) of the lithiated  $\text{Li}(\text{Mn}_{1-x}\text{Ni}_x)_2\text{O}_4$  structures, compared to the de-lithiated  $(\text{Mn}_{1-x}\text{Ni}_x)_2\text{O}_4$  end state and the lowest energy state of pure Li. For the body-centred cubic (*bcc*) Li metal we calculated an optimized lattice constant *a* = 3.35 Å, which is in excellent agreement with experiments.<sup>27,28</sup> Li<sup>+</sup> is preferentially intercalated into tetrahedral sites.

Figure 5 shows the average intercalation voltage for the lowest energy configurations at each Ni concentration, where the voltage changes with the composition of the material. The spinel-structured LiMn<sub>1.5</sub>Ni<sub>0.5</sub>O<sub>4</sub> (LMNO) has attracted significant attention, owing to its higher working voltage (4.7 V) in comparison with already commercialized cathode materials, such as LiFePO<sub>4</sub> (3.4 V), LiCoO<sub>2</sub> (3.9 V) and LiMn<sub>2</sub>O<sub>4</sub> (4.1 V).<sup>29,30</sup> Figure 5 shows how the average voltage of intercalation in Li(Mn<sub>1-x</sub>Ni<sub>x</sub>)<sub>2</sub>O<sub>4</sub> is related to the energies of the end states (charged and discharged) and varies with Ni concentration. The most energetically stable spinel structure LiMn<sub>1.5</sub>Ni<sub>0.5</sub>O<sub>4</sub> has a calculated average voltage of 4.866 V, which agrees very well with the experimental operating voltage of ~4.8 V.<sup>1,31</sup> Up to 1000 K, only three configurations have a probability of occurrence that is larger than zero for the most stable Ni composition



**Figure 5** Voltage of Li intercalation for Li( $Mn_{1-x}Ni_{x}/O_4$  as a function of the Ni concentration and (inset) average voltage of lithium intercalation as a function of temperature.

(LiMn<sub>1.5</sub>Ni<sub>0.5</sub>O<sub>4</sub>), leading to an average Li intercalation voltage that has negligible temperature dependence.

#### 4. Conclusions

We have employed DFT calculations to investigate the effect of substituting nickel for manganese in the  $\text{LiMn}_2\text{O}_4$  cathode material. Our results indicate that any small change in the amount of Ni will be reflected in the stability of the  $\text{Li}(\text{Mn}_{1-x}$  $\text{Ni}_x)_2\text{O}_4$  spinel. The most stable Ni concentration corresponds to the  $\text{LiMn}_{1.5}\text{Ni}_{0.5}\text{O}_4$  stoichiometry, which displays the largest average voltage for the Li intercalation reaction. We found no change in the average voltage for the Li intercalation reaction as the temperature is increased. Our calculations indicate that the lithium intercalation voltage can be modified *via* the control of the Ni concentration in the Li( $\text{Mn}_{1-x}\text{Ni}_x$ )<sub>2</sub>O<sub>4</sub> spinel.

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#### **Supplementary Material**

Supplementary information is provided in the online supplement.

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# Supplementary material to:

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# Controlling the lithium intercalation voltage in the Li(Mn<sub>1-x</sub>Ni<sub>x</sub>)<sub>2</sub>O<sub>4</sub> spinel via tuning of the Ni concentration: a Density Functional Theory study

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1. Optimised lattice geometry and ionic positions for  $\text{Li}\text{Mn}_2\text{O}_4$ 

