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Analyzing relationships between color, emotion and music using Bayes' rule in Bach's Well-Tempered Clavier Book 1

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Abstract

A probabilistic approach to the perception of emotion and color in music is proposed and the application of Bayes' rule to predict previously collected data is investigated. Specifically, performances of Bach's Well-Tempered Clavier Book I were analyzed in terms of mode, tempo and intensity. Estimates of probabilistic relationships between features and emotion dimensions were used to predict listeners' associations with the music in terms of emotion and color. Predictions were particularly successful for emotion perception, although color was also reliably predicted for 14 out of 24 Preludes. If color was predicted directly from emotion perception, reliable prediction increased to 18 out of 24 Preludes.

1 Introduction

Perception of emotion and meaning in music is to a great extent probabilistic rather than deterministic: the presence of certain musical features in music may increase the likelihood that listeners will perceive the music to have a certain quality, but it is by no means guaranteed. Moreover, associations between music and other domains may function in a probabilistic manner; if participants perceive a certain emotion in music, they may also associate a certain color with it [1-2], or, if the music suggests a certain motion or gesture, this may contribute to the perception of emotional attributes [3].
Several existing models of emotion perception in music assume probabilistic emotion perception. For example, in explaining cue utilization and the well-known lens model of emotion communication, Juslin [4] states "because the cues are only probabilistically related to the performer's expressive intention, the achievement cannot be more than probabilistic." (p. 1799) In this paper, I argue for a specific probabilistic approach, using Bayesian statistics. I describe what can be seen as the need and potential benefits of adopting this approach, and provide a proof of concept by reanalyzing a dataset that investigated relationships between color and emotion perception in music [5].

2 Modelling the probability of emotion and meaning in music

Bayes' rule offers a simple heuristic to relate the probability of an outcome to the presence of certain cues in the environment. In cognitive science, it has been shown to be a powerful means of modeling perception and cognition including probabilistic perception of structure in music [6-7]. Bayes' rule states the probability of a hypothesis (in our case e.g. an emotion) given an event (in our case a musical property or cue) is equal to the probability of that event given the hypothesis, times the prior probability of that hypothesis and divided by the prior probability of the event across different hypotheses [8]. In our case, we are considering the hypothesis of an emotion j, in the context of a cue, as in (1),

$$p(Emotion_j \mid Cue) = (p(Cue \mid Emotion_j) * p(Emotion_j))/p(Cue) \quad (1)$$

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where the prior probability of a cue is the summed probability of the cue in the context of any emotion, weighted by the probability of the emotion, as in (2).

$$p(Cue) = \sum_i^n p(Cue \mid Emotion_i)P(Emotion_i) \quad (2)$$

Here, we consider two applications of Bayes' rule to music, emotion and meaning. The first is to find the probability of an emotional response given features of the music, such as mode, tempo, and intensity. This approach is based on the distributions of musical features across emotions and the probability of emotions in the context of music. The advantage over a simple multiple regression model that linearly links cues to perceived emotion (as in [4, 9]) is that Bayes' rule includes competition between emotions and competition between features; in certain musical contexts and corpora, certain emotions will be more commonly perceived than others. Similarly, in certain contexts and corpora, certain features are more definitive than others.
The second application of Bayes' rule concerns cross-domain associations, for example between music and color. We can consider the probability of a certain cross-modal correspondence [10-11], in the context of certain musical features. What is the probability of an association to green or blue given specific cues in the music? This would depend on the probability of associating color with the musical features, on the probability of associating that color with music, and the probability of the musical feature.

In this paper, a first exploration of this approach is conducted by looking at the prediction of the probability of emotion and color association in the context of music with certain properties. This analysis is complemented by an analysis of the probabilistic relationship between emotion and color: given an association with emotion, what is the probability of a color association?

3 Predicting the perception of emotion based on musical cues

The first exploration of a Bayesian approach presented here is the potential to predict the probability of emotions to be perceived in a musical excerpt with certain musical properties. Data collected previously were reanalyzed for this purpose [5]. The data comes from three experiments. In Experiment 1, participants listened to the first 15 seconds of all preludes from Bach's Well-Tempered Clavier Book I in random order and choose the color (1 of 8 options) that they perceived to best fit the music. In Experiment 2, participants were presented the 8 colors and gave an emotion rating for each of them. Each color obtained 9 emotion ratings. In Experiment 3, participants listened to the music excerpts as in Experiment 1, and gave a rating for each excerpt on 9 emotion scales. Different participant groups participated in each experiment.

