This is a repository copy of *Multi-criteria decision making using Fuzzy Logic and ATOVIC with application to manufacturing*.

White Rose Research Online URL for this paper:
https://eprints.whiterose.ac.uk/166731/

Version: Accepted Version

Multi-criteria decision making using Fuzzy Logic and ATOVIC with application to manufacturing

Hesham Yusuf
Automatic Control & Systems Engineering
University of Sheffield
Sheffield, UK
hahyusuf1@sheffield.ac.uk

George Panoutsos
Automatic Control & Systems Engineering
University of Sheffield
Sheffield, UK
g.panoutsos@sheffield.ac.uk

Abstract—In this paper multi-criteria decision making (MCDM) is investigated as a framework for classification of part quality in a manufacturing process. The importance of linguistic interpretability of decisions is highlighted, and a new framework relying on the integration of Fuzzy Logic and an existing MCDM method is proposed. ATOVIC, previously developed as a TOPSIS-VIKOR-based MCDM framework is enhanced with a Fuzzy Logic framework for decision making - Fuzzy-ATOVIC. This research work demonstrates how to add linguistic interpretability to decisions made by the MCDM framework. This contributes to explainable decisions, which can be crucial on numerous domains, for example on safety-critical manufacturing processes. The case study presented is the one of ultrasonic inspection of plastic pipes, where thermomechanical joining is a critical part of the manufacturing process. The proposed framework is used to classify (take decisions) on the quality of manufactured parts using ultrasonic images around the joint region of the pipes. For comparison, both the original and the Fuzzy Logic-enhanced MCDM methods are contrasted using data from manufacturing trials and subsequent ultrasonic testing. It is shown, that Fuzzy-ATOVIC provides a framework for linguistic interpretability while the performance is the same or better compared to the original MCDM framework.

Index Terms—multi-criteria decision making, ATOVIC, Fuzzy Logic, classification, manufacturing, ultrasonic testing.

I. Introduction

Interpretability in Machine Learning (ML) and data-driven systems in general, has been an area of interest among researchers [1]–[7]. The potential to trace and explain data, predictions and decisions is one of the main motivations for this research interest. The interpretation, for example, of the outputs of a data-driven model can be crucial for safety-critical applications such as defect classification in advanced manufacturing. Locating and characterising part defects can be a critical step in the manufacturing life-cycle, for example when manufacturing gas pipes, aerospace parts, medical parts

etc. Black-box models and ML frameworks that do not provide inherent interpretability traits can be challenging for users to derive justifications, hence limiting interpretability to meta-analysis of model performance. There are two main categories for model interpretability, model-based (inherent) and meta-analysis (post-hoc) of model behaviour [3].

The next section, Section II entails a review of research work demonstrating the use of Multi-Criteria Decision Making (MCDM) for classification, as well as how interpretability is defined and assessed within a data modelling context. The section after that, Section III describes the proposed Fuzzy-ATOVIC framework after a summary of the original ATOVIC framework proposed by Baccour in 2018 [8]. Section IV describes the results of using a MCDM-based technique - Amended fused TOPSIS-VIKOR for Classification (ATOVIC) - for the classification of ultrasonic images. The ATOVIC model is then modified to include a Fuzzy Inference System (FIS) in an attempt to enhance the interpretability of the model. The discussion in Section IV is on whether the modifications are an improvement on the original ATOVIC model defined by Baccour in 2018 [8]. The final section, Section V, describes the conclusion and future work.

II. Literature Review

A. Multi-criteria Decision Making

MCDM are a set of computational and mathematical techniques for assessing a set of alternatives, based on often several conflicting *criteria* such as various costs and benefits. Despite the varying complexity level of different MCDM methods, they are often simpler than Artificial Neural Networks (ANN) and other complex ML techniques. MCDM techniques are used in several applications, a review disclosed that the largest number of review literature on MCDM was from energy fuels while the second largest was operations research management science [9]. In the same reference, in advanced manufacturing there is an example that reviews the use of TOPSIS. Despite popularity of TOPSIS for advanced manufacturing, it was rarely used for classification [11]–[14]. There are two literatures that demonstrate application of TOPSIS for classification [8], [15]. Baccour's literature proposing Amended Fused TOPSIS-VIKOR for Classification (ATOVIC) illustrates convincing results where ATOVIC in several cases

