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# Highlights

# Spanwise wake development of a bottom-fixed cylinder subjected to vortex-induced vibrations

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- Proper Orthogonal Decomposition can extract main coherent cylinder motions
- Coherent motion of the maximum cylinder response is an elliptical-type trajectory
- Wake and motion spanwise synchronisation maximised at the highest cylinder response
- Bottom-up desynchronisation develops after the maximum cylinder response is achieved
- 2S and 2P vortex modes observed along the cylinder span and across flow rates

# Spanwise wake development of a bottom-fixed cylinder subjected to vortex-induced vibrations

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#### ABSTRACT

This study analyses the spanwise wake dynamics and structural response of a bottom-fixed cylinder subjected to a range of open-channel fully developed turbulent flows. The experiments were performed with a Reynolds number ranging between  $4.5 \times 10^2$  and  $1 \times 10^3$ . The cylinder free end response and the flow velocity in the wake were measured using Particle Image Velocimetry and image-based tracking techniques. The cylinder had a significant modulated response, from which a Proper Orthogonal Decomposition revealed a clockwise elliptical-type trajectory at the maximum cylinder response. Wake dynamic analysis showed that the maximum response is achieved when the cylinder motion and vortex shedding frequencies are equal (i.e. synchronised) to the natural frequency of the structure measured in still water, and when this equivalence is preserved along the span of the cylinder. As the flow velocity increases, a spanwise bottom-up desynchronisation region strength along the span of the cylinder, reducing the maximum structural response and eventually changing the vortex shedding pattern. Despite the highly three-dimensional experimental conditions under significant turbulent incoming flows, the findings of previous studies based on simpler experimental models can still be used to broadly explain the observed desynchronisation process.

#### 1. Introduction

Vortex induced vibrations (VIV) is a non-linear, self-1 governed, multi-degree-of-freedom (DOF) phenomenon 2 that occurs due to the interaction between the vortex formation behind a body and its structural response (Williamson and Govardhan (2004)). VIV can be an important contrib-5 utor to fatigue damage on numerous engineering problems, 6 such as marine risers, large chimneys, heat exchanger tubes, to name a few. Thus, this phenomena has been the sub-8 ject of constant research in the last decades (see, for example, the extensive reviews of Gabbai and Benaroya (2005); 10 Williamson and Govardhan (2004); Sarpkaya (2004)). An 11 important part of experimental VIV research involves the 12 study of simplified cylindrical models. These experiments 13 usually consist on rigid cylinders forced or free to vibrate in 14 their crossflow direction and subjected to a range of uniform 15 and low-turbulence flows. These simplified models have 16 provided substantial insights into the nature of VIV. Kha-17 lak and Williamson (1996, 1997) showed that the mass ra-18 tio  $m^*$  (ratio between the oscillating structure and displaced 19 fluid mass) has an impact on the range of reduced velocities 20  $U_{\rm r}$  in which the cylinder response and the vortex shedding 21 frequency are equal, called synchronisation region. Here, 22  $U_{\rm r} = U_{\rm inlet}/(f_{\rm water}D)$ , where  $U_{\rm inlet}$  is the mean incoming 23 flow velocity,  $f_{\text{water}}$  is the natural frequency of the struc-24 ture measured in still water, and D is the diameter of the 25 cylinder. Cylinders with low  $m^*$  develop three distinctive 26

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regimes within the synchronisation range: the initial branch, 27 the upper branch, where the maximum response is achieved, 28 and the lower branch. Govardhan and Williamson (2000) 29 related each branch to different vortex shedding patterns us-30 ing flow visualisation techniques. Williamson and Roshko 31 (1988) used forced-vibration experiments to map these vor-32 tical structures as a function of the maximum cylinder dis-33 placement and  $U_{\rm r}$ . The researchers observed three vortex 34 patterns or modes: 2S mode (two single vortices per cycle) 35 measured in the initial branch, 2P (two pairs of vortices at 36 every oscillation) observed in the upper and lower branch, 37 and a wake pattern presented only in forced vibration stud-38 ies, called P+S (single and a pair of vortices per cycle of 39 body motion). Later, Morse and Williamson (2009) found 40 a new vortex mode in the upper branch which they called 41 2Po. In this mode, a weaker vortex in each pair per cycle 42 was observed. 43

