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Graph Colour-based Resource Allocation for Relay-Assisted D2D Underlay Communications

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Abstract: Relay-assisted Device-to-Device (D2D) communications has been proposed as a supplement for direct D2D communications to enhance traffic offloading capacity. In this paper, we propose a joint mode selection, relay selection and resource allocation for relay-assisted D2D communications. We aim at maximizing the overall system throughput while guaranteeing the power limitation and signal-to-noise-and-interference ratios (SINRs) of all cellular and active D2D links. Since this optimization is NP-Hard, we then propose a Graph Colour-based Resource Allocation (GCRA) algorithm to effectively solve it. Simulation results show that our proposed algorithm can produce close-to-optimal performance with acceptable computational complexity.

1 Introduction

Device to device (D2D) communication is a promising technique to improve spectral and energy efficiency (EE) in 5G and beyond cellular networks [1]. Many works have been done on resource allocation for D2D communications [2]-[5], where D2D pair communicating with each other directly in direct mode (DM). However, solely using direct mode restricts the system from taking full advantage of the D2D transmission. D2D transmitter (DUT) and receiver (DUR) may not be able to perform direct communications due to the long separation distance or poor channel condition between them [6]. In such cases, network-assisted transmission through relays could be adopted, namely, relay-assisted D2D communications and defined as the relay mode (RM). Thus, D2D links can operate in either DM or RM determined by a mode selection procedure. Moreover, the D2D link can select the best relay node to assist RM D2D communication, which is referred to as relay selection.

Authors in [7] propose a new radio protocol architecture (RPAs) to implement the relay-assisted D2D communications, and propose a joint scheduling in mode selection and resource allocation for D2D links without considering spectrum reuse. Authors in [8] [9] propose joint mode selection and resource allocation algorithm for D2D underlay communications without considering relay selection. Although relay selection scheme is investigated in [10] [11], the D2D links always operate in RM. The relay selection is considered in [12], and the Genetic Algorithm (GA) is applied to solve this combination problem, which is insufficient to achieve the close-to-optimal performance.

Authors in [13] and [14] consider a number of cellular users (CUEs) and unmanned aerial vehicles (UAVs) coexisting in a cell, where two transmission modes are considered i.e. UAV-to-network (U2N) and UAV-to-UAV (U2U) communications. [13] proposed a channel allocation algorithm to maximize the system sum rate, but did not consider power allocation, and thus can not guarantee the QoS of links. Paper [14] assumes that the channels for cellular network have been pre-allocated. Although the authors in [15] considered power allocation to guarantee the quality of services (QoS), it did not consider mode selection and assumed that D2D pairs only operate in Direct Mode. [16] proposes the channel allocation algorithm to maximize the system sum rate, without considering the power allocationso it can not guarantee the QoS of links. [16] [17] investigate relay-based UASs, where UAVs are the relay nodes to assist the CUE communications. The channel resource is allocated exclusively to the CUE communications, and cannot be reused by D2D pairs.

[18] investigates power allocation for a simple network, where only one CUE and one relay node are considered. [19] [20] investigate the resource allocation for D2D networks based on graph coloring. However, in [19] the D2D links are allocated dedicated channel resource, which cannot be used by CUEs at the same time. In [20], authors assume that the D2D pair can only operate in Direct Mode.

As explained above, most of the existing paper simplified their scenario to some extent. Whereas in this paper, we comprehensive a more general scenario where mode selection, relay selection and resource allocation including channel resources and power for both cellular and D2D links is jointly considered. The joint consideration is more realistic and comprehensive than the existing works. Therefore the low-complexity solution to this joint optimization greatly improves the overall system throughput. In our system, D2D pairs reuse the same channel resource with existing CUEs, and operate in either direct or relay mode depending on which mode produces better sum rate. Moreover, effective power and channel allocation algorithm is proposed to control the mutual interference and maintain the quality of services.

However, this leads to a complicated problem and requires significantly high computational complexity. Therefore, we formulate the resource reusing relationship among different cellular users (CUEs), D2D pairs and relay nodes as a colour-based graph and propose a graph colour-based resource allocation (GCRA) algorithm to effectively obtain a close-to-optimal solution with low complexity. Instead of associating each attribute with an individual communication link as in some existing schemes [21] [22], we creatively introduce three individual attributes for each channel resource, namely cellular link attribute, channel assignment attribute and power allocation attribute. Thanks to the use of these new attributes, the proposed iterative GCRA algorithm can achieve near optimal performance. Simulation results confirm that the proposed algorithm achieves the close-to-optimal performance in terms of the network throughput with much lower complexity, in comparison with optimal resource allocation obtained via exhaustive search method.

The rest of paper is organized as follows. Section 2 introduces the system model for relay-assisted D2D communications. Section 3 shows the problem formulation. The proposed algorithm is explained in section 4. The optimal power allocation is in 5. Simulation results and complexity analysis are presented in section 6. Section 7 concludes this paper.

Table 1 The list of symbols

Symbols	Meanings
$p^k_{C_{m,n}}$	The transmission power of CUE m, when it shares the channel k with D2D pair n in DM
$\tilde{p}^k_{C_{m,n}}$	The optimal power of CUE m, when it shares the channel k with D2D pair n in DM
$p^k_{D_{m,n}}$	The transmission power of DUT n, when it reuses the channel k with CUE m in DM
$\tilde{p}^k_{D_{m,n}}$	The optimal power of DUT n, when it shares the channel k with D2D pair n in DM
$R^k_{C_{m,n}}$	The data rate of cellular link m, when it shares the channel k with D2D link n in DM
$R^k_{D_{m,n}}$	The data rate of D2D link n, when it reuses the channel k with cellular link m in DM
$R^k_{m,n}$	The total data rate on channel k in DM
$\tilde{R}^k_{m,n}$	The optimal total data rate on channel k in DM
$p^{k,r}_{C_{m,n}}$	The transmission power of CUE m, when it shares the channel k with D2D link n in RM
$\tilde{p}^{k,r}_{C_{m,n}}$	The optimal power of CUE m, when it shares the channel k with D2D link n in RM
$p^{k,r}_{D_{m,n}}$	The transmission power of DUT n, when it reuses the channel k with cellular link m in RM
$\tilde{p}^{k,r}_{D_{m,n}}$	The optimal power of DUT n, when it shares the channel k with D2D link n in RM
$p^{k,r}_{R_{m,n}}$	The transmission power of relay node r, when it supports D2D link n in RM
$\tilde{p}^{k,r}_{R_{m,n}}$	The optimal power of relay node r, when it supports D2D link n in RM
$R^{k,r}_{C_{m,n}}$	The data rate of cellular link m, when it shares the channel k with D2D link n in RM
$R^{k,r}_{D_{m,n}}$	The data rate of D2D link n, in RM
$R^{k,r}_{m,n}$	The total data rate on channel k, when it reuses the channel k with cellular link m in RM
$\tilde{R}^{k,r}_{m,n}$	The optimal total data rate on channel k in RM

