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Block-Sparsity Log-Sum-Induced Adaptive Filter for Cluster Sparse System Identification

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ABSTRACT In this work, an effective adaptive block sparsity log-sum least mean square (BSLS-LMS) algorithm is proposed to improve the convergence performance of cluster sparse system identification. The main idea of the proposed scheme is to add a new block-sparsity-induced term into the cost function of the LMS algorithm. We utilize the norm of the adaptive tap weights and log-sum as a mixed constraint. Via optimizing the cost function through the gradient descent method, the proposed adaptive filtering method can iteratively move the identified signals towards the optimal solutions and finally identify the unknown system accurately. The cluster-sparse system response, with a certain block length and arbitrary average sparsity, is generated by a Markov-Gaussian (M-G) model. For the white Gaussian input data, the theoretical formulas of the steady-state misadjustment and the convergence behaviors of the BSLS-LMS are derived in a general sparse system and a block sparse system, respectively. Numerical experiments demonstrate that the proposed adaptive BSLS-LMS algorithm achieves much better convergence behavior than conventional sparse signal recovery solutions. The experimental study also verifies the consistency between the simulation results and the theoretical analysis.

**INDEX TERMS** Block-Sparse System, Channel Estimation, Identification, Log- Sum, LMS, Performance Analysis.

I. INTRODUCTION

Signal reconstruction technologies have attracted much attention in the channel estimation, image recovery, and sparse unknown system identification fields [1-6]. In many cases, sparse unknown systems have only a few nonzero entries, and these limited nonzero or large coefficient responses appear independently in different locations over a long pulse. Different from the general sparse signal, another type of sparse signal is called the block-structured sparse signal, and it has a special cluster structure in the form of nonzero coefficients [7]. Nonzero coefficients can be located randomly in general sparse systems, but for a block sparse system, the impulse response typically consists of one or several clusters in which nonzero coefficients gather. Multiple-input multiple-output (MIMO) communication networks, the satellite-linked path and other practical applications are typical examples of multiclustering sparse structures.

 A number of block-sparsity signal recovery algorithms based on conventional sparse signal recovery methods have been designed. Convex optimization algorithms have been applied to block sparse signal reconstruction and studied [7-9]. Mixed program block-sparse signal recovery was proposed in [7, 8]. The block version of the zero-point attracting projection algorithm employing an approximate norm as the cost function was proposed in [9]. Convex relaxations equivalent to the original nonconvex formulation using the norm were studied in [10]. Greedy pursuit algorithms have been proposed for block-sparse systems [11, 12]. The block version of the orthogonal matching pursuit (BOMP) algorithm that identifies block-sparse signals successfully via a mixed optimization approach was investigated in [8]. Model-based compressive sampling matching pursuit (CoSaMP) was proposed in [12], and Bayesian Compressed Sensing framework-based algorithms were developed in [13-15]. The Stochastic Taps Normalized Least Mean Squares (STNLMS) [16], the Select and Queue with a Constraint (SELQUE) [17] and improved M-SELQUE algorithm [18] were applied to identify an unknown system by estimating the scattered region. The aforementioned algorithms can reconstruct unknown block sparse signals using prior knowledge of the block partition and cluster structure. To recover a block signal accurately, the block structure is fairly important. However, the key prior knowledge of the block partition of sparse coefficients is not always practically available. In fact, the locations and group sizes of the sparse clusters in an unknown system are random and entirely unknown.

 Recently, block sparse least mean squares adaptive filtering has received extensive attention [19-24]. These algorithms can recover the block-sparse signals of unknown clustering sparse systems without prior knowledge of the block-structured nonzero coefficients. The sparse adaptive algorithms have been shown to outperform other algorithms due to their low computational complexity and robustness against noise. To accelerate the convergence process of the coefficients, zero-attracting block-sparsity induced LMS algorithms have been proposed and applied to recover block-sparse impulse responses in previous studies [20-22]. Since most of the partitions in block-sparse systems are zero-coefficient groups, zero-attraction algorithms generally converge faster than other algorithms. The main idea of zero-attracting block sparsity-aware algorithms is to insert the constraint of block sparsity into the cost function of the standard LMS algorithm, such as the norm LMS and block-sparse LMS algorithms [23], a mixed norm, or an approximate mixed norm of the uniformly divided filter tap-weight vector [24].