Different researchers have been trying to determine the 44 applicability of simplified models to cases of increasing 45 complexity, which are closer to engineering applications. 46 Morse and Williamson (2009) satisfactorily predicted the 47 maximum response and wake mode of a free-vibration cylin-48 der using their vortex mode map. Nevertheless, they noted a 49 random component on the cylinder response that could not 50 be reproduced in its forced-vibration counterpart. The effect 51 of the Reynolds number on VIV has been found to be more 52 significant than previously thought. High Reynolds number 53 experiments are characterised by a broader synchronisation 54 range and higher maximum cylinder response (Raghavan 55 and Bernitsas (2011); Wanderley and Soares (2015)). Jau-56 vtis and Williamson (2004) analysed the effects of allowing 57 an additional DOF (crossflow and streamwise response) on 58

an elastically mounted rigid cylinder in terms of peak ampli-59 tudes, response branches, and vortex shedding modes. They 60 found a new branch (called super-upper branch) for cylinders 61 with  $m^* < 6$ . Within this branch, peak crossflow amplitudes 62 reached up to 1.5D, and a new vortex mode called 2T was 63 observed. This new vortex pattern consists on a triplet of 64 vortices each half-cycle of body motion. Additionally, the 65 tested cylinder traced eight-type trajectories throughout its 66 synchronisation range. Other studies observed different mo-67 tion patterns, such as elliptical-type (Oviedo-Tolentino et al. 68 (2014)), eight-type, or a combination of both (Kang and Jia 69 (2013)). Kheirkhah et al. (2012, 2016) showed that the tra-70 jectory type depends on the structural coupling between the 71 streamwise and crossflow motion. The spanwise variability 72 of tapered, pinned and cantilever cylinders haven been used 73 to study complex fluid-structure interactions. Labbé and 74 Wilson (2007) simulated a three-dimensional cylinder and 75 showed the importance of the spanwise length to capture the 76 main features of the flow. Franzini et al. (2014) simultane-77 ously tested a two-DOF cantilever cylinder, and a one-DOF 78 elastically mounted rigid cylinder. Both cylinders had simi-79 lar mass ratios, diameters, lengths, and damping values. The 80 cantilever cylinder had a broader synchronisation region and 81 a higher maximum crossflow amplitude of 1.15D compared 82 to 0.9D of the rigid cylinder. Flemming and Williamson 83 (2005) studied the response of a free-vibration, two-DOF 84 pinned cylinder. For small streamwise amplitudes, the re-85 sponse agreed qualitatively well with the one-DOF cylin-86 der of Govardhan and Williamson (2000). However, at large 87 streamwise motions, peak crossflow amplitudes of approxi-88 mately 1.5D and a new vortex mode called 2C, composed of 89 two co-rotating vortices per half-cycle, were observed in the 90 upper branch. In addition, Flemming and Williamson (2005) 91 observed the simultaneous existence of two vortex patterns 92 along the span of the cylinder. Hybrid modes were also 93 found by Techet et al. (1998) on tapered cylinders. Voorhees 94 et al. (2008) studied flow three-dimensionality on a one-05 DOF free-vibration cylinder. Flow visualisation at differ-96 ent heights along the span of the cylinder showed significant 97 differences with the predictions obtained using the vortex 98 modes map of Morse and Williamson (2009). 99

The multi-DOF, variable-amplitude, free-vibration ex-100 perimental work on VIV has shown substantial differences 101 with the simplified cylindrical models. The two-DOF 102 variable-amplitude bottom-fixed cylinder subjected to VIV 103 provides a compelling case to study how changes in the wake 104 dynamics along the span of the cylinder affect its structural 105 response, a condition commonly found in numerous engi-106 neering problems. Previous studies on bottom-fixed cylin-107 ders observed and characterised a motion history with a 108 highly dominant coherent response around its maximum dis-109 placement and measured the vortex shedding frequency at 110 discrete horizontal planes along the span of the cylinder (see, 111 for example, Franzini et al. (2014); Oviedo-Tolentino et al. 112 (2014)). The identification and characterisation of these co-113 herent responses when the cylinder motion is highly modu-114 lated and the variability between the structural response and 115

vortex shedding synchronisation along the span of the cylin-116 der has not been fully addressed. For this, flow measure-117 ments were performed using a two-dimensional Particle Im-118 age Velocimetry (PIV) system at one vertical plane along the 119 span of the cylinder and four horizontal planes at different 120 water depths. This high-resolution system allowed to mea-121 sure the wake dynamics and spanwise vortex shedding as the 122 cylinder is subjected to a range of turbulent flows. Each PIV 123 measurement was synchronised with a camera that recorded 124 the free end displacement of the cylinder. The recordings 125 were analysed with image-based tracking techniques to de-126 termine the spatiotemporal displacement of the cylinder. 127

This paper is organised as follows: a summary of the 128 snapshot POD method is presented in Section 2. Experi-129 mental setup of a bottom-fixed cylinder subjected to a range 130 of turbulent flows is given in Section 3. The results are sepa-131 rated into two Subsections: Subsection 4.1 is focused on the 132 characterisation of the cylinder in terms of maximum ampli-133 tude, main frequency of oscillation, and coherent trajectory 134 identification using POD. Subsection 4.2 presents the wake 135 dynamics and spanwise vortex shedding variability along the 136 span of the cylinder at different flow velocities. Conclusions 137 are given in Section 5. 138