2 System model

A spectrum sharing relay-assisted D2D underlay network is shown in Fig. 1, where we consider M CUEs in the set $\mathcal{M} = \{1, ...m, ...M\}$, N D2D pairs in the set $\mathcal{N} = \{1, ...n, ...N\}$. We assume each D2D pair includes a DUT and a DUR. $\mathcal{R} = \{1, 2, ..., r, ..., R\}$ is the set of relay nodes, and $\mathcal{K} = \{1, ...k, ...K\}$ is the set of channels in the system. We consider a fully loaded cellular network, in which M active cellular links occupy K orthogonal channels and no spare spectrum dedicated to D2D links (i.e. $M = K$). Each D2D link can only reuse no more than one uplink channel resource and each relay node can only support one D2D link. Moreover, each D2D link can operate in either DM or RM. $h^c_{a,b}$ denotes the channel gain from transmitter or node a to receiver or node b on channel c.

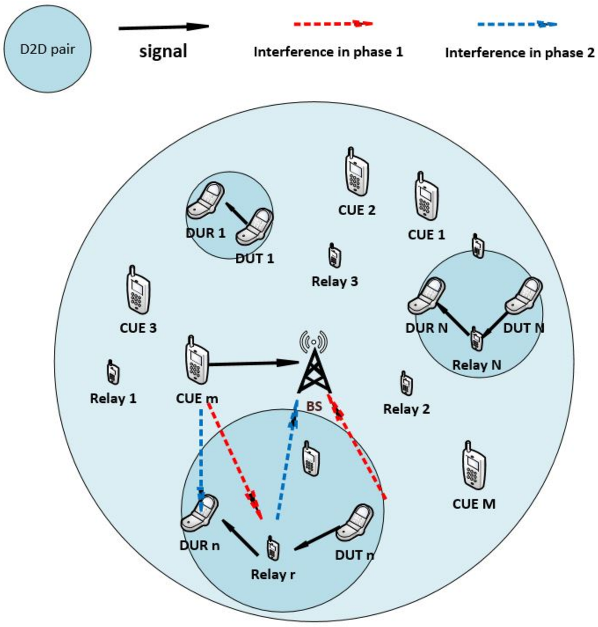

Fig. 1: A single cell system with M cellular links, N D2D links and R relay nodes.

2.1 Direct Mode

In DM, the D2D pair n communicates with each other directly. The signal to interference and noise ratios (SINRs) of cellular link m and D2D links n on channel k can be expressed as

$$\gamma^k_{C_{m,n}} = \frac{p^k_{C_{m,n}} h^k_{m,B}}{\sigma^2 + p^k_{D_{m,n}} h^k_{n,B}}, \tag{1}$$

$$\gamma^k_{D_{m,n}} = \frac{p^k_{D_{m,n}} h^k_{n,n}}{\sigma^2 + p^k_{C_{m,n}} h^k_{m,n}}, \tag{2}$$

in which $p^k_{C_{m,n}}$ and $p^k_{D_{m,n}}$ are the transmission power of CUE m and DUT n on channel k in DM. $h^k_{m,B}$ is the channel gain between CUE m and the BS, and $h^k_{n,B}$ is the interfering channel gain from DUT n to the BS on channel k. $h^k_{n,n}$ and $h^k_{m,n}$ are the channel gain between D2D pair n and the interfering channel gain from CUE m to DUR n on channel k. The data rates in bits per second per hertz of cellular link m and D2D link n on channel k are expressed as

$$R^k_{C_{m,n}} = \log_2(1 + \gamma^k_{C_{m,n}}), \tag{3}$$

$$R^k_{D_{m,n}} = \log_2(1 + \gamma^k_{D_{m,n}}). \tag{4}$$

Therefore, the total data rates on channel k in DM is

$$R^k_{m,n} = R^k_{C_{m,n}} + R^k_{D_{m,n}}. \tag{5}$$

2.2 Relay Mode

When D2D link n operates in RM, D2D link n selects relay node r and shares the same channel k with the cellular link m. The Decode and Forward (DF) relaying strategy is employed, where each communication period is divided into two equal intervals corresponding to the DUT to relay node communication phase (phase 1) and relay node to DUR communication phase (phase 2). Also, we assume that communications in phase 1 and phase 2 use the same channel resource.

In phase 1, the SINRs of cellular link m and D2D link n can be expressed as

$$\gamma 1^{k,r}_{C_{m,n}} = \frac{p^{k,r}_{C_{m,n}} h^k_{m,B}}{\sigma^2 + p^{k,r}_{D_{m,n}} h^k_{n,B}}, \tag{6}$$

$$\gamma 1^{k,r}_{D_{m,n}} = \frac{p^{k,r}_{D_{m,n}} h^k_{n,r}}{\sigma^2 + p^{k,r}_{C_{m,n}} h^k_{m,r}}. \tag{7}$$

Similarly, in phase 2, the SINRs of cellular link m and D2D link n can be expressed as

$$\gamma 2_{C_{m,n}}^{k,r} = \frac{p_{C_{m,n}}^{k,r} h_{m,B}^{k}}{\sigma^2 + p_{R_{m,n}}^{k,r} h_{r,B}^{k}}, \tag{8}$$

$$\gamma 2_{D_{m,n}}^{k,r} = \frac{p_{R_{m,n}}^{k,r} h_{r,n}^{k}}{\sigma^2 + p_{C_{m,n}}^{k,r} h_{m,n}^{k}}, \tag{9}$$

where $p_{C_{m,n}}^{k,r}$, $p_{D_{m,n}}^{k,r}$ and $p_{R_{m,n}}^{k,r}$ are the transmission power of CUE m, DUT n and relay node r on channel k in RM. $h_{n,r}^{k}$ is the channel gain from the DUT n to relay node r and $h_{m,r}^{k}$ is the interfering channel gain from CUE m to relay node r on channel k. The signal channel gain from relay node r to DUR n receiver is denoted as $h_{r,n}^{k}$.