outperforming state-of-the-art classification techniques such as Naive Bayes, Augmented Nonogram and Logistic Regression [8]. Baccour presents ATOVIC performance results for nine different experiments, in comparison with results from literature for state-of-the-art classifiers. The first application was the CLEVELAND dataset in which three different experiments were conducted with variations of how the data subsets were selected. Experiment one compared ATOVIC to ANN and Neuro-fuzzy System. The accuracy of ATOVIC was the second best for the models compared, at 81.9%, while ANN and Neuro-fuzzy models had accuracies of 82.3% and 67.2% respectively. The ATOVIC accuracy for experiment one was comparable to ANN which is impressive because, as opposed to ANN, it does not use any optimisation or training.

Piegat et. al compares TOPSIS, AHP and a new proposed MCDM-based classification method called Characteristics Object Method (COMET) [15]. COMET outperforms the other techniques at 96.6%. However, when comparing COMET and ATOVIC, the latter was supported by a greater deal of evidence with results from more experiments. In addition, Baccour's method ATOVIC was simpler and tailored significantly for data-driven classification.

Interpretability in modelling has two categories: model-based and post-hoc [3]. The sources of interpretability are illustrated in Fig. 1. As defined by Lipton, in [3], Model-based interpretability consists of algorithmic transparency, decomposability and simulatability. Algorithmic transparency becomes useful for the model training as it enables more understanding of how the model learns from data and is constructed. On the other hand, decomposability and simulatability are useful for model execution. When a model is decomposable, its different components can be divided into different parts, the purpose of each is understood. Hence, data from different decomposable components can be used to extract explanation for a certain output. When a model is simulatable, it means the complexity of its components is low enough for a human to be able to understand how the models variables and parameters are used to determine the output. Simulatability becomes vital for humans trying to construct the models.

III. PROPOSED METHODOLOGY: FUZZY-ATOVIC

Amended fused TOPSIS-VIKOR for Classification (ATOVIC) as referred to by researcher Leila Baccour is an MCDM-based classification framework envisaged from TOPSIS and VIKOR; both MCDM methodologies. The ATOVIC framework, as described by Baccour, was developed in two main steps: fusion and amendment. In the first step, fusion, Baccour combines two techniques: TOPSIS and VIKOR. Both techniques are MCDM-based techniques and have the same five steps:

1) Define decision matrix X
2) Determine weights of criteria w
3) Categorise criteria as either *cost* or *benefit*
4) Calculate positive and negative ideal solutions (f^-, f^+)
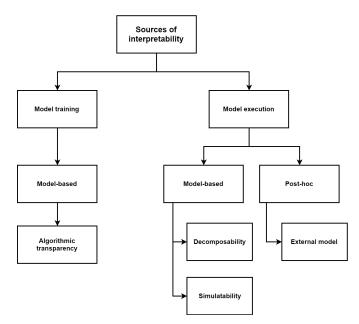

Fig. 1. Illustration of difference between ATOVIC and Fuzzy-ATOVIC models' structures.

5) Find best ranked alternative (solution) using a combination of several distancing techniques and logical operations.

Baccour fuses TOPSIS and VIKOR by combining the strong suits of both techniques to form TOPSIS-VIKOR. The fusion results in a TOPSIS-VIKOR technique not tailored for classification hence step two: amendment. In step two, ATOVIC is formed by amending TOPSIS-VIKOR to formulate it for the classification problem by creating relevant variable names *features* and *objects* as opposed to *criteria* and *alternatives*. Step five is altered in ATOVIC to perform classification based on the values of the three distancing *measures*: Q, R and S. In addition, the structure was extended to allow for binary or multi-class classification by repeating steps 3-5 of the model for each class.

ATOVIC works by using a decision matrix X which contains the *features* of the *objects*. The decision matrix X is then normalised. The weights for the different *features* are assigned manually or using a mathematical formula. The decision matrix X is then divided into a reference and test dataset, X^r and X_t respectively. For fitting the model, X_r is used, while X_t is used to test the performance. The fitting process entails calculating all the model parameters including the weights, normalisation factors and ideal solutions. Model execution utilises the model parameters to classify the *objects*. Equations (1-8) in Section IV define the model parameters described as per Baccour's original ATOVIC.

