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**Article:**

Allulakshmi, K, Vinod, JS, Heitor, A et al. (2021) DEM Study on the Instability Behaviour of Granular Materials. *Geotechnical and Geological Engineering*, 39. pp. 2175-2185. ISSN: 0960-3182

<https://doi.org/10.1007/s10706-020-01617-7>

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## DEM Study on the Instability Behaviour of Granular Materials

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### Abstract

This paper presents the Discrete Element Method (DEM) simulations on the instability behaviour of granular materials during Constant Shear Drained condition (CSD). CSD condition was implemented by decreasing mean effective stress on an assembly of particles under strain controlled loading. In this study, the instability condition was predicted at the particle scale level using particle second order work increment (Nicot et al., 2012). The DEM contact parameters have been calibrated to capture the macroscopic responses and the instability behaviour consistently with the laboratory experimental observations. Using the same contact parameters the effect of different range of initial states at the beginning of CSD condition such as different initial mean effective stress ( $p'_0$ ), void ratio ( $e_0$ ) and deviatoric stress ( $q$ ) on the instability behaviour were analysed. In addition, the micromechanical parameters such as coordination number, anisotropic coefficients (geometric, mechanical) have been extracted to assist in characterising the instability behaviour during CSD condition. The initial stress state of the soil (i.e. at the onset of CSD) condition has shown a significant influence on the evolution of anisotropic coefficients, an evident behaviour change was noted once the CSD condition is imposed. A continuous increase of geometric anisotropy, and a gradual decrease of mechanical anisotropy was observed after the instability condition is reached.

**Keywords:** discrete element method, constant shear stress, particle scale instability, anisotropy.

## Introduction

Water ingress in soils through infiltration can trigger instability leading to failures in slopes and embankments under drained conditions. Subsequent investigations on such failures have shown that the infiltration of water in soils can cause a reduction in effective stress leading to the instability of slopes and embankments (e.g. Eckersley 1990; Olson et al. 2000; Dong et al. 2015; Lashkari et al. 2019). Brand (1981) and Sasitharan et al. (1993) have suggested constant shear drained (CSD) triaxial test to predict the instability caused by the reduction in effective stress. In the recent past, several researchers have conducted CSD laboratory tests to investigate the instability behaviour of different soils (e.g. Skopek et al. 1994, Anderson and Riemer 1995; Zhu and Anderson 1998; Gajo et al. 2000; Chu et al. 2003, 2012; Nicot et al. 2011; Dong et al. 2015; Rabbi et al. 2019).

Instability in CSD tests indicates the sudden development of large plastic strains correlated with the inability of the soil to sustain the imposed load (Skopek et al. 1994; Chu et al. 2003; Sawicki and Swidzinski 2010; Dong et al. 2015). Hill (1958) proposed the instability criterion in terms of second order work as a function of stress-strain ( $d^2W = d\sigma' d\varepsilon$ ) has been widely used to identify the instability behaviour during CSD conditions (e.g. Darve et al. 2004, 2007; Sibille et al. 2007; Daouadji et al. 2010; Nicot et al. 2011; Dong et al. 2015). Based on Hill's criterion, the material is stable if the second increment of work is strictly positive ( $d^2W > 0$ ) for all changes in stresses and strains (Darve et al. 2004). In addition, many constitutive models have also incorporated Hill's instability criterion to capture the instability during CSD tests (Ramos et al. 2012; Alipour and Lashkari 2017; Lashkari et al. 2019). Recently, Nicot et al. (2012) have proposed a microscopic formulation for the second order work as a combination of contact forces and resultant particle forces. This approach enables the identification of the instability point at the particle scale level and provides more insight to address the instability from the micromechanical perspective with the local governing variables (Hadda et al. 2013).

The Discrete Element Method (DEM) developed by Cundall and Strack (1979) is an alternative to the continuum mechanics approach for modelling and analysing the soil mechanical behaviour at a particle scale level. Even though many experimental CSD analyses have been conducted on the instability of granular materials, only, limited studies were performed to simulate the CSD test using micromechanical based approaches such as DEM (Ning et al. 2013; Perez et al. 2016; Lashkari et al. 2019). Perez et al. (2016) and Lashkari et al. (2019) have adopted a stress controlled loading in their analyses and identified the instability behaviour

based on Hill's particle scale criterion and strain rate increment. Furthermore, they simulated the evolution of micromechanical quantities (coordination number, anisotropic coefficients) and highlighted the behavioural change of micro parameters post the instability point.

It is evident from the literature that all the DEM studies on CSD had adopted a stress controlled loading condition. However, in this study, CSD condition was simulated under strain controlled loading and performed a detailed micromechanical analysis of soil specimen under constant shear load. The effect of different type of initial stress state at the beginning of the CSD condition (i.e.  $q, p', e_0$ ) on the onset of instability condition has been studied. The macroscopic responses obtained from the numerical results (in the  $q - p'$  space) are compared with the experimental investigations reported by Dong et al. (2015) on Toyoura sand. In addition, the instability behaviour at the particle scale level, has been identified using Nicot et al. (2012) second order work increment. Furthermore, the response of micromechanical parameters (e.g. coordination number, anisotropic coefficients) and their evolution under these controlled loading conditions has been investigated.