Therefore, the data rates of cellular link m and D2D link n in RM can be calculated as

$$R_{C_{m,n}}^{k,r} = \frac{1}{2}\log_2(1 + \gamma 1_{C_{m,n}}^{k,r}) + \frac{1}{2}\log_2(1 + \gamma 2_{C_{m,n}}^{k,r}), \tag{10}$$

$$R_{D_{m,n}}^{k,r} = \frac{1}{2}\log_2(1 + \min\{\gamma 1_{D_{m,n}}^{k,r}, \gamma 2_{D_{m,n}}^{k,r}\}). \tag{11}$$

The total data rate on channel k in RM is

$$R_{m,n}^{k,r} = R_{C_{m,n}}^{k,r} + R_{D_{m,n}}^{k,r}. \tag{12}$$

In addition, when cellular links do not experience any co-channel interference from D2D links, the maximum throughput could be achieved when cellular links transmit with their maximum power (i.e. p_{max}^{C}). Thus, the data rate of cellular link m on channel k without being reused can be calculated as

$$R_{C_m}^{k} = \log_2(1 + \frac{p_{max}^{C} h_{m,B}^{k}}{\sigma^2}). \tag{13}$$

For convenience, the frequently used symbols in this paper are listed in Table 1.

3 Problem Formulation

In this section, we formulate the joint mode selection, relay selection and resource allocation problem, which aims to maximize the sum data rate of all communication links while guaranteeing the required minimum SINRs of cellular and active D2D links. Three decision variables x_m^k, $y_{m,n}^k$ and $z_{m,n}^{k,r}$ corresponding to cellular links, D2D links and relay nodes are introduced:

$$x_m^k = \begin{cases} 1, & \text{if celluar link } m \text{ exclusively occupies channel } k, \\ 0, & \text{otherwise;} \end{cases} \tag{14}$$

$$y_{m,n}^k = \begin{cases} 1, & \text{if D2D link } n \text{ operates in DM by reusing the same} \\ & \text{channel } k \text{ with cellualr link } m, \\ 0, & \text{otherwise;} \end{cases} \tag{15}$$

$$z_{m,n}^{k,r} = \begin{cases} 1, & \text{if D2D link } n \text{ operates in RM supported by relay} \\ & \text{node } r \text{ and reuses the channel } k \text{ with cellualr link } m, \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Thus, the problem is expressed as

$$\max_{P,X,Y,Z} \{\sum_{k=1}^{K}\sum_{m=1}^{M}(x_m^k R_{C_m}^k + \sum_{n=1}^{N}(y_{m,n}^k R_{m,n}^k + \sum_{r=1}^{R} z_{m,n}^{k,r} R_{m,n}^{k,r}))\} \tag{17}$$

s.t.

$$\gamma_{C_{m,n}}^{k} \geq \gamma_{min}^{C}; \gamma 1_{C_{m,n}}^{k,r}, \gamma 2_{C_{m,n}}^{k,r} \geq \frac{1}{2}\gamma_{min}^{C}, \forall m \in \mathcal{M}, \tag{17a}$$

$$\gamma_{D_{m,n}}^{k} \geq \gamma_{min}^{D}; \gamma 1_{D_{m,n}}^{k,r}, \gamma 2_{D_{m,n}}^{k,r} \geq \frac{1}{2}\gamma_{min}^{D}, \forall n \in \mathcal{N}, \tag{17b}$$

$$0 \leq p_{C_{m,n}}^{k}, p_{C_{m,n}}^{k,r} \leq p_{max}^{C}, \forall m \in \mathcal{M}, \tag{17c}$$

$$0 \leq p_{D_{m,n}}^{k}, p_{D_{m,n}}^{k,r} \leq p_{max}^{D}, \forall n \in \mathcal{N}, \tag{17d}$$

$$0 \leq p_{R_{m,n}}^{k,r} \leq p_{max}^{R}, \forall r \in \mathcal{R}, \tag{17e}$$

$$\{\sum_{m=1}^{M}(x_m^k + \sum_{n=1}^{N}(y_{m,n}^k + \sum_{r=1}^{R} z_{m,n}^{k,r}))\} = 1, \forall k \in \mathcal{K}, \tag{17f}$$

$$\{\sum_{k=1}^{K}(x_m^k + \sum_{n=1}^{N}(y_{m,n}^k + \sum_{r=1}^{R} z_{m,n}^{k,r}))\} = 1, \forall m \in \mathcal{M}, \tag{17g}$$

$$\{\sum_{k=1}^{K}\sum_{m=1}^{M}(y_{m,n}^k + \sum_{r=1}^{R} z_{m,n}^{k,r})\} \leq 1, \forall n \in \mathcal{N},, \tag{17h}$$

$$\{\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N} z_{m,n}^{k,r}\} \leq 1, \forall r \in \mathcal{R}, \tag{17i}$$

$$x_m^k, y_{m,n}^k, z_{m,n}^{k,r} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall n \in \mathcal{N}, \forall r \in \mathcal{R}, \tag{17j}$$

where P is the power allocation matrix. X, Y and Z are the channel allocation matrices. In (17), the first term is the sum data rate on channels which are exclusively occupied by cellular links. The second and the third terms are the sum data rate of channels while D2D link operating in DM and RM, respectively.

Constraints $(17a)$-$(17b)$ show the minimum SINR requirement of each link in all transmission intervals. Constraints $(17c)$-$(17e)$ express the limited transmission power of cellular, active D2D transmitters and relay node. Constraint $(17f)$ shows that each channel resource can only be either used exclusively by one cellular link or reused by one cellular link and one active D2D link. It also reveals that each D2D link can operate in DM or RM. Constraint $(17g)$ shows that each cellular link can only use one channel resource, either exclusively or reuse with one active D2D link. Constraint $(17h)$ shows that each active D2D link can only reuse no more than one channel resource. Constraint $(17i)$ shows that each relay node can support no more than one active D2D link. The final constraint $(17j)$ means the channel allocation indicator is binary.

The optimal solution to (17) can be obtained through exhaustive search of all possible choices of P, X, Y and Z subject to constraints given from $(17a)$-$(17j)$, but it requires extremely high computational complexity. Problem in (17) is clearly NP-hard, so we propose a GCRA algorithm to effectively obtain a near-optimal solution with much lower complexity.