#### 2. Proper Orthogonal Decomposition

The Proper Orthogonal Decomposition is a statistical tech-140 nique based on the decomposition of spatiotemporal data 141 into a linear combination of spatial basis functions or modes 142 ( $\Phi$ ) and their time-dependent modal coefficients ( $\alpha$ ). POD 143 provides a mathematical definition of energy-relevant struc-144 tures, arranged in descending order, and a method for their 145 extraction (Brevis and García-Villalba (2011)). As it will be 146 shown later, the cylinder exhibits a single-frequency linear-147 elastic response along its span due mainly to its low deforma-148 tion across  $U_r$ . Thus, the POD technique is suitable to extract 149 the main coherent trajectories associated with the most dom-150 inant frequencies when the cylinder response is highly mod-151 ulated. A review of this technique can be found in Berkooz 152 et al. (1993); Chatterjee (2000). Here, the snapshot POD 153 method is briefly described in the context of the cylinder mo-154 tion history. Detailed information about this technique can 155 be found in Sirovich (1987). 156

The spatiotemporal position of the cylinder is expressed 157 in vector form as  $\mathbf{x}_{c}(t_{d}) = (x(t_{d}), y(t_{d}))$ , where  $t_{d} =$ 158 [1, 2, ..., N] and N is the number of data points. Like-159 wise, the fluctuating part of the cylinder response  $x_c'$  is 160 obtained after removing its temporal average  $\overline{(\cdot)}$  position, 161  $\mathbf{x}_{c}' = (x', y')$ , where  $x'(t_{d}) = x(t_{d}) - \overline{x}$  and  $y'(t_{d}) = y(t_{d}) - \overline{y}$ . 162 The fluctuating part  $x_c'$  is separated in k vectors of equal size 163 L(KL = N) and arranged in matrix form as 164

$$\mathbf{X} = \begin{bmatrix} x'_1 & x'_2 & \dots & x'_K \\ y'_1 & y'_2 & \dots & y'_K \end{bmatrix}$$
(1)

The dimensions of the assembled matrix are 2LxK. Each column in Eq. 1 is considered a snapshot and represents the 166



(a) PIV vertical plane.

(b) PIV horizontal plane.

**Figure 1:** Experimental setup of a bottom-fixed cylinder subjected to a range of turbulent flows. Cylinder oscillations in transverse (y-axis) and longitudinal (x-axis) directions. The z-axis lies along the span of the cylinder. a) Vertical PIV plane. b) Horizontal PIV plane.

<sup>167</sup> trajectory traced by the cylinder over *L* data points. Thus, <sup>168</sup> the goal of using POD is to find the underlying coherent mo-<sup>169</sup> tion across all snapshots. Given a number of spatial modes <sup>170</sup>  $\Phi_n$  and their corresponding modal coefficient  $\alpha_n$ , the POD <sup>171</sup> method finds the best fit for *X* in a least-square sense

$$\left\| \boldsymbol{X} - \sum_{n=1}^{K} \alpha_n \boldsymbol{\Phi}_n \right\| \tag{2}$$

172

where  $\|\cdot\|$  is the L<sub>2</sub>-norm, and n = [1, 2, ..., K]. The POD method solves Eq. 2 through the solution of the following Eigenvalue problem

$$C\Phi_n = \lambda_n \Phi_n \tag{3}$$

176

where  $C = X^T X$  is the autocovariance matrix, and  $\lambda_n$ are the eigenvalues. The POD modes are orthonormal to each other and the eigenvalues represent the contribution of mode *n* to the total variance. The POD modes are usually arranged in descending order based on their corresponding  $\lambda_n$  to identify dominant patterns in the data. The relative of the i-th POD modal value is defined as

$$\epsilon_i = \frac{\lambda_i}{\sum_{n=1}^K \lambda_n} \tag{4}$$

#### **3. Experimental Setup**

The experiments were performed at the Civil and Structural Engineering water laboratory, University of Sheffield, United Kingdom. The flume was covered with clear cast acrylic sheets, leaving a squared cross-sectional area with width 486 mm and a longitudinal fixed slope of 0.001 m/m. A water depth of  $H_w = 347$  mm was fixed using a computer-190 controlled system and a control gate located at the end of the 191 flume. The Reynolds number  $R_e$  ranged between  $4.5 \times 10^2$ 192 and  $1 \times 10^3$ , which corresponds to the maximum flow rate of 193 the facility. Here,  $R_e = U_{inlet}D/v$ , where  $U_{inlet}$  is the mean 194 incoming flow velocity, D is the diameter of the cylinder, and 195 v is the kinematic viscosity of water. The incoming turbulent 196 intensity was measured at 5% for all tested flow velocities. 197 A 5 mm diameter hole with 10 mm depth was drilled on a 198 squared acrylic base with width 165 mm. A cylinder made 199 of clear cast acrylic (elastic modulus of  $3.2 \times 10^4$  Kg cm<sup>-2</sup>) 200 was inserted into the hole and chemically welded fabricating 201 a fixed end. The model was placed 10.5 m downstream, en-202 suring that the centre of the cylinder coincided with the mid-203 dle of the flume's width. The cylinder configuration allowed 204 the opposite end to vibrate in both the in-line (x-axis) and 205 transverse (y-axis) flow directions. The z-axis lies along the 206 span of the cylinder. The cylinder had a diameter of 5 mm, 207 length of 491 mm, and  $m^*$  of 1.41. Figure 1 shows a sketch 208 of the experimental setup. 209