The proposed ATOVIC-based methodology uses a FIS in the final step of the classification [8]. The final step of the ATOVIC framework makes use of several *distance variables* to classify the object. The ATOVIC framework although naturally transparent, does not provide a framework for linguistic interpretation. One of the techniques known for interpretability is

Mamdani-type Fuzzy Logic [16]. Hence the proposed Fuzzy-ATOVIC framework was designed to utilise a Mamdani-type FIS to classify the objects using the ATOVIC's *distance variables*. ATOVIC generates three sets of distance variables: S, R and Q. The distancing method for the variables are Manhattan, Chebyshev and weighted sum of both respectively. All distance calculations are normalised. The length of each distance set is the number of classes k squared. To keep things simple and tracable, the Fuzzy-ATOVIC model uses the weighted sum distance Q. For binary classification, the length of Q is four which minimises the number of possible rule combinations, hence, satisfying the decomposability and simulatability guidelines. Fig. 2 illustrates the difference in structure between ATOVIC and Fuzzy-ATOVIC. In ATOVIC, the model classifies the objects using the measures directly, however, in Fuzzy-ATOVIC the measures are used by an FIS to produce the class output. Implementation of Fuzzy-ATOVIC entails constructing an Mamdani-type FIS with human-designed rules.
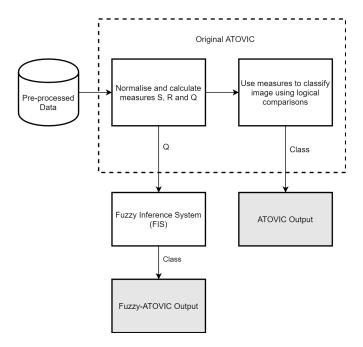

Fig. 2. Illustration of difference between ATOVIC and Fuzzy-ATOVIC models' structures.

IV. RESULTS AND DISCUSSION

The section presents and discusses performance results of the ATOVIC model before and after adding a fuzzy component in the final step of the model.

A. Ultrasonic Testing of Weld joints

The dataset consists of ultrasonic images from Butt-fusion (BF) weld joints in polyethylene pipes of 110mm diameter. The data was collected by NDT experts using a Phased Array Ultrasonic Testing (PAUT) technique which is a multi-probe Ultrasonic Testing (UT) technology capable of generating scans of the internals of the pipe. Although the images are not a exact reflection of what is inside the material, patterns and shapes in the images can be used to recognise different objects in the pipe, some of which can be defects. In this case study, the aim of the modelling work was to classify the images which contain a key indication - the bead indication. The bead is an important object to locate in the images because it acts as a reference for the position of the pipe in the image. The file for each weld consisted of 60 to 70 images each at a different scanning angle. The image type used was B-Scan which combines the A-Scans from the same angle across the scanning circumference of the pipe.

B. Data pre-processing

The ultrasonic pipe weld data was pre-processed to convert it from the raw A-Scan format to B-Scan format which was required for the modelling work. As opposed to complex ML techniques, ATOVIC cannot handle image data directly, hence feature extraction had to be performed. The feature extraction had two steps, initially to extract the features from the images by edge detection. Secondly, to extract the features from the edge data by segmenting the images and calculating statistical variables such as: mean, maximum, minimum and standard deviation.

The weld data consisted of the following number of welds. Hence, it was balanced to ensure both classes had approximately the same number of images for the reference and testing dataset.

- Number of welds: 30
- Number of images per weld: 60-70
- Total number of images: 2030
    - Class 0 (no bead): 1492
    - Class 1 (bead): 538

C. Image Classification Models

Two models were developed and compared, both of which were based on the ATOVIC framework defined by Baccour in 2018 [8]. The first model was implemented according the definition of ATOVIC by Baccour while the second one was an improved algorithm that included a Fuzzy Inference System (FIS) for the final step of the classification. To distinguish it from ATOVIC, the improved model will be referred to as Fuzzy-ATOVIC. Fig. 2, shows the difference in structure between the ATOVIC and Fuzzy-ATOVIC models. The key difference in Fuzzy-ATOVIC is in the final step of classification. In ATOVIC, the measures S, R and Q are used in the final step to determine the class of the image. The process involves a series of logical operations and comparison that make use of all the measures. On the other hand, Fuzzy-ATOVIC uses a Mamdani-type FIS with the Q measure, a weighted normalised sum of the S and R measures, as an input. The S and R measures are defined as the weighted normalised Manhattan and Chebyshev distances respectively. They are calculated in reference to the ideal solutions (positive and negative). A set of ideal solutions exist for each class in the model. In this case study two classes exist: no bead (0) and bead (1). The image data was labelled with one of the two classes. ATOVIC

is data-driven and is fitted using a labelled reference dataset in one iteration.