4 The proposed GCRA algorithm

4.1 Graph Construction

To perform the joint resource allocation, relay and mode selection for D2D relay-assisted communication based on a graph colouring approach, we construct the graph corresponding to the network topology. Constraint $(17h)$ means that each D2D link can operate in DM or RM and constraint $(17i)$ shows that each D2D link can be only supported by one relay node when D2D link operates in RM. To graphically express the constraints, we introduce a virtual relay node R_0 and a virtual D2D link D_0. Note that both the virtual link and the virtual node are only used to indicate inactive links, so we allow multiple connections and thus one virtual D2D link and one virtual relay node are sufficient. For convenience, we also define the set $\mathcal{R}_R = \mathcal{R} \cup \{R_0\}$ and $\mathcal{N}_D = \mathcal{N} \cup \{D_0\}$.

In the constructed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, the set of elements called vertexes is defined as $\mathcal{V} = \{V_m, V_n, V_r | \forall m \in \mathcal{M}, \forall n \in \mathcal{N}_D, \forall r \in \mathcal{R}_R\}$,

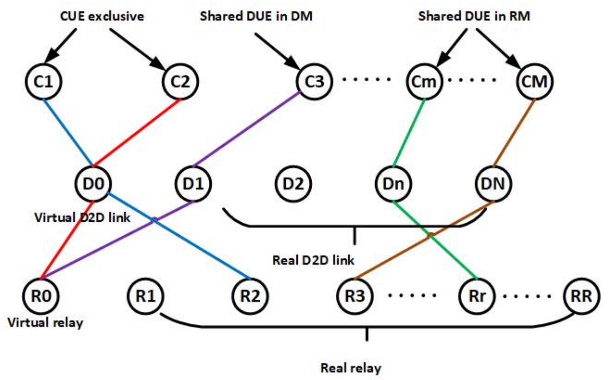

Fig. 2: Illustrative example of the channel allocation.

and the subsets of $\mathcal{V}$ called edges $\mathcal{E} = \{e = (V_m, V_n, V_r) | \forall m \in \mathcal{M}, \forall n \in \mathcal{N}_D, \forall r \in \mathcal{R}_R\}$ represents the communication relationship between three different kinds of vertexes from different clusters i.e. cellular users, D2D pairs and relay nodes. Moreover, each channel resource is represented by one colour.

The resource allocation is conducted by using these K colours to dye K colours to dye the vertices in the graph. It is clear that if the edges connecting cellular link m, virtual D2D link D_0 and any relay node $\forall r \in \mathcal{R}_R$ are dyed the colour k, it means cellular link m uses channel k exclusively. Note that any relay node is fine in this case since it is not active when connected to a virtual D2D link. If the edges connecting cellular link m, actual D2D link n, and virtual relay node R_0 is dyed colour k, it means D2D link n operates in DM by reusing channel k with cellular link m. Moreover, if the edges connecting cellular link m, actual D2D link n, and actual relay node r are dyed colour k, then D2D link n operates in RM, supported supported by relay node r through and reuses channel k with cellular link m.

An example is illustrated in Fig. 2, where the virtual D2D link D_0 are connected with cellular link C_1 and C_2, respectively. It means that both cellular link C_1 and C_2 exclusively use the corresponding channel resource. Cellular links (i.e. C_3, C_m and C_M) are connected with actual D2D links (such as D_1, D_n and D_N), respectively. More specifically, D2D link D_1 operates in DM shown by connecting with a virtual relay node. D2D links D_n and D_N operate in RM and are supported by the actual relay nodes R_r and R_3, respectively. The same colour of the edges indicates these links are sharing the same channel resource.

Each vertex in the constructed graph has one attribute, namely, type of links, which indicates which set it belongs to, i,e. $V_m \in \mathcal{M}$ or $V_n \in \mathcal{N}_D$ or $V_r \in \mathcal{R}_R$. Moreover, each colour in the designed graph has three attributes, namely, cellular link attribute, channel assignment vector attribute and power allocation vector attribute.

1. The channel link attribute contains two parameters, namely, the cellular vertex list L^k and the current interested vertex index l_1^k. The vector L^k contains cellular vertex indices, which are arranged in order of decreasing SNR values at the BS. The l_1^k is the first element of L^k.
2. The channel assignment vector a_k is a $1 \times (M + N + R + 2)$ vector, which is expressed as

$$a_k = [a_k(V_m), a_k(V_n), a_k(V_r) | \forall V_m \in \mathcal{M}, \forall V_n \in \mathcal{N}_D, \forall V_r \in \mathcal{R}_R].$$

This vector indicates that channel k is allocated to these vertexes for data transmission. If V_m, V_n and V_r are chosen from different vertex types to share the same channel k, then $a_k(V_m) = a_k(V_n) = a_k(V_r) = 1$; otherwise, $a_k(V_m) = a_k(V_n) = a_k(V_r) = 0$.

3. The power allocation vector, expressed as:

$$p_k = [p_k(V_m), p_k(V_n), p_k(V_r) | \forall V_m \in \mathcal{M}, \forall V_n \in \mathcal{N}_D, \forall V_r \in \mathcal{R}_R],$$

indicates that the transmission power of communication links on channel k. In fact, if $p_k(V_m) = p_k(V_n) = p_k(V_r) = 0$, then $a_k(V_m) = a_k(V_n) = a_k(V_r) = 0$, *vice versa*. Note that when the virtual vertex D_0 or R_0 is selected, the corresponding transmission power $p_k(D_0)$ or $p_k(R_0)$ is zero.

4.2 GCRA Algorithm

Based on the above colouring-based graph construction, we now start to dye the vertices with colours. In our system, the cellular links always have the highest priority to access the channel resource. D2D link becomes active only when its reuse can improve the total throughput and all links meet the minimum SINR requirements. To proceed with the proposed GCRA algorithm, we define:

(1) Let set C represent unused colour, and its complementary set C^* is $\mathcal{K} - C$.

(2) The channel cluster $\mathcal{B}_k = \{V_m, V_n, V_r\}$ as the set of communication links that share channel k for individual data transmission. Based on this, colours used in the graph can be represented by a vector $\mathcal{B} = [\mathcal{B}_1, \mathcal{B}_2, ..., \mathcal{B}_K]$.

(3) The throughput value $v_t(\mathcal{B}_k)$ defines the sum capacity of all communication links that belong to set $\mathcal{B}_k$. The value of $v_t(\mathcal{B}_k)$ is given as

$$v_t(\mathcal{B}_k) = \begin{cases} \widetilde{R}^k_{C_m}, & \text{when } V_n = D_0, \\ \widetilde{R}^k_{m,n}, & \text{when } V_n \neq D_0, V_r = R_0, \\ \widetilde{R}^{k,r}_{m,n}, & \text{when } V_n \neq D_0, V_r \neq R_0, \end{cases} \tag{18}$$

where the $\widetilde{R}^k_{C_m}$, $\widetilde{R}^k_{m,n}$ and $\widetilde{R}^{k,r}_{m,n}$ are the optimal transmission data rate on channel k in different channel reuse scenarios, which will be discussed in section 5.