 It is reported that log-sum minimization requires fewer measurements to recover sparse signals than minimization [25]. This advantage of using the log-sum penalty function has been proved in the Reweighted Zero- Attracting LMS (RZA-LMS) algorithm [26]. Inspired by the RZA-LMS and new developments in compressive sensing [27, 28], we propose a new log-sum penalty LMS to identify unknown sparse systems and theoretically verify the adoption of the proposed new penalty as an alternative sparsity-aware function. In the proposed novel block sparse LMS algorithm, we insert a mixed norm and log-sum penalty for the coefficients of the unknown system into the cost function so that the attractor selectively promotes the zero taps instead of uniformly promoting the zeros on all the taps. In this paper, the M-G model [23] is applied to generate and describe block-sparse systems. Then, the optimal group size of the sparse structure is determined using the M-G model and the proposed algorithm. It is demonstrated that the behavior of the BSLS-LMS algorithm is better than that of the block-sparse LMS (BS-LMS) when the group size is properly selected. The convergence behaviors of the algorithm are assessed using the mean square deviation (MSD). The experimental results show that the proposed BSLS-LMS algorithm performs better than the BS-LMS and other block sparse filtering algorithms. In addition, when the number of blocks in the system increases, the block sparsity log-sum LMS is robust and the performance loss is very small compared with the standard. The main contributions of this paper are summarized as follows:

*•* We propose a new block-sparse system identification algorithm to reconstruct the system response. We present theoretical expressions of the steady-state misadjustment and transient convergence behavior of the proposed algorithm with an appropriate group partition size for white Gaussian input data. Then, we theoretically demonstrate that the BSLS-LMS has much better convergence behavior than previous methods [16-18], [23].

• We propose a new cost function by utilizing the norm of the adaptive tap weights and the log-sum as a mixed penalty. The proposed adaptive filtering method can iteratively move the identified signals towards the optimal solutions and finally identify the unknown system accurately. The proposed adaptive algorithm has low computational complexity and has much better convergence behavior than conventional sparse signal recovery solutions.

The rest of the paper is organized as follows. In section 2, the log-sum block sparse algorithm designed to improve reconstruct performance is proposed and its computational complexity is analyzed. In section 3, the theoretical results of the steady-state performance and convergence of the BSLS-LMS are analyzed. In section 4, several experiments are implemented to verify the theoretical derivation, and the effectiveness of the BSLS-LMS is evaluated by comparing it with existing block sparse algorithms. Finally, Section 5 concludes.

II. BLOCK SPARSITY LOG-SUM LEAST MEAN SQUARES ALGORITHM

The unknown system coefficients to be identified are denoted as , and the input signal at time *n* is , where is the length of the unknown system, both and are real-valued vectors, and represents the transposition. The output signal of the unknown system at time ***n*** is defined as

, (1)

where is the additive white Gaussian noise of the unknown system.

We denote as the adaptive tap-weights. In other words, isthe reconstructed signal of the unknown coefficients vector**.** As a result, the estimated error at time *n* between the desired output of the unknown system and the output of the adaptive filter is

**,** (2)

where represents the adaptive tap- weights vector. In many practical scenarios, the nonzero coefficients of the unknown systems appear in the form of clusters rather than randomly distributed. To define the block- sparsity, we consider as a concatenation of blocks; throughout this paper, we assume that there are block groups in the unknown system and that the group partition size length of every block is , where. The block sparse signal is represented as

. (3)

We adopt the mixed norm and log-sum constraint to evaluate the block sparsity of as

, (4)

where denotes the group of . We assume that the signals in vector can always be evenly divided into M groups with a length of G by adding zero taps at the end of . Aiming to study the unknown systems by utilizing the block-sparsity, we propose a new cost function that combines the expectation of the estimated error, the mixed norm, the log-sum constraint of the tap-weighted vector and the balance parameters, i.e.,

, (5)

where () and () are positive constants that guarantee that the cost function is well defined and that better sense the sparsity of the block-sparse signal, respectively. The regularization parameter is a positive factor that regulates the strength between the block-sparsity penalty and the mean squared error, and the parameter should be appropriately chosen to ensure satisfactory performance [22]. It is noted that this idea differs from the RZA-LMS. The constraint function in the RZA-LMS is, while we define as a positive constant. By using gradient descent, the new recursive formula of the adaptive tap-weights can be defined as

**,** (6)

where, is the step size, and denotes a zero attraction parameter that balances the strength of the block-sparse penalty term for a given step size and group zero attraction,

,

and

**,** (7)

where is a sign function.

The proposed algorithm and comparisons of the computational costs are detailed in Table I and Table II, respectively.