The first analysis concerns the data collected in Experiment 3. The ratings on emotion scales were transformed into probabilities. This was done using an exponential transform of the linear data followed by a normalization across emotion terms per prelude to get probabilities. Previous work has shown that exponential functions offer a useful approximation of probabilistic variables [9, 6].
Next, each excerpt of Bach's Well-tempered Clavier Book I was analyzed in terms of tonal mode, tempo and intensity. Tempo was measured in Sonic Visualiser by tapping along to the music and taking the average inter-tap-interval. The intensity was analyzed using the intensity function in PRAAT.
The average tempo and intensity were then categorized into five levels based on their distribution (from low to high), which is not essential but was done to facilitate determination of probabilities in the next step (see Table III). Table I shows the raw measures and categorized measures for each of the preludes. The last columns show the emotion that obtained highest ratings and colour that was most frequently chosen for each prelude in the original study (Experiment 3 [5]).

Next, condition probabilities of cues in the context of emotions were estimated. To operationalize this, emotions were defined in terms of two dimensions – valence and arousal (see Table II), following Russell's circumplex model [12]. The conditional probabilities of cues in the context of emotions that were negative, neutral or positive in valence and low medium or high in arousal were derived using two simple rules: a linear relationship between cue and emotion dimension was predicted and this relationship was estimated to either be strong, moderate or weak. In case of a strong relationship, the probabilities ranged from 0.1 to 0.9 (increments of 0.2). For moderate relationships, the probabilities ranged from 0.3 to 0.7 (increments of 0.1), and for weak relationships, the probabilities ranged from 0.4 to 0.6 (increments of .05), see top rows of Table III. The exception was a non-linear prediction of a relatively high probability of major mode compared to minor mode in the context of neutral valence (.6 vs. .4 instead of .5 vs .5, as neutral valence is associated with major mode [13]). These probabilities were normalized across cue levels to add up to a total probability of 1, resulting in the probabilities shown in the bottom rows of Table III.

TABLE I: MODE, TEMPO AND INTENSITY OF THE START OF EACH PRELUDE (15 SEC. EXCERPTS). RAW AND CATEGORIZED SCORES ARE PROVIDED FOR TEMPO (T) AND INTENSITY (I). LAST COLUMNS INDICATE THE EMOTION THAT RECEIVED HIGHEST RATINGS AND COLOR THAT WAS MOST FREQUENTLY CHOSEN [5].

Prelude	Mode	Tempo	T-Category	Intensity	I-Category	Emotion	Color
A Maj	2	81.4	1	69.9	3	Lively	Char.*
A Min	1	92.3	3	7.5	4	Lively	Red
Ab Maj	2	108.3	4	7.2	3	Happy	Yellow
B Maj	2	88.9	3	66.4	2	Lively	Yellow
B Min	1	81.6	2	62.5	0	Sad	Blue
Bb Maj	2	141.1	4	64.5	0	Lively	Yellow
Bb Min	1	82.8	2	66.4	1	Sad	Blue
C Maj	2	84.6	2	66.5	2	Calm	Cyan
C Min	1	105.1	3	7.6	4	Strong	Red
C# Maj	2	78.0	1	65.2	1	Lively	Yellow
C# Min	1	33.4	0	65.2	1	Calm	Blue
D Maj	2	13.4	4	67.9	3	Lively	Yellow
D Min	1	104.4	3	66.6	2	Calm	Purple
E Maj	2	78.3	1	65.8	1	Calm	Green^
E Min	1	57.9	0	66.4	2	Sad	Blue
Eb Maj	2	72.7	1	7.6	4	Positive	Orange
Eb Min	1	41.2	0	61.3	0	Sad	Blue
F Maj	2	78.6	1	7.9	4	Lively	Yellow
F Min	1	46.7	0	65.9	1	Sad	Blue
F# Maj	2	84.5	2	65.1	0	Happy	Chart.*
F# Min	1	115.4	4	7.8	4	Lively	Red
G Maj	2	91.0	3	68.6	3	Lively	Yellow
G Min	1	84.0	2	66.8	3	Sad	Blue
G# Min	1	37.9	0	66.7	2	Sad	Blue

* Chartreuse, ^ Green and Cyan were equally often chosen.