(4) The total network throughput T can be calculated as

$$T = \sum_{k=1}^{K} v_t(\mathcal{B}_k). \tag{19}$$

The basic idea of the graph colouring based algorithm is to iteratively dye the uncoloured vertices and gather them to the corresponding edge set $\mathcal{B}_k$ to maximize the total network throughput T.

As described in TABLE I, at the beginning of the resource allocation process, the graph that describes the situation of the network is constructed, in which vertices that indicate different communication links are uniformly generated in the area of cellular network. First, each channel's individual information is initialized based on the SNR of the cellular links. All the elements in the vertex assignment and power allocation attributes are initialized as zeros. Next, the unused colour set C and each channel's cluster set $\mathcal{B}_k$ are initialized as empty. The throughput value $v_t(\mathcal{B}_k)$ is initialized as zero.

In the graph colouring based resource allocation algorithm, i.e. colouring process, we first select an unused colour from C^* and its corresponding current interested vertex index. Then, use this colour to dye the vertex in set $\mathcal{B}_k^{'} = \{V_m, D_0, R_0\}$, and update the throughput value $v_t(\mathcal{B}_k)$. Here the optimal power allocation is considered to obtain the throughput value $v_t(\mathcal{B}_k)$ according to (18). Then, an uncoloured vertex V_n is selected to replace D_0. If its replacement can not increase the cluster throughput, we then continue replace another type vertex R_0 in $\mathcal{B}_k^{'}$ with uncoloured V_r. After tried all the feasible vertexes, we update the throughput value with the largest $v_t(\mathcal{B}_k)$, and colour the corresponding vertexes, which means $a_k(V_m) = a_k(V_n) = a_k(V_r) = 1$ (if vertexes V_m, V_n and V_r are selected). Meanwhile, the power allocation p_k can be obtained according to corresponding power allocation algorithms. This iterative procedure will continue until all the colours are used. This procedure is described in Table 2.

The cellular and D2D links can transmit data in the allocated channel with permitted transmission power.

As explained in Table 2, the proposed scheme can approach the close-to-optimal network performance because communication links