### **Numerical simulation program**

A cylindrical specimen having 33,480 spherical particles (see Fig.1 (b)) was created to mimic the particle size distribution shown in Figure 1 (a). A non-linear Hertz-Mindlin (H-M) contact model was used to capture all the particle interactions. The elastic particles in three-dimensional assemblies are expected to interact in a non-linear fashion, a non-linear contact model is considered as a simple contact model for soils and often used in many DEM simulations to capture the characteristic behaviour of soil (e.g. Cui and O'Sullivan 2006; Sitharam et al. 2008; O'Sullivan 2011). The Hertz-Mindlin (H-M) contact model, used in these simulations is derived by considering the interaction of two elastic particles, and the particle stiffness values are determined using material properties. The material properties are particle shear modulus of 12 GPa, particle Poisson's ratio of 0.15, contact friction value of 0.3, particle density of 2650 kg/m<sup>3</sup>, local damping coefficient of 0.1, gravity was zero in these simulations. All simulations were carried out using PFC3D code (Itasca 2016).

After generating particles in the cylindrical chamber, the assembly was brought to an equilibrium state before applying any loading. Then, the system was subjected to isotropic compression up to a desired initial mean effective stress ( $p'_0$ ). During the consolidation stage, the specimen was loaded in a strain-controlled manner, with constant velocities imposed on the boundary walls using a servo-control mechanism until a target confining stress was achieved

(Itasca 2016). The void ratio obtained at the end of isotropic compression was ( $e_0 = 0.562$ ). After reaching the initial stress state, the specimens were sheared under drained conditions to specific deviatoric stresses ( $q$ ) of 120, 150 and 180 kPa. Then a CSD stress path was implemented in the same manner as suggested by (Ning et al. 2013), where the mean effective stress was decreased gradually, this procedure enables a constant shear load (constant ( $q$ ) path) throughout a CSD test. The strain rate used in the current study was of strain  $0.005 \text{ s}^{-1}$ . As the loading conditions are strain controlled it is always ensured that the system is in quasi-static regime by monitoring the inertial number  $I = \dot{\epsilon}d\sqrt{\rho/p'}$  in which  $p'$  is mean effective stress,  $d$  is mean particle size,  $\rho$  is particle density and  $\dot{\epsilon}$  is strain rate (MiDi 2004). Based on the simulation configuration considered in this work, the inertial number was calculated, and it was less than ( $I < 3 \times 10^{-3}$ ), thereby ensuring all the simulations presented in the current study are under quasi-static loading conditions. The stress ratio ( $\eta = q/p'$ ) at the onset of CSD was ( $\eta_{CSD} = 0.5, 0.59$  and  $0.69$ ). In this study, the stresses within the assembly were calculated using Eq. 1 (Bagi 1996).

$$\sigma_{ij} = \frac{1}{V} \sum_1^{N_c} f_i l_j \quad (1)$$

where,  $\sigma_{ij}$  is the stress tensor,  $V$  is the volume of the cylindrical chamber,  $l_j$  is the  $j^{\text{th}}$  component of the branch vector  $l$  that joins the centres of two particles in contact,  $f_i$  is the  $i^{\text{th}}$  component of the contact force vector  $f$ , and  $N_c$  is the total number of contacts in the domain.

In this study, the rigid-wall boundary condition was adopted for all the simulations, and the boundary effect arises due to this condition was minimised by following certain specific criteria's (e.g. maintaining a high ratio of sample diameter to maximum particle size, calculation of stress responses internally with particle-particle contacts), as mentioned in the literature (e.g. Head 1994; Marketos and Bolton 2010; Fu and Dafalias 2011; Huang et al. 2014). Head (1994) suggested that the ratio of sample diameter to maximum particle size should be higher than 10. This criteria was adopted in this study, where at the end of consolidation stage the ratio was 10. Moreover, to avoid the rigid-boundary constrain, the stress responses have been measured internally with particle-particle contacts using Eq. (1) rather than at the boundaries. In addition, the deviatoric fabric ( $\Phi_d$ ) is the difference of maximum and minimum eigenvalues was calculated using fabric tensor of contact normals defined by Satake (1982). The ( $\Phi_d$ ) value at the beginning of the shearing stage was of 0.004, ensuring an isotropic packing of the sample, amidst rigid boundaries.

## Calibration of the model with laboratory experimental data

For the reference purpose, recent experimental investigations of Dong et al. (2015) on loose Toyoura sand subjected to CSD loading conditions are considered here, and the DEM model has been calibrated to the laboratory results reported in Dong et al. (2015). The material parameters mentioned in the above section were calibrated properties. Simulation results of three CSD tests, along with experimental results, are shown in Figure 2. Here ‘diamond’ markers indicate the onset of CSD loading, whereas ‘cross’ markers relate to the onset of instability. The same notation is followed in all the following figures. It is to be noted that diffuse failure mode under constant shear loading and the associated instability behaviour is considered in this study.

It is evident from Figure 2 that the DEM model has captured the CSD test qualitatively compared to the experimental results. After instability, the specimen shows a gradual decrease of  $q$ , indicating the inability to withstand the imposed constant deviatoric stress. The stress ratio at instability ( $\eta_{IL}$ ) predicted from the DEM model using Nicot et al. (2012) particle scale second order work is in the range of ( $\eta_{IL} = 0.7 - 0.75$ ), which is in good agreement with the experimental-instability of ( $\eta_{IL} = 0.72 - 0.74$ ) (Dong et al. 2015).