TABLE II: CHARACTERIZING OF EACH EMOTION IN THE STUDY IN TERMS OF VALENCE AND AROUSAL, AND PRIOR PROBABILITY OF THE EMOTION AS ESTIMATED FROM THE COLLECTED DATA.

Emotion	Valence	Arousal	Prior probability
Angry	0	2	0.008
Strong	1	1	0.094
Negative	0	1	0.017
Sad	0	0	0.234
Calm	2	0	0.149
Weak	1	0	0.011
Positive	2	1	0.105
Happy	2	2	0.090
Lively	1	2	0.292

TABLE III: ESTIMATED PROBABILITY OF A CUE, GIVEN AN EMOTIONAL CONTEXT. CUES RELATE TO MAJOR (1) OR MINOR (-1) MODE, TEMPO (0-4) OR INTENSITY (0-4). EMOTIONAL CONTEXT IS DEFINED USING THE TWO DIMENSIONS OF VALENCE (VAL.) AND AROUSAL (AR.). TOP ROWS PROVIDE RAW PROBABILITIES AND BOTTOM ROWS NORMALIZED PROBABILITIES.

Cue	Mode		Tempo					Intensity				
	1	-1	0	1	2	3	4	0	1	2	3	4
Val.												
0	0.1	0.9	0.6	0.55	0.5	0.45	0.4	0.6	0.55	0.5	0.45	0.4
1	0.6	0.4	0.5	0.55	0.6	0.55	0.5	0.5	0.55	0.6	0.55	0.5
2	0.9	0.1	0.4	0.45	0.5	0.55	0.6	0.4	0.45	0.5	0.55	0.6
Ar.												
0	0.4	0.6	0.9	0.7	0.5	0.3	0.1	0.7	0.6	0.5	0.4	0.3
1	0.5	0.5	0.5	0.7	0.9	0.7	0.5	0.5	0.6	0.7	0.6	0.5
2	0.6	0.4	0.1	0.3	0.5	0.7	0.9	0.3	0.4	0.5	0.6	0.7
Val.												
0	0.1	0.9	0.24	0.22	0.2	0.18	0.16	0.24	0.22	0.2	0.18	0.16
1	0.6	0.4	0.185	0.204	0.22	0.204	0.185	0.185	0.204	0.22	0.204	0.185
2	0.9	0.1	0.16	0.18	0.2	0.22	0.24	0.16	0.18	0.2	0.22	0.24
Ar.												
0	0.4	0.6	0.36	0.28	0.2	0.12	0.04	0.28	0.24	0.2	0.16	0.12
1	0.5	0.5	0.152	0.212	0.272	0.212	0.152	0.172	0.207	0.241	0.207	0.172
2	0.6	0.4	0.04	0.12	0.2	0.28	0.36	0.12	0.16	0.2	0.24	0.28

Based on previous literature [13-15], strong relationships were expected between valence and mode, and between arousal, tempo and intensity. Weak relationships were expected between valence, tempo and intensity, and between arousal and mode.

It will be of interest to fine-tune these estimates in future research from empirical observations, and to explore the option of directly associating emotion categories with musical features rather than emotion dimensions.

As each excerpt was defined in terms of three cues, the conditional probability of a set of cues given valence or arousal was estimated as the product of conditional probabilities of each cue. This was done for each level of valence as in (3) and each level of arousal. The conditional probability of cues given an emotion was defined as the combined conditional probability of cues given a specific level of arousal and given a specific level of valence.

$$p\left(Cue_i \middle| Valence_j\right) = \prod_{i=1}^{n} p\left(Cue_i \middle| Valence_j\right) \quad (3)$$

For example, for A Maj Prelude and Anger, the conditional probability of the set of cues for that prelude (major mode, low to medium tempo and medium to high intensity), given the hypothesis of anger is the product of the probabilities of each cue given negative valence and given high arousal.
Finally, Bayes' rule was applied: the conditional probability of the set of cues given a level of valence and arousal (i.e. given an emotion) was multiplied by the prior probability of the emotion and divided by the product of the prior probabilities of the three cues. The prior probability of an emotion was calculated as the average probability of that emotion across musical excerpts, using the transformed data of Experiment 3 [5]. The prior probability of a cue was calculated as the proportion of excerpts featuring a particular cue. The resulting probabilities of emotions, given three cues were normalized across emotions, leading to the predicted probabilities per emotion and prelude listed in Table IV.