The ATOVIC fitting process includes the following [8]:

- Normalisation of reference matrix according to (1,2). Where θ is the normalised term and x is the non-normalised term coming from the reference matrix X^r. The letter r denotes reference matrix while i and j are matrix positions (row and column). The letter p is the class number and ranges from 1 to number of classes k. The eigenvector $h^r_{j_p}$ is calculated for each class p using (2).
- Calculation of weights w using (3). The weight of each feature corresponds to its standard deviation divided by the sum of all standard deviations for all the features.
- Categorising features as either a *cost* or *benefit*. Where C and B store the indexes of the costs and benefits respectively. These were determined after examining graphical data and observing how features vary for different classes. If a feature increases for a particular class, then it is a benefit for the class. Meanwhile, if a feature decreases for a particular class then it is a cost for the class.
- Calculation of positive and negative ideal solutions f_p using (5).

$$\theta^r_{ij_p} = \frac{x^r_{ij}}{h^r_{j_p}} \tag{1}$$

$$h^r_{j_p} = \sqrt{\sum_{i=1}^{m^r}(x^r_{ij_p})^2} \tag{2}$$

$$w^r_j = \frac{\sigma^r_j}{\sum_{j=1}^{n}\sigma^r_j} \tag{3}$$

$$f^+_p = \left\{\theta^{r^+}_1, \theta^{r^+}_2, \ldots, \theta^{r^+}_n\right\}$$
$$= \left\{(max_i\theta^r_{ij_p}/j \in B), (min_i\theta^r_{ij_p}/j \in C)\right\} \tag{4}$$

$$f^-_p = \left\{\theta^{r^-}_1, \theta^{r^-}_2, \ldots, \theta^{r^-}_n\right\}$$
$$= \left\{(min_i\theta^r_{ij_p}/j \in B), (max_i\theta^r_{ij_p}/j \in C)\right\} \tag{5}$$

The measures S, R and Q are calculated by (6-8). The equations (6-8) use the positive and negative ideal solutions f, to determine the weighted normalised Manhattan and Chebyshev distances, S and R respectively. The weights w specify how much effect the different features have on the distance calculation. Q is the normalised weighted sum of S and R adjusted by the weighted sum variable ρ.

$$S_{c_i} = \sum_{j=1}^{n} w_j * (f^+_{ij_c} - \theta^t_{ij_c})/(f^+_{ij_c} - f^-_{ij_c}), \ S_{c_i} \in [0,1] \tag{6}$$

$$R_{c_i} = \max_j \left[w_j*(f^+_{ij_c} - \theta^t_{ij_c})/(f^+_{ij_c} - f^-_{ij_c}) \right], \ R_{c_i} \in [0,1] \tag{7}$$

$$Q_{c_i} = \rho\frac{S_{c_i} - S^-_c}{S^+_c - S^-_c} + (1-\rho)\frac{R_{c_i} - R^-_c}{R^+_c - R^-_c} \tag{8}$$

The ATOVIC model utilised two sub-models, one for each class. Therefore, each of the measures consists of four distances:

- Using model optimised for class one (no bead):
  - Distance to class one (1,1)
  - Distance to class two (1,2)
- Using model optimised for class two (bead):
  - Distance to class one (2,1)
  - Distance to class two (2,2)

The rules for the fuzzy component of Fuzzy-ATOVIC were selected based an understanding of what the values of Q mean. The understanding was translated into rules that the FIS can use to take a decision. Table I shows the settings used for the FIS for the different fuzzy methods. All MFs were of the same type: trapezoidal. The FIS used in the Fuzzy-

TABLE I
MAMDANI FIS CONFIGURATION

Param	Value
MF types	Trapezoidal
And method	Minimum
Or method	Maximum
Implication method	Minimum
Aggregation method	Maximum
Defuzzification method	Centroid