TABLE I

PROPOSED ALGORITHM

|  |
| --- |
| Input: Output: 1. Initialize ,
2. for *n*=0,1,2,┄
3. Calculate instantaneous error vector
4. ;

for *i* ;end for1. for

 Update step size and balance factor;1. Update estimation equation

;1. end for
2. end for
 |

According to the recovery process of the log-sum penalty LMS, as summarized in Table I, the computational complexity of this new algorithm is *O*(*L*). The detailed calculations per iteration are listed and compared with those of the -LMS and BS-LMS. As shown in Table II, the number of computations of the BSLS-LMS is less than that of the BS-LMS [17]. In practical applications, since unknown systems vary slowly, partial updates can be used to save calculations.

In fact, literature [25] shows that when, the log-sum penalty is virtually the same as the -norm. Therefore, it is plausible that when is very small, the behavior of the log-sum penalty function, as shown in equation (6), is similar to that of the-norm. Moreover, we define as , which guarantees better performance of the log-sum penalty block LMS than the approximate-norm LMS.

TABLE II

COMPUTATIONAL COSTS

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Algorithm | MAC | Mul | Div | Com | SR |
| LMS-LMSBS-LMSBSLS-LMS | 2*L*4*L*5*L*4*L* | *−**L**L**L* | *−**−**L*/*G**L*/*G* | *−*2*L**L*/*G**−* | *−**−* *L*/*G**−* |

Mul: Multiplication, Div: Division, Com: Comparison, and SR: Square Root

III. PERFORMANCE ANALYSIS OF THE BSLS-LMS ALGORITHM IN GENERAL SPARSE SYSTEMS

In the proposed BSLS-LMS algorithm, the sparse constraint term adopts a mixed norm and has a modified log-sum penalty on the impulse response coefficients of the unknown system, which is similar to the -pseudo-norm of the coefficient vector and forces the solution of the proposed algorithm to be sparse. The basic assumptions of the system are as follows: (a) the input signal follows an i.i.d. zero-mean Gaussian distribution; (b) the input vector, tap-weights, and additive white noise are independent of each other; and (c) the variance of is, and all the tap-weights can be modeled by Gaussian variables. To provide a theoretical basis for this novel induced term as an alternative sparsity-aware function, we analyze the mean and mean squared performances of the BSLS-LMS based on the above assumptions in this section.

The rationale of making the above assumptions is the following: in the standard LMS, the tap-weights of can converge to their optimal values uniformly under an i.i.d. Gaussian input signal. In the proposed block sparse log-sum LMS, due to the group zero-attracting in (5), uniform convergence exists globally. Therefore, the strength of the temporary tap-weight is very close to that of the unknown coefficient in the group. In fact, numerical experiments have verified that this assumption is valid.

A. MEAN PERFORMANCE BASED ON THE PROPOSED ALGORITHM

The misalignment vector of identification is defined as

. (8)

Combining (1), (2), and (6), the update equation of the misalignment vector can be formulated as

. (9)

Taking the expectation for (9) and using the assumption (c), when goes to infinity, and converge, as o

 (10)

The upper bound of the derivation is

**,** (11)

where **1** is the vector with all single entries, which means that the proposed algorithm has a stability condition for the convergence of the coefficient misalignment vector.

B. MEAN SQUARED STEADY STATE ANALYSIS OF THE BSLS-LMS ALGORITHM

In this section, we derive the steady-state mean squared deviation (MSD) between the original signal and the estimated signal, and then, we deduce a criterion for zero attraction parameter selection for the proposed algorithm to outperform other block sparse algorithms.

 We multiply both sides of (9) by their respective transposes:

. (12)

We denote as the second-order moment matrix of the coefficient misalignment vector and as the MSD at iteration *n*.

 (13)

 (14)

Substituting (9) into (13), taking the expectations of both sides of (13) and utilizing the independence assumption, we obtain

, (15)

Using the fact that and taking the trace of both sides of (15), we obtain

. (16)

From (16), we obtain the condition of convergence as follows:

.

Using (16) and considering the diagonal element, we obtain

. (17)

To derive, we classify the index set into two groups according to the unknown coefficients. We define  and as the index sets of the adaptive nonzero tap-weights and zero tap-weighs, respectively.

. (18)

When , we consider that the tap-weight has the same sign as the corresponding unknown coefficient .