The measurements were taken using a three-camera PIV 210 system, consisting of a double-pulsed 532 nm wavelength 211 Nd: YAG compact Laser of 200 mJ maximum power output, 212 three MX 4M cameras of 2048x2048 pixel resolution, and 213 a Programmable Time Unit (PTU) used to synchronise the 214 cameras and laser trigger times. Higham and Brevis (2018) 215 used the same water channel and PIV equipment to measure 216 the wake around multiple obstacles. The water was seeded 217 with Polyamide 12 of 100 µm mean particle size and 1.06 218  $g \, cm^{-3}$  density. Two cameras recorded the spatio-temporal 219 motion of these particles. Simultaneously, a third camera 220 recorded the free end response of the cylinder. After ad-221 justing the cameras and laser position, a calibration plate 222 LaVision model 309-15 was placed on the desired measure-223 ment plane, and an image was taken. The markers within 224 the calibration plate were used to correct the measurements 225 for optical distortions and to establish a correspondence be-226



**Figure 2:** Cylinder free end response at  $4.5 \times 10^2 \le R_e \le 1 \times 10^3$ . a) Spatiotemporal displacement. b) Maximum displacement in the crossflow direction. c) Maximum displacement in the streamwise direction.

tween a Pixel and real-world coordinates. The wake zone 227 and cylinder response were measured at six flow rates at 70 228 Hz for two minutes. The PIV measurement consisted of a 229 vertical plane through the cylinder centreline and four hori-230 zontal planes at (x, y, z) = (x, y, [20, 34, 52, 60]D). Figure 1 231 shows a sketch of a vertical (Figure 1a) and horizontal (Fig-232 ure 1b) measurement. A multiple-pass correlation process 233 with subpixel accuracy of 0.1 pixels were used to determine 234 the flow velocity field from the flow images. An initial inter-235 rogation window of 64x64 pixel with two passes, followed 236 by a 32x32 pixel window with three passes were employed. 237 In each correlation, an overlap of 75% between interrogation 238 windows was used to increase the resolution of the velocity 239 field. The maximum spatial resolution was 0.39 mm. The 240 images of the cylinder free end response were calibrated us-241 ing the same procedure but with a smaller calibration plate 242 LaVision model 058-5. The cylinder response was estimated 243 using the image-based tracking technique described in Mella 244 et al. (2019). 245

A free decay test was conducted on the bottom-fixed cylin-246 der to determine its damping ratio and the natural frequency 247 measured in air  $f_{air}$  and still water  $f_{water}$ . The cylinder was 248 subjected to a uni-dimensional displacement parallel and per-249 pendicular to the flow direction. A PS3 Eye Camera was 250 placed on the free end of the cylinder and recorded its re-251 sponse. Each video sequence was taken at 187 Hz with a 252 320x240 pixel resolution. The images were calibrated us-253 ing the same procedure described for the cylinder free end 254 response. The results show that the natural frequency mea-255 sured in air and still water is equal in both directions, with 256 values of  $f_{air} = 6.4$  Hz and  $f_{water} = 5.3$  Hz respectively. 257 Considering a logarithmic decay response, the damping ratio 258 measured in air was estimated at 4%. The free decay test was repeated after all the tests were completed with no degrada-260 tion on the dynamical properties of the cylinder. 261