TOPSIS contained 16 rules for all the possible combinations of antecedents. The inputs to the FIS is the Q measure while the output is the image class as a continuous output approximately ranging from 0 to 1. The fuzzy output is saturated between 0 and 1, then rounded to a whole number to determine class. When storing the classes, class one is 0 and class two is 1. Table II lists the rules for the FIS. There are two membership functions (MFs) for each input, one for *high* values and the other for *low*. Three output MFs were used, one for each class and one for uncertain results. The uncertain MF will still result in a classification however it will be closer to the threshold region and can be either class depending on the inputs. For the input MFs, when the *low* is fired, it means the image is likely to be a member of that class. For instance, the most certain case of an image being a member of class one (no bead) is when both $Q_{1,1}$ and $Q_{2,1}$ fire *low*, and everything else fires *high* - as in rule 6. The high certainty case for class two (bead) is rule 11. The second level of certainty is when both models agree on *low* distance for a certain class, however do not agree on *high* distance for the opposite class or vice versa. In this case, the consequent is set to the class which both models agree on having *low* distance.

Table III shows the accuracy performance of ATOVIC and Fuzzy-ATOVIC. ATOVIC had more consistent values with all standard deviations lower than 8%. Furthermore, the Fuzzy-ATOVIC model had larger standard deviations that tend to increase with higher mean accuracy - the highest at 15.2%. When further examining the mean accuracies of both models, they indicate that ATOVIC works better the lower the ρ. Meanwhile, it is the opposite case for Fuzzy-ATOVIC. The

TABLE II
FIS RULES FOR FUZZY-ATOVIC MODEL

Rule no.	Antecedents (input MF)				Consequent (output MF)
	$Q_{1,1}$	$Q_{1,2}$	$Q_{2,1}$	$Q_{2,2}$	
1	Lo	Lo	Lo	Lo	Uncertain
2	Lo	Lo	Lo	Hi	No bead
3	Lo	Lo	Hi	Lo	Bead
4	Lo	Lo	Hi	Hi	No bead
5	Lo	Hi	Lo	Lo	No bead
6	Lo	Hi	Lo	Hi	No bead
7	Lo	Hi	Hi	Lo	Uncertain
8	Lo	Hi	Hi	Hi	No bead
9	Hi	Lo	Lo	Lo	Uncertain
10	Hi	Lo	Lo	Hi	Bead
11	Hi	Lo	Hi	Lo	Bead
12	Hi	Lo	Hi	Hi	Bead
13	Hi	Hi	Lo	Lo	Bead
14	Hi	Hi	Lo	Hi	No bead
15	Hi	Hi	Hi	Lo	Bead
16	Hi	Hi	Hi	Hi	Uncertain

Key:
Hi: represents the high MF for values ranging from 0.5 and above.
Lo: represents the low MF for values ranging from 0.5 and below.

Fig. 3. Plot of Fuzzy-ATOVIC prediction vs. actual class.

trend indicates that ATOVIC and Fuzzy-ATOVIC are working better with a Q measure biased towards S and R measures respectively. The optimal value of ρ varies with different applications as demonstrated by Baccour [8].

Param (ρ)	ATOVIC ACC (%) $\mu \pm \sigma$	Fuzzy-ATOVIC ACC (%) $\mu \pm \sigma$
0.0	77.6 ± 2.8	61.7 ± 2.4
0.1	72.8 ± 6.7	63.8 ± 5.0
0.2	70.8 ± 7.2	65.6 ± 6.8
0.3	66.1 ± 4.7	72.9 ± 5.0
0.4	63.7 ± 7.4	72.6 ± 11.9
0.5	61.9 ± 7.6	74.9 ± 13.3
0.6	60.1 ± 8.1	80.4 ± 12.0
0.7	59.9 ± 6.0	78.9 ± 13.6
0.8	60.2 ± 6.8	75.1 ± 15.2
0.9	61.6 ± 6.7	76.4 ± 12.5
1.0	61.6 ± 5.4	75.6 ± 12.5

The Fuzzy-ATOVIC model provides a continuous output ranging from approximately 0 and 1. A continuous output when provided by a FIS can enable explanation. The further the Fuzzy-ATOVIC output is from the 0.5 threshold between the two classes, the more certain the model is of the classification result. For example, a value of 0.9 is more certain to be class one than say a value of 0.6 despite both resulting in the same classification.