Table 2 The proposed GCRA algorithm

```
1. Initialization
* Construct the $M + N + R$ vertices, with $M$ cellular vertices, $N$
  D2D vertices, and $R$ relay node vertices.
* Initialize each channel's individual attributes i.e. $L^k$ and $l_1^k$.
* Initialize each channel's vertex assignment and power allocation vector $a_k = [a_k(1), a_k(2), ..., a_k(M + N + R + 2)]$ is zero.
* Initialize each channel's vertex set $\mathcal{B}_k = \phi$, then $v_t(\mathcal{B}_k) = 0$, $\forall k \in \mathcal{K}$.
* Initialize $C = \phi$, then $C^* = \mathcal{K}$.
2. REPEAT
 -Select an unused colour $k$ from $C$, i.e. $C^*$, and its corresponding current interested vertex
    index $V_m = l(k)$.
 -Set $\mathcal{B}_k^{'} = \{V_m, D_0, R_0\}$, and store the current value $v_t^{'}(\mathcal{B}_k)$.
    Repeat
      -select the uncoloured D2D vertex $V_n$.
      -replace $D_0$ with $V_n$ in $\mathcal{B}_k$, i.e. $\mathcal{B}_k = \{V_m, V_n, R_0\}$.
      - update the value of $v_t(\mathcal{B}_k)$ according to (18).
      - If the value of $v_t(\mathcal{B}_k) \leq v_t^{'}(\mathcal{B}_k)$, then
          Repeat
            -Select an uncoloured relay vertex $V_r$ to replace $R_0$ in
             $\mathcal{B}_k$, i.e. $\mathcal{B}_k = \{V_m, V_n, V_r\}$.
            -Update the value of $v_t(\mathcal{B}_k)$ according to (18), and
             record the result.
            -Delete the vertex $V_r$ from $\mathcal{B}_k$, and recover the value
             of $v_t(\mathcal{B}_k)$ and relay vertex to uncoloured state.
          Until all the uncoloured relay vertices have been tried or
             the value of $v_t(\mathcal{B}_k)$ is not greater than that of last cycle.
        -Choose the largest result from all the recorded values of
           $v_t(\mathcal{B}_k)$. If $v_t(\mathcal{B}_k) \geq v_t^{'}(\mathcal{B}_k)$, colour the corresponding
           optimum vertex group $\{V_m, V_n, V_r\}^{opt}$ using colour $k$
           and set $\mathcal{B}_k = \{V_m, V_n, V_r\}$, else $\mathcal{B}_k = \mathcal{B}_k^{'}$.
        -Update the value of $v_t(\mathcal{B}_k)$, and record the result
      -Else record the value of $v_t(\mathcal{B}_k)$
      -Delete the vertex $V_n$ from $\mathcal{B}_k$, and recover the D2D vertex
         to uncoloured state or delete the vertex $V_n$ and $V_r$ from
         $\mathcal{B}_k$, and recover the D2D and relay vertex to uncoloured
         state.
    Until all the uncoloured D2D vertices have been tried or the
     value of $v_t(\mathcal{B}_k)$ is not greater than that of the cycle.
 -Choose the largest result from all the recorded values of
   $v_t(\mathcal{B}_k)$, and colour the corresponding optimum vertex group
   $\{V_m, D_0, R_0\}$ or $\{V_m, V_n, R_0\}$ or $\{V_m, V_n, V_r\}$ using colour $k$. Set
   $\mathcal{B}_k = \{V_m, D_0, R_0\}$ or $\mathcal{B}_k = \{V_m, V_n, R_0\}$ or $\mathcal{B}_k = \{V_m, V_n, V_r\}$
 -Update the value of $v_t(\mathcal{B}_k)$
UNTIL $C^* = \phi$ or the value of $v_t(\mathcal{B}_k)$ is not greater than that of
the last cycle. Then $\mathcal{B}$ is obtained and the total $T = \sum_{k=1}^{K} v_t(\mathcal{B}_k)$
converges.
3. The power and channel allocations are given as $[p_1, p_2, ..., p_k]$
and $[a_1, a_2, ..., a_k]$.
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will be selected only if they can further improve the network capacity performance. Once a link is selected, it will be omitted in the following iterations. Thus, the computational complexity is reduced.

5 Optimal Power Allocation

In this section, we illustrate the power allocation algorithms for different scenarios shown in (18).

5.1 When $V_n = D_0$

In this case, the cellular link exclusively occupies the channel resource. The optimal data rate on channel k can be obtained when the cellular link transmits on its maximum power, which is calculated as

$$\widetilde{R}_{C_m}^k = R_{C_m}^k. \tag{20}$$

5.2 When $V_n \neq D_0, V_r = R_0$

In this case, the cellular link shares the same channel with a D2D link operating in DM. Thus, the power allocation problem can be expressed as

$$(\tilde{p}_{C_{m,n}}^k, \tilde{p}_{D_{m,n}}^k) = \underset{(p_{C_{m,n}}^k, p_{D_{m,n}}^k)}{\arg\max} \; R_{m,n}^k \tag{21}$$

s.t.

$$\gamma_{C_{m,n}}^k \geq \gamma_{min}^C, \tag{21a}$$

$$\gamma_{D_{m,n}}^k \geq \gamma_{min}^D, \tag{21b}$$

$$0 \leq p_{C_{m,n}}^k \leq p_{max}^C, \tag{21c}$$

$$0 \leq p_{D_{m,n}}^k \leq p_{max}^D. \tag{21d}$$

It has been proved that either $\tilde{p}_{C_{m,n}}^k$ or $\tilde{p}_{D_{m,n}}^k$ are equal to the maximum transmit power in order to achieve the optimal sum data rate.

When $\tilde{p}_{C_{m,n}}^k = p_{max}^C$, problem (21) is then convex in the range of $0 \leq p_{D_{m,n}}^k \leq p_{max}^D$. The same happens when $\tilde{p}_{D_{m,n}}^k = p_{max}^D$. Hence, $(\tilde{p}_{C_{m,n}}^k, \tilde{p}_{D_{m,n}}^k)$ can be selected from a set of feasible solutions Ω as explained in [4]:

$$(\tilde{p}_{C_{m,n}}^k, \tilde{p}_{D_{m,n}}^k) = \begin{cases} (0,0) & \Omega = \phi, \\ \underset{(p_{C_{m,n}}^k, p_{D_{m,n}}^k) \in \Omega}{\arg\max} \; R_{m,n}^k & \text{otherwise}, \end{cases} \tag{22}$$

where

$$\Omega = \Omega 1 \cup \Omega 2 \tag{23}$$

$$\Omega 1 = \begin{cases} \{(P_{max}^C, P1), (P_{max}^C, P2)\}, & P1 \leq P2 \\ \phi, & \text{otherwise}. \end{cases} \tag{24}$$

$$\Omega 2 = \begin{cases} \{(P3, P_{max}^D), (P4, P_{max}^D)\}, & P3 \leq P4 \\ \phi, & \text{otherwise}. \end{cases} \tag{25}$$

$$P1 = \max\left\{0, \frac{\gamma_{min}^D(\sigma^2 + P_{max}^C h_{m,n}^k)}{h_{n,n}^k}\right\}, \tag{26}$$

$$P2 = \min\left\{P_{max}^D, \frac{(P_{max}^C h_{m,B}^k - \gamma_{min}^C \sigma^2)}{h_{n,B}^k \gamma_{min}^C}\right\}, \tag{27}$$

$$P3 = \max\left\{0, \frac{\gamma_{min}^C(\sigma^2 + P_{max}^D h_{n,B}^k)}{h_{m,B}^k}\right\}, \tag{28}$$

$$P4 = \min\left\{P_{max}^C, \frac{P_{max}^D h_{n,n}^k - \gamma_{min}^D \sigma^2}{\gamma_{min}^D h_{m,n}^k}\right\}. \tag{29}$$

Thus, the optimal sum data rate on channel k can be obtained as