From Eq. (8), when approaches infinity, converges to

, (19)

. (20)

 has a small enough positive value that for every, we have [5]

. (21)

Therefore, it can be shown that

. (22)

When , we have

. (23)

Thus, we can have

. (24)

According to assumption (c), from the property of the Gaussian distribution, the following results are obtained:

, (25)

where .

 (26)

For , substituting (25) and (26) into (17), we obtain

, (27)

where.

For , combining (25), (26) and (17), we obtain

, (28)

where denotes, and for simplicity. We summarize (27) and (28) for alls and consider that

, (29)

where denotes the number of tap-weights of the nonzero group partitions, that is, . It can be further derived that

, (30)

where , and . Then, we define . Combining (27) and (28), it can be concluded that can be defined by the following equation:

. (31)

After solving the quadratic equation of (31), the final mean squared deviation of the log-sum block sparse LMS is

, (32)

where , ,,

.

In the steady-state MSD formula shown in Eq. (32), the first item on the right hand side is the steady-state MSD of the traditional LMS, and the latter two terms are caused by group zero attraction. If , then the other parts disappear accordingly and the block sparse log-sum LMS becomes the standard LMS. When the group zero-attraction is negative, the proposed block sparse algorithm has a smaller steady-state MSD, which means that it has a better steady-state performance than the standard LMS. Therefore, we derive the condition that the block sparse log-sum LMS is superior to other adaptive LMS algorithms in the steady state as

.

The log-sum penalty function has greater sparsity-encouraging potential than the approximate norm [20]. As shown in (32), the final MSD is proportional to the zero attraction parameter and the power of the measured noise, which means that a large will lead to a large MSD and a smallwill mean a weak zero attraction. Then, a weak zero attraction will slow the convergence. Therefore, in particular applications, the parameteris determined by a trade-off between the convergence rate and signal estimation accuracy. The proposed algorithm will obtain the minimum steady-state MSD when is

. (33)

Substituting (32) and (33), the minimum steady-state MSD is

. (34)

IV. PERFORMANCE OF THE LOG-SUM LMS FOR BLOCK SPARSE SYSTEMS

The performance of the log-sum block sparse LMS in block- sparse applications is further studied by using the M-G model. An appropriate assumption, , is adopted, which means that the partition size is very small relative to the filter length to ensure that the system response is still sparse in the group partition. The principle behind this assumption is that is an important predefined parameter that needs to be carefully selected. In addition, it is acceptable that the log-sum LMS penalizes sparsity in the group partition, and a too large will definitely destroy the sparsity.

In this paper, we study the performance of the BSLS-LMS for an unknown system response generated by the M-G model proposed in [20]. Following the approach in [20], we define as

In block sparse systems, the other parameters in (26) are defined as

, ,

, .

Here, we present some approximations that. Utilizing and, we have

, (35)

, (36)

, (37)

 (38)

We denote as the minimum transient MSD. Utilizing (35)-(38) in (33), we achieve the temporary result that

. (39)

In a sparse unknown system, we assume that, and then, the above equation can be simplified as

. (40)

We denote the optimal group partition size as, which can be found numerically by

. (41)

The above equation shows the selection criteria of the optimal partition size.

V. SIMULATION RESULTS

In this section, we design three experiments to verify the effectiveness of the proposed algorithm. The first two experiments verify the performance of the log-sum sparse LMS for a general sparse system and a block sparse system under different signal-to-noise ratios, and the last experiment theoretically analyzes the BSLS-LMS. Finally, all the simulation results prove that the convergence rate of the BSLS-LMS is remarkably superior to those of other classical algorithms when the group partition size is close to its optimal value.

1. THE CHANNEL ESTIMATION PERFORMANCE OF THE BSLS-LMS FOR A COOPERATIVE SYSTEM

In this section, the particular application considered is that of estimating the channel state information (CSI) for a communication system. We apply the proposed algorithm to study its channel estimation performance in a cooperative communication network. Channel estimation is a system identification problem that seeks to identify the CSI of the unknown system. Suppose the cooperation channel model is , as in [29]. is the unknown channel impulse response, corresponding to in Eq.(1).Denoteas the baseband channel between the source nodeand the relay node and as the channel impulse response from the relay nodeto the destination node. (with a length of ) is the cascaded channel that is the convolution between and .

Two cases are designed to prove the estimation performance of the block sparse log-sum constraint LMS algorithm through comparison with several existing classical algorithms, including the conventional LMS, the Zero-Attracting LMS (ZA-LMS), the RZA-LMS, the reweighted norm penalized LMS [26-28] and the reweighted norm constrained LMS [29]. The nonzero coefficients of the channel impulse response are Gaussian variables with a zero mean and unit variance and their positions are randomly selected. The input signal and additive noise are zero mean Gaussian sequences with various SNRs.

