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Resource Allocation for D2D Underlay Communications With Proportional Fairness Using Iterative-Based Approach

MIAOMIAO LIU, (Member, IEEE), AND LI ZHANG, (Senior Member, IEEE)
School of Electrical and Electronic Engineering, Institute of Communication and Power Networks, University of Leeds, Leeds LS2 9JT, U.K.
Corresponding author: Li Zhang (l.x.zhang@leeds.ac.uk)

ABSTRACT In this paper, we develop a novel resource allocation scheme that aims to improve the system fairness, with minimum SINR (Signal to Interference and Noise Ratio) constraints and power limits for the Device-to-Device (D2D) underlay communications. Since the evaluation of the system fairness is different for $t = 1$ and $t \geq 2$, where t is the scheduling period, we divide our joint optimization problem into two cases: $t = 1$ and $t \geq 2$. For each case, we decompose the joint optimization problem into two subproblems: power and channel allocations. We then propose the corresponding power allocation algorithm for each case. By introducing virtual D2D links, we model the channel allocation as a 3-D (3-Dimensional) assignment problem, which is effectively solved by our proposed 2-D (2-Dimensional) iterative method. Simulation results show that our proposed iterative method can produce the close-to-optimal performance with low computational complexity. Moreover, comparing with existing schemes, our proposed scheme can not only enhance the system fairness but also improve the overall throughput.

INDEX TERMS D2D underlay communications, V2V communications, resource allocation, proportional fairness (PF).

I. INTRODUCTION

Device-to-Device (D2D) communications is identified as one of the technology complements for 5G communications system [1]. In D2D underlay cellular network, a D2D pair can directly communicate with each other by sharing the same resource used by the traditional cellular user to enhance the network spectral efficiency and energy efficiency (EE) [2]–[5]. The mutual interference among cellular and D2D links is one of the most critical issues in such systems. Therefore, a well-designed power allocation and channel allocation scheme is required for D2D underlay communication system. A number of previous publications have looked into this. Works in [6] and [7] prove that the maximum throughput can be achieved when at least one of the users transmit at its maximum power. Authors in [6] and [8] allocate the channel by maximizing a weight bipartite matching problem, which can be solved by the well-known *Kuhn-Munkres* method (also called *Hungary* method in some works). Wang *et al.* transform the power allocation into a D.C. programming (Difference of Convex functions programming), which can be solved by the standard convex algorithm [9]. Based on the assumption that the cellular transmission power is fixed, [10] uses the convex approximation technique to formulate the power allocation into a solvable convex optimization problem. Utilising the properties of fractional programming, [11] and [12] transform the original non-convex energy efficiency (EE) problem into an equivalent optimization problem with subtractive form, which is solved by the well-known *Dinkelbach* method. In [13], Hoang *et al.* propose three channel allocation algorithms: dual-based, BnB (Branch-and-Bound) and RBR (Relaxation-based Rounding) algorithms with different computational complexity levels.

However, most of the above works focus on improving the total system throughput or EE when allocating the power and channel resource while completely ignoring the fairness aspect. In order to to ensure all users have equal chance to access the system resource, fairness should be considered in resource allocation [14]. Generally, the links with good channel condition which can improve the whole system throughput are usually chosen to communicate. The links with poor channel gain will have lower or no chance to access

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the channel resource. This will lead to an unfair system and is not acceptable for some applications such as Vehicle-to-Everything (V2X) communications [15], [16].

Thus, in this paper, we will propose a novel resource allocation scheme aiming to improve the system fairness to make sure all users have the same priority to access the system resource. Normally, the increase of system fairness lead to the decrease of throughput, *vice versa*. The proportional fairness (PF) scheduler can offer a better trade-off between system throughput and fairness comparing with other schedulers (e.g. Round Robin and Max-Min schedulers) [17], so the PF scheduler is adopted in our proposed scheme.

There are few studies conducted on how the PF scheduler is applied in D2D underlay communications. Authors in [18] optimize the system sum rate considering the fairness among D2D links only. Work in [19] transforms the PF scheduler scheme as an assignment problem by applying the *Maclaurin* series expansion. But same transmission power is allocated to all users and the QoS requirements of all communication links can not be guaranteed, which will lead to harmful mutual interference between cellular and D2D links. To simply the system model, work in [20] assumes that the system is completely fair when allocating the transmission power for both cellular and D2D links, which is unrealistic and will lead to an unfair system. Our work in [21] enhances the system fairness by taking into account the previous average data rates into power allocation.

In most of the above works, the channel allocation for cellular links is assumed to be pre-allocated. However, joint channel resource allocation for both cellular and D2D links can result in a better system performance [7]. Therefore, to further enhance system performance, we consider a more general scheme that the base station (BS) needs to simultaneously allocate the channel resource to both cellular and D2D links.

Thus, in this paper, we formulate the joint power and channel allocation for both cellular and D2D links to maximize the system fairness while guaranteeing the QoS requirements and power limits of communication links for all scheduling periods. Since the joint problem in each scheduling period is a MINLP (Mixed Integer Non-Linear Programming) problem, which can not be solved in polynomial-time. We divide it into two sub-problems: power allocation and channel allocation, and solve them sequentially.

To the best of our knowledge, this is the first work that optimizes the joint power and channel allocations of the mobile cellular and D2D links aiming at maximizing the system fairness. The power allocation is related to control the transmission powers of the cellular link and D2D link to maximize system fairness while avoiding harmful mutual interference. The channel allocation is to allocate the appropriate channel resources using proposed PF scheduler to enhance the system fairness.

Our main contributions are summarised as follows.

- We propose the PF scheduler for D2D underlay communications aiming to maximize the system fairness and deduce it into a solvable form using *Talyor* series.
- When $t = 1$, the system fairness is modelled as the sum the logarithm of current data rates for the cellular and D2D links. The optimal power allocation for one D2D link and one cellular link which reuse the a certain channel resource can be obtained by comparing at most 4 potential solutions.
- When $t \geq 2$, the system fairness problem is transformed into maximizing the sum of the ratios of current data rate to average data rate. The optimal power allocation for cellular link and D2D link can be obtained by comparing at most 6 potential solutions when they share the same spectrum resource.
- Based on the above power allocation results, we introduce the concept of virtual D2D links and model the channel allocation as a 3-D assignment problem. A novel iterative 2-D assignment (*I2-DA*) algorithm is proposed to solve it efficiently. Specifically, in each iteration, we solve a selected 2-D assignment problem by using the *Kuhn-Munkres* method. In this way, the system computational complexity is reduced dramatically.

Simulation results show that our proposed scheme can not only enhance the system fairness but also increase the overall system throughput comparing with the existing schemes. Moreover, the numerical results reveal that our proposed iterative method can produce the close-to-optimal performance with low computational complexity.

The rest of paper is organized as follows. Section II introduces the system model and the expressions of PF scheduling scheme for D2D communications. Problem formulations and the proposed algorithms for $t = 1$ and $t \geq 2$ are shown in section III and IV, respectively. The proposed *I2-DA* algorithm is described in section V. Simulation results and analysis are presented in section VI. Section VII concludes this paper.

II. SYSTEM MODEL AND PF SCHEDULER

A. SYSTEM MODEL

We consider an open-air-festival scenario occurred within a single cell system with a BS in the centre, where K cellular equipments (CUEs) in the set $\mathcal{K} = \{1, \dots i, \dots K\}$, and L predefined and configured D2D pairs in the set $\mathcal{L} = \{1, \dots j, \dots L\}$. Each D2D pair includes a transmitter (DUT) and a receiver (DUR) as shown in Fig. 1.

In this paper, the available channel resource is 2-D, i.e. including both frequency and time domains as shown in Fig. 2, where the uplink frequency bandwidth is divided into N subbands, with their indices collected in set $\mathcal{N} = \{1, \dots n, \dots N\}$. One subband over one time slot is defined as one resource block (RB). We assume that each RB can be allocated to at most one cellular link and one D2D link. Moreover, we consider a dense system that there is no spare RB to be allocated to D2D link exclusively, hence D2D links

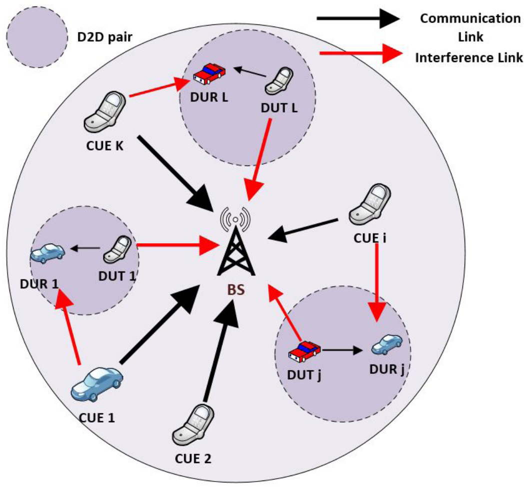

FIGURE 1. The system model of the dynamic D2D underlay communications.

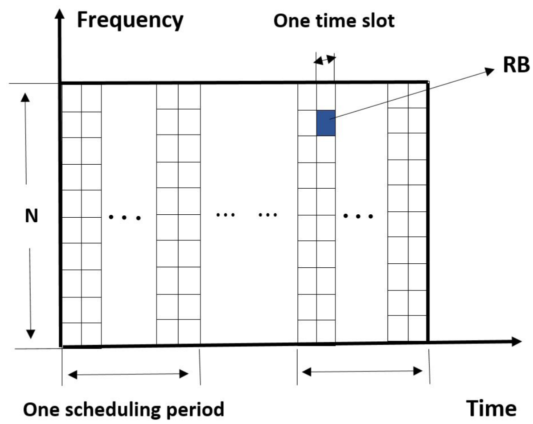

FIGURE 2. The Two-dimensional RBs for channel allocation.

can only reuse the RBs with CUEs. Without loss of generality, the total number of RBs can be assumed to be equal to the number of cellular links (i.e. $N = K$).

In the design of our scheme, we also consider the overhead in practical use. In this work, a centralised resource planning and scheduling architecture is employed, assuming BS has the capability of acquiring perfect channel state informations (CSIs) of all communication links, which have to be estimated at the receivers and then fed back to the BS. However, due to the mobility of vehicles, it is extremely hard to track the real-time small-scale channel fading coefficients and updating CSIs every time slot will generate too much overhead and consume excessive bandwidth. Considering the channel coherence time is usually much larger than a time slot even in fast time-varying scenario [22], it is reasonable to set the scheduling period much longer than the time slot, as shown in Figure 2, such as a few hundred milliseconds. In this way, the CSIs can be updated in every scheduling period instead of in every time slot, so the overhead can be greatly reduced. In addition, the investigation in [23] shows that the system performance degradation caused by averaging out the small-scale fading is acceptable. Accordingly, we only consider the large-scale fading phenomenon [24]. Therefore, the channel gain between node a and b on RB c in scheduling period t is modelled as

$$h^t_{a,b,c} = d^{-\alpha}_{a,b}\kappa, \tag{1}$$

where κ is the shadow fading gain whose distribution is lognormal. α is the path-loss exponent and $d_{a,b}$ is the distance between node a and b in scheduling period t.

When D2D pair j reuses the same RB n with CUE i, the SINRs (Signal to Interference and Noise Ratios) of cellular link i and D2D link j on RB n in scheduling period t can be expressed as

$$\gamma^t_{C_{i,j,n}} = \frac{p^t_{C_{i,j,n}} h^t_{i,B,n}}{\sigma^2 + p^t_{D_{i,j,n}} h^t_{j,B,n}}, \quad t = 1, \ldots. T, \tag{2}$$

$$\gamma^t_{D_{i,j,n}} = \frac{p^t_{D_{i,j,n}} h^t_{j,n}}{\sigma^2 + p^t_{C_{i,j,n}} h^t_{i,j,n}}, \quad t = 1, \ldots. T, \tag{3}$$

in which $p^t_{C_{i,j,n}}$ and $p^t_{D_{i,j,n}}$ are the transmission powers of CUE i and DUT j in scheduling period t on RB n, respectively. $h^t_{i,B,n}$ is the channel gain between CUE i and BS and $h^t_{j,B,n}$ is the interfering channel gain from DUT j to BS in scheduling period t on RB n. $h^t_{j,n}$ is the channel gain between D2D pair j in scheduling period t on RB n. $h^t_{i,j,n}$ is the interfering channel gain from CUE i to DUR j in scheduling period t on RB n. σ^2 is the noise power. T is the total scheduling periods.

Therefore, the data rates of cellular link i (i.e. $r^t_{C_{i,j,n}}$) and D2D link j (i.e. $r^t_{D_{i,j,n}}$) on RB n can be expressed as

$$r^t_{C_{i,j,n}} = \log_2(1 + \gamma^t_{C_{i,j,n}}), \quad t = 1, \ldots. T, \tag{4}$$

$$r^t_{D_{i,j,n}} = \log_2(1 + \gamma^t_{D_{i,j,n}}), \quad t = 1, \ldots. T, \tag{5}$$

where the data rate is calculated in b/s/Hz, which is normalized by channel bandwidth.