## Instability point using particle scale second order work proposed by Nicot et al. (2012)

Many studies (e.g. Darve et al. 2004, 2007; Daouadji et al. 2010; Nicot et al. 2011; Dong et al. 2015; Rabbi et al. 2019) have successfully identified the instability point using Hill (1958) second order work criterion in CSD tests. However, in this study, the instability points were identified at the particle scale from the ‘sign change of second order work’ (occurrence of negative values in  $W_2^P$ ) proposed by Nicot et al. (2012) given in Eq. 2.

$$W_2^P = \sum_{c \in V} \delta f_i^c \delta l_i^c + \sum_{p \in V} \delta f_i^p \delta x_i^p \quad (2)$$

Where,  $l^c$  is the branch vector joins the centres of two particles in contact,  $f^c$  the inter-particle contact force,  $x^p$  the position of particle ‘p’ and ,  $f^p$  the resultant force applied for the particle ‘p’.

It captures the changes in contact force and resultant forces acting on particles and can, therefore, provide more insight into the instability at the particle scale level. The results of three CSD simulations at stress ratios ( $\eta_{CSD} = 0.5, 0.59$  and  $0.69$ ) predicting second order work, deviatoric stress and the ratio of incremental stress ratio  $dq/dp'$  versus time are plotted in

Figures 3 (a), (b), (c) respectively. The deviatoric stress response shows that the specimen is in a stable condition and can withstand the imposed constant deviatoric load. Later, the specimen shows the instability behaviour with the decrease in deviatoric stress, as it becomes difficult to sustain the imposed load. The instability behaviour can be understood as the inability of the particle assembly to withstand the continuously imposed static loads. The fluctuations (negative values) of particle scale second order work ( $W_2^P$ ) represents the onset of instability within the particle assembly. The breakage of force chains along with the particles rearrangement post the onset of instability are the main contributing factors for the fluctuations in second order work. In addition, the incremental stress ratio ( $dq/dp'$ ) identifies the behavioural change in deviatoric stress throughout the test. The ratio was approximately zero until the instability point, after that as the sample experiences change in deviatoric stress, the  $dq/dp'$  ratio starts developing and increases gradually beyond the onset of instability. From Fig 3 (a), (b), (c), it can be observed that the negative values of  $W_2^P$  exactly coincides with the decline of deviatoric stress ( $q$ ), the sudden increase in  $dq/dp'$  during the CSD shearing stage. The noticeable change in  $dq/dp'$ , negative values of  $W_2^P$ , corresponding to the reduction in  $q$  occur concurrently and consistently with time for each stress ratio ( $\eta_{CSD}$ ), indicating the onset of instability.

### **Effect of $q$ on the instability point**

Figure 4 (a), (b), (c) shows the macroscopic behaviour of the sample during CSD test of different stress ratios ( $\eta_{CSD} = 0.5, 0.59$  and  $0.69$ ). The response of deviator stress ( $q$ ), axial strain ( $\epsilon_a$ ) and void ratio ( $e$ ) are plotted against mean effective stress  $p'$  respectively. It is evident from Figure 4 that ( $\eta_{CSD}$ ) has a significant influence on the onset of instability. After the onset of CSD loading,  $q$  remains constant with a corresponding reduction of  $p'$ . However, once the reduction of  $p'$  reaches a particular value,  $q$  decreases gradually, highlighting that the sample cannot sustain the imposed constant  $q$ , thus indicating instability. From the axial strain behaviour, it can be noticed that after imposing CSD loading, the axial strain development was clearly influenced until the onset of instability. After that, the increase in axial strain rate has begun indicating instability, and axial strain has shown a steady increase with the corresponding decrease in  $p'$ . Similar response of  $\epsilon_a$  against  $p'$  was also shown by Lashkari et al. (2019). Whereas, the  $e - p'$  plot shows an instant change in the behaviour once the CSD is imposed, the void ratio starts developing due to the rebound (relaxation) phenomena experienced by the soil with the decrease of mean effective stress under constant deviatoric

stress. And the void ratio response remains unaffected even after the onset of instability. Overall, regardless of the stress ratio at the onset of CSD, the void ratios and axial strains evolve in the same way.

### **Effect of $p'$ on the instability behaviour**

To investigate the effect of  $p'$  on the instability behaviour, two other different mean effective stress conditions are also considered ( $p'_0 = 75, 500$  kPa). Total three specimens were prepared by subjecting to different initial confining stresses of  $p'_0 = 75, 200$  and  $500$  kPa. And an initial void ratio of ( $e_0 = 0.56$ ), was maintained for all three CSD simulations following isotropic compression. Before imposing CSD loading, the specimens were sheared to different deviatoric stress level in each test say for ( $p'_0 = 75$  kPa;  $q = 49$  kPa), ( $p'_0 = 200$  kPa;  $q = 150$  kPa) and ( $p'_0 = 500$  kPa;  $q = 240$  kPa). From Fig 5 (a) it is inferred that regardless of initial  $p'$  and deviatoric stress applied prior to CSD loading, the soil undergoes instability and has got a unique instability line applicable during CSD loading. The instability line drawn from the origin connecting the instability points has a slope of ( $\eta_{IL} = 0.75$ ). Similar observations of unique instability line in  $q - p'$  plane for Hostun sand has been reported by Daouadji et al. (2010).