Format: VASP POSCAR file

| Li Mn O                                 |   |   |
|---|---|---|
| 2 22050178802050                        | $\gamma$                                | 0000 0 00000000000000000000000000000000 |
| 8.33950178895056                        |   |   |
| 0.0000000000000000000000000000000000000 | 8.339501/88930                          |   |
| -0.000000000000000                      | 0.0000000000000000000000000000000000000 | 8.3395017889305052                      |
| Li Mn O                                 |   |   |
| 8 16 32                                 |   |   |
| Direct                                  |   |   |
| 0.12500000000000000                     | 0.12500000000000000                     | 0.1250000000000000                      |
| 0.6250000000000000                      | 0.12500000000000000                     | 0.6250000000000000                      |
| 0.12500000000000000                     | 0.6250000000000000                      | 0.6250000000000000                      |
| 0.6250000000000000                      | 0.6250000000000000                      | 0.1250000000000000                      |
| 0.8750000000000000                      | 0.37500000000000000                     | 0.3750000000000000                      |
| 0.8750000000000000                      | 0.87500000000000000                     | 0.8750000000000000                      |
| 0.3750000000000000                      | 0.37500000000000000                     | 0.8750000000000000                      |
| 0.3750000000000000                      | 0.8750000000000000                      | 0.3750000000000000                      |
| 0.5000000000000000                      | 0.5000000000000000                      | 0.500000000000000                       |
| 0.2500000000000000                      | 0.75000000000000000                     | 0.000000000000000                       |
| 0.7500000000000000                      | 0.0000000000000000                      | 0.250000000000000                       |
| 0.0000000000000000                      | 0.2500000000000000                      | 0.750000000000000                       |
| 0.7500000000000000                      | 0.2500000000000000                      | 0.000000000000000                       |
| 0.2500000000000000                      | 0.0000000000000000                      | 0.750000000000000                       |
| 0.0000000000000000                      | 0.7500000000000000                      | 0.250000000000000                       |
| 0.5000000000000000                      | 0.0000000000000000                      | 0.000000000000000                       |
| 0.2500000000000000                      | 0.2500000000000000                      | 0.500000000000000                       |
| 0.7500000000000000                      | 0.5000000000000000                      | 0.750000000000000                       |
| 0.7500000000000000                      | 0.7500000000000000                      | 0.500000000000000                       |
| 0.2500000000000000                      | 0.5000000000000000                      | 0.250000000000000                       |
| 0.0000000000000000                      | 0.5000000000000000                      | 0.000000000000000                       |
| 0.5000000000000000                      | 0.2500000000000000                      | 0.250000000000000                       |
| 0.5000000000000000                      | 0.7500000000000000                      | 0.750000000000000                       |
| 0.0000000000000000                      | 0.0000000000000000                      | 0.500000000000000                       |
| 0.2621894965390581                      | 0.2621894965390581                      | 0.2621894965390581                      |
| 0.4878105034609419                      | 0.9878105034609419                      | 0.7621894965390581                      |
| 0.9878105034609419                      | 0.7621894965390581                      | 0.4878105034609419                      |
| 0.7621894965390581                      | 0.4878105034609419                      | 0.9878105034609419                      |
| 0.0121894965390581                      | 0.5121894965390581                      | 0.2378105034609419                      |
| 0.7378105034609419                      | 0.7378105034609419                      | 0.7378105034609419                      |
| 0.5121894965390581                      | 0.2378105034609419                      | 0.0121894965390581                      |
| 0.2378105034609419                      | 0.0121894965390581                      | 0.5121894965390581                      |
| 0.5121894965390581                      | 0.0121894965390581                      | 0.2378105034609419                      |
| 0.0121894965390581                      | 0.2378105034609419                      | 0.5121894965390581                      |
| 0.2378105034609419                      | 0.5121894965390581                      | 0.0121894965390581                      |
| 0.9878105034609419                      | 0.4878105034609419                      | 0.7621894965390581                      |
| 0.4878105034609419                      | 0.7621894965390581                      | 0.9878105034609419                      |
| 0.7621894965390581                      | 0.9878105034609419                      | 0.4878105034609419                      |
| 0.2621894965390581                      | 0.7621894965390581                      | 0.7621894965390581                      |
| 0.4878105034609419                      | 0.4878105034609419                      | 0.2621894965390581                      |
| 0.9878105034609419                      | 0.2621894965390581                      | 0.9878105034609419                      |
| 0.0121894965390581                      | 0.0121894965390581                      | 0.7378105034609419                      |
| 0.7378105034609419                      | 0.2378105034609419                      | 0.2378105034609419                      |
| 0.5121894965390581                      | 0.7378105034609419                      | 0.5121894965390581                      |
| 0.5121894965390581                      | 0.5121894965390581                      | 0.7378105034609419                      |
| 0.0121894965390581                      | 0.7378105034609419                      | 0.0121894965390581                      |
| 0.9878105034609419                      | 0.9878105034609419                      | 0.2621894965390581                      |
| 0.4878105034609419                      | 0.2621894965390581                      | 0.4878105034609419                      |
| 0.7621894965390581                      | 0.2621894965390581                      | 0.7621894965390581                      |
| 0.2621894965390581                      | 0.4878105034609419                      | 0.4878105034609419                      |
| 0.2378105034609419                      | 0.7378105034609419                      | 0.2378105034609419                      |