In case 1, we assume that the channel vectors of the cooperative relay system have the same length,, so the length of the convolution channel vectors is. There are four large coefficients of uniformly distributed, and the rest are all zeroes; hence, the sparsity of the system is. In case 2, the concatenated channel has the same length,, so the convolution channel vectors length is. Eight random tap-weights of are nonzero in case 2, and the sparsity of the cooperative channel is. The convergences of the proposed algorithm in the cases of a low SNR of 10 dB and a high SNR of 20 dB are tested. The step size parameter is set as for all algorithms. The average estimated mean squared errors (MSEs) between the actual channel state information and the estimated CSI are shown in Figure 1 and Figure 2.

The parameters of the proposed BSLS-LMS channel estimation algorithm are set as follows: , and . For the reference algorithms, all the adjustment parameters are properly chosen to obtain the fastest convergence speed. For the ZA-LMS and RZA-LMS, we set the series zero-attraction parameters as and.

The MSE results of the estimated channel impulse response in sparsity case 1 are shown in Figures 1(a) ~ (b), and the estimated result of sparsity case 2 are shown in Figures 2 (a) ~ (b). It can be seen from Figures 1 and 2 that the convergence of the sparse-aware parameter estimation algorithms decreases as the channel sparsity increases. By comparing the convergence curves of all algorithms, it is concluded that the block sparsity log-sum LMS algorithm is generally superior to the other algorithms. However, when the SNR = 10 dB, the performance of the reweighted norm constrained sparse filtering algorithm is closer to that of the log-sum penalty LMS algorithm. In both cases, the proposed algorithm achieves much better MSE performance when the SNR is large.

According to Figure 3, the channel length is the same as the system shown in Figure. 2. As the channel sparsity increases, the convergence performances of the sparse parameter estimation algorithms decrease accordingly. Through analysis of the convergence curves, it is concluded that the performance of the proposed algorithm is superior to those of all the other algorithms in the cases of the SNR=10 dB and SNR=20 dB. We also plotted the MSE performance against the number of iterations in the scenario of the density channel in Figure 4. It is clear that the block sparse log-sum algorithm has a better performance than the traditional algorithm and the other sparse-aware constrained adaptive filter algorithms, which is due to the upper bound of in (11).

The MSEs between the coefficients of the BSLS-LMS algorithm and the CSI are shown in Figure 1~Figure 4. The proposed BSLS-LMS algorithm converges faster and has better steady-state performance than all the reference algorithms. When there are very few nonzero coefficients in the impulse response of the unknown system, all the sparse-aware adaptive filters have faster convergence speeds and better steady-state performances than the traditional LMS. In addition, it can be observed that the proposed method achieves a lower MSE than the other algorithms. After 500iterations, when the number of nonzero coefficients increases, the BSLS-LMS still achieves the best convergence among the algorithms for both low and high SNRs.

1. REGARDING THE PERFORMANCE OF THE BLOCK LOG-SUM LMS

In the second experiment, we test the convergence performance of the BSLS-LMS algorithm by identifying block-sparse systems and comparing its performance with the performances of the reference algorithms, including the BS-LMS [23], -LMS [27], STNLMS [16], SELQUE [17], and M-SELQUE [18]. The unknown systems have the same length, L = 800, and the impulse response of the unknown system is generated by the M-G model. Based on assumption (a), the input signal and the additive noise are independent zero-mean Gaussian sequences, and the variance of the input signal is unit, i.e.,. In this experiment, the SNRs are set as 40 dB and 20 dB. The simulation results are averaged over 10 independent tests for each unknown system. One hundred unknown systems are generated and identified, and then, the learning curves of identifying these systems are averaged. For the BSLS-LMS and BS-LMS, the step size *μ* is always set as half of the maximum value and the zero attraction parameter and the group partition size are set as 8*.*66e-7 and **4**, respectively. For the reference algorithms, all the parameters are properly adjusted to obtain their fastest convergence speed and their best performance.

The simulation results corresponding to the unknown systems whose impulse responses are plotted in Figure 5(a) are plotted in Figure 6. When there are four clusters in the system response, the BSLS-LMS and BS-LMS achieve the fastest steady-states at 20 dB and 40 dB, respectively. The convergence performance of the BSLS-LMS is better than those of other block sparsity recovery algorithms.