$$\widetilde{R}_{m,n}^k = \log_2(1 + \frac{\widetilde{p}_{C_{m,n}}^k h_{m,B}^k}{\sigma^2 + \widetilde{p}_{D_{m,n}}^k h_{n,B}^k}) + \log_2(1 + \frac{\widetilde{p}_{D_{m,n}}^k h_{n,n}^k}{\sigma^2 + \widetilde{p}_{C_{m,n}}^k h_{m,n}^k}). \tag{30}$$

Note that $\Omega = \phi$ means the transmission power of both cellular and D2D links can not meet all the constraints in (21), so the cellular and D2D links will not be allowed to share the same channel. Therefore, both transmit power pair and the optimal data rate is 0 for this channel.

5.3 When $V_n \neq D_0, V_r \neq R_0$

In this case, the cellular link shares the same channel with the D2D link operating in RM with relay node r. Thus, the power allocation problem can be expressed as

$$(\tilde{p}^{k,r}_{C_{m,n}}, \tilde{p}^{k,r}_{D_{m,n}}, \tilde{p}^{k,r}_{R_{m,n}}) = \underset{(p^{k,r}_{C_{m,n}}, p^{k,r}_{D_{m,n}}, p^{k,r}_{R_{m,n}})}{\arg\max} \quad R^{k,r}_{m,n} \tag{31}$$

$$\gamma 1^{k,r}_{C_{m,n}}, \gamma 2^{k,r}_{C_{m,n}} \geq \frac{1}{2}\gamma^{C}_{min}, \tag{31a}$$

$$\gamma 1^{k,r}_{D_{m,n}}, \gamma 2^{k,r}_{D_{m,n}} \geq \frac{1}{2}\gamma^{D}_{min}, \tag{31b}$$

$$0 \leq p^{k,r}_{C_{m,n}} \leq p^{C}_{max}, \tag{31c}$$

$$0 \leq p^{k,r}_{D_{m,n}} \leq p^{D}_{max}, \tag{31d}$$

$$0 \leq p^{k,r}_{R_{m,n}} \leq p^{R}_{max}. \tag{31e}$$

Since problem in (31) involves the power allocation for a relay, a D2D transmitter and a cellular user on a channel resource, we can reformulate it in the following concise form where for brevity the channel index k is omitted and the subscript symbols "*c*", "*d*", and "*r*" denote cellular user, D2D transmitter and relay node, respectively.

[8] has proved that problem (31) can achieve its optimum if $p_c = p^{C}_{max}$ or $p_d = p^{D}_{max}$ or $p_r = p^{R}_{max}$, and $\gamma 1^{k,r}_{D_{m,n}} = \gamma 2^{k,r}_{D_{m,n}}$. In addition, [23] verified that the system performance is not related to the p^{D}_{max} and p^{R}_{max} because the cellular links have the highest priority to communicate than the D2D links. Hence, only consider $p_c = p^{C}_{max}$ is enough to achieve the near optimal power allocation*.

Thus, the problem in (31) can be simplified as

$$(p^{C}_{max}, \widetilde{p}_d, \widetilde{p}_r) = \underset{(p_d, p_r)}{\arg\max}\, f_c(p^{C}_{max}, p_d, p_r) \tag{32}$$

s.t.

$$\frac{p_d h_{dr}}{\sigma^2 + p^{C}_{max} h_{cr}} = \frac{p_r h_{rd}}{\sigma^2 + p^{C}_{max} h_{cd}}, \tag{32a}$$

$$\frac{p_d h_{dr}}{\sigma^2 + p^{C}_{max} h_{cr}} \geq \frac{1}{2}\gamma^{D}_{min}, \tag{32b}$$

$$\frac{p^{C}_{max} h_{cB}}{\sigma^2 + p_d h_{dB}} \geq \frac{1}{2}\gamma^{C}_{min}, \tag{32c}$$

$$\frac{p^{C}_{max} h_{cB}}{\sigma^2 + p_r h_{rB}} \geq \frac{1}{2}\gamma^{C}_{min}, \tag{32d}$$

$$0 \leq p_d \leq p^{D}_{max}, 0 \leq p_r \leq p^{R}_{max}, \tag{32e}$$

where $f(p^{C}_{max}, p_d, p_r) = (1 + \frac{p^{C}_{max} h_{cB}}{\sigma^2 + p_d h_{dB}})(1 + \frac{p^{C}_{max} h_{cB}}{\sigma^2 + p_r h_{rB}})(1 + \frac{p_d h_{dd}}{\sigma^2 + p^{C}_{max} h_{cr}})$. The first constraint in (32a) suggests that p_d can be expressed as a linear function of p_r

$$p_d = \frac{h_{rd}(\sigma^2 + p^{C}_{max} h_{cr})}{h_{dr}(\sigma^2 + p^{C}_{max} h_{cd})} p_r. \tag{33}$$

Therefore, the objective function in (32) can be expressed as a function of p_r, which is the ratio between one cubic polynomial and one quadratic polynomial

$$f_r = \frac{\zeta(p_r)}{g(p_r)} = \frac{a_3 p_r^3 + a_2 p_r^2 + a_1 p_r + a_0}{b_2 p_r^2 + b_1 p_r + b_0}, \tag{34}$$

Table 3 Simulation Parameters

Maximum distance between D2D pairs d_{max} (m)	300 (if fixed)
Number of channel resource K	20
Number of cellular links M	20
Number of D2D links N ($N \leq M$)	10 (if fixed)
Number of relay nodes R	30
Maximum cellular transmission power P^{C}_{max}(dBm)	20
Maximum D2D transmission power P^{D}_{max}(dBm)	20
Maximum relay transmission power P^{D}_{max}(dBm)	20
SINR requirements of cellular links γ^{C}_{min} (dB)	10 (if fixed)
SINR requirements of D2D links γ^{D}_{min} (dB)	15
Noise power σ^2 (dBm)	-110
Pathloss exponent for relay communications $\alpha 1$	3
Pathloss exponent for other communications $\alpha 2$	4

where the coefficients $a_3, a_2, a_1, a_0, b_2, b_1$ and b_0 can be obtained by the Matlab *expand* function. Moreover, constraints (32b)-(32e) imply that $p_r \in [P_r^{min}, P_r^{max}]$, where the P_r^{min} and P_r^{max} are the lower and upper bounds. The optimal solution i.e. $(p^{C}_{max}, \tilde{p}_d, \tilde{p}_r)$ and the optimal value i.e. $\tilde{R}^{k,r}_{m,n}$ of problem (34) can be obtained by applying the *Dinkelbach* method [24].

6 Simulation Results and Complexity Analysis

Monte-Carlo simulations are conducted in this section to evaluate the efficiency of the proposed GCRA scheme. An isolated cell is considered here with radius of $500m$, where the BS is located at the centre. The CUEs and the relay nodes are uniformly distributed. Each D2D pair (includes one DUT and one DUR) are uniformly distributed in a randomly located cluster with radius d_{max}. The channel gain in our proposed model is modelled as $h_{a,b} = d_{a,b}^{-\alpha}\chi$ for all communication links, where $d_{a,b}$ is the distance between node a and b, α is the distance-dependent pathloss exponent*; χ represents the Rayleigh fading gain, which has zero means and unit variance. The simulation parameters summarized in Table 3, are chosen from [4] for the purpose of comparison.

6.1 Simulation Results

We name our proposed method as the *proposed GCRA* algorithm, and compare it with the following schemes:

Tra-Cellular: In traditional cellular network, the CUEs are allocated orthogonal channel resource to transmit data through the BS directly.

Only-Direct: In [4], the D2D links communicate with each other directly by reusing the channel resources of CUEs. Only DM is considered.