Figure 5 (b) shows the axial strain response against mean effective stress for  $p'_0 = 75, 200, 500$  kPa at  $q = 49, 150, 240$  kPa. In the case of high initial confining pressure ( $p'_0 = 500$  kPa and  $e_0 = 0.56$ ), when the CSD loading is imposed, the specimen shows dilation behaviour, where the sample extends in the axial direction and reaches to  $0.06\%$  from  $0.065\%$  strain. After that, the specimen contracts and continues to show contraction behaviour throughout the test. A similar type of response in terms of reversal of  $\epsilon_1$  is consistent with the laboratory experimental findings reported by Nicot et al. (2011). Even though there is small dilation behaviour, no disturbance is observed in deviatoric stress response. The sample has sustained the imposed load by maintaining constant  $q$  (Fig 5(a)). This dilation tendency is only observed at high confining stresses. Whereas, the specimens at  $p'_0 = 200$  and  $75$  kPa show typical contraction behaviour in axial direction after the onset of CSD condition and remains the same throughout the test.

### **Effect of void ratio on the instability of granular materials**

To investigate the influence of void ratio on the CSD instability subjected to similar loading conditions another particle assembly was created having a slightly higher void ratio of ( $e_0 = 0.585$ ) at the end of the isotropic compression stage. The effect of void ratio in  $q - p'$  stress

plane is shown in Figure 6. It is evident that the instability point is significantly influenced by ( $e_0$ ). For the  $q = 120$  kPa condition, sample having ( $e_0 = 0.562$ ) follows a longer stress path and shows a decrease in  $q$  at lower  $p'$  compared to the higher void ratio ( $e_0 = 0.585$ ) specimen. Similarly, at a high magnitude of deviatoric stress ( $q = 180$  kPa),  $q$  decreases instantly at the onset of CSD loading in the high void ratio ( $e_0 = 0.585$ ) specimen indicating that the sample cannot withstand large magnitude constant  $q$  load. Similar behaviour of deviatoric stress for different void ratio samples during CSD loading was reported by Perez et al. (2016) and Lashkari et al. (2019).

### Microscopic response during CSD shearing

The micromechanical behaviour of the granular assembly during a CSD stress path was captured with particle scale descriptors, namely average coordination number and structural anisotropy. The quantifying sources of structural anisotropy are geometrical anisotropy and mechanical anisotropy (Sitharam and Vinod 2010; Guo and Zhao 2013). Geometrical anisotropy is characterised based on the distribution of contact normals corresponding to the local orientation of the contact plane, while, mechanical anisotropy occurrence is due to contact forces which in turn are related to contact plane orientations (Iwashita and Oda 1999). The fabric tensor relation proposed by Satake (1982) is used in this study.

$$\Phi_{ij} = \frac{1}{N_c} \sum_1^{N_c} n_i n_j \quad (3)$$

where,  $n_i$  is the unit contact normal and  $N_c$  is the total number of contacts. The deviatoric fabric  $\Phi_{ij}$  characterises the fabric anisotropy. It is reported that the stress ratio is the main element that can be associated for causing the overall anisotropic behaviour in a soil, (Rothenburg and Bathurst, 1989). The geometrical anisotropy, which is expressed based on the distribution of contact normals can also be termed as the contact anisotropy ( $a_c = 15/2(\Phi_1 - \Phi_3)$ ), where  $\Phi_1$  and  $\Phi_3$  are the maximum and minimum eigenvalues of  $\Phi_{ij}$ . The mechanical anisotropy which is dependent on the force vector can be distinguished into, normal force anisotropy  $a_n$  (due to normal contact forces) and tangential force anisotropy  $a_t$  (due to tangential contact forces). Perez et al. (2016) reported that the  $a_n$ ,  $a_c$  anisotropic coefficients are more influenced by stress ratio under CSD loading. Hence these anisotropic coefficients have been emphasized in this study. Rothenburg and Bathurst (1989), Guo and Zhao (2013) defined normal force anisotropy  $a_n$  as follows:

$$\begin{aligned}
F_{ij}^n &= \frac{1}{4\pi} \int \bar{f}_n(V) n_i n_j dV \\
&= \frac{1}{N_c} \sum_1^{N_c} \frac{f_n n_i n_j}{1 + (15/2) \phi'_{kl} n_k n_l}
\end{aligned} \tag{4}$$

$$\bar{f}_n(V) = \bar{f}^0 [1 + a_{ij}^n] \tag{5}$$

where,  $a_n$  is the second invariant of  $a_{ij}^n = (15/2) \left( \frac{F_{ij}^n}{\bar{f}^0} \right)$ , here  $\bar{f}^0 = F_{ii}^n$  is the average normal contact force calculated over volume  $V$ , different from the mean normal contact force overall contacts.