2. Optimised lattice geometry and ionic positions for  $\text{LiNi}_2\text{O}_4$ 

Format: VASP POSCAR file

| Li Ni O             |                     |                         |
|---------------------|---------------------|-------------------------|
| 1.00000000000000000 |                     |                         |
| 8.03174983210274    | 72 0.000000000000   | 0000 0.0000000000000000 |
| -0.00000000000000   | 00 8.031749832102   | 7472 0.0000000000000000 |
| -0.00000000000000   | -0.000000000000     | 0000 8.0317498321027472 |
| Li Ni O             |                     |                         |
| 8 16 32             |                     |                         |
| Direct              |                     |                         |
| 0.12500000000000000 | 0.12500000000000000 | 0.1250000000000000      |
| 0.62500000000000000 | 0.12500000000000000 | 0.6250000000000000      |
| 0.12500000000000000 | 0.6250000000000000  | 0.6250000000000000      |
| 0.6250000000000000  | 0.6250000000000000  | 0.1250000000000000      |
| 0.87500000000000000 | 0.37500000000000000 | 0.3750000000000000      |
| 0.87500000000000000 | 0.87500000000000000 | 0.8750000000000000      |
| 0.37500000000000000 | 0.3750000000000000  | 0.875000000000000       |
| 0.37500000000000000 | 0.8750000000000000  | 0.375000000000000       |
| 0.50000000000000000 | 0.5000000000000000  | 0.500000000000000       |
| 0.25000000000000000 | 0.7500000000000000  | 0.00000000000000        |
| 0.75000000000000000 | 0.0000000000000000  | 0.250000000000000       |
| 0.00000000000000000 | 0.2500000000000000  | 0.750000000000000       |
| 0.7500000000000000  | 0.2500000000000000  | 0.00000000000000        |
| 0.2500000000000000  | 0.0000000000000000  | 0.750000000000000       |
| 0.0000000000000000  | 0.7500000000000000  | 0.250000000000000       |
| 0.5000000000000000  | 0.0000000000000000  | 0.00000000000000        |
| 0.2500000000000000  | 0.2500000000000000  | 0.500000000000000       |
| 0.7500000000000000  | 0.5000000000000000  | 0.750000000000000       |
| 0.7500000000000000  | 0.7500000000000000  | 0.500000000000000       |
| 0.2500000000000000  | 0.5000000000000000  | 0.250000000000000       |
| 0.0000000000000000  | 0.5000000000000000  | 0.00000000000000        |
| 0.5000000000000000  | 0.2500000000000000  | 0.250000000000000       |
| 0.5000000000000000  | 0.7500000000000000  | 0.750000000000000       |
| 0.0000000000000000  | 0.0000000000000000  | 0.500000000000000       |
| 0.2627472841061365  | 0.2627472841061365  | 0.2627472841061365      |
| 0.4872527158938635  | 0.9872527158938634  | 0.7627472841061366      |
| 0.9872527158938634  | 0.7627472841061366  | 0.4872527158938635      |
| 0.7627472841061366  | 0.4872527158938635  | 0.9872527158938634      |
| 0.0127472841061365  | 0.5127472841061366  | 0.2372527158938635      |
| 0.7372527158938634  | 0.7372527158938634  | 0.7372527158938634      |
| 0.5127472841061366  | 0.2372527158938635  | 0.0127472841061365      |
| 0.2372527158938635  | 0.0127472841061365  | 0.5127472841061366      |
| 0.5127472841061366  | 0.0127472841061365  | 0.2372527158938635      |
| 0.0127472841061365  | 0.2372527158938635  | 0.5127472841061366      |
| 0.2372527158938635  | 0.5127472841061366  | 0.0127472841061365      |
| 0.9872527158938634  | 0.4872527158938635  | 0.7627472841061366      |
| 0.4872527158938635  | 0.7627472841061366  | 0.9872527158938634      |
| 0.7627472841061366  | 0.9872527158938634  | 0.4872527158938635      |
| 0.2627472841061365  | 0.7627472841061366  | 0.7627472841061366      |
| 0.4872527158938635  | 0.4872527158938635  | 0.2627472841061365      |
| 0.9872527158938634  | 0.2627472841061365  | 0.9872527158938634      |
| 0.0127472841061365  | 0.0127472841061365  | 0.7372527158938634      |
| 0.7372527158938634  | 0.2372527158938635  | 0.2372527158938635      |
| 0.5127472841061366  | 0.7372527158938634  | 0.5127472841061366      |
| 0.5127472841061366  | 0.5127472841061366  | 0.7372527158938634      |
| 0.0127472841061365  | 0.7372527158938634  | 0.0127472841061365      |
| 0.9872527158938634  | 0.9872527158938634  | 0.2627472841061365      |
| 0.4872527158938635  | 0.2627472841061365  | 0.4872527158938635      |
| 0.7627472841061366  | 0.2627472841061365  | 0.7627472841061366      |
| 0.2627472841061365  | 0.4872527158938635  | 0.4872527158938635      |
| 0.2372527158938635  | 0.7372527158938634  | 0.2372527158938635      |