**FIGURE 6**. **MSD performance comparison of different recovery algorithms when the unknown systems whose impulse responses are those plotted in FIGURE 5 (a).**

In the third experiment, there are more than four clusters in the identified various unknown block-sparse system whose impulse responses are plotted in Figure 5(b). The experimental results are shown in Figure 7, where the BSLS-LMS and M-G models show their advantages. It is noted that the BSLS-LMS, similar to the BS-LMS, does not need to detect the active regions of the unknown system, and thus, its performance is better than the others shown in Figure 6. The convergence speeds of the SELQUE and STNLMS are obviously reduced because the latency between the two clusters and other active regions are regarded as a long active region [17]; meanwhile, the other two algorithms cannot detect the active region effectively. Although the M-SELQUE algorithm can detect all the regions, the convergence of the unknown system is still poor when it has multiple clusters.

**FIGURE 7**.**MSD performance comparison of different recovery algorithms when the unknown systems have the impulse responses plotted in FIGURE 5(b)**.

According to Figure 7, the BSLS-LMS is always the best at identifying the various block-sparse systems generated by the M-G model with different parameter settings. Utilizing the block-sparsity prior, the BSLS-LMS converges faster than the BS-LMS and -LMS. Although the M-SELQUE algorithm performs better than other active region detection algorithms, its convergence speed is still slower than that of the proposed algorithm. The main reason for this slower convergence speed is that more iterations will be needed to determine the location of the nonzero coefficients when there are more clusters and scattered clusters; hence, the convergence rate will decrease correspondingly, which is very likely to be caused by using the M-G model. From the learning curves, the convergence performances of the SELQUE and STNLMS are poor because they are not suitable for the multicluster system. In summary, we can infer that the BSLS-LMS is more robust than other classical algorithms, especially in multiple distributed cluster sparse systems.

In the fourth experiment, the steady-state MSDs of the BSLS-LMS with various group partition sizes versus are tested. The results of this experiment corresponding to the unknown systems shown in Figure 5(c) are plotted in Figure 8. The step size is set as. is set as 1, 5, and 15. For each, the variation range of is from to. It can be seen from Figure 8 that the theoretical steady-state performance of the log-sum LMS is in good agreement with the simulation results. For each group partition size, the steady-state MSD of the BSLS-LMS decreases when increases from, which means that the appropriate group zero-attraction helps to decrease the amplitudes of the coefficients in. Meanwhile, the stronger group zero attraction enhances the deviation of the coefficients in as parameter continues to increase. The minimum steady-state MSD and the corresponding optimal are different for different group partition sizes. It is concluded that the simulation result of the optimal agrees well with the theoretical, as shown by the solid square in Figure 8.

**FIGURE 8**. **MSDs of the BSLS-LMS with different group sizes versus.**

VI. CONCLUSIONS

In this paper, we proposed a new block sparse LMS algorithm for unknown system identification. Specifically, we utilized the norm and the log-sum of the adaptive tap weights as a mixed constraint in the cost function algorithm. The Markov-Gaussian model was adapted to generate the impulse response of the cluster-sparse unknown system. In addition, based on the expressions of the mean squared deviation, the performance of the block sparse log-sum LMS was theoretically analyzed, and the results proved that the proposed algorithm is theoretically superior to other algorithms. Finally, several experiments were designed to verify the effectiveness of the theoretical results, and the simulation results demonstrated the superior performance of the proposed algorithm. Moreover, the proposed BSLS-LMS algorithm has low computational complexity, which makes the method practical for channel estimation and system identification. Therefore, we can conclude that the theoretical results well predict the trend for the MSD when the signal-to-noise ratio is 20 dB/40 dB, and the simulations results agree well with the analytical values.

VII. ABBREVIATIONS

Please see Table III.

TABLE III

 LIST OF ABBREVIATIONS

|  |  |
| --- | --- |
| Abbreviations | Full name |
| LMSBSLS-LMSM-GMIMOBOMPSTNLMSSELQUEZA-LMSRZA-LMSBS-LMSSNRMSD | Least Mean SquaresBlock Sparsity Log-Sum LMSMarkov-Gaussian model Multiple Input Multiple OutputBlock version of Orthogonal Matching PursuitStochastic Taps Normalized Least Mean SquaresSelect and Queue with a ConstraintZero-Attracting LMSReweighted Zero-Attracting LMSBlock-Sparse LMSSignal-to-Noise RatioMean Squared Deviation |