The evolution of coordination numbers against time for three stress ratios ( $\eta_{CSD} = 0.5, 0.59$  and  $0.69$ ) are illustrated in Figure 7 (a), (b) and (c) respectively. The coordination number  $Z = 2N_c/N_p$ , where  $N_p, N_c$  are the number of particles and contacts, respectively. As shearing progresses, the coordination number begins to decrease continuously after the onset of CSD loading. As all the samples had the same initial void ratio ( $e_0$ ), regardless of stress state ( $\eta$ ) at the onset of CSD, the rate of decrease in coordination number remained uniform. The  $Z$  value at the instability point is approximately 4.8 irrespective of load (constant  $q$ ) magnitude.

Figures 7 (a), (b) and (c) also presents the evolution of normal force anisotropy  $a_n$ , and contact anisotropy  $a_c$  against time for three stress ratios ( $\eta_{CSD} = 0.5, 0.59$  and  $0.69$ ) respectively. It can be observed from figure 7 that the evolution of  $a_n$  is immediate upon shearing and increases linearly with time prior to CSD loading. After the onset of CSD, regardless of  $\eta$ , a nonlinear growth of  $a_n$  is observed, and its evolution clearly depends on deviatoric stress applied. After the instability point  $a_n$  reaches a peak value and then decreases gradually indicating the lack of ability to withstand the load while  $p'$  is decreasing.

However, the growth of  $a_c$  is different from  $a_n$ , where it is slow at the beginning until the onset of CSD, and the rate of increase in  $a_c$  is varied after CSD loading is imposed. The increase in  $a_c$  continues even after the instability point, which indicates that rearrangement of contacts in the loading direction to sustain the load (Perez et al. 2016; Lashkari et al. 2019). The continuous growth of  $a_c$  also suggests that fabric contacts are getting oriented in a more anisotropic way. The overall behaviour of  $a_c$  with time remained almost the same for all the stress states.

## **Comparison of stress ratio at instability with laboratory experimental data**

The stress ratio and the corresponding void ratio at instability ( $e_{IL}$ ) are plotted in Figure 8. The experimental data reported by Dong et al. (2015), Reid and Fourie (2019) are included for comparison. It is evident that the DEM simulations are able to predict the stress ratio at instability accurately and closely match the experimental results. This further strengthens the DEM model developed in this study as it could able to predict the instability of granular material during CSD loading qualitatively.

## **Conclusions**

The results of the DEM simulations on the instability behaviour of granular materials during CSD testing conditions are reported in this paper. The particle-scale second order work relation, which was employed to identify the instability during CSD tests has captured the onset of instability, and it was consistent with the macroscopic behavioural change of deviatoric stress and axial strain. The axial strain shows steady growth and deviatoric stress decreases gradually post the instability point. Before the onset of CSD, even though all the samples are under different stress states and densities, all have become unstable during CSD loading. In this study, the decrease of deviatoric stress was also captured by the ratio of incremental stress ratio  $dq/dp'$ , where an abrupt rise in the incremental stress ratio was noticed at the onset of instability. In addition, the DEM model has captured the dilation behaviour of axial strain after the onset of CSD condition at higher confining stresses. The stress ratio at the onset of CSD condition has shown influence on the evolution of micro-mechanical quantities,  $a_n$  and  $a_c$ . The rate of increase in  $a_c$  varied after imposing CSD condition and its growth continued beyond the instability due to the rearrangement of contacts in the loading direction. Whereas, the response of  $a_n$  changed from linear to nonlinear increments after imposing CSD loading and after the instability point  $a_n$  decreased indicating the lack of ability to withstand the load while  $p'$  is decreasing. Overall, the DEM model successfully predicted the instability point similar to the experimental data and qualitatively captured the macroscopic response.

## **Acknowledgement**

The authors acknowledge the assistance of all university partners (University of Western Australia (lead university), University of New South Wales, University of South Australia, and University of Wollongong) involved in this project. This work is part of TAILLIQ (Tailings

Liquefaction) project, which is an Australian Research Council (ARC) Linkage Project supported by Anglo American, BHP, Freeport-McMoRan, Newmont, Rio Tinto, and Teck.

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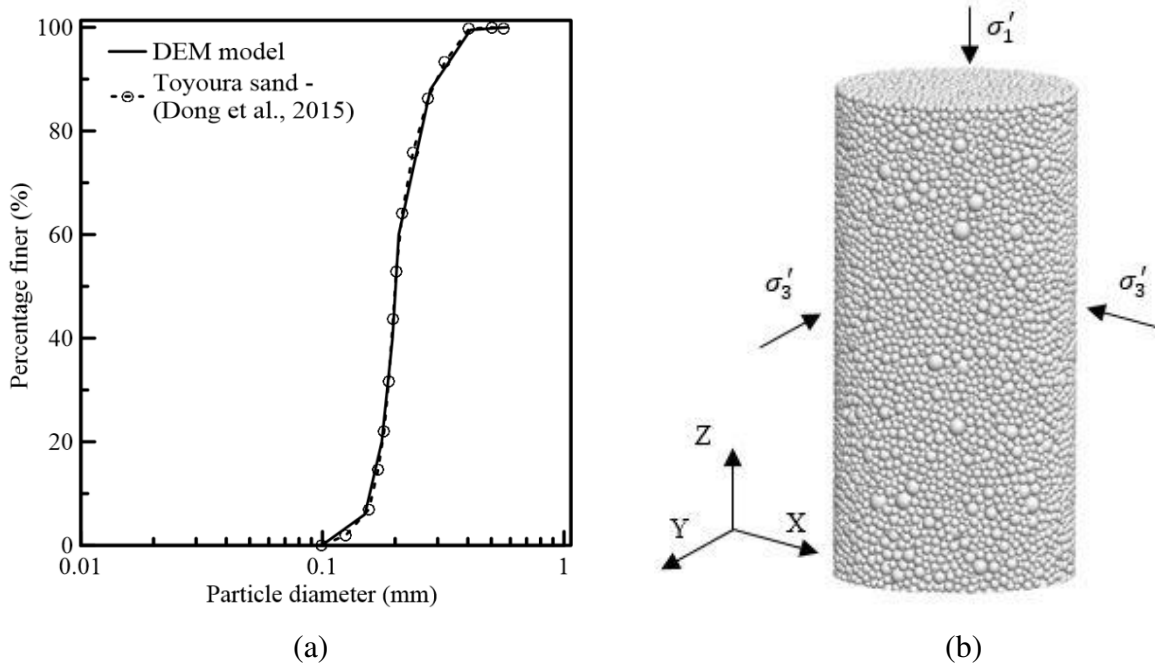