When cellular link does not experience any co-channel interference from D2D links, the maximum throughput could be achieved when cellular link transmits with its maximum transmission power (i.e. p^C_{max}). The data rate of cellular link i on RB n in scheduling period t without reusing can be expressed as[1]

$$r^t_{C_{i,n}} = \log_2(1 + \frac{p^C_{max} h^t_{i,B,n}}{\sigma^2}), \quad t = 1, \ldots. T. \tag{6}$$

For convenience, the frequently used symbols in this paper are listed in TABLE 1.

B. PF SCHEDULING

As mentioned above, a system is fair if it provides equal average data rates to all links over a long-term service and each link is activated only if the minimum SINR requirement is satisfied. According to [25], the PF scheduler scheme F

[1] We assume that the cellular links without reusing always meet the minimum SINR constraints.

TABLE 1. The list of symbols.

Symbols	Meanings
$p_{C_{i,j,n}}^{t}$	The transmission power of CUE i, when it shares the RB n with D2D pair j at scheduling period t
$p_{D_{i,j,n}}^{t}$	The transmission power of DUT j, when it reuses the RB n with CUE i at scheduling period t
$\gamma_{C_{i,j,n}}^{t}$	The SINR of cellular link i, when it shares the RB n with D2D pair j at scheduling period t
$\gamma_{D_{i,j,n}}^{t}$	The SINR of D2D link j, when it reuses the RB n with CUE i at scheduling period t
$r_{C_{i,j,n}}^{t}$	The data rate of cellular link i, when it shares the RB n with D2D pair j at scheduling period t
$r_{D_{i,j,n}}^{t}$	The data rate of D2D link j, when it reuses the RB n with CUE i at scheduling period t
$p_{C_{i,j,n}}^{t^*}$	The optimal transmission power of CUE i, when it shares the RB n with D2D pair j at scheduling period t
$p_{D_{i,j,n}}^{t^*}$	The optimal transmission power of DUT j, when it reuses the RB n with CUE i at scheduling period t
$r_{C_{i,j,n}}^{t^*}$	The optimal data rate of cellular link i, when it shares the RB n with D2D pair j at scheduling period t
$r_{D_{i,j,n}}^{t^*}$	The optimal data rate D2D link j, when it reuses the RB n with CUE i at scheduling period t
$pd_{low,i,j,n}^{t}$	The low bound of feasible set $p_{D_{i,j,n}}^{t}$ when CUE i transmits at it maximum power
$pd_{up,i,j,n}^{t}$	The up bound of feasible set $p_{D_{i,j,n}}^{t}$ when CUE i transmits at it maximum power

can be expressed as:

$$F = \arg\max_{S} \sum_{m\in U} \ln R_m^{(S)}, \tag{7}$$

where U is the user set, and $R_m^{(S)}$ is the average data rate of user m achieved by scheduling scheme S.

In D2D Underlay communication system, (7) can be equally transformed into

$$F = \begin{cases} \arg\max_{S}(\sum_{i\in\mathcal{K}} \ln r_{i,t}^{(S)} + \sum_{j\in\mathcal{L}} \ln r_{j,t}^{(S)}), & \text{for } t=1, \\ \arg\max_{S}(\Pi_{i\in\mathcal{K}}(1+\frac{r_{i,t}^{(S)}}{(t-1)R_{i,t-1}}) \\ \quad\times\Pi_{j\in\mathcal{L}}(1+\frac{r_{j,t}^{(S)}}{(t-1)R_{j,t-1}})), & \text{for } t\geq 2, \end{cases} \tag{8}$$

where $r_{i,t}^{(S)}$ and $r_{j,t}^{(S)}$ are the current data rates of CUE i and D2D pair j achieved by scheduling scheme S in scheduling period t, respectively. $R_{i,t-1}^{(S)}$ and $R_{j,t-1}^{(S)}$ are the average date rates of of CUE i and D2D pair j during previous $(t-1)$ scheduling periods. The deviation of (8) is given in Appendix A. As seen in (8), the scheduling scheme F has different expressions for $t=1$ and $t\geq 2$. Thus, they will be solved separately in the following sections. Noted that the PF scheduler scheme in $t=1$ is the initialization of the PF scheduler scheme when $t\geq 2$.

III. PROBLEM FORMULATION AND PROPOSED ALGORITHM FOR $t=1$

As shown in section II, the PF scheduler scheme for $t=1$ is modelled to maximize the sum of the logarithm of the current data rates for all links. Since the channel allocation of cellular and D2D links affect each other, we introduce two channel allocation matrices $\boldsymbol{\Phi}^1$ and $\boldsymbol{\chi}^1$ for cellular and D2D links,[2] respectively, where

$$\Phi_{i,n}^1 = \begin{cases} 1, & \text{if cellular link } i \text{ occupies RB } n \\ & \text{exclusively}, \\ 0, & \text{otherwise}, \end{cases} \tag{9}$$

[2]Here, the superscript 1 means $t=1$.

$$\chi_{i,j,n}^1 = \begin{cases} 1, & \text{if cellular link } i \text{ and D2D link } j \text{ reuse} \\ & \text{the same RB } n, \\ 0, & \text{otherwise}. \end{cases} \tag{10}$$

Then, our problem in (8) for $t=1$ taking into account the RB allocation can be reformulated into

$$\begin{aligned} &(\boldsymbol{P}_*^1, \boldsymbol{\Phi}_*^1, \boldsymbol{\chi}_*^1) \\ &= \arg\max_{(\boldsymbol{P}^1,\boldsymbol{\Phi}^1,\boldsymbol{\chi}^1)} \sum_{n=1}^{N}\{(\sum_{i=1}^{K}\Phi_{i,n}^1[\ln r_{C_{i,n}}^1]) \\ &\quad + \sum_{i=1}^{K}\sum_{j=1}^{L}\chi_{i,j,n}^1([\ln r_{C_{i,j,n}}^1] + [\ln r_{D_{i,j,n}}^1])\} \end{aligned} \tag{11}$$

$$\text{s.t. } \gamma_{C_{i,j,n}}^1 \geq \gamma_{min}^C, \quad 0 \leq p_{C_{i,j,n}}^1 \leq p_{max}^C, \ \forall i\in\mathcal{K}, \tag{11a}$$

$$\gamma_{D_{i,j,n}}^1 \geq \gamma_{min}^D, \quad 0 \leq p_{D_{i,j,n}}^1 \leq p_{max}^D, \ \forall j\in\mathcal{L}, \tag{11b}$$

$$\sum_{n=1}^{N}(\sum_{j=1}^{L}\chi_{i,j,n}^1 + \Phi_{i,n}^1) = 1, \quad \forall i\in\mathcal{K}, \tag{11c}$$

$$\sum_{n=1}^{N}\sum_{i=1}^{K}\chi_{i,j,n}^1 \leq 1, \quad \forall j\in\mathcal{L}, \tag{11d}$$

$$\sum_{i=1}^{K}(\sum_{j=1}^{L}\chi_{i,j,n}^1 + \Phi_{i,n}^1) = 1, \quad \forall n\in\mathcal{N}, \tag{11e}$$

$$\Phi_{i,n}^1, \chi_{i,j,n}^1 \in \{0,1\}, \quad \forall i\in\mathcal{K}, \forall j\in\mathcal{L}, \forall n\in\mathcal{N}, \tag{11f}$$

in which $\boldsymbol{\Phi}^1$ is a $K\times N$ channel allocation matrix for cellular links. Both $\boldsymbol{P}^1$ and $\boldsymbol{\chi}^1$ are the $K\times L\times N$ power and channel allocation matrices. $P_{i,j,n}^1 = [(p_{C_{i,j,n}}^{1^*}, p_{D_{i,j,n}}^{1^*})]$ is the optimal power allocated to cellular link i and D2D link j when they reuse the RB n in the first scheduling period. γ_{min}^C and γ_{min}^D are the minimum SINR requirements of cellular and D2D links, respectively. p_{max}^C and p_{max}^D are the maximum transmission power of cellular and D2D transmitters. In (11), the first term is the sum of logarithm cellular link data rates without reusing, and the second term is the sum of logarithm data rates of the cellular links and D2D links when they reuse the same RB.

Constraints $(11a)$ and $(11b)$ show that the minimum SINR requirement and power limit of individual cellular link and active D2D link in all transmission intervals. Constraint $(11c)$ shows that each cellular link is allocated one RB and constraint $(11d)$ indicates that each active D2D link can only reuse no more than one RB. Constraint $(11e)$ shows that each RB can be either exclusively allocated to one cellular link or reused by one cellular and one active D2D links. The final constraint $(11f)$ means the value of channel allocation indicators are binary.

In order to solve (11) efficiently, we first derive the optimal power allocation on a given RB for one reuse pair of cellular and D2D links, which enables us to determine the data rate of each RB if it is allocated. Based on the power allocation results, we then transform the original resource allocation into a 3-D channel allocation problem.

A. POWER ALLOCATION FOR $t = 1$

The power allocation is to allocate the transmission power to one D2D link and one cellular link which share the same RB by optimizing the sum function while meeting their minimum SINR requirements and power limits. Therefore, we will present our proposed optimal power allocation algorithm on one RB basis. Note that this procedure will be repeated for all channel allocation possibilities that cellular and D2D links reuse all different RBs.

Mathematically, when cellular link i reuses the RB n with D2D link j, the power allocation can be simplified as

$$\begin{aligned}
&(p^{1*}_{C_{i,j,n}}, p^{1*}_{D_{i,j,n}}) \\
&\quad = \underset{(p^1_{C_{i,j,n}}, p^1_{D_{i,j,n}})}{\arg\max} \; \{\ln r^1_{C_{i,j,n}} + \ln r^1_{D_{i,j,n}}\} \\
&\quad = \underset{(p^1_{C_{i,j,n}}, p^1_{D_{i,j,n}})}{\arg\max} \; \{r^1_{C_{i,j,n}} \times r^1_{D_{i,j,n}}\} \\
&\quad = \underset{(p^1_{C_{i,j,n}}, p^1_{D_{i,j,n}})}{\arg\max} \; \{\log(1+\gamma^1_{C_{i,j,n}}) \\
&\qquad\qquad \times \log(1+\gamma^1_{D_{i,j,n}})\} \\
&\quad = \underset{(p^1_{C_{i,j,n}}, p^1_{D_{i,j,n}})}{\arg\max} \; \{\ln \gamma^1_{C_{i,j,n}} \times \ln \gamma^1_{D_{i,j,n}}\} \qquad (12) \\
&\qquad \text{s.t. } \gamma^1_{C_{i,j,n}} \geq \gamma^C_{min}, \quad \gamma^1_{D_{i,j,n}} \geq \gamma^D_{min}, \qquad (12a) \\
&\qquad\quad 0 \leq p^1_{C_{i,j,n}} \leq p^C_{max}, \quad 0 \leq p^1_{D_{i,j,n}} \leq p^D_{max}. \qquad (12b)
\end{aligned}$$

Constrain $(12a)$ makes sure the SINRs of both cellular and active D2D links satisfy the minimum requirements, which are around 10dB in most cases. $(12b)$ is the transmission power constraints for both links.

It has been proved in [6] that at least one of the cellular and D2D links transmit at its maximum power will lead to the optimal performance, it also can be proved that the following lemma holds.

Lemma 1: At least one of $p^1_{C_{i,j,n}}$ or $p^1_{D_{i,j,n}}$ needs to reach its maximum value in order to maximize the product in (12).

Proof: See Appendix B. ■

Lemma 1 reduces the computational complexity of searching the feasible set by fixing the value of one variable in each case at $t = 1$. We define $\Omega^1_{i,j,n}$ as the feasible set of problem in (12), $\Omega 1^1_{i,j,n}$ and $\Omega 2^1_{i,j,n}$ are the feasible sets when cellular and D2D users transmit the maximum power, respectively. Thus, we have the following proposition.

Proposition 1. If the problem in (12) is feasible, its optimal power allocation solution belongs to the set $\Omega^1_{i,j,n} = \Omega 1^1_{i,j,n} \cup \Omega 2^1_{i,j,n}$; otherwise, the set is empty ($\Omega^1_{i,j,n} = \phi$).