#### 4. Results

#### 4.1. Cylinder response and modal decomposition

Figure 2 summarises the cylinder response over the 264 range of tested  $U_{\rm r}$ . Figure 2a shows the spatiotemporal free 265 end displacement at  $U_r = [3.38, 5.15, 7.5]$  obtained with 266 the image-based tracking technique described in Mella et al. 267 (2019). The mean streamwise free end position of the cylin-268 der moves downstream as  $U_r$  increases, reaching a maxi-269 mum constant displacement of 2.5D and an inclination of 270 approximately 1.5° at the maximum flow velocity. Figure 271 2a suggests that the total cylinder response is composed of 272 a main coherent motion with a superimposed random com-273 ponent. The visualisation of this large-scale pattern will be 274 addressed later by means of the POD technique. Figures 275 2b and 2c show the maximum displacement in the stream-276 wise  $A_x$  and crossflow  $A_y$  direction. Following the defini-277 tion of Hover et al. (1998), the maximum displacement in 278 a given direction is the mean value of the highest 10% of 279 the recorded response. The maximum displacement in both 280 directions increases with  $U_r$ , reaching a maximum value of 281  $A_{\rm v} = 0.74D$  and  $A_{\rm x} = 0.04D$  at  $U_{\rm r} = 5.15$ . Then, at fur-282 ther increments of  $U_{\rm r}$ , the maximum displacement decreases 283 reaching  $A_v = 0.43D$  and  $A_x = 0.03D$  at  $U_r = 7.5$ . The ra-284 tio  $A_v/A_x$  ranges from 8 at  $U_r = 3.18$  to 19 at  $U_r = 6.04$ , 285 indicating an overall predominance of the crossflow motion 286 over its streamwise counterpart. 287

Eight minutes of displacement data were separated in 288 vectors of equal length L and arranged in matrix form as 289 described in section 2. Given a fixed measurement time, the 290 selection of L is a trade-off between the number of snap-291 shots used for POD and the cylinder trajectory traced within 292 each snapshot. Convergence of the first (average) and sec-293 ond (variance) order statistics is achieved at L = 560 data 294 points. Thus, snapshots of L = 700 (53 motion cycles on av-295 erage) were selected for analysis. The relative modal value 296  $\varepsilon_i$  of the first eight modes (i = [1, ..., 8]) is shown in Figure 297 3a. The first two modes capture an important part of the total 298

262



**Figure 3:** Relative modal values of the cylinder response. a)  $\varepsilon_i$ , where i = [1, 2, ..., 8]. b)  $\varepsilon_1 + \varepsilon_2$ 

variance at higher cylinder responses  $(U_r > 5)$  as opposed 299 to lower displacements ( $U_r < 5$ ), from which the relative 300 energy of higher-order modes are significant. It is expected 301 that the first two modes contain relevant trajectory patterns 302 underlying the total displacement for  $U_r > 5$  and that the mo-303 tion history for  $U_r < 5$  has no coherent spatial mode. Figure 304 3b shows the contribution of  $\varepsilon_1 + \varepsilon_2$  at different  $U_r$ .  $\varepsilon_1 + \varepsilon_2$  in-305 creased up to 45% at the maximum cylinder response. Then, 306 it jumped to 71% when the maximum crossflow displace-307 ment decreased from its maximum value of  $A_v = 0.74D$  to 308  $A_v = 0.7D$ . Lastly, it decreased to 63% when  $\dot{A_v} = 0.43D$  at 309 the highest tested  $U_r$ . This increment in the predominance 310 of the trajectory pattern to the total body motion after the 311 cylinder reached its maximum response could be explained 312 by a transition to the lower branch, characterised by a higher 313 periodic motion compared to the upper branch (Khalak and 314 Williamson (1999)). 315

Figure 4 shows the reconstructed displacement signal 316 adding the spatial modes  $\Phi_1 + \Phi_2$ . Higher order modes 317 only contributed to the irregularity of the trajectory pat-318 terns for all  $U_{\rm r}$ . Irregular modal shapes were obtained for 319  $U_r < 5$ . Clockwise elliptical-type trajectories were identi-320 fied for  $U_r = 5.15$  and  $U_r = 6.04$ . Elliptical trajectories 321 are associated with strong structural coupling between the 322 streamwise and transverse motion (Kheirkhah et al. (2012, 323 2016)). Similar findings were observed in Oviedo-Tolentino 324 et al. (2014) for a bottom-fixed cylinder with  $m^* = 8.13$ . 325 Pure elliptical-type trajectories were observed for  $U_r > 5$ , 326 while irregular shapes were obtained at lower reduced veloc-327 ities. The researchers suggested that irregular shapes could 328 be associated with a high dependence of the added mass to 329 low  $U_r$  values. A deviation from a pure elliptical-type tra-330 jectory is observed at  $U_r = 7.5$ . A Power Spectral Density 331 (PSD) analysis of its second spatial mode showed that the 332 energetic value of the first harmonic in the streamwise di-333 rection is 83% of its main frequency. The influence of this 334 harmonic explains this particular combination between an 335 elliptical- and eight-type trajectory. Despite the unclear tra-336

Table 1Streamwise and crossflow normalised frequencies of the firsttwo modes at different  $U_r$ 