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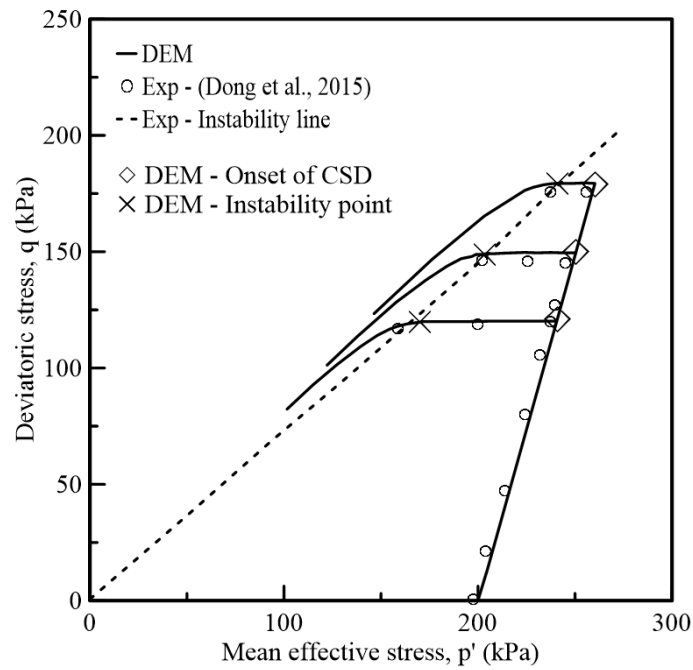


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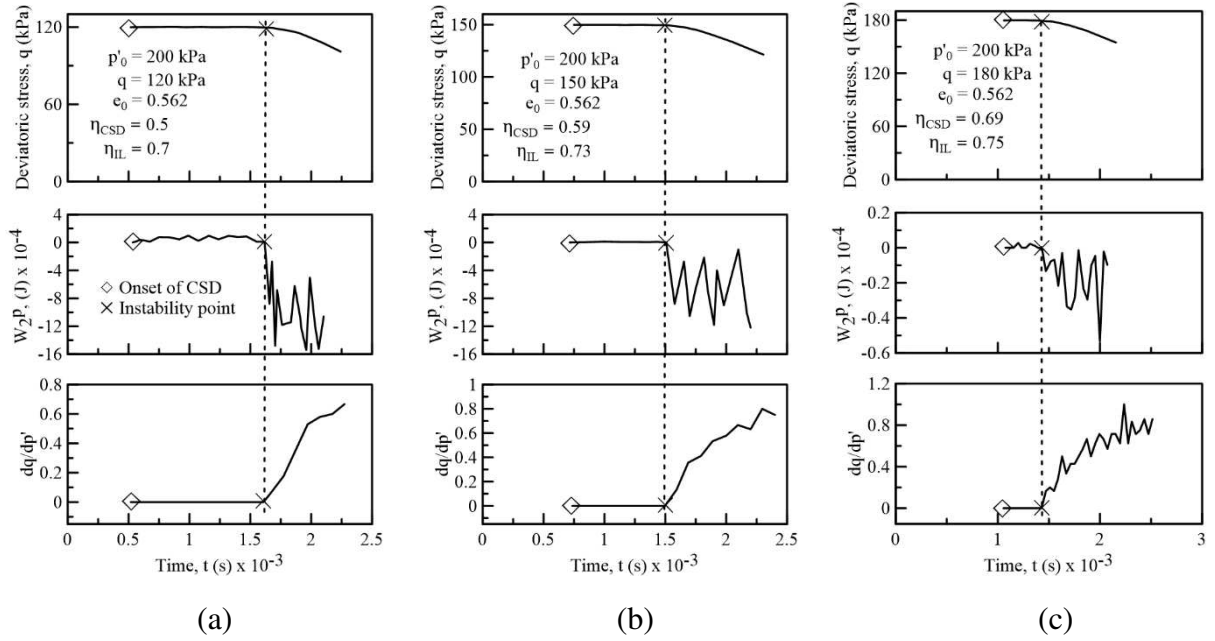