We first assume $p^1_{C_{i,j,n}} = p^C_{max}$, then problem in (12) becomes

$$(p^C_{max}, p^{1*}_{D_{i,j,n}}) = \underset{(p^C_{max}, p^1_{D_{i,j,n}})}{\arg\max} \; f(p^C_{max}, p^1_{D_{i,j,n}}) \qquad (13)$$

$$\text{s.t. } \frac{p^C_{max} h^1_{i,B,n}}{\sigma^2 + p^1_{D_{i,j,n}} h^1_{j,B,n}} \geq \gamma^C_{min}, \qquad (13a)$$

$$\frac{p^1_{D_{i,j,n}} h^1_{j,n}}{\sigma^2 + p^C_{max} h^1_{i,j,n}} \geq \gamma^D_{min}, \qquad (13b)$$

$$0 \leq p^1_{D_{i,j,n}} \leq p^D_{max}, \qquad (13c)$$

where

$$f(p^C_{max}, p^1_{D_{i,j,n}}) = \ln\left(\frac{p^C_{max} h^1_{i,B,n}}{\sigma^2 + p^1_{D_{i,j,n}} h^1_{j,B,n}}\right) \ln\left(\frac{p^1_{D_{i,j,n}} h^1_{j,n}}{\sigma^2 + p^C_{max} h^1_{i,j,n}}\right).$$

Constraint $(13a)$ shows the minimum SINR requirements of cellular and active D2D links and constraint $(13c)$ shows the D2D transmission power should be positive and less than the maximum power (i.e. p^D_{max}).

From constraints $(13a)$-$(13c)$, the continuous closed and bounded feasible set of $p^1_{D_{i,j,n}}$ is obtained as $[pd^1_{low,i,j,n}, pd^1_{up,i,j,n}]$, where the lower and upper bounds $pd^1_{low,i,j,n}$ and $pd^1_{up,i,j,n}$ can be calculated as:

$$\begin{aligned}
pd^1_{low,i,j,n} &= \max\{0, \frac{\gamma^D_{min}(\sigma^2 + p^C_{max} h^1_{i,j,n})}{h^1_{j,n}}\}, \\
pd^1_{up,i,j,n} &= \min\{p^D_{max}, \frac{(p^C_{max} h^1_{i,B,n} - \gamma^C_{min}\sigma^2)}{h^1_{j,B,n}\gamma^C_{min}}\}, \qquad (14)
\end{aligned}$$

respectively. Note that the set $\Omega 1^1_{i,j,n}$ is valid only when $pd^1_{low,i,j,n} \leq pd^1_{up,i,j,n}$; otherwise, $\Omega 1^1_{i,j,n}$ is empty (i.e. $\Omega 1^1_{i,j,n} = \phi$).

As derived in Appendix C, only two solutions are included in $\Omega 1^1_{i,j,n}$:

$$\Omega 1^1_{i,j,n} = \{(p^C_{max}, pd^1_{low,i,j,n}), (p^C_{max}, pd^1_{up,i,j,n})\}. \qquad (15)$$

Since $\Omega 2^1_{i,j,n}$ can be obtained in the similar way, the deviation of $\Omega 2^1_{i,j,n}$ is omitted due to the space limitation.

Thereby, the optimal power allocation $(p^{1*}_{C_{i,j,n}}, p^{1*}_{D_{i,j,n}})$ can be performed by comparing at most 4 feasible solutions in set $\Omega^1_{i,j,n}$, which maximizes (13). The data rates of cellular link i and D2D link j on RB n can be calculated by

$$r^{1*}_{C_{i,j,n}} = \log_2(1 + \frac{p^{1*}_{C_{i,j,n}} h^1_{i,B,n}}{\sigma^2 + p^{1*}_{D_{i,j,n}} h^1_{j,B,n}}),$$
$$r^{1*}_{D_{i,j,n}} = \log_2(1 + \frac{p^{1*}_{D_{i,j,n}} h^1_{j,n}}{\sigma^2 + p^{1*}_{C_{i,j,n}} h^1_{i,j,n}}). \quad (16)$$

In the case that $\Omega 1^1_{i,j,n} = \phi$, we set $r^{1*}_{C_{i,j,n}} = r^{1*}_{D_{i,j,n}} = Q$, where Q is a extremely small value, which means RB n can not be reused by cellular link i and D2D link j.

B. CHANNEL ALLOCATION FOR $t = 1$

After the power allocation is performed by considering all reusing possibilities, the channel allocation is modelled as

$$(\mathbf{\Phi}^1_*, \boldsymbol{\chi}^1_*) = \arg\max_{(\mathbf{\Phi}_1, \boldsymbol{\chi}_1)} \sum_{n=1}^{N} \{(\sum_{i=1}^{K} \Phi^1_{i,n}[\ln r^1_{C_{i,n}}]) + \sum_{i=1}^{K}\sum_{j=1}^{L} \chi^1_{i,j,n}([\ln r^{1*}_{C_{i,j,n}}] + [\ln r^{1*}_{D_{i,j,n}}])\} \quad (17)$$

with the same constraints ($11c$)-($11f$). Note that $r^{1*}_{C_{i,j,n}}$ and $r^{1*}_{D_{i,j,n}}$ are obtained from the above power allocation.

Although problem in (17) can be optimally solved by the standard ILP (Integer Linear Programming) methods (such as interior-point method and Balas method in [26]), the complexity (i.e. $O(KLN)^{3.5}$) is extremely high. It is necessary to propose an effective solution to solve the above channel allocation.

We introduce $(K - L)$ virtual D2D links in set $\mathcal{L}_{virt} = \{L + 1, L + 2, \ldots, K\}$ to construct a cubic $K \times K \times K$ 3-D channel allocation problem with equal dimension, i.e. the total number of D2D links is K now and $N = K$. In this way, all the possible channel allocation can be modelled uniformly. Specifically, the cellular link which exclusively uses a particular RB can be treated as sharing its channel with a virtual D2D link. Otherwise, cellular link reuses a RB with an actual D2D link.

In order to distinguish the above actual D2D link index j, we now use a new index $l \in \{\mathcal{L}, \mathcal{L}_{virt}\}$ to uniformly indicate the actual and virtual D2D links in the rest of this subsection. This new 3-D allocation matrix allows us to apply the proposed iterative method with a single channel allocation index $\Gamma^1_{i,l,n}$, where

$$\Gamma^1_{i,l,n} = \begin{cases} 1, & \text{if cellular link } i \text{ and D2D link } l \text{ reuse} \\ & \text{the same RB } n \text{ in } t = 1, \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

As a result, the problem in (17) is restructured into

$$\mathbf{\Gamma}^*_1 = \arg\max_{\mathbf{\Gamma}_1} \sum_{n=1}^{K}\sum_{i=1}^{K}\sum_{l=1}^{K} \Gamma^1_{i,l,n}\Psi^1_{i,l,n} \quad (19)$$

$$\text{s.t.} \sum_{n=1}^{N}\sum_{l=1}^{L} \Gamma^1_{i,l,n} = 1, \quad \forall i \in \mathcal{K}, \quad (19a)$$
$$\sum_{n=1}^{N}\sum_{i=1}^{K} \Gamma^1_{i,l,n} = 1, \quad \forall l \in \{\mathcal{L}, \mathcal{L}_{virt}\}, \quad (19b)$$
$$\sum_{i=1}^{K}\sum_{l=1}^{L} \Gamma^1_{i,l,n} = 1, \quad \forall n \in \mathcal{N}, \quad (19c)$$
$$\Gamma^1_{i,l,n} \in \{0, 1\}, \quad \forall i \in \mathcal{K}, \ \forall l \in \{\mathcal{L}, \mathcal{L}_{virt}\}, \ \forall n \in \mathcal{N}, \quad (19d)$$

where

$$\Psi^1_{i,l,n} = \begin{cases} [\ln r^{1*}_{C_{i,j,n}}] + [\ln r^{1*}_{D_{i,j,n}}], & \text{for } l \in \mathcal{L}, \\ [\ln r^1_{C_{i,n}}], & \text{for } l \in \mathcal{L}_{virt}. \end{cases} \quad (20)$$

Constraint ($19a$) ensures that each cellular link occupies one RB. Constraint ($19b$) shows each D2D link can reuse one RB with cellular link and constraint ($19c$) means that each RB can be reused by one cellular link and one D2D link. Constraint ($19d$) shows the channel allocation index should be binary. Note that three constraints ($19a$)-($19c$) correspond to the ($11c$)- ($11e$). Moreover, (20) makes sure that when $\Gamma^1_{i,l,n} = 1$, if $l \in \mathcal{L}$, cellular and the actual D2D links share the same RB, otherwise (i.e. $l \in \mathcal{L}_{virt}$), cellular link occupies the RB exclusively. In this way, problem in (19) can be efficiently solved by our proposed *I2-DA* algorithm, which will be presented in Algorithm 3 in section V.

Algorithm 1 presents the operational procedure of the proposed joint power and channel allocation algorithm scheme for $t = 1$. As shown in *Step20* the average data rates of all cellular and actual D2D links are obtained according to (33) in Appendix A, and this will be used as the initialization of the subsequent scheduling periods.

IV. PROBLEM FORMULATION AND PROPOSED ALGORITHM FOR $t \geq 2$

Applying *Talyor* theorem, the objective function in (8) can be converted to maximize the sum of the ratios of the current data rate to the average data rate for $t \geq 2$ (see Appendix D). Thus, in a given scheduling period t, the joint problem can be expressed as

$$\max_{\boldsymbol{P}^t, \boldsymbol{\Phi}^t, \boldsymbol{\chi}^t} \sum_{n=1}^{K}\{\sum_{i=1}^{K} \Phi^t_{i,n}\frac{r^t_{C_{i,n}}}{R_{i,t-1}} + \sum_{i=1}^{K}\sum_{j=1}^{L} \chi^t_{i,j,n}\frac{r^t_{C_{i,j,n}}}{R_{i,t-1}} + \sum_{j=1}^{L}\sum_{i=1}^{K} \chi^t_{i,j,n}\frac{r^t_{D_{i,j,n}}}{R_{j,t-1}}\} \quad (21)$$

$$\text{s.t. } \gamma^t_{C_{i,j,n}} \geq \gamma^C_{min}, \quad 0 \leq p^t_{C_{i,j,n}} \leq p^C_{max}, \ \forall i \in \mathcal{K}, \quad (21a)$$
$$\gamma^t_{D_{i,j,n}} \geq \gamma^D_{min}, \quad 0 \leq p^t_{D_{i,j,n}} \leq p^D_{max}, \ \forall j \in \mathcal{L}, \quad (21b)$$
$$\sum_{n=1}^{N}(\sum_{j=1}^{L} \chi^t_{i,j,n} + \Phi^t_{i,n}) = 1, \quad \forall i \in \mathcal{K}, \quad (21e)$$

Algorithm 1 Joint Power and Channel Allocation Algorithm When $t = 1$

Power Allocation for $t = 1$**:**
for all $i \in \mathcal{K}, j \in \mathcal{L}, n \in \mathcal{N}$ **do**
  Obtain $pd^1_{low,i,j,n}$ and $pd^1_{up,i,j,n}$ from (14).
  if $pd^1_{low,i,j,n} \leq pd^1_{up,i,j,n}$ **then**
    $\Omega 1^1_{i,j,n} = \{(p^C_{max}, pd^1_{low,i,j,n}), (p^C_{max}, pd^1_{up,i,j,n})\}$ according to (15)
  else
    $\Omega 1^1_{i,j,n} = \phi$
  end if
  Obtain $\Omega 2^1_{i,j,n}$ in the similar way.
  Obtain $\Omega^1_{i,j,n}$ according to **Proposition 1**
  if $\Omega^1_{i,j,n} = \phi$ **then**
    $r^{1*}_{C_{i,j,n}} = r^{1*}_{D_{i,j,n}} = Q$
  else
    $r^{1*}_{C_{i,j,n}}$ and $r^{1*}_{D_{i,j,n}}$ can be obtained by (16), where $(p^{1*}_{C_{i,j,n}}, p^{1*}_{D_{i,j,n}}) = \underset{(p^1_{C_{i,j,n}}, p^1_{D_{i,j,n}}) \in \Omega^1_{i,j,n}}{\arg\max} (r^1_{C_{i,j,n}} \times r^1_{C_{i,j,n}})$
  end if
  $r^1_{C_{i,n}} = \log_2(1 + \frac{p^C_{max} h^1_{i,B,n}}{\sigma^2}), \forall i \in \mathcal{K}, \forall n \in \mathcal{N}$
end for
Channel Allocation for $t = 1$:
Based on above power allocation, $r_{i,1}$ and $r_{j,1}$, $\forall i \in \mathcal{K}, \forall j \in \mathcal{L}$ can be obtained by solving problem in (19) through our proposed *I2-DA* algorithm in Algorithm 3.
Obtain $R_{i,1} = r_{i,1}, R_{j,1} = r_{j,1}, i \in \mathcal{K}, j \in \mathcal{L}$.

$$\sum_{n=1}^{N}\sum_{i=1}^{K} \chi^t_{i,j,n} \leq 1, \quad \forall j \in \mathcal{L}, \tag{21f}$$
$$\sum_{i=1}^{K}(\sum_{j=1}^{L} \chi^t_{i,j,n} + \Phi^t_{i,n}) = 1, \quad \forall n \in \mathcal{N}, \tag{21g}$$
$$\Phi^t_{i,n}, \chi^t_{i,j,n} \in \{0, 1\}, \quad \forall i \in \mathcal{K}, \ \forall j \in \mathcal{L}, \ \forall n \in \mathcal{N}. \tag{21h}$$

where $\boldsymbol{P}^t$ and $\boldsymbol{\chi}^t$ are the $K \times L \times N$ power and channel allocation matrices at time t. $\boldsymbol{\Phi}^t$ is the $K \times N$ channel allocation matrix. The channel allocation index $\Phi^t_{i,n} = 1$, if cellular link i occupies RB n exclusively, otherwise, $\Phi^t_{i,n} = 0$ in scheduling period t. $\chi^t_{i,j,n} = 1$, if cellular link i and D2D link j reuse the same RB n, otherwise, $\chi^t_{i,j,n} = 0$ in scheduling period t. In (21), the first term is the sum of the ratios for cellular links without reusing; the second term is the sum of the ratios for cellular links with channel reusing, and the last term is the sum of the ratios for all D2D links. Constraints (21a)-(21h) in scheduling period t are equivalent to (11a)-(11f).