Fig. 4-6 show examples of TP, TN and FP cases respectively. For the true cases, the Fuzzy-ATOVIC model provided an output of 0.67 for the positive case (bead detected) and 0.37 for the negative case (no bead detected). As shown in Fig. 4, the bead indication is characterised by a yellow ribbon in the same direction as the x-axis at around the 200 A-Scan point. As shown in Fig. 5, similar indications may appear at different
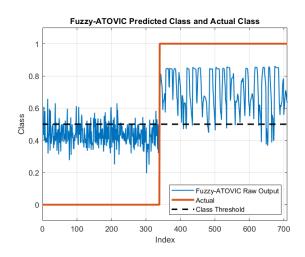
positions but are not from the bead.

For the FN case the output was 0.45 which was relatively close to the threshold. The output being close to the threshold means the model was not as certain as it was for the TP and TN cases. The image in Fig. 6 shows a faint bead indication along the 200 A-Scan position, which explains why the Fuzzy-ATOVIC model's output was only 0.05 lower than the threshold.
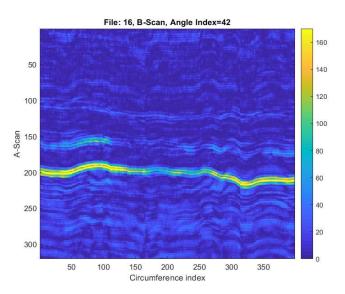

Fig. 4. Example TP case for file 16. Image classified as bead detected correctly.

Fig. 7-9 show screenshot from the rule-firing visualisation tool in Matlab. For the TP case in Fig. 7, the antecedents were high, low, high and low for Q inputs in order. Consequently, rule 11 was fired the output MF was *bead*. Looking at the antecedents it can be confirmed that both sub-models have a consensus on what should be the class of the image, which means there is more confidence of the result. In contrast, the antecedents for the TN case were low-low-low-high. Both

models predicted higher distance to class two (bead detected), which means naturally the FIS should select class one (bead detected), which it did despite the MFs fired were not ideal. Rule 2 was fired which had a consequent of *no bead*.

Fig. 5. Example TN case for file 16. Image classified as no bead detected correctly.
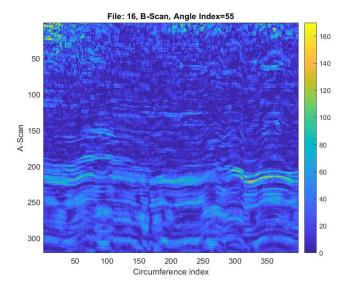

Fig. 6. Example FN case for file 16. Image classified as no bead detected incorrectly.

In the FN case, the antecedents were similar to the TN case: low-low-low-high, which also resulted in a firing of rule 2, hence the negative classification. When looking closely at the FIS inputs, it was noticed that not only are the first two inputs both *low*, but for this case $Q_{1,2}$ was smaller than $Q_{1,1}$. Hence, the continuous output was close to the threshold of 0.5 at 0.447. The output being close to the threshold signifies that although the image was classified as a negative case,

it is quite similar to a positive case. When looking closely at the image, it can be confirmed that the bead signal can be distinguished manually, however it is much fainter than a typical bead indication.
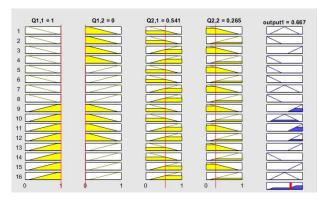

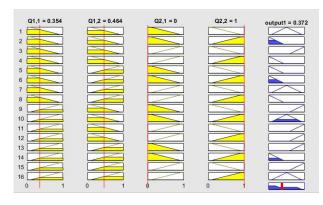Fig. 7. Rule fired GUI shows which MFs where fired at which value for the TP case.

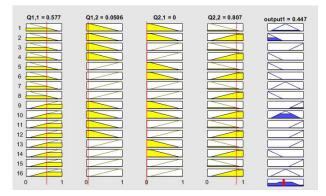Fig. 8. Rule fired GUI shows which MFs where fired at which value for the TN case.

Fig. 9. Rule fired GUI shows which MFs where fired at which value for the FN case.