Axis	Mode	U <sub>r</sub>					
		3.38	4.03	4.5	5.15	6.04	7.5
Х	1	0.76	0.86	0.90	1.01	1.02	1.04
	2	0.76	1.68	0.90	1.01	1.02	1.03
Y	1	0.76	0.84	0.89	1.01	1.02	1.03
	2	0.76	0.84	0.89	1.01	1.02	1.03
	$f_{\rm c}$	0.76	0.85	0.90	1.01	1.02	1.04

jectories in Figure 4, the main vibration frequencies of the 337 first two spatial modes were successfully extracted using a 338 PSD analysis. The results are summarised in Table 1. The 339 main frequency in the streamwise  $f_x$  and crossflow  $f_y$  di-340 rection was normalised by  $f_{\rm water}.$  Here, the average of the 341 first mode between  $f_x$  and  $f_y$  is defined as the main peak frequency of the cylinder  $f_c$ . The results show that  $f_x$  and 342 343  $f_{\rm v}$  have similar values throughout  $U_{\rm r}$ , which is consistent 344 with an elliptical-type trajectory.  $f_{\rm c}$  ranges between 0.76 to 345 1.03 as  $U_r$  increases. The cylinder achieves its maximum re-346 sponse when  $f_c \approx 1$  at  $U_r = 5.15$ , which is consistent with 347 the findings of Oviedo-Tolentino et al. (2014). 348

#### 4.2. Spanwise synchronisation region

Spanwise wake dynamics were analysed using two-350 dimensional PIV measurements at a vertical plane through 351 the cylinder centreline and four horizontal planes at 352 (x, y, z) = (x, y, [20, 34, 52, 60]D). The lowest flow mea-353 surements ( $U_r = 3.38$  and  $U_r = 4.03$ ) are not presented in 354 this section as the results are similar to  $U_{\rm r}=4.5$ . Figure 5 355 shows the reduced velocity profile at 8D upstream from the 356 centre of the cylinder  $U_r(x = -8D, y, z)$ . Dashed lines in-357 dicate the average incoming reduced velocity. The velocity 358 profile resembles a parabolic distribution as expected from 350



Wake development of a bottom-fixed cylinder subjected to VIV

Figure 4: Reconstructed cylinder response from  $\Phi_1 + \Phi_2$ . From top left to bottom right:  $U_r = [3.38, 4.03, 4.5, 5.15, 6.04, 7.5]$ .



**Figure 5:** Spanwise reduced velocity profile at 8*D* upstream from the centre of the cylinder.  $U_r(x = -8D, 0, z)$ .  $\Leftrightarrow$ : 4.5,  $- \circ$ : 5.15,  $- \circ$ : 6.04,  $- \circ$ : 7.5. Dashed lines are the average incoming reduced velocity calculated using  $U_{inlet}$ 

open-channel flows. Approximately 90% of the average incoming reduced velocity is achieved at z > 18D. As will be shown later, the velocity gradient at z < 18D had a limited impact on the spanwise synchronisation between the vortex shedding and cylinder main motion frequency.

The spatiotemporal variability of the streamwise velocity component u(x, y, z, t) was analysed to compare its vor-

tex shedding frequency  $f_v$  to the main peak frequency of 367 the cylinder  $f_c$ . The streamwise velocity is decomposed 368 as  $u(x, y, z, t) = u'(x, y, z, t) + \overline{U}(x, y, z)$ , where u' is the 369 fluctuating component, and U is the time-averaged veloc-370 ity. The PSD of u'(4.8D, 0, z, t) was calculated at each grid 371 point within z. Figure 7 shows the main peak frequency of 372 the PSD normalised by  $f_c$ . Across all reduced velocities, 373 there is a region between 9D and 16D from the bed surface 374 where  $f_v < f_c$ . This low  $f_v$  region is the result of the in-375 teraction between a sheared incoming flow velocity, which 376 is produced by the parabolic velocity profile of the open-377 channel shown in Figure 5, and the presence of the cylin-378 der with a small response amplitude in that region. Analy-379 sis of the mean streamwise velocity field downstream of the 380 cylinder showed that  $\overline{U}(4.8D, 0, z) \approx U_{\text{inlet}}/2$  at z < 16D381 which is in line with the observed reduction in the vortex 382 shedding frequency in that region. Figure 7a and 7b show 383 good agreement between  $f_v$  and  $f_c$  at z > 16D for  $U_r = 4.5$ 384 and  $U_r = 5.15$  respectively. Specifically, Figure 7b shows 385 that the maximum cylinder response is achieved when the 386 equivalence  $f_{\rm c} = f_{\rm v} = f_{\rm water}$  is preserved along the span 387 of the cylinder, i.e. when the spanwise synchronisation re-388 gion is maximal. It is worth noting that, despite the parabolic 389 distribution of the incoming velocity profile, the synchroni-390 sation region extends from z = 16D up to the free surface. 391 The constrained desynchronised region of z < 16D for all 392



**Figure 6:** Normalised time series of  $u'(4.8D, 0, z, t)/\overline{U}(4.8D, 0, z)$ .



Figure 7: Normalised vortex shedding frequency measured at 4.8D downstream from the cylinder centre.