Problem in (21) is to allocate the appropriate transmission power and RBs for links which have high current data rates and low previous average data rates. Similar with the case in $t = 1$, we divide it into two sub-problems: power and channel allocations, then solve them sequentially.

A. POWER ALLOCATION FOR $t \geq 2$

When D2D link j shares the same RB n with cellular link i, the power allocation in a given scheduling period t is transformed to:

$$\begin{aligned}
&(p^{t*}_{C_{i,j,n}}, p^{t*}_{D_{i,j,n}}) \\
&= \underset{(p^t_{C_{i,j,n}}, p^t_{D_{i,j,n}})}{\arg\max} \ (\frac{r^t_{C_{i,j,n}}}{R_{i,t-1}} + \frac{r^t_{D_{i,j,n}}}{R_{j,t-1}}) \\
&= \underset{(p^t_{C_{i,j,n}}, p^t_{D_{i,j,n}})}{\arg\max} \ \frac{1}{R_{i,t-1}}\{\log_2(1 + \gamma^t_{C_{i,j,n}}) \\
&\qquad + \xi \log_2(1 + \gamma^t_{D_{i,j,n}})\} \\
&= \underset{(p^t_{C_{i,j,n}}, p^t_{D_{i,j,n}})}{\arg\max} \ (1 + \gamma^t_{C_{i,j,n}})(1 + \gamma^t_{D_{i,j,n}})^{\xi}
\end{aligned} \tag{22}$$
$$\text{s.t. } \gamma^t_{C_{i,j,n}} \geq \gamma^C_{min}, \quad \gamma^t_{D_{i,j,n}} \geq \gamma^D_{min}, \tag{22a}$$
$$0 \leq p^t_{C_{i,j,n}} \leq p^C_{max}, \quad 0 \leq p^t_{D_{i,j,n}} \leq p^D_{max}, \tag{22b}$$

where $\xi = \frac{R_{i,t-1}}{R_{j,t-1}}$. Note that the values of $R_{i,t-1}$ and $R_{j,t-1}$ are available in scheduling period t.

We define $\Omega^t_{i,j,n}$ as the feasible set of problem in (22). $\Omega 1^t_{i,j,n}$ and $\Omega 2^t_{i,j,n}$ are the feasible sets when cellular and D2D links transmit their maximum power, respectively. Then, we have the following lemma and proposition.

Lemma 2: At least one of $p^t_{C_{i,j,n}}$ and $p^t_{D_{i,j,n}}$ needs to reach its maximum value to maximize the objective function in (22).

Proof: Since the proof is similar with $t = 1$, which is given in Appendix B, we omit it in this paper. ∎

Similarly, *Lemma 2* can reduce the computational complexity of searching the feasible set by fixing the value of one variable in each case at $t \geq 2$.

Proposition 2. If the problem in (22) is feasible, its optimal power allocation solution belongs to the set $\Omega^t_{i,j,n} = \Omega 1^t_{i,j,n} \cup \Omega 2^t_{i,j,n}$; otherwise $\Omega^t_{i,j,n} = \phi$.

As derived in in Appendix E, we get set $\Omega 1^t_{i,j,n}$

$$\Omega 1^t_{i,j,n} = \begin{cases} \{(p^C_{max}, pd1^t_{i,j,n})\}, \\ \text{if } \Delta^t_{D,i,j,n} \geq 0, pd1^t_{i,j,n} \in [pd^t_{low,i,j,n}, pd^t_{up,i,j,n}], \\ \{(p^C_{max}, pd^t_{low,i,j,n}), (p^C_{max}, pd^t_{up,i,j,n})\}, \\ \text{if } \Delta^t_{D,i,j,n} \geq 0, pd1^t_{i,j,n} \notin [pd^t_{low,i,j,n}, pd^t_{up,i,j,n}], \\ \{(p^C_{max}, pd^t_{up,i,j,n})\}, \\ \text{if } \Delta^t_{D,i,j,n} < 0. \end{cases} \tag{23}$$

Since $\Omega 2^t_{i,j,n}$ can be obtained in the similar way, its deviation is omitted due to the space limitation. Then we can obtain the feasible set of power allocation as $\Omega^t_{i,j,n} = \Omega 1^t_{i,j,n} \cup \Omega 2^t_{i,j,n}$. Thereby, the optimal power allocation $(p^{t*}_{C_{i,j,n}}, p^{t*}_{D_{i,j,n}})$ can be performed by comparing at most 6 feasible power pairs in set $\Omega^t_{i,j,n}$, which can maximize (22). The optimal data rates of cellular link i and D2D link j on RB n can then be calculated as

$$r^{t*}_{C_{i,j,n}} = \log_2(1 + \frac{p^{t*}_{C_{i,j,n}} h^t_{i,B,n}}{\sigma^2 + p^{t*}_{D_{i,j,n}} h^t_{j,B,n}}),$$

$$r^{t*}_{D_{i,j,n}} = \log_2(1 + \frac{p^{t*}_{D_{i,j,n}} h^t_{j,n}}{\sigma^2 + p^{t*}_{C_{i,j,n}} h^t_{i,j,n}}). \quad (24)$$

In cases that $\Omega^t_{i,j,n} = \phi$, we set $r^{t*}_{C_{i,j,n}} = r^{t*}_{D_{i,j,n}} = Q$, indicating that D2D link j and cellular link i can not reuse the same RB n in scheduling period t.

B. CHANNEL ALLOCATION FOR $t \geq 2$

Once the power allocation is performed, the channel allocation can be modelled as:

$$(\mathbf{\Phi}^t_*, \boldsymbol{\chi}^t_*) = \arg\max_{\boldsymbol{\chi}_t} \sum_{n=1}^{K}\{\sum_{i=1}^{K} \Phi^t_{i,n} \frac{r^t_{C_{i,n}}}{R_{i,t-1}} + \sum_{i=1}^{K}\sum_{j=1}^{L} \chi^t_{i,j,n} \frac{r^{t*}_{C_{i,j,n}}}{R_{i,t-1}} + \sum_{j=1}^{L}\sum_{i=1}^{K} \chi^t_{i,j,n} \frac{r^{t*}_{D_{i,j,n}}}{R_{j,t-1}}\} \quad (25)$$

with constraints $(21e)$-$(21h)$.

Similarly, with the use of virtual D2D links, problem in (25) can also be restructured into

$$\mathbf{\Gamma}^*_t = \arg\max_{\mathbf{\Gamma}^t} \sum_{n=1}^{K}\sum_{i=1}^{K}\sum_{l=1}^{K} \Gamma^t_{i,l,n} \Psi^t_{i,l,n} \quad (26)$$

$$\text{s.t.} \sum_{n=1}^{N}\sum_{l=1}^{L} \Gamma^t_{i,l,n} = 1, \quad \forall i \in \mathcal{K}, \quad (26a)$$

$$\sum_{n=1}^{N}\sum_{i=1}^{K} \Gamma^t_{i,l,n} = 1, \quad \forall l \in \{\mathcal{L}, \mathcal{L}_{virt}\}, \quad (26b)$$

$$\sum_{i=1}^{K}\sum_{l=1}^{L} \Gamma^t_{i,l,n} = 1, \quad \forall n \in \mathcal{N}, \quad (26c)$$

$$\Gamma^t_{i,l,n} \in \{0, 1\}, \quad \forall i \in \mathcal{K},\ \forall l \in \{\mathcal{L}, \mathcal{L}_{virt}\}, \forall n \in \mathcal{N}, \quad (26d)$$

where $\Gamma^t_{i,l,n} = 1$, if cellular link i and the D2D link l reuse the same RB n, otherwise, $\Gamma^t_{i,l,n} = 0$. And

$$\Psi^t_{i,l,n} = \begin{cases} \frac{r^{t*}_{C_{i,j,n}}}{R_{i,t-1}} + \frac{r^{t*}_{D_{i,j,n}}}{R_{j,t-1}}, & \text{for } l \in \mathcal{L}, \\ \frac{r^t_{C_{i,n}}}{R_{i,t-1}}, & \text{for } l \in \mathcal{L}_{virt}. \end{cases} \quad (27)$$

Constraints $(26a)$-$(26d)$ and (27) are equivalent to $(19a)$-$(19d)$ and (20). In this way, problem in (26) can be solved by the proposed *I2-DA* algorithm effectively, which will be discussed in the following section.

Algorithm 2 presents the operational procedure of the proposed resource allocation scheme for scheduling period $t \geq 2$. We set $T = 20$, as commonly used for PF scheduling in practical systems [27]. Note that all the following results are presented and analysed in the scheduling period $t = 20$ if not otherwise specified.

Algorithm 2 Joint Power and Channel Allocation Algorithm When $t \geq 2$

1: **Initialization**: $R_{i(j),1} = r_{i(j),1}, \forall i \in \mathcal{K}, \forall j \in \mathcal{L}$ according to Algorithm 1.
2: **for all** t = 2:T **do**
3: **Power Allocation for** $t \geq 2$**:**
4: **for all** $i \in \mathcal{K}, j \in \mathcal{L}, n \in \mathcal{N}$ **do**
5: **Obtain** $\Omega^t_{i,j,n}$ according to **Proposition 2**
6: **if** $\Omega^t_{i,j,n} = \phi$ **then**
7: $r^{t*}_{C_{i,j,n}} = r^{t*}_{D_{i,j,n}} = Q$
8: **else**
9: $r^{t*}_{C_{i,j,n}}$ and $r^{t*}_{D_{i,j,n}}$ can be obtained by (24), where $(p^{C*}_{i,j,t}, p^{t*}_{D_{i,j,n}}) = \arg\max_{(p^t_{C_{i,j,n}}, p^t_{D_{i,j,n}}) \in \Omega^t_{i,j,n}} \{(1+\gamma^t_{C_{i,j,n}})(1+\gamma^t_{D_{i,j,n}})^{\xi}\}$
10: **end if**
11: **end for**
12: $r^t_{C_{i,n}} = \log_2(1 + \frac{p^C_{max} h^t_{i,B,n}}{\sigma^2}), \forall i \in \mathcal{K}, \forall n \in \mathcal{N}$
13: **Channel Allocation for** $t \geq 2$**:**
14: Based on above power allocation, $r_{i,t}$ and $r_{j,t}$, $\forall i \in \mathcal{K}, \forall j \in \mathcal{L}$ can be obtained by solving problem in (26) through our proposed *I2-DA* algorithm in Algorithm 3.
15: **Obtain** $R_{i,t}$ and $R_{j,t}$, $\forall i \in \mathcal{K}, \forall j \in \mathcal{L}$ according to (33) in Appendix A.
16: **end for**

In Algorithm 2, the average data rates of all links are initialized by the results obtained from Algorithm 1. The joint power and channel allocation for each subsequent scheduling period is then conducted. Specifically, in each subsequent scheduling period, we divide the problem in (21) into two sub-problems: power allocation and channel allocation, and solve them sequentially as discussed above. After that, the current and average data rates of all links in scheduling period t can be obtained as shown in *Step14-15*.

V. I2-DA ALGORITHM

The 3-D assignment problem can be solved by our proposed iterative three-stage 2-D assignment algorithm. In Stage1, we group the cellular and D2D links[3] which share the same RB as a couple. Then, we reallocate RBs to all the couples to maximize the system performance. Based on these results, the D2D link and its RB are treated as a new couple in Stage2. We then rematch the cellular links with all those new couples. In Stage3, the cellular link and its RB are coupled, then remapping is done between the D2D links and those couples. These three-stage allocation is repeated iteratively for Y iterations. According to our simulation, the convergence achieves after 3 iterations.

[3] In this section, the D2D links include both the actual and virtual D2D links.