3. Optimised lattice geometry and ionic positions for  $LiMn_{1.5}Ni_{0.5}O_4$ 

## Format: VASP POSCAR file

| li Ni Mn O          |                     |                         |
|---------------------|---------------------|-------------------------|
| 1,00000000000000000 |                     |                         |
| 8.20619083167936    | 66 0.025948911013   | 3812 0.0000000000000000 |
| 0.02594891101338    | 12 8.206190831679   | 3666 0.0000000000000000 |
| -0.0000000000000000 | 00 -0.0000000000000 | 0000 8.4146280562520612 |
| li Ni Mn O          |                     |                         |
| 8 4 12              | 32                  |                         |
| Direct              |                     |                         |
| 0.1208566457535635  | 0.1214847681639917  | 0.1207376360905351      |
| 0.6291433542464364  | 0.1285152318360083  | 0.6207376360905351      |
| 0.1287343998670981  | 0.6245290363758530  | 0.6212999452454613      |
| 0.6212656001329019  | 0.6254709636241470  | 0.1212999452454613      |
| 0.8714847681639917  | 0.3708566457535635  | 0.3792623639094578      |
| 0.8785152318360083  | 0.8791433542464364  | 0.8792623639094649      |
| 0.3745290363758530  | 0.3787343998670981  | 0.8787000547545387      |
| 0.3754709636241470  | 0.8712656001329019  | 0.3787000547545387      |
| 0.5009602047960844  | 0.4990397952039228  | 0.500000000000000       |
| 0.2490397952039157  | 0.7509602047960844  | 0.000000000000000       |
| 0.7469006805754552  | 0.9969006805754552  | 0.250000000000000       |
| 0.0030993194245448  | 0.2530993194245448  | 0.750000000000000       |
| 0.7471693900688734  | 0.2528306099311267  | 0.000000000000000       |
| 0.2528200239625895  | 0.0025174945206890  | 0.7516578649458704      |
| 0.9974825054793111  | 0.7471799760374105  | 0.2483421350541297      |
| 0.5004609503732514  | 0.0025482092504695  | 0.9947603169566815      |
| 0.2495390496267486  | 0.2474517907495305  | 0.4947603169566815      |
| 0.7525174945206889  | 0.5028200239625895  | 0.7483421350541296      |
| 0.7525482092504695  | 0.7504609503732514  | 0.5052396830433185      |
| 0.2466956790159850  | 0.4966956790159850  | 0.2500000000000000      |
| 0.9974517907495305  | 0.4995390496267486  | 0.0052396830433185      |
| 0.49/1/99/603/4105  | 0.24/4825054/93110  | 0.2516578649458703      |
| 0.5033043209840150  | 0.7533043209840150  | 0.7500000000000000      |
| 0.0028306099311266  | 0.9971693900688734  | 0.5000000000000000      |
| 0.2594500564245756  | 0.2578/11892518081  | 0,203238/800402411      |
| 0.4905055415750242  | 0.9921200107401910  | 0.7052587800402411      |
| 0.3693271303721110  | 0.7505511547551520  | 0.4021093304024004      |
| 0.0078711892518081  | 0.4990000492000000  | 0.2367612139537589      |
| 0.7421288107481918  | 0.7405633415756242  | 0.7367612139537589      |
| 0.5069311547991920  | 0.2393271365721110  | 0.0178906635975336      |
| 0 2430688452008079  | 0 0106728634278961  | 0 5178906635975337      |
| 0.5055918570291146  | 0.0090507566140311  | 0.2274533675383855      |
| 0.0191372323491933  | 0.2328835331710633  | 0.4951661426746514      |
| 0.2299453295942063  | 0.5003605027529520  | 0.0212307912301326      |
| 0.9909492433859689  | 0.4944081429708854  | 0.7725466324616145      |
| 0.4996394972470480  | 0.7700546704057937  | 0.9787692087698674      |
| 0.7671164668289367  | 0.9808627676508067  | 0.5048338573253486      |
| 0.2703903077351128  | 0.7685280740030340  | 0.7524991481922543      |
| 0.4796096922648872  | 0.4814719259969660  | 0.2524991481922543      |
| 0.9828835331710633  | 0.2691372323491933  | 0.0048338573253486      |
| 0.0236448852389072  | 0.0154758265708955  | 0.7555768004382646      |
| 0.7263551147610928  | 0.2345241734291045  | 0.2555768004382646      |
| 0.5200546704057937  | 0.7496394972470480  | 0.5212307912301326      |
| 0.5185280740030340  | 0.5203903077351127  | 0.7475008518077457      |
| 0.9987147862501721  | 0.7326411559537384  | 0.0216673639060260      |
| 0.9845241734291045  | 0.9763551147610928  | 0.2444231995617354      |
| 0.4826411559537384  | 0.2487147862501720  | 0.4783326360939740      |
| 0.7654758265708955  | 0.2/36448852389072  | 0./44423199561/354      |
| 0.250360502/529520  | 0.4/99453295942134  | 0.4/8/69208/6986/4      |
| 0.2314/19259969660  | 0./2900909226488/3  | 0.24/30003100//45/      |