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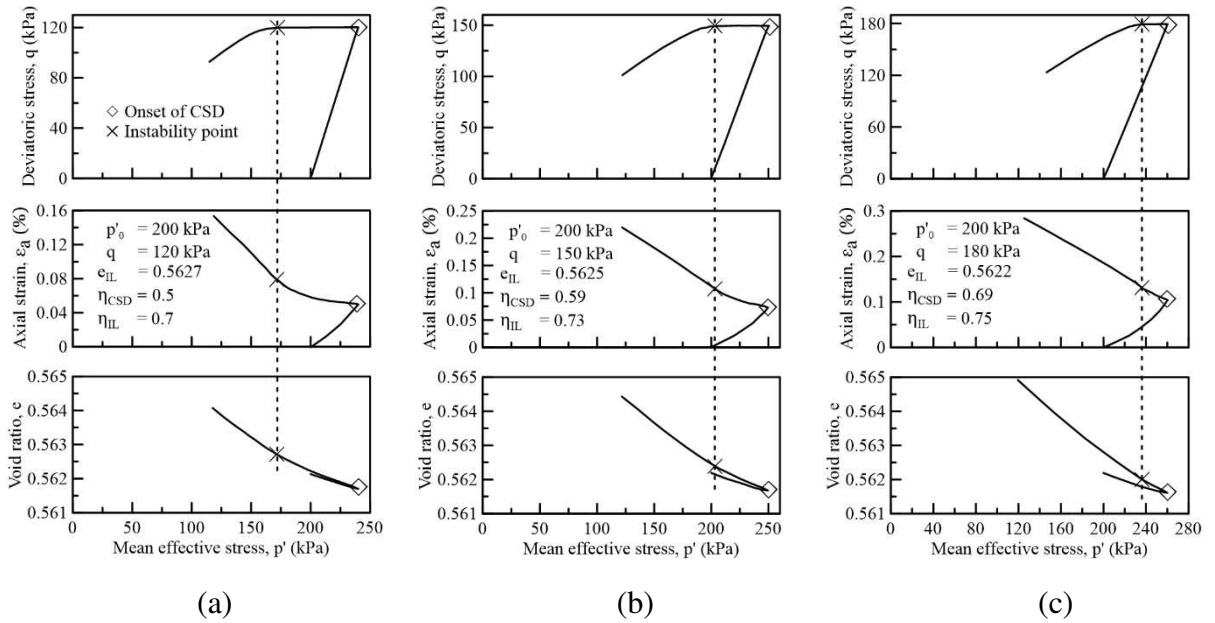


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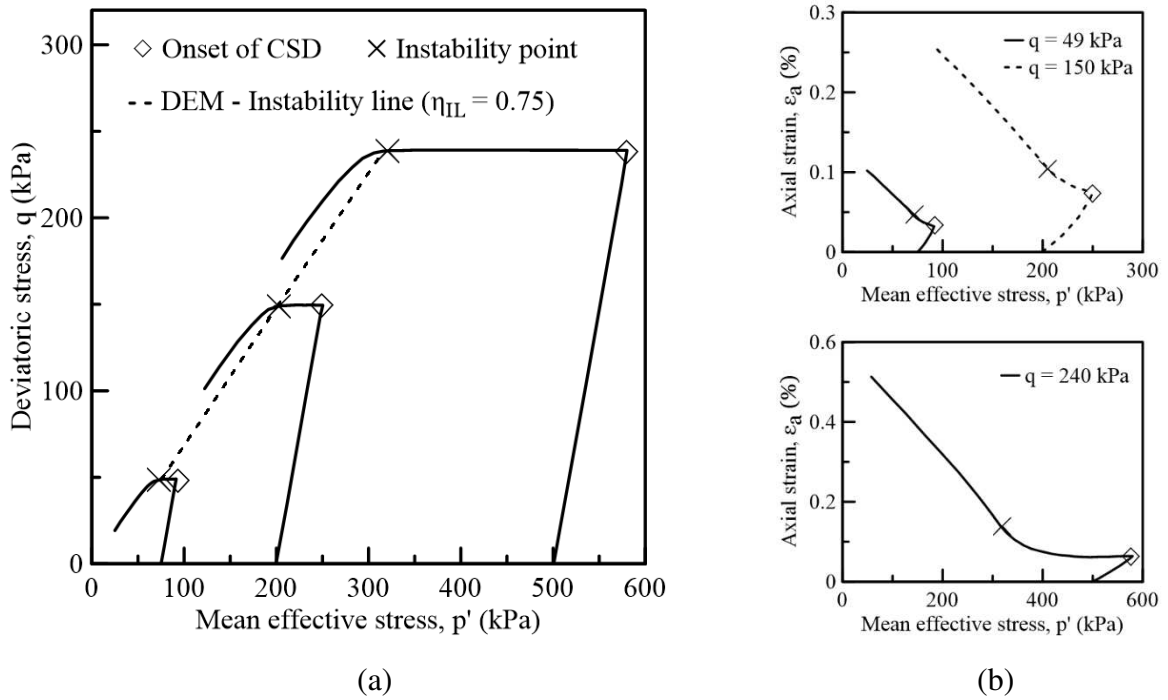