In section III and IV, we have modelled the channel allocation problem in (19) for $t = 1$ and (26) for $t \geq 2$ as the same 3-D assignment problem, so the scheduling period indices are omitted in this section for brevity. The detailed procedure of our proposed algorithm is given below.

Initialization: Initialize the allocation matrix as $\mathbf{\Gamma}_{K\times K\times K} = \mathbf{\Gamma}_0^*$. Note that this initial allocation matrix will affect the performance of *I2-DA* algorithm. Thus, we apply the heuristic channel allocation algorithm in [28] for this initialization.

Stage1: Let index $\tau = i$ denote the cellular and D2D links couple $(i, l) \in \Delta_1$, where $\Delta_1 = \{(i, l)|[\mathbf{\Gamma}_0^*]_{i,l,n} = 1, \forall i, l, n\}$. We adopt a 2-D matching matrix $U_{K\times K} = [q_{\tau,n}]$, where $q_{\tau,n} = 1$ if the RB n is assigned to the reusing couple τ, otherwise, $q_{\tau,n} = 0$. Then the design problem becomes

$$U^* = \arg\max_{U} \sum_{\tau=1}^{K}\sum_{n=1}^{K} q_{\tau,n}\Psi 1_{\tau,n} \tag{28}$$

$$\text{s.t.} \sum_{n}^{N} q_{\tau,n} = 1, \quad \forall \tau; \sum_{\tau}^{K} q_{\tau,n} = 1, \quad \forall n, \tag{28a}$$

where $\Psi 1_{\tau,n} = \{\Psi_{i,l,n}|(i, l) \in \Delta_1\}$. Constraint (28$a$) shows each cellular link is allocated one RB. This is a 2-D matching problem, which can be solved by the *Kuhn-Munkres* algorithm directly. Now $\mathbf{\Gamma}_1^* = (\Delta_1, U^*)$ is a solution to the 3-D channel allocation problem.

Stage2: Let $\theta = l$ denote the D2D link and RB couple $(l, n) \in \Delta_2$, where $\Delta_2 = \{(l, n)|[\mathbf{\Gamma}_1^*]_{i,l,n} = 1, \forall i, l, n\}$. We adopt a 2-D matching matrix $Z_{K\times K} = [z_{\theta,i}]$, where $z_{\theta,i} = 1$ if the cellular link i is allocated to the couple θ, otherwise, $z_{\theta,i} = 0$. The design problem is

$$Z^* = \arg\max_{Z} \sum_{\theta=1}^{K}\sum_{i=1}^{K} z_{\theta,i}\Psi 2_{\theta,i} \tag{29}$$

$$\text{s.t.} \sum_{\theta}^{K} z_{\theta,i} = 1, \quad \forall i; \sum_{i}^{K} z_{\theta,i} = 1, \ \forall \theta, \tag{29a}$$

where $\Psi 2_{\theta,i} = \{\Psi_{i,l,n}|(l, n) \in \Delta_2\}$. Constraint (29$a$) shows that each D2D link reuses one RB with cellular link. Similarly, problem in (29) can be solved by the *Kuhn-Munkres* algorithm and returns the feasible set $\mathbf{\Gamma}_2^* = (\Delta_2, Z^*)$ to the 3-D channel allocation problem.

Stage3: Let $\upsilon = i$ denote the cellular link and RB couple $(i, n) \in \Delta_3$, where $\Delta_3 = \{(i, n)|[\mathbf{\Gamma}_2^*]_{i,l,n} = 1, \forall i, l, n\}$. We adopt the 2-D matching matrix $S_{K\times K} = [s_{\upsilon,l}]$, where $s_{\upsilon,l} = 1$ if D2D link l is allocated to the couple υ, otherwise, $s_{\upsilon,l} = 0$. The design problem is

$$S^* = \arg\max_{S} \sum_{\upsilon=1}^{K}\sum_{l=1}^{K} s_{\upsilon,l}\Psi 3_{\upsilon,l} \tag{30}$$

$$\text{s.t.} \sum_{\upsilon}^{K} s_{\upsilon,j} = 1, \quad \forall l; \sum_{j}^{K} s_{\upsilon,l} = 1, \ \forall \upsilon, \tag{30a}$$

where $\Psi 3_{\upsilon,l} = \{\Psi_{i,l,n}|(i, n) \in \Delta_3\}$. As discussed above, constraint (30a) indicates each D2D link reuses a RB with cellular link. By solving problem in (30), we can obtain $\mathbf{\Gamma}_3^* = (\Delta_3, S^*)$, which is the feasible set of the 3-D channel allocation problem. This three-stage procedure will be repeated Y times with the updated $\mathbf{\Gamma}_0^* = \mathbf{\Gamma}_3^*$.

Algorithm 3 shows the details of the proposed *I2-DA* algorithm. After all the iterations, the algorithm can converge to at least a local optimum. The proof of its convergence can refer to [29]. In Algorithm 3, the final current data rates of cellular and actual D2D links are obtained from *Step14-21*. It means when the actual D2D link l reuses the same RB n with cellular link i, the current data rates of both links can be obtained directly. Otherwise, it means cellular link does not share its RB with any D2D links. If the cellular and D2D links with Q data rates are selected, it means the cellular link occupies the channel exclusively. In such case, the D2D link is inactive, so its data rate is set as Q, as shown in *Step16*. Meanwhile, in this way, those inactive D2D pairs will have higher priorities to obtain the RBs in the subsequent scheduling periods.

Algorithm 3 *I2-DA* Algorithm

1: **Input**: The $\Psi_{i,l,n}$.
2: **Output**: r_i and r_j, $\forall i \in \mathcal{K}$, $\forall j \in \mathcal{L}$.
3: **Initialization**: Suppose $\mathbf{\Gamma}_0^*$
4: **for all** $y = 1{:}Y$ **do**
5:     **Stage 1**:
6:     **Obtain** U^* by solving problem in (28), $\mathbf{\Gamma}_1^* = (\Delta_1, U^*)$.
7:     **Stage 2**:
8:     **Obtain** Z^* by solving problem in (29), $\mathbf{\Gamma}_2^* = (\Delta_2, Z^*)$.
9:     **Stage 3**:
10:     **Obtain** S^* by solving problem in (30), $\mathbf{\Gamma}_3^* = (\Delta_3, S^*)$.
11:     Then, go back to Step 5 with updated $\mathbf{\Gamma}_0^* = \mathbf{\Gamma}_3^*$
12: **end for**
13: **Get** $\mathbf{\Gamma}^* = \mathbf{\Gamma}_3^*$.
14: **for all** $[\mathbf{\Gamma}^*]_{i,l,n} = 1, i \in \mathcal{K}, l \in \{\mathcal{L}, \mathcal{L}_{virt}\}, n \in \mathcal{N}$ **do**
15:     **if** $l \in \mathcal{L}$ and $r^*_{C_{i,j,n}} = Q$ **then**
16:         $r_i = r_{C_{i,n}}, r_j = Q$
17:     **else if** $r^*_{C_{i,j,n}} > Q$ **then**
18:         $r_i = r^*_{C_{i,j,n}}, r_j = r_l = r^*_{D_{i,j,n}}$
19:     **else**
20:         $r_i = r_{C_{i,n}}$
21:     **end if**
22: **end for**

VI. SIMULATION RESULTS AND COMPLEXITY ANALYSIS

A. SIMULATION SETUP

Monte Carlo simulation is used to evaluate the performance of our proposed algorithms. We consider test case 9 [30] defined by METIS, which describes an open-air-festival environmental model. In our system, the entire region is a cell with radius of 500 m. The BS is located in the centre, and the cellular users and D2D transmitters are distributed uniformly in the cell. The D2D receivers are distributed uniformly in a disk centred by the corresponding D2D transmitters with a radius of d_{max}. The cellular users and D2D pairs move in every scheduling period following the random-waypoint mobility model, where users choose their speeds and directions in the range [0 10] (m/s) and [0 2π] randomly. Our simulation parameters as summarized in TABLE 2, are chosen according to [20] for the purpose of comparison.

TABLE 2. Simulation parameters [20].

Maximum distance between D2D pairs d_{max} (m)	(20,...,500)
Number of channels N	20
Number of cellular links K	20
Number of D2D links L ($L \leq K$)	15 (if fixed)
Maximum cellular transmission power p^C_{max}(W)	0.5
Maximum D2D transmission power p^D_{max} (W)	0.5
SINR requirements of cellular links γ^C_{min} (dB)	5 (if fixed)
SINR requirements of D2D links γ^D_{min} (dB)	15
Noise power σ^2 (dBm)	-110
Pathloss exponent for all communications α	3

We use Jain's fairness index to measure the long-term fairness between different users in terms of their average data rate at scheduling period t, which is expressed as:

$$J_t = \frac{|\sum_{i=1}^{K} R_{i,t} + \sum_{j=1}^{L} R_{j,t}|^2}{(K+L)(\sum_{i=1}^{K} R_{i,t}^2 + \sum_{j=1}^{L} R_{j,t}^2)}. \quad (31)$$

J_t takes the values between 0 and 1. Value 1 means completely fair at scheduling period t (i.e. all average data rates are equal), and value 0 means absolutely unfair at time t (i.e. the divergence of all average data rates is very large). The decrease in divergence of all average data rates leads to the increase of J_t.

B. RESULTS AND ANALYSIS

We label the proposed joint power allocation and the *I2-DA* algorithm scheme as the *Iterative-scheme*, and compare it with the following three schemes.

Existing-scheme: The system fairness is researched in [20], where the RBs of cellular links are pre-allocated. However, the fairness is only considered in channel allocation in $t \geq 2$. Channel allocation in $t = 1$ and power allocation in all scheduling periods are converted to maximize the system throughput without considering the effect of average data rates on system fairness. This leads to an unfair system.

Improved-scheme: In [21], we assume that the RBs of cellular links are pre-allocated as did in [20]. The fairness is taken into account for both power and channel allocation in all scheduling periods. In this way, the *Improved-scheme* brings better performance than the *Existing-scheme* in any scheduling periods. The RB allocation in both the *Existing-scheme* and the *Improved-scheme* are solved by the *Kuhn-Munkres* algorithm.

Optimal-scheme: Transmit power is allocated by our proposed optimal power allocation in all scheduling periods. And the standard ILP method is applied to optimally solve the joint cellular and D2D links channel allocation problem.

Note that both the *Existing-scheme* and the *Improved-scheme* assume that the RBs for cellular links have been pre-allocated, and the *Optimal-scheme* jointly allocate the RBs for both cellular and D2D links. Moreover, both the *Improved-scheme* and the *Optimal-scheme* use the proposed optimal power allocation in all scheduling periods. Hence, the *Improved-scheme* is a special case of the *Optimal-scheme*. The system performance is evaluated in terms of system sum rate, the number of active D2D links and the Jain's fairness index against different parameters including the maximum distance between D2D pairs d_{max}, the total number of D2D links L, and the cellular link's minimum SINR requirement γ^C_{min}.

1) JAIN'S FAIRNESS INDEX

Fig. 3 shows J_t of various schemes for different scheduling periods. It is shown that J_t of any schemes increases with the scheduling period t when($t \leq 10$). That is because the links with low average data rates have more chance to improve their current data rates during the first few scheduling periods, so the divergence in the links' average data rates is reduced. This increase slows down and converges when $t \geq 10$.

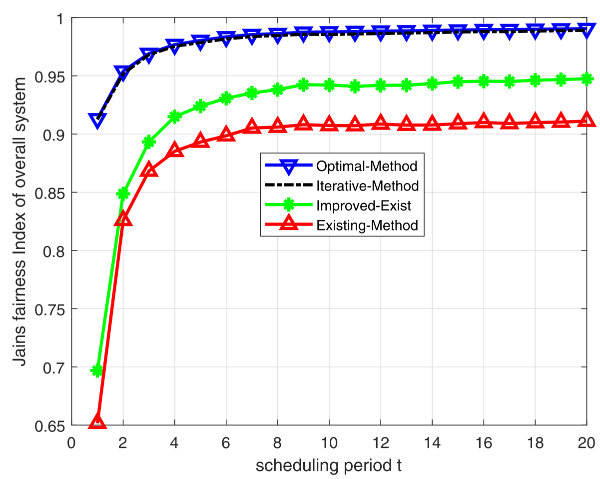

FIGURE 3. Jain's fairness index J_t of overall system versus different scheduling period t with $d_{max} = 20m$, $L = 10$.