TABLE IV: PREDICTED PROBABILITIES OF PERCEIVED EMOTION PER PRELUDE. PROBABILITIES ARE HIGHLIGHTED IN BOLD THAT WERE PREDICTED AND OBSERVED TO BE HIGHEST. CORRELATIONS BETWEEN PREDICTED AND OBSERVED PROBABILITIES ARE GIVEN IN THE RIGHT COLUMN.

Prelude	Angry	Strong	Negative	Sad	Calm	Weak	Positive	Happy	Lively	Correla-tion
A Maj	0.002	0.116	0.003	0.021	0.119	0.006	0.187	0.167	**0.379**	0.881
A Min	0.029	0.105	0.033	0.094	0.012	0.003	0.041	0.067	**0.617**	0.831
Ab Maj	0.002	0.037	0.001	0.001	0.011	0.018	0.088	0.331	0.511	0.645
B Maj	0.002	0.105	0.003	0.009	0.061	0.003	0.173	0.200	**0.445**	0.947
B Min	0.001	0.052	0.025	**0.836**	0.04	0.015	0.011	0.001	0.019	0.998
Bb Maj	0.002	0.120	0.004	0.021	0.121	0.005	0.227	0.171	**0.329**	0.808
Bb Min	0.004	0.095	0.038	**0.706**	0.041	0.015	0.021	0.005	0.074	0.990
C Maj	0.001	0.156	0.004	0.017	0.098	0.006	0.213	0.138	0.367	0.242
C Min	0.029	0.105	0.033	0.094	0.012	0.003	0.041	0.067	0.617	0.089
C# Maj	0.001	0.146	0.005	0.074	0.285	0.018	0.192	0.074	0.204	0.416
C# Min	0.001	0.026	0.015	0.897	0.035	0.013	0.006	0.000	0.007	0.635
D Maj	0.002	0.038	0.001	0.001	0.011	0.000	0.090	0.337	**0.520**	0.863
D Min	0.014	0.153	0.050	0.398	0.035	0.010	0.042	0.021	0.278	0.297
E Maj	0.001	0.146	0.005	0.074	**0.285**	0.018	0.192	0.074	0.204	0.735
E Min	0.001	0.052	0.025	**0.836**	0.040	0.015	0.011	0.001	0.019	0.983
Eb Maj	0.002	0.079	0.002	0.006	0.045	0.002	0.153	0.246	0.464	0.463
Eb Min	0.000	0.013	0.009	**0.933**	0.029	0.012	0.003	0.000	0.002	0.994
F Maj	0.002	0.079	0.002	0.006	0.045	0.002	0.153	0.246	**0.464**	0.916
F Min	0.001	0.026	0.015	**0.897**	0.035	0.013	0.006	0.000	0.007	0.945
F# Maj	0.001	0.165	0.006	0.083	0.320	0.02	0.216	0.050	0.139	0.102
F# Min	0.033	0.068	0.021	0.028	0.004	0.001	0.032	0.093	**0.720**	0.780
G Maj	0.002	0.065	0.002	0.004	0.035	0.001	0.128	0.267	**0.496**	0.872
G Min	0.014	0.153	0.050	0.398	0.035	0.01	0.042	0.021	0.278	0.753
G# Min	0.001	0.052	0.025	**0.836**	0.040	0.015	0.011	0.001	0.019	0.712

This analysis led to a median correlation between predicted and observed emotion probabilities of 0.79: for 16 out of 24 preludes, the prediction was reliable, showing correlations of 0.67 ($p < 0.05$) or higher (correlation of 0.88 on average). Lively and Sad were most reliably predicted and most often predicted, and in one instance Calm. For eight preludes, the correlation between predicted and observed probabilities was not significant. It will be of interest to look into the reasons for the weak predictions in future research as well as to compare with other predictive models.