<sup>393</sup>  $U_r$  is expected to have a small impact on the maximum cylin-<sup>394</sup> der response due to its proximity to the cylinder fixed end. <sup>395</sup> At higher  $U_r$ , Figure 7c shows an increment of  $f_v$  between <sup>396</sup> 1.2 and 1.3 at  $9D \le z \le 25D$ . Likewise, at the maximum <sup>397</sup> tested  $U_r$ ,  $f_v$  jumped from 1.01 to between 1.65 and 1.70 <sup>398</sup> at  $13D \le z \le 59D$ . In terms of maximum crossflow dis-<sup>399</sup> placement,  $A_v$  reaches its maximum value of 0.74*D* when the synchronisation region is maximal. Then,  $A_y$  decreased 5.4% and 39.7% when the desynchronised region extended 17D and 53D, respectively. Figure 6 shows the time series of u'(4.8D, 0, z, t), normalised by  $\overline{U}(4.8D, 0, z)$ , for  $U_r = 5.15$ and  $U_r = 6.04$ . Here, the normalised time was defined as  $t^* = tU_{inlet}/D$ . Wake patterns are clearly visible for both Ur. The extension of these wake patterns, delimited by hori-



(a)  $U_{\rm r}$ .  $- \bullet$ : 4.5,  $- \bullet$ : 5.15,  $- \bullet$ : 6.04,  $- \bullet$ : 7.5.

**Figure 8:**  $L_{\rm f}$  and maximum  $R_{11}$  at different water depths.

<sup>407</sup> zontal lines, coincided with the regions where  $f_v = f_c$ . The <sup>408</sup> additional horizontal line in Figure 6b delimits an interme-<sup>409</sup> diate region where  $f_v > f_c$ . Overall, Figure 6 and 7 show <sup>410</sup> the development of a bottom-up desynchronisation process, <sup>411</sup> starting at z = 16D and moving towards the free surface as <sup>412</sup>  $U_r$  increases.

Wake dynamics as the desynchronisation process devel-413 ops are analysed in terms of fluctuating velocity fields. Fig-414 ure 8 shows the vortex formation length  $L_{\rm f}$  and the max-415 imum normal Reynold Stress  $R_{11} = \overline{u'u'}/U_{\text{inlet}}^2$  along the 416 wake centreline at (x, y, z) = [20, 34, 52, 60]D. The for-417 mation length corresponds to the downstream distance from 418 the cylinder surface where the maximum  $R_{11}$  is achieved. 419  $L_{\rm f}$  and the maximum  $R_{11}$  are plotted against its correspond-420 ing local maximum crossflow cylinder displacement  $A_{y}(z)$ , 421 which was estimated assuming a linear-elastic response. A 422 general reduction of  $L_{\rm f}$  is observed as  $A_{\rm v}(z)$  increases in 423 the synchronised region (see  $U_r = [4.5, 5.15]$  in Figure 424 8a). In contrast, the maximum  $R_{11}$  increases with  $A_v(z)$ . 425 These results are in line with Bearman (1984), which in-426 dicated that higher cylinder displacements lead to stronger 427 and shorter vortex formation near the cylinder with a subse-428 quent stronger vortex shedding. At  $U_r = 6.04$ ,  $L_f$  increases 429 from 2.16D to 2.35D as the plane of measurement moves 430 from a desynchronised region (z = 18D) to a synchronised 431 one (z = 34D). Then,  $L_{\rm f}$  restores it inverse relationship 432 with  $A_{\rm v}(z)$  for z > 34D in the synchronised region. At 433  $U_{\rm r} = 7.5$ , where only  $z \ge 59D$  is synchronised,  $L_{\rm f}$  ranges 434 between 2.2D and 2.5D and no clear trend is observed. In 435 contrast, the maximum  $R_{11}$  slowly increases from 0.26 to 436 0.28 throughout its desynchronised region (z < 59D) and 437 then jumps to 0.36 at z = 60D, where the cylinder and wake 438 are still synchronised. A significant reduction in Lf and max-439 imum  $R_{11}$  is observed at z = 60D for  $U_r = 4.5$ . The  $R_{11}$  dis-440 tribution along the wake centreline shows a double peak in 441 its vortex formation region which suggest the confluence be-442 tween two vortex pattern configurations. Nevertheless, fur-443 ther research needs to be conducted to analyse this particular 444 case. 445



(b)  $U_{r}$ .  $- \bullet$ : 4.5,  $- \bullet$ : 5.15,  $- \bullet$ : 6.04,  $- \bullet$ : 7.5.