Optimal-Relay: The optimization programming is applied to obtain the optimal performance of the joint problem. Specifically, for both DM and RM, all the reuse possibilities of CUEs and D2D links on each channel resource are considered in the power allocation process. Then, the channel allocation problem among multiple CUEs and D2D links are solved by the exhaustive search method.

Fig. 3 shows that the system sum data rate achieved by various schemes for different N when $d_{max} = 300$. We observe that the sum data rate increases with the increase of the total number of D2D links for the *Only-Direct*, *proposed GCRA* algorithm and the *Optimal-Relay* algorithms, due to the increasing number of active D2D links. Moreover, the *proposed GCRA* algorithm achieves close-to-optimal performance and significantly outperforms the *Only-Direct* and *Tra-Cellular* methods.

Fig. 4 shows that the system sum data rate of different schemes with varying d_{max}. As the increase of d_{max}, the sum data rate decreases in all schemes except in the traditional cellular networks. This is because the channel gain between D2D links is

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*More details can be found in [8] and [23].

To demonstrate the benefit that relay can bring us, we will choose different pathloss exponent value for DUTs to relay nodes and relay nodes to DURs.

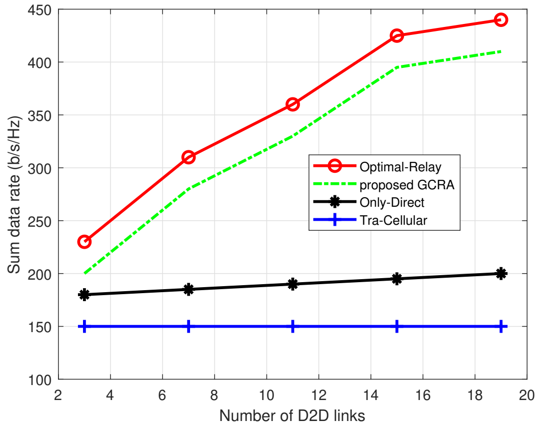

Fig. 3: The system sum rate with various schemes for different N.

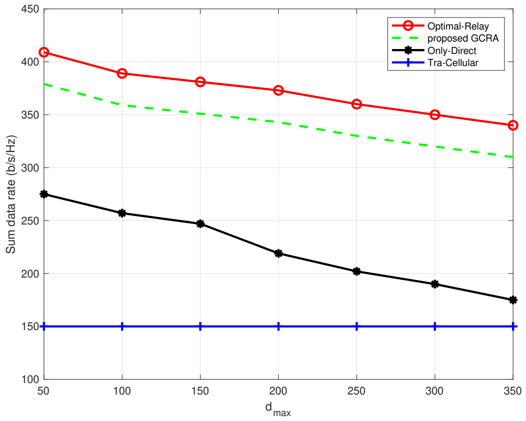

Fig. 4: System sum rate with various schemes for different d_{max}.

decreasing with the increase of d_{max}, and this can lead to deteriorating system performance in the *Only-Direct*, proposed GCRA and *Optimal-Relay* algorithms.

In addition, when d_{max} is small, most of the D2D pairs tend to operate in DM. Thus, with the increase of d_{max}, decrease of the sum rate have the same slope in both *Only-Direct* and *proposed GCRA* schemes. But, when d_{max} is big enough to some point such as $d_{max} \geq 150m$, in the *proposed GCRA* schemes, the decrease of sum data rate slows down since some D2D transmission via RM are established. The proposed GCRA closely approaches the optimal-relay and notably outperforms the other two methods due to the jointly consideration of he channel allocation for both cellular and D2D links.

Fig. 5 shows that the system sum rate of various schemes for different γ^C_{min}. With the increase of γ^C_{min}, the sum data rate of the *Tra-Cellular* and the *Only-Direct* methods decrease. The sum rates of the *proposed GCRA* and the *Optimal-Relay* scheme also decrease, but moderately with the increase of γ^C_{min}. This is because the number of both active cellular links and D2D links decrease with the increase of γ^C_{min}, and hence system sum data rate decreases . Due to the assistance of relay nodes, the sum data rate of the *proposed GCRA* and the *Optimal-Relay* schemes decrease slowly with high γ^C_{min}.

All the above results show that the proposed algorithm produces notably better performance than the existing works. This is because our proposed scheme not only applies the relay selection for D2D links to select the best relay node but also considers the joint channel allocation for both cellular and D2D links. Moreover, thanks to the the optimal power allocation in each iteration, the proposed GCRA algorithm produces the close-to-optimal results.

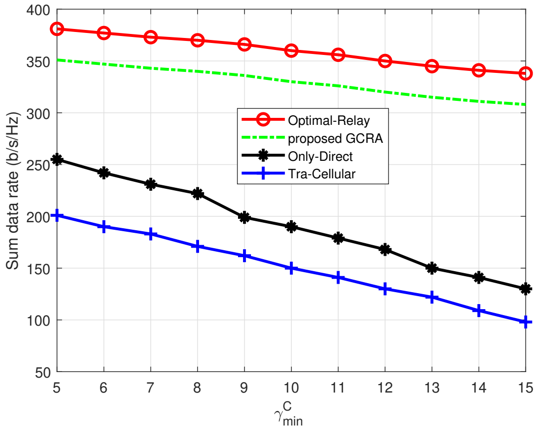

Fig. 5: System sum rate with various schemes for different γ^C_{min}.

6.2 Complexity Analysis

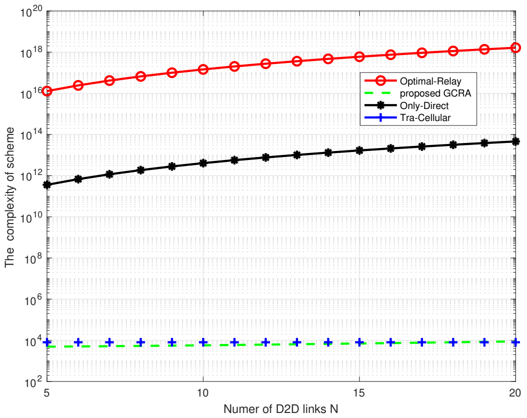

Fig. 6: Comparison complexity of different schemes.

The computational complexity is analysed by counting the number of operations required in the power and channel allocations. Although the optimal resource allocation includes the power and channel allocations, but the complexity of the power allocation is indeed negligible. The complexity of the *Optimal-Relay* scheme considering all the channel allocation possibilities is $O(MN + MNK + MNKR)^{3.5} = O(MNKR)^{3.5}$. The complexity of the *Only-Direct* scheme considering all the channel allocation possibilities is $O(MNK)^{3.5}$. The channel allocation in *Tra-Cellular* scheme is formulated as a job assignment problem, then solved by the Hungary method. Thus, the complexity of the *Tra-Cellular* scheme is $O((\max\{M, K\})^3)$.

According to Algorithm 1, the proposed GCRA algorithm is operated iteratively. Different initial states of the graph constructed based on the current investigated network will lead to different numbers of iterations to obtain the final solution. Therefore, we focus on the worst case complexity of GCRA, which can sufficiently verify the efficiency of the proposed algorithm.

In GCRA, the core idea of solving the resource allocation problem is to iteratively replace the vertexes in channel cluster with the available D2D or relay vertexes, until one available vertex can improve throughput. Thus, the worst case complexity can be readily calculated considering the case where all the channel cluster tested by all the available D2D and relay vertexes. A certain channel cluster, a cellular link, the virtual D2D link and the virtual relay link are selected in each iteration. The worst case complexity of the proposed GCRA algorithm can be then given as

$$\begin{aligned} C =& O\{K(M + \frac{N(N+1)}{2} + \frac{R(R+1)}{2})\} \\ =& O\{K(\max\{N, R\}^2)\} \\ =& O\{KR^2\}. \end{aligned} \tag{35}$$

From Fig. 6, we can conclude that the proposed algorithm can largely reduce the computational complexity with slightly degraded system performance.

7 Conclusion

In this paper, a joint mode selection, relay selection and resource allocation for both cellular and D2D links is formulated. Since the joint optimization is NP-Hard, we then proposed a GCRA algorithm to effectively solve it. Simulation results show that the proposed algorithm can produce the close-to-optimal performance with lower computational complexity.

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