4 Predicting colour perception

The second exploration of a Bayesian approach concerned the prediction of associated colour with music, as assessed in Experiment 1 of the original study [5]. Following previous research, an emotion-mediation hypothesis was employed to predict colour association with music [1]. Specifically, the blue-yellow colour dimension has been found to correlate with a sad-happy axis, which varies both in valence (negative to positive) and arousal (or activity – low to high). The green-red dimension has been found to correlate with strength (weak to strong). Note that the correlation of green-red with strength is a simplification of the specific results of the study, as the strongest correlation was found with the axes blue-red rather than green-red [1]. Each color was categorized in this two-dimensional space, where one dimension combines two correlated dimensions, as shown in Table V.

TABLE V: CHARACTERIZATION OF A COLOR IN TERMS OF VALENCE/AROUSAL AND STRENGTH. DIMENSION LEVELS RANGE FROM 0-2 IN FIVE LEVELS, REPRESENTING LOW, LOW-MID, MID, MID-HIGH AND HIGH. THE LAST COLUMN SHOWS THE PRIOR PROBABILITY OF THE COLOR AS ESTIMATED FROM PREVIOUSLY COLLECTED DATA [5].

Color	Valence/Arousal	Strength	Prior probability
Red	1	2	0.107
Orange	1.5	1.5	0.123
Yellow	2	1	0.150
Chartreuse	1.5	.5	0.114
Green	1	0	0.101
Cyan	.5	.5	0.110
Blue	0	1	0.159
Purple	.5	1.5	0.136

The same cues were used to characterize the properties of each musical excerpt as in the above prediction of perceived emotion, namely mode (2 levels), tempo (5 levels) and intensity (5 levels). Using simple linear relationships between cues and color dimension, the conditional probability was estimated of the presence of a cue in the context of a color dimension. As above, either strong, moderate or weak linear relationships were predicted, which determined the range of the probabilities (from 0.1-0.9, 0.3-0.7 or 0.4-0.6). The same relationships with mode, tempo and intensity were predicted as above for valence and arousal. For strength, previous research was followed that examined relationships of tempo and dynamics with external force [16]. Based on these previous observations, a strong relationship was predicted with intensity, and a moderate relationship with tempo. Furthermore, we predicted a weak relationship between strength and mode. These estimated probabilities were normalized to add up to 1 across cue levels.

TABLE VI: PROBABILITIES OF A CUE GIVEN A COLOR DIMENSION THAT IS LOW, LOW-MEDIUM, MEDIUM, MEDIUM-HIGH OR HIGH IN STRENGTH OR VALENCE/AROUSAL. PROBABILITIES DEPEND ON WHETHER THE RELATIONSHIPS ARE EXPECTED TO BE STRONG, MODERATE OR WEAK.

	Cue level	0	1	2	3	4
Relationship	Color dimension level					
Strong	Low	0.36	0.226	0.152	0.097	0.04
	Low-Mid	0.28	0.29	0.212	0.161	0.12
	Mid	0.2	0.226	0.273	0.226	0.2
	Mid-High	0.12	0.161	0.212	0.29	0.28
	High	0.04	0.097	0.152	0.226	0.36
Moderate	Low	0.28	0.214	0.172	0.143	0.12
	Low-Mid	0.24	0.25	0.207	0.179	0.16
	Mid	0.2	0.214	0.241	0.214	0.2
	Mid-High	0.16	0.179	0.207	0.25	0.24
	High	0.12	0.143	0.172	0.214	0.28
Weak	Low	0.16	0.17	0.185	0.208	0.24
	Low-Mid	0.18	0.189	0.204	0.226	0.22
	Mid	0.2	0.208	0.222	0.208	0.2
	Mid-High	0.22	0.226	0.204	0.189	0.18
	High	0.24	0.208	0.185	0.17	0.16

For clarity Table VI shows these normalized probabilities for strong, moderate and weak relationships in case of 5 cue levels. For mode, the probabilities ranged from 0.1 to 0.9 in increments of 0.2 for valence, and from 0.4 to 0.6 in increments of 0.05 for arousal and strength.