Fig. 4 shows the J_t of overall system with various schemes for different parameters. Fig. 4 (a) shows the comparison of J_t of various schemes for different d_{max}. We observe that J_t first increases and then decreases with the increase of d_{max} for all schemes. This is because the current data rates of D2D links are larger than that of cellular links due to the short transmission distance when d_{max} is small. This leads to a large data rate divergence between cellular and D2D links. However, with the increase of d_{max}, the current data rates of D2D links decrease. When $d_{max} = 100m$, the current data rates of D2D and cellular links are close to each other. Therefore, J_t of those various schemes reach the peak values at this point. With the continuous increase of d_{max}, the current

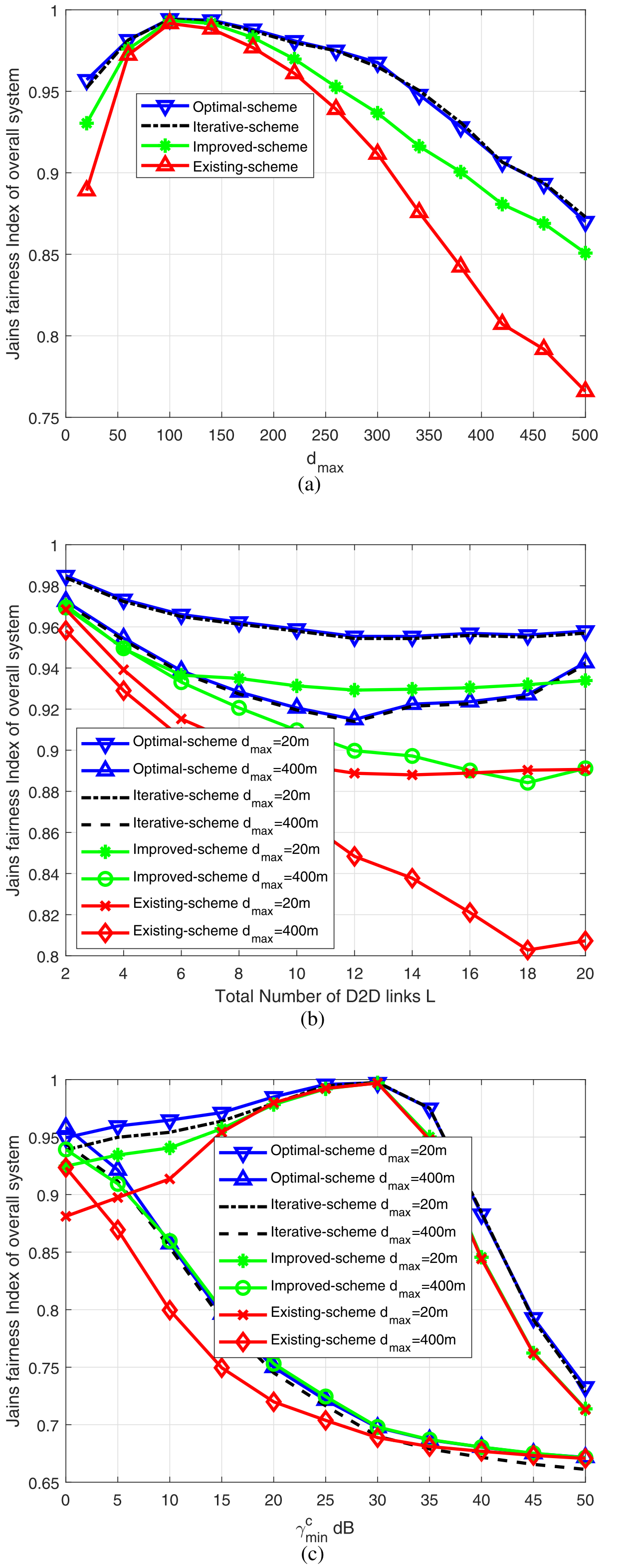

FIGURE 4. J_t **of overall system with various schemes (a) versus different** d_{max} **when** $L = 15$**; (b) versus different** L **with** $d_{max} = 20m$ **and** $d_{max} = 400m$**; (c) versus different** γ^C_{min} **with** $d_{max} = 20m$ **and** $d_{max} = 400m$**, when** $L = 15$**.**

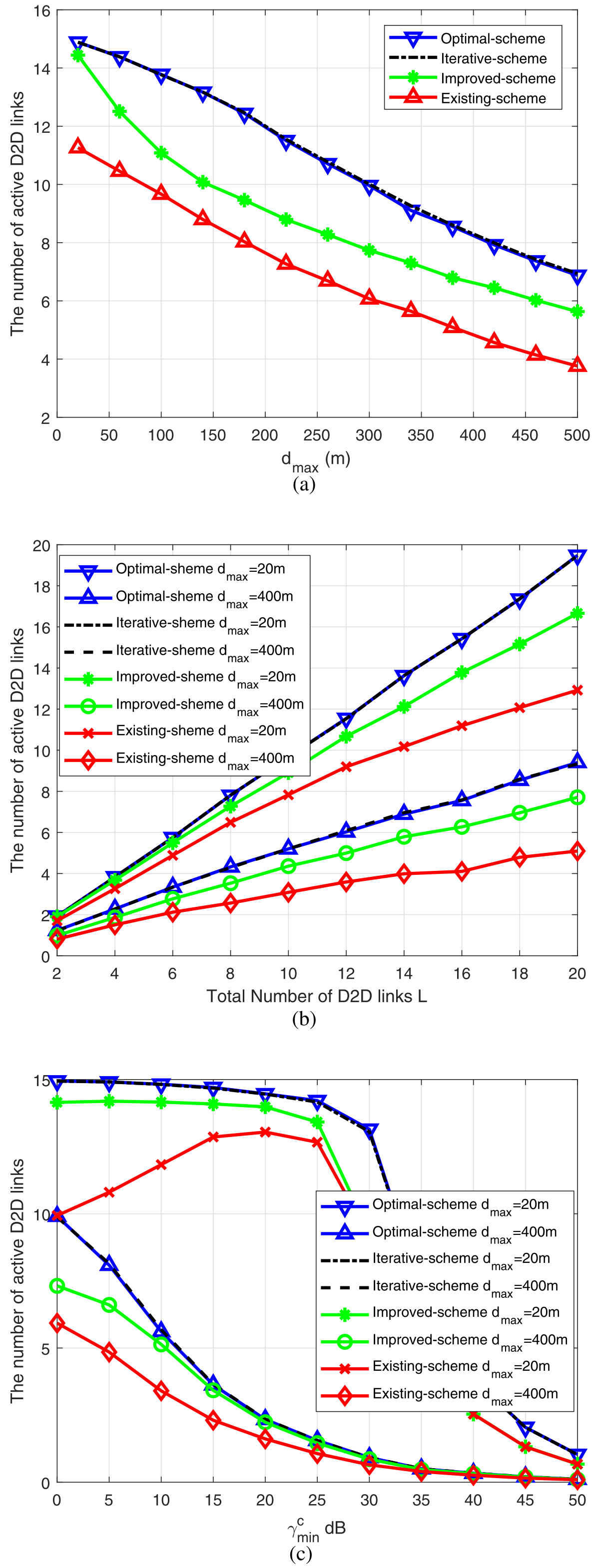

FIGURE 5. The number of active D2D links with various schemes (a) versus different d_{max} **when** $L = 15$**; (b) versus different** L **when** $d_{max} = 20m$ **and** $d_{max} = 400m$**; (c) versus different** γ^C_{min} **when** $L = 15$ **under** $d_{max} = 20m$ **and** $d_{max} = 400m$**.**

data rates of active D2D links become smaller than that of cellular links. In addition, the number of active D2D links also decreases with the increase of d_{max} (as proved in Fig. 5 (a)). Thus, the difference between the average data rates of cellular and D2D links increases again, leading to the decrease of J_t.

Fig. 4 (b) shows J_t decreases slightly with the increase of L when $d_{max} = 20m$. This is because more D2D links are active with the increase of L, and the active D2D links have higher current data rate than that of cellular links due to the short transmission distance. However, J_t decreases dramatically when $d_{max} = 400m$. This is because more D2D links are inactive when d_{max} is large. Thus, the ratio of active D2D links and inactive D2D links becomes small with the increase of L, leading to high divergence of all links.

Fig. 4 (c) shows the comparison of J_t between various schemes for different γ^C_{min}. With the increase of γ^C_{min}, J_t first increases and then decreases when $d_{max} = 20m$, and decreases dramatically when $d_{max} = 400m$. When $d_{max} = 20m$, the current data rates of cellular links get close to that of D2D links with the increase of γ^C_{min}. Thus, with large number of active D2D links, the divergence of current data rates between cellular and D2D links becomes smaller. When γ^C_{min} becomes large enough, the number of active D2D links decreases dramatically as shown in Fig.5 (c), which directly results in lower J_t. When $d_{max} = 400m$, the increase of γ^C_{min} leads to the decrease of active D2D links. This results in the increased divergence of current data rates between cellular and D2D links.

All the above results show clearly that our proposed *Iterative-scheme* produces higher system fairness than that of the *Improved-scheme* and the *Existing-Method* in any scenarios, and approaches the performance of the *Optimal-scheme* closely.

2) SYSTEM SUM RATE

Fig. 6 shows the system sum rate in various schemes for different parameters. As shown in Fig. 6 (a), with the increase of d_{max}, the system sum rate decreases. This is because the contribution of D2D links' current data rates become smaller with the increase of d_{max} due to the poor channel gain. Also, the number of active D2D links decreases with the increase of d_{max} as shown in Fig. 5 (a).

Fig. 6 (b) shows that when $d_{max} = 20m$, the system sum data rate increases dramatically for various schemes with the increase of L. Observing this result together with Fig.5 (b), we find that when $d_{max} = 20m$, the number of active D2D links increases notably with the increase of L, leading to the dramatical increase in system sum data rate. However, when $d_{max} = 400m$ it decreases slightly for the *Iterative-scheme* and keeps stable for both the *Improved-scheme* and *Existing-Method*. This is due to the low ratio of active D2D links and inactive D2D links. Moreover, the proposed *Iterative-scheme* sacrifices the system sum data rate to maintain the high system fairness,

Fig. 6 (c) shows that the system sum data rate decreases with the increase of γ^C_{min} when $d_{max} = 20m$. This is because fewer cellular links can meet the minimum SINR requirement with the increase of γ^C_{min}. This results in the decrease of the number of active D2D links as shown in Fig. 5 (c). However, the system sum rate first increases slightly and then stays stable with the increase of γ^C_{min} when $d_{max} = 400m$. Since we

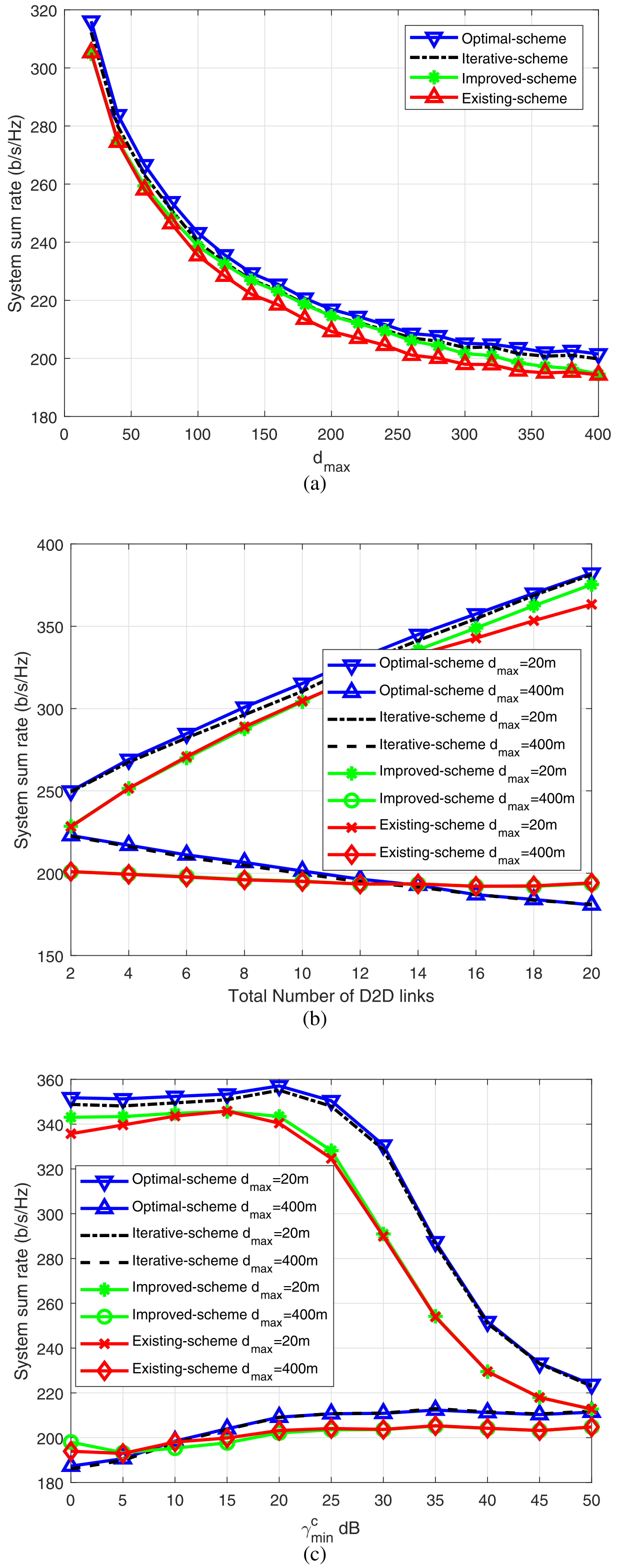

FIGURE 6. The system sum rate with various schemes (a) versus different d_{max}; (b) versus different L with $d_{max} = 20m$ and $d_{max} = 400m$; (c) versus different γ^C_{min} with $d_{max} = 20m$, $d_{max} = 400m$.

have assumed that cellular links without reusing always meet the QoS requirements, the number of cellular links which exclusively use the RBs increase with the increase of the γ^C_{min}, leading to the increase of the system sum rate. In addition, with the continuous increase of γ_{min}^{C}, none of the D2D links is activated. Therefore, the network only contains cellular users leading to stable sum data rate.