The desynchronisation process described above can be 446 partially explained by the particular characteristics of the 447 tested bottom-fixed cylinder. Flemming and Williamson 448 (2005) showed that, for small streamwise amplitudes, cylin-449 ders with two- and one-DOF free-vibration agree qualita-450 tively well. Furthermore, Williamson and Roshko (1988) in-451 dicated that the synchronisation range, dependent on the tim-452 ing between the cylinder-fluid acceleration, increases with 453 the cylinder displacement. The bottom-fixed cylinder has a 454 dominant crossflow response up to 19 times its streamwise 455 counterpart. Consequently, the range of crossflow response 456 acceleration along the span of the cylinder, which dimin-457 ishes to zero approaching its fixed end, is largely responsi-458 ble for the range of  $U_r$  in which synchronisation occurs. As 459  $U_{\rm r}$  increases and the cylinder motion is significant, the syn-460 chronisation region along the span of the cylinder is maxi-461 mal, and the vortex shedding frequency is locked-in to the 462 cylinder motion frequency. This relationship is preserved 463 at higher  $U_r$  until  $f_c = f_{water}$ , where the maximum cylin-464 der displacement is achieved. Then, at further increments of 465  $U_{\rm r}$ , the cylinder displacement near its fixed end is not able 466 to reach the needed increment in acceleration to sustain a 467 synchronised condition, and desynchronisation occurs. As a 468 consequence, the vortex region strength along the span of 469 the cylinder decreases with a subsequent reduction in the 470 cylinder response. This desynchronisation process is en-471 hanced as the desynchronised region develops towards the 472 water surface, with a higher percentage of the cylinder re-473 sponse along its span being unable to sustain lock-in, reduc-474 ing the overall strength of the vortices, and causing a sys-475 tematic reduction in the cylinder response. Further changes 476 are observed in the vortex pattern as the desynchronisation 477 region progresses. Figure 9 shows the contours of normal 478 Reynold stress  $R_{11} = \overline{u'u'}/U_{\text{inlet}}^2$  at z = [34, 52, 60]D. Im-479 portant changes are observed as z increases, specifically at 480 z = 60D. Following the contour distribution in Govardhan 481 and Williamson (2001), a transition from 2S vortex mode 482 (Figure 9a) to 2P (Figures 9b and 9c) is observed. Then, 483 Figure 9d suggest a transition back to a 2S vortex pattern 484

Wake development of a bottom-fixed cylinder subjected to VIV



**Figure 9:** Contours of  $\overline{u'u'}/U_{\text{inlet}}^2$  (contour interval = 0.035) at z = 60D (a-d), z = 52D (e-h) and z = 34D (i-l). Grey line: maximum contour value.

<sup>485</sup> at the highest  $U_r$ . A 2S-2P dual-mode configuration is ob-<sup>486</sup> served across the span of the cylinder for  $U_r = 5.15$  (Figures <sup>487</sup> 9b, 9f, and 9j) and  $U_r = 6.04$  (Figures 9c, 9g, and 9k).

### 5. Conclusions

This study analyses the spanwise vortex shedding dynamics and structural response of a bottom-fixed cylinder subjected to a range of open-channel, fully developed turbulent flows. Planar PIV measurements and a synchronised

single camera were used to capture the flow around the wake 493 region and the cylinder free end response. The results showed 494 that the POD technique successfully uncovers the main tra-495 jectory patterns in cases where the cylinder response is highly 496 modulated. The patterns uncovered for the tested cylinder 497 correspond to a clockwise elliptical-type trajectory for  $U_r >$ 498 5. A deviation from a pure elliptical-type trajectory was 499 found at  $U_r = 7.5$ , which is given by the relation between 500 the main streamwise frequency of its second spatial mode 501 and its first harmonic. Wake dynamic analysis showed that 502 the maximum response is achieved when the cylinder mo-503 tion and vortex shedding frequencies are equal (i.e. synchro-504 nised) to the natural frequency of the structure measured in 505 still water, and when this equivalence is preserved along the 506 span of the cylinder. As  $U_r$  increases, the cylinder displace-507 ment near its fixed end is not able to reach the needed in-508 crement in acceleration to sustain a synchronised condition, 509 and desynchronisation occurs. As a consequence, the vor-510 tex strength is reduced across the span of the cylinder with 511 a subsequent decrement in the cylinder response. As  $U_r$  is 512 further increased, the desynchronised region progresses to-513 wards the water surface alongside further decrements in vor-514 tex strength and a systematic reduction in the cylinder re-515 sponse. Changes in the wake dynamics as the desynchro-516 nised region progresses and the cylinder response decreases 517 showed a transition from a 2S-2P dual-mode configuration 518 at the highest cylinder response to a predominant 2S mode. 519 The results showed that, despite the highly three-dimensional 520 experimental conditions under significant turbulent incom-521 ing flows, the findings of previous studies based on simpler 522 experimental models can still be used to broadly explain the 523 observed bottom-up desynchronisation process. 524

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