V. CONCLUSION AND FUTURE WORK

In summary, the ATOVIC and Fuzzy-ATOVIC models were compared for different values of ρ. Accuracy performance

did not distinguish the models greatly. Moreover, for interpretability the Fuzzy-based ATOVIC model provided a more useful output that was continuous which can be used, as demonstrated, to determine how *strong* the classification result is depending on how far it is from the threshold. The same can be done with the ATOVIC model, however more numbers will have to be examined to come to the same conclusion. The FIS output simplified the process of explanation.

In conclusion, employing an FIS did not impact accuracy performance greatly but provided a means for interpretability conventional ATOVIC does not provide. In future work, the Fuzzy-ATOVIC model will be tested further using more datasets. In addition, the Fuzzy-ATOVIC model will be extended further to include a component that can generate human-understandable explanation. Although ATOVIC performed well for the above case study, more work needs to be done to optimise it for large data-driven problems.

REFERENCES

[1] L. H. Gilpin, D. Bau, B. Z. Yuan, A. Bajwa, M. Specter, and L. Kagal, "Explaining Explanations: An Overview of Interpretability of Machine Learning," Massachusetts Institute of Technology, Tech. Rep., 2019.

[2] C. Molnar, Interpretable Machine Learning. Christoph Molnar, 2018.

[3] Z. C. Lipton, The Mythos of Model Interpretability. 2017.

[4] B. Kim, "Interactive and interpretable machine learning models for human machine collaboration", PhD thesis, Massachusetts Institute of Technology, 2015.

[5] A. Vellido, J. D. Martín-Guerrero, and P. J. G. Lisboa, "Making machine learning models interpretable," in ESANN 2012 proceedings, European Symposium on Artificial Neural Networks, Computational Intelligence and Machine Learning, 2012, pp. 163–172, ISBN: 9782874190490.

[6] M. T. Ribeiro, S. Singh, and C. Guestrin, "Model-Agnostic Interpretability of Machine Learning," Tech. Rep., 2016.

[7] F. Doshi-Velez and B. Kim, "Towards A Rigorous Science of Interpretable Machine Learning," Tech. Rep., 2017.

[8] L. Baccour, "Amended fused TOPSIS-VIKOR for classification (ATOVIC) applied to some UCI data sets," Expert Syst. Appl., vol. 99, pp. 115–125, 2018.

[9] E. K. Zavadskas, Z. Turskis, and S. Kildiene, "State of art surveys of overviews on MCDM/MADM methods," Technological and Economic Development of Economy, vol. 20, no. 1. Taylor and Francis Ltd., pp. 165–179, 2014.

[10] M. Behzadian, S. Khanmohammadi Otaghsara, M. Yazdani, and J. Ignatius, "A state-of the-art survey of TOPSIS applications," Expert Syst. Appl., vol. 39, no. 17, pp. 13051–13069, 2012.

[11] G. Athanasopoulos, C. R. Riba, and C. Athanasopoulou, "A decision support system for coating selection based on fuzzy logic and multi-criteria decision making," Expert Syst. Appl., vol. 36, no. 8, pp. 1084––10853, Oct. 2009.

[12] R. Gamberini, A. Grassi, and B. Rimini, "A new multi-objective heuristic algorithm for solving the stochastic assembly line re-balancing problem," Int. J. Prod. Econ., vol. 102, pp. 226–243, 2006.

[13] D. E. Koulouriotis and M. K. Ketipi, "A Fuzzy Digraph Method for Robot Evaluation and Selection," Expert Syst. Appl., vol. 38, no. 9, pp. 11901–11910, Sep. 2011.

[14] A. Majumdar, B. Sarkar, and P. K. Majumdar, "Determination of quality value of cotton fibre using hybrid AHP-TOPSIS method of multi-criteria decision-making," J. Text. Inst., vol. 96, no. 5, pp. 303–309, Oct. 2005.

[15] A. Piegat and W. Sałabun, "Comparative Analysis of MCDM Methods for Assessing the Severity of Chronic Liver Disease," 2015, pp. 228–238.

[16] M. J. Gacto, R. Alcalá, and F. Herrera, "Interpretability of linguistic fuzzy rule-based systems: An overview of interpretability measures," Inf. Sci. (Ny)., vol. 181, no. 20, pp. 4340–4360, Oct. 2011.

[1]TWI Ltd, Cambridge, UK.