In summary, the system sum data rate of the *Iterative-scheme* is significantly better than that of both the *Improved-scheme* and the *Existing-Method*, and approaches closely to the performance of the *Optimal-scheme*.

C. COMPLEXITY ANALYSIS

The computational complexity is analysed in terms of number of operations required in the power allocation and channel allocation. The complexity of the power allocation is to select among a few solutions, so it is indeed negligible in this paper. Since the computational complexity of the *Kuhn-Munkres* method is $\mathcal{O}(K^3)$ for a 2-D assignment problem, then the complexity of the proposed three-stage *I2-DA* algorithm in each iteration is $\mathcal{O}(3 \times K^3)$.

Therefore, the overall complexity of the proposed algorithm is $\mathcal{O}(Y \times 3 \times K^3) = \mathcal{O}(K^3)$. The standard ILP method used in the *Optimal-scheme* requires $O(KLN)^{3.5}$ [31], which is significantly higher than the *I2-DA* algorithm ($\mathcal{O}(KLN)^{3.5} \gg \mathcal{O}(K^3)$), even with a small L value. For example, in a typical D2D underlay communication system, with $K = 20, L = 10, N = 20$, the complexity of the proposed algorithm is $K^3 = 20^3$, and the *Optimal-scheme* requires $O(KLN)^{3.5} = (4000)^{3.5}$.

VII. CONCLUSION

In this paper, we have formulated the joint power and channel allocation for D2D underlay communications aiming to maximize the system fairness while guaranteeing the QoS of all cellular and active D2D links. Based on the optimal power allocation solutions for all possibles of cellular and D2D links on any channels, we introduced virtual D2D links and constructed a 3-D channel allocation with equal dimensions, and proposed the *I2-DA* algorithm to efficiently solve the channel allocation with greatly reduced computational complexity. The performance results reveal that the proposed channel allocation algorithm outperforms the existing method and produces the close-to-optimal performance with low computational complexity. Moreover, numerical results show that the proposed scheme not only dramatically enhance the system fairness but also improve the system throughput comparing with the existing schemes.

APPENDIX A
THE DERIVATION OF (8)

According to (7) a PF scheduler scheme F at scheduling period t should satisfy

$$\sum_{i\in\mathcal{K}} \ln R_{i,t}^{(F)} + \sum_{j\in\mathcal{L}} \ln R_{j,t}^{(F)} \geq \sum_{i\in\mathcal{K}} \ln R_{i,t}^{(S)} + \sum_{j\in\mathcal{L}} \ln R_{j,t}^{(S)}, \quad (32)$$

where $R_{i,t}^{(F)}$ and $R_{j,t}^{(F)}$ are the average date rate of cellular user i and D2D link j in scheduling period t with scheduling scheme F, $R_{i,t}^{(S)}$ and $R_{j,t}^{(S)}$ are the average date rate of cellular user i and D2D link j in scheduling period t with scheduling scheme S. $R_{i,t}^{(S)}$ and $R_{j,t}^{(S)}$ can be obtained as

$$R_{i,(j),t}^{(S)} = \begin{cases} r_{i(j),t}^{(S)}, & t = 1, \\ \dfrac{(t-1)R_{i,(j),t-1} + r_{i(j),t}^{(S)}}{t}, & t \geq 2. \end{cases} \quad (33)$$

For $t = 1$, (32) can be rewritten as

$$\sum_{i\in\mathcal{K}} \ln r_{i,t}^{(F)} + \sum_{j\in\mathcal{L}} \ln r_{j,t}^{(F)} \geq \sum_{i\in\mathcal{K}} \ln r_{i,t}^{(S)} + \sum_{j\in\mathcal{L}} \ln r_{j,t}^{(S)}. \quad (34)$$

For $t \geq 2$, (32) can be transformed to

$$\Pi_{i\in\mathcal{K}} R_{i,t}^{(F)} \times \Pi_{j\in\mathcal{L}} R_{j,t}^{(F)} \geq \Pi_{i\in\mathcal{K}} R_{i,t}^{(S)} \times \Pi_{j\in\mathcal{L}} R_{j,t}^{(S)}. \quad (35)$$

According to (33), (35) can be rewritten as

$$\Pi_{i\in\mathcal{K}} \frac{(t-1)R_{i,t-1} + r_{i,t}^{(F)}}{t} \times \Pi_{j\in\mathcal{L}} \frac{(t-1)R_{j,t-1} + r_{j,t}^{(F)}}{t} \geq \Pi_{i\in\mathcal{K}} \frac{(t-1)R_{i,t-1} + r_{i,t}^{(S)}}{t} \times \Pi_{j\in\mathcal{L}} \frac{(t-1)R_{j,t-1} + r_{j,t}^{(S)}}{t}. \quad (36)$$

By multiplying $\Pi_{i\in\mathcal{K}} \frac{t}{(t-1)R_{i,t-1}} \times \Pi_{j\in\mathcal{L}} \frac{t}{(t-1)R_{j,t-1}}$ on both sides of (36), we can obtain

$$\Pi_{i\in\mathcal{K}}(1 + \frac{r_{i,t}^{(F)}}{(t-1)R_{i,t-1}}) \times \Pi_{j\in\mathcal{L}}(1 + \frac{r_{j,t}^{(F)}}{(t-1)R_{j,t-1}}) \geq \Pi_{i\in\mathcal{K}}(1 + \frac{r_{i,t}^{(S)}}{(t-1)R_{i,t-1}}) \times \Pi_{j\in\mathcal{L}}(1 + \frac{r_{j,t}^{(S)}}{(t-1)R_{j,t-1}}). \quad (37)$$

In summary, the PF scheduler scheme F for D2D communication underlay network can be expressed as (8).

APPENDIX B

Let $\alpha > 1$, we have the following inequality:

$$\begin{aligned} &\ln(\frac{\alpha p_{C_{i,j,n}}^{1} h_{i,B,n}^{1}}{\sigma^2 + \alpha p_{D_{i,j,n}}^{1} h_{j,B,n}^{1}}) \times \ln(\frac{\alpha p_{D_{i,j,n}}^{1} h_{j,n}^{1}}{\sigma^2 + \alpha p_{C_{i,j,n}}^{1} h_{i,j,n}^{1}}) \\ &= \ln(\frac{p_{C_{i,j,n}}^{1} h_{i,B,n}^{1}}{\frac{\sigma^2}{\alpha} + p_{D_{i,j,n}}^{1} h_{j,B,n}^{1}}) \times \ln(\frac{p_{D_{i,j,n}}^{1} h_{j,n}^{1}}{\frac{\sigma^2}{\alpha} + p_{C_{i,j,n}}^{1} h_{i,j,n}^{1}}) \\ &\geq \ln(\frac{p_{C_{i,j,n}}^{1} h_{i,B,n}^{1}}{\sigma^2 + p_{D_{i,j,n}}^{1} h_{j,B,n}^{1}}) \times \ln(\frac{p_{D_{i,j,n}}^{1} h_{j,n}^{t}}{\sigma^2 + p_{C_{i,j,n}}^{1} h_{i,j,n}^{1}}). \end{aligned} \quad (38)$$

From the inequality, larger α leads to larger multiplication. Therefore, maximum multiplication can be achieved when at least one of $p_{C_{i,j,n}}^{1}$ and $p_{D_{i,j,n}}^{1}$ achieves its maximum value.

APPENDIX C
THE DERIVATION OF (15)

The maximum value of $f(p_{max}^{C}, p_{D_{i,j,n}}^{1})$ can be obtained by solving the following equation

$$f'(p_{max}^{C}, p_{D_{i,j,n}}^{1}) = 0,$$

which always has two roots

$$pd1^1_{i,j,n} = \frac{h^1_{i,j,n}p^C_{max} + \sigma^2}{h^1_{j,n}},$$
$$pd2^1_{i,j,n} = \frac{h^1_{i,B,n}p^C_{max} - \sigma^2}{h^1_{j,B,n}}.$$

Combining with (14), we find that $pd1^1_{i,j,n} \leq pd^1_{low,i,j,n}$ and $pd2^1_{i,j,n} \geq pd^1_{up,i,j,n}$ always hold. It means these two roots do not belong to the set $[pd^1_{low,i,j,n}, pd^1_{up,i,j,n}]$, where $pd^1_{low,i,j,n} \leq pd^1_{up,i,j,n}$. Thus, the $f(p^C_{max}, p^1_{D_{i,j,n}})$ is monotonous in the set $[pd^1_{low,i,j,n}, pd^1_{up,i,j,n}]$. The maximum value of $f(p^C_{max}, p^1_{D_{i,j,n}})$ is obtained on the boundary i.e. $pd1^1_{i,j,n}$ or $pd2^1_{i,j,n}$, so there are only two possible solutions in the set $\Omega 1^1_{i,j,n}$.

APPENDIX D

From (8), the optimal PF scheduling for $t \geq 2$ can be rewritten as

$$F = \arg\max_S (\sum_{i\in\mathcal{K}} \ln(1 + \frac{r^S_{i,t}}{(t-1)R_{i,t-1}}) + \sum_{j\in\mathcal{L}} \ln(1 + \frac{r^S_{j,t}}{(t-1)R_{j,t-1}})). \quad (39)$$

Since *Maclaurin* series is a special case of *Taylor* series centred at 0, and as t increases, $[\frac{r^S_{i,t}}{(t-1)R_{i,t-1}}]$ and $[\frac{r^S_{j,t}}{(t-1)R_{j,t-1}}]$ will approach 0. Thus, (39) can be further written as

$$F = \arg\max_S \{\sum_{i\in\mathcal{K}}\sum_{m=0}^{\infty} \frac{(-1)^m}{(m+1)}[\frac{r^S_{i,t}}{(t-1)R_{i,t-1}}]^{(m+1)} + \sum_{j\in\mathcal{L}}\sum_{m=0}^{\infty} \frac{(-1)^m}{(m+1)}[\frac{r^S_{j,t}}{(t-1)R_{j,t-1}}]^{(m+1)}\}, \quad (40)$$

with infinite terms. In this paper we only take the first significant term to express F (i.e. $m = 0$). It can be simplified as

$$F \approx \arg\max_S \{\sum_{i\in\mathcal{K}}[\frac{r^S_{i,t}}{(t-1)R_{i,t-1}}] + \sum_{j\in\mathcal{L}}[\frac{r^S_{j,t}}{(t-1)R_{j,t-1}}]\} = \arg\max_S \{\sum_{i\in\mathcal{K}} \frac{r^S_{i,t}}{R_{i,t-1}} + \sum_{j\in\mathcal{L}} \frac{r^S_{j,t}}{R_{j,t-1}}\}. \quad (41)$$

APPENDIX E

We assume $p^t_{C_{i,j,n}} = p^C_{max}$, so the problem in (22) with constraints can be written as

$$(p^C_{max}, p^{t*}_{D_{i,j,n}}) = \arg\max_{(p^C_{max}, p^t_{D_{i,j,n}})} f(p^C_{max}, p^t_{D_{i,j,n}}) \quad (42)$$

$$\text{s.t.} \frac{p^C_{max}h^t_{i,B,n}}{\sigma^2 + p^t_{D_{i,j,n}}h^t_{j,B,n}} \geq \gamma^C_{min}, \quad (42a)$$

$$\frac{p^t_{D_{i,j,n}}h^1_{j,n}}{\sigma^2 + p^C_{max}h^t_{i,j,n}} \geq \gamma^D_{min}, \quad (42b)$$

$$0 \leq p^t_{D_{i,j,n}} \leq p^D_{max}, \quad (42c)$$

in which $f(p^C_{max}, p^t_{D_{i,j,n}}) = \{(1 + \frac{p^C_{max}h^t_{i,B,n}}{\sigma^2 + p^t_{D_{i,j,n}}h^t_{j,B,n}}) \times (1 + \frac{p^t_{D_{i,j,n}}h^t_{j,n}}{\sigma^2 + p^C_{max}h^t_{i,j,n}})^\xi\}$.