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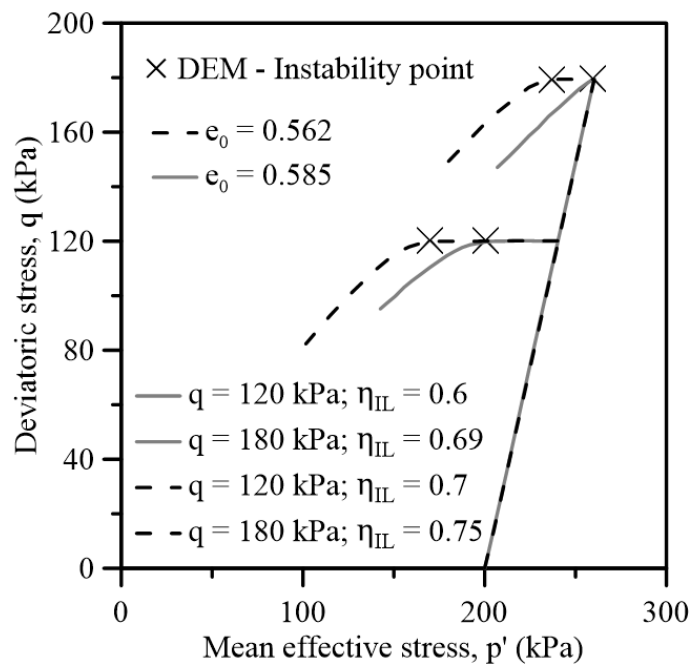


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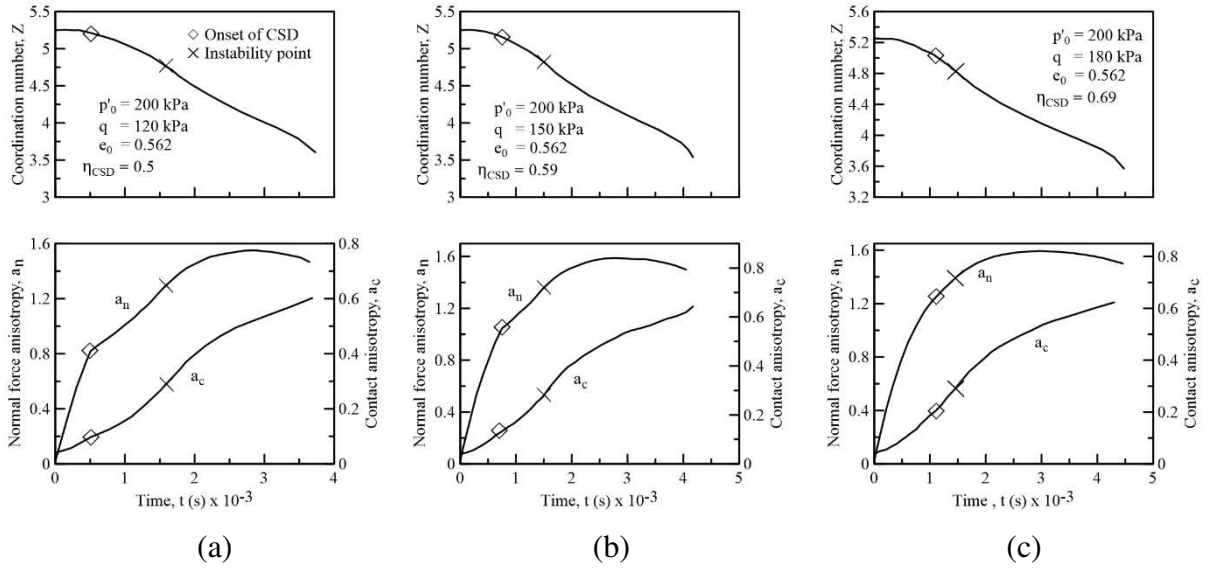


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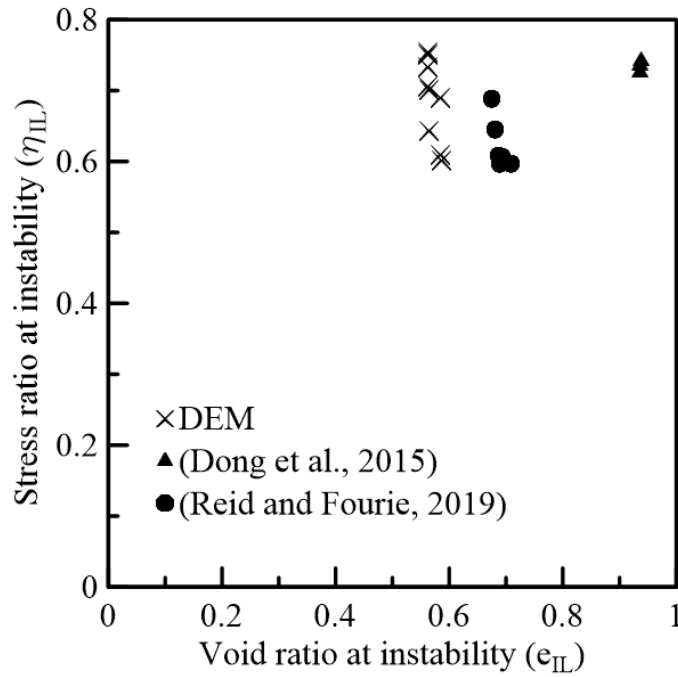


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