According to constraints (42*a*)-(42*c*), we can get the continuous closed and bounded feasible set of $p^t_{D_{i,j,n}}$, which is $[pd^t_{low,i,j,n}, pd^t_{up,i,j,n}]$, where

$$pd^t_{low,i,j,n} = \max\{0, \frac{\gamma^D_{min}(\sigma^2 + p^C_{max}h^t_{i,j,n})}{h^t_{j,n}}\}, \quad (43)$$

$$pd^t_{up,i,j,n} = \min\{p^D_{max}, \frac{(p^C_{max}h^t_{i,B,n} - \gamma^C_{min}\sigma^2)}{h^t_{j,B,n}\gamma^C_{min}}\}. \quad (44)$$

Problem in (42) is valid only when $pd^t_{low,i,j,n} \leq pd^t_{up,i,j,n}$; otherwise, the set $\Omega 1^t_{i,j,n}$ is empty.

When problem in (42) is valid, the optimal value of $f(p^C_{max}, p^t_{D_{i,j,n}})$ can be found by solving the following equation

$$f'(p^C_{max}, p^t_{D_{i,j,n}}) = \frac{A^t_{D,i,j,n}(p^t_{D_{i,j,n}})^2 + B^t_{D,i,j,n}p^t_{D_{i,j,n}} + V^t_{D,i,j,n}}{W^t_{D,i,j,n}} = 0, \quad (45)$$

where

$$\begin{aligned} A^t_{D,i,j,n} &= \xi h^t_{j,n}(h^t_{j,B,n})^2, \\ B^t_{D,i,j,n} &= (\xi - 1)p^C_{max}h^t_{i,B,n}h^t_{j,B,n}h^t_{j,n} + 2\xi h^t_{j,n}\sigma^2 h^t_{j,B,n}, \\ V^t_{D,i,j,n} &= \xi h^t_{j,n}\sigma^2(\sigma^2 + p^C_{max}h^t_{i,B,n}) - p^C_{max}h^t_{i,B,n}h^t_{j,B,n}(\sigma^2 + p^C_{max}h^t_{i,j,n}), \\ W^t_{D,i,j,n} &= (\sigma^2 + p^C_{max}h^t_{i,j,n})^\xi(\sigma^2 + p^t_{D_{i,j,n}}h^t_{j,B,n})^2. \end{aligned} \quad (46)$$

If $\Delta^t_{D,i,j,n} = (B^t_{D,i,j,n})^2 - 4\,A^t_{D,i,j,n}V^t_{D,i,j,n} \geq 0$, then (45) has two solutions:

$$pd1^t_{i,j,n} = \frac{-B^t_{D,i,j,n} - \sqrt{\Delta^t_{D,i,j,n}}}{2A^t_{D,i,j,n}}, \quad (47)$$

$$pd2^t_{i,j,n} = \frac{-B^t_{D,i,j,n} + \sqrt{\Delta^t_{D,i,j,n}}}{2A^t_{D,i,j,n}}. \quad (48)$$

$A^t_{D,i,j,n}$ is always positive, so $pd1^t_{i,j,n}$ and $pd2^t_{i,j,n}$ are the local maximum and minimum points of function $f(p^C_{max}, p^t_{D_{i,j,n}})$, respectively. If $pd1^t_{i,j,n} \in [pd^t_{low,i,j,n}, pd^t_{up,i,j,n}]$, $pd1^t_{i,j,n}$ is the optimal solution of function $f(p^C_{max}, p^t_{D_{i,j,n}})$. If not, the bounds $pd^t_{up,i,j,n}$ or $pd^t_{low,i,j,n}$ is the optimal solution. This is because $f(p^C_{max}, p^t_{D_{i,j,n}})$ is a convex function in $[pd^t_{low,i,j,n}, pd^t_{up,i,j,n}]$ when $pd1^t_{i,j,n} \notin [pd^t_{low,i,j,n}, pd^t_{up,i,j,n}]$. Therefore, the optimal solution of $f(p^C_{max}, p^t_{D_{i,j,n}})$ can be either $pd^t_{up,i,j,n}$ or $pd^t_{low,i,j,n}$.

If $\Delta^t_{D,i,j,n} < 0$, it means $f'(p^C_{max}, p^t_{D_{i,j,n}})$ is always positive, so $f(p^C_{max}, p^t_{D_{i,j,n}})$ increases monotonically in $[pd^t_{low,i,j,n}, pd^t_{up,i,j,n}]$. Therefore, upper bound $pd^t_{up,i,j,n}$ is the optimal solution.

REFERENCES

[1] K. Doppler, M. Rinne, C. Wijting, C. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-advanced networks," *IEEE Commun. Mag.*, vol. 47, no. 12, pp. 42–49, Dec. 2009.

[2] X. Zhu, S. Wen, G. Cao, X. Zhang, and D. Yang, "QoS-based resource allocation scheme for device-to-device (D2D) radio underlaying cellular networks," in *Proc. 19th Int. Conf. Telecommun. (ICT)*, Apr. 2012, pp. 1–6.

[3] B. Peng, C. Hu, T. Peng, Y. Yang, and W. Wang, "A resource allocation scheme for D2D multicast with QoS protection in OFDMA-based systems," in *Proc. IEEE 24th Annu. Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, Sep. 2013, pp. 12383–12387.

[4] P. Phunchongharn, E. Hossain, and D. I. Kim, "Resource allocation for device-to-device communications underlaying LTE-advanced networks," *IEEE Wireless Commun.*, vol. 20, no. 4, pp. 91–100, Aug. 2013.

[5] Z. Liu and Y. Ji, "Intercell interference coordination under data rate requirement constraint in LTE-advanced heterogeneous networks," in *Proc. IEEE 79th Veh. Technol. Conf. (VTC Spring)*, May 2014, pp. 1–5.

[6] D. Feng, L. Lu, Y. Yuan-Wu, G. Y. Li, G. Feng, and S. Li, "Device-to-device communications underlaying cellular networks," *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3541–3551, Aug. 2013.

[7] T. D. Hoang, L. B. Le, and T. Le-Ngoc, "Resource allocation for D2D communication underlaid cellular networks using graph-based approach," *IEEE Trans. Wireless Commun.*, vol. 15, no. 10, pp. 7099–7113, Oct. 2016.

[8] C. Gao, X. Sheng, J. Tang, W. Zhang, S. Zou, and M. Guizani, "Joint mode selection, channel allocation and power assignment for green device-to-device communications," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2014, pp. 178–183.

[9] F. Wang, C. Xu, L. Song, and Z. Han, "Energy-efficient resource allocation for device-to-device underlay communication," *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2082–2092, Apr. 2015.

[10] R. Yin, C. Zhong, G. Yu, Z. Zhang, K. K. Wong, and X. Chen, "Joint spectrum and power allocation for D2D communications underlaying cellular networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 4, pp. 2182–2195, Apr. 2016.

[11] Y. Jiang, Q. Liu, F. Zheng, X. Gao, and X. You, "Energy-efficient joint resource allocation and power control for D2D communications," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6119–6127, Aug. 2016.

[12] Z. Zhou, M. Dong, K. Ota, G. Wang, and L. T. Yang, "Energy-efficient resource allocation for D2D communications underlaying Cloud-RAN-Based LTE–A networks," *IEEE Internet Things J.*, vol. 3, no. 3, pp. 428–438, Jun. 2016.

[13] T. D. Hoang, L. B. Le, and T. Le-Ngoc, "Energy-efficient resource allocation for D2D communications in cellular networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 6972–6986, Sep. 2016.

[14] S. Timotheou and I. Krikidis, "Fairness for non-orthogonal multiple access in 5G systems," *IEEE Signal Process. Lett.*, vol. 22, no. 10, pp. 1647–1651, Oct. 2015.

[15] A. Hisham, W. Sun, E. G. Strom, and F. Brannstrom, "Power control for broadcast V2 V communications with adjacent carrier interference effects," in *Proc. IEEE Int. Conf. Commun. (ICC)*, May 2016, pp. 1–6.

[16] F. Zhang, J. Xi, and R. Langari, "Real-time energy management strategy based on velocity forecasts using V2 V and V2I communications," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 2, pp. 416–430, Feb. 2017.

[17] S. A. AlQahtani and M. Alhassany, "Comparing different LTE scheduling schemes," in *Proc. 9th Int. Wireless Commun. Mobile Comput. Conf. (IWCMC)*, Jul. 2013, pp. 264–269.

[18] T. D. Hoang, L. B. Le, and T. Le-Ngoc, "Resource allocation for D2D communications under proportional fairness," in *Proc. IEEE Global Commun. Conf.*, Dec. 2014, pp. 1259–1264.

[19] J. Gu, S. J. Bae, S. F. Hasan, and M. Y. Chung, "Heuristic algorithm for proportional fair scheduling in D2D-cellular systems," *IEEE Trans. Wireless Commun.*, vol. 15, no. 1, pp. 769–780, Jan. 2016.

[20] X. Li, R. Shankaran, M. A. Orgun, G. Fang, and Y. Xu, "Resource allocation for underlay D2D communication with proportional fairness," *IEEE Trans. Veh. Technol.*, vol. 67, no. 7, pp. 6244–6258, Jul. 2018.

[21] M. Liu, L. Zhang, and Y. You, "Joint power and channel allocation for underlay D2D communications with proportional fairness," in *Proc. 15th Int. Wireless Commun. Mobile Comput. Conf. (IWCMC)*, Jun. 2019, pp. 1333–1338.

[22] P. Sun and L. Zhang, "Low complexity pilot aided frequency synchronization for OFDMA uplink transmission," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3758–3769, Jul. 2009.

[23] X. Cheng, L. Yang, and X. Shen, "D2D for intelligent transportation systems: A feasibility study," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 4, pp. 1784–1793, Aug. 2015.

[24] Y. Ren, F. Liu, Z. Liu, C. Wang, and Y. Ji, "Power control in D2D-based vehicular communication networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 12, pp. 5547–5562, Dec. 2015.

[25] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Trans. Netw.*, vol. 8, no. 5, pp. 556–567, Oct. 2000.

[26] T. Weise, *Global Optimization Algorithms-Theory and Application*. Self-Published Thomas Weise, 2009.

[27] H. Y. Lee, M. Kang, Y. J. Sang, and K. S. Kimy, "The modified proportional fair scheduling algorithms for real-time applications in multiuser multicarrier systems," in *Proc. IEEE Mil. Commun. Conf. (MILCOM)*, Oct. 2009, pp. 1–6.

[28] M. Liu and L. Zhang, "Joint power and channel allocation for relay-assisted Device-to- device communications," in *Proc. 15th Int. Symp. Wireless Commun. Syst. (ISWCS)*, Aug. 2018, pp. 1–5.

[29] T. Kim and M. Dong, "An iterative hungarian method to joint relay selection and resource allocation for D2D communications," *IEEE Wireless Commun. Lett.*, vol. 3, no. 6, pp. 625–628, Dec. 2014.

[30] (2013). *Simulation Guidelines, ICT-317669-Metis/D6.1, Metis Deliverable D6.1*. [Online]. Available: https://www.metis2020.com/documents/deliverables/

[31] S. Arora, *Optimization: Algorithms and Applications*. Wilmington, NC, USA: CSC Press, 2015.

MIAOMIAO LIU (Member, IEEE) received the B.Sc. degree in communications engineering from Qufu Normal University, Qufu, China, in 2013, and the M.Sc. degree in communications and signal processing from the School of Electronic and Electrical Engineering, University of Leeds, Leeds, U.K., in 2015, where she is currently pursuing the Ph.D. degree. Her research interests include resource allocation, interference management in device-to-device (D2D) communications, and optimization and graph theory. She received the Best Student Paper Award from the International Symposium on Wireless Personal Multimedia Communications (WPMC2019), in 2019.

LI ZHANG (Senior Member, IEEE) received the Ph.D. degree in communications from the University of York, in 2003. She was a Postdoctoral Research Fellow with the Communication Research Group, University of York, from March 2003 to March 2004. From April 2004 to August 2004, she was a Research Engineer in ECIT with Queen's University, Belfast, U.K. She is currently a Senior Lecturer in communications with the School of Electronic and Electrical Engineering, University of Leeds, U.K. Her current research interests include wireless communications and signal processing techniques, including massive MIMO, spatial modulation, heterogeneous networks, D2D communications and 5G systems, and so on. She was an EPSRC Panel Member and has been a member of the prestigious EPSRC (the U.K.'s main agency for funding research in engineering and the physical sciences) Peer Review College, since 2006. She has been a Technical Programme Committees of the most major IEEE conferences in communications, since 2006. She received the Nuffield Award for a Newly Appointed Lecturer, in 2005, and the IEEE Exemplary Reviewer, in 2005. She has been a Reviewer of the major IEEE journals and conferences, since 2006.