As each excerpt was defined in terms of three cues, the conditional probability of a set of cues given valence or arousal was estimated as the product of conditional probabilities of each cue. As the probability of a cue in the context of certain levels of valence or arousal are different (e.g. weak relationship between valence and tempo and strong relationship between arousal and tempo), both were included as separate dimensions. This was done for each level of valence as in (3), each level of arousal. The conditional probability of cues given an emotion was defined as the combined conditional probability of cues given a specific level of valence, arousal and strength.

Colour perception was finally predicted using Bayes' rule: the probability of a colour in the context of music was set to be equal to the probability of that cue in the context of the colour dimension (4) times the prior probability of the colour divided by the prior probability of the cue (following (1)). The prior probability of a colour was calculated from the data – the frequency with which a colour was chosen across excerpts (Experiment 1 [5]). The prior probabilities of cues were estimated from the musical feature analysis and were multiplied, following the same procedure as for the prediction of emotion, above. The outcome was normalized across colours to get the predicted probability per musical excerpt, see Table VII.

TABLE VII: PREDICTED PROBABILITIES OF COLOR PER PRELUDE. PROBABILITIES ARE HIGHLIGHTED IN BOLD THAT WERE PREDICTED AND OBSERVED TO BE HIGHEST. CORRELATIONS BETWEEN PREDICTED AND OBSERVED PROBABILITIES ARE GIVEN IN THE RIGHT COLUMN.

Prelude	Red	Orange	Yellow	Chartreuse	Green	Cyan	Blue	Purple	Correlation
A Maj	0.130	0.261	0.23	0.155	0.053	0.052	0.018	0.102	0.163
A Min	**0.246**	0.286	0.110	0.100	0.026	0.044	0.041	0.147	0.833
Ab Maj	0.101	0.329	**0.438**	0.093	0.012	0.005	0.001	0.021	0.874
B Maj	0.136	0.257	**0.318**	0.140	0.057	0.026	0.011	0.055	0.823
B Min	0.011	0.009	0.002	0.024	0.14	0.223	**0.494**	0.097	0.862
Bb Maj	0.06	0.177	**0.249**	0.209	0.146	0.071	0.020	0.069	0.835
Bb Min	0.04	0.022	0.008	0.046	0.134	0.27	**0.328**	0.152	0.627
C Maj	0.156	0.183	0.283	0.140	0.098	0.047	0.022	0.071	-0.401
C Min	0.246	0.286	0.110	0.100	0.026	0.044	0.041	0.147	0.697
C# Maj	0.067	0.081	0.097	0.155	0.147	0.245	0.062	0.146	0.332
C# Min	0.011	0.005	0.001	0.016	0.087	0.271	**0.507**	0.101	0.767
D Maj	0.093	0.304	**0.482**	0.086	0.011	0.005	0.001	0.019	0.862
D Min	0.125	0.121	0.039	0.098	0.118	0.125	0.196	0.177	0.496
E Maj	0.067	0.081	0.097	0.155	0.147	0.245	0.062	0.146	0.501
E Min	0.032	0.014	0.003	0.023	0.106	0.183	**0.516**	0.123	0.943
Eb Maj	0.175	0.232	0.364	0.106	0.018	0.028	0.007	0.070	-0.086
Eb Min	0.003	0.002	0.000	0.009	0.093	0.191	**0.647**	0.055	0.844
F Maj	0.173	0.232	**0.368**	0.106	0.018	0.027	0.006	0.068	0.899
F Min	0.011	0.005	0.001	0.016	0.087	0.271	**0.507**	0.101	0.794
F# Maj	0.048	0.091	0.083	0.162	0.271	0.156	0.088	0.101	0.569
F# Min	**0.284**	0.266	0.197	0.087	0.019	0.029	0.015	0.103	0.858
G Maj	0.097	0.395	0.327	0.119	0.017	0.009	0.003	0.033	0.83
G Min	0.151	0.138	0.037	0.087	0.092	0.111	0.183	0.202	0.613
G# Min	0.041	0.021	0.005	0.037	0.136	0.191	**0.440**	0.129	0.961

This analysis led to a median correlation between predicted and observed color probabilities of 0.81: for 14 out of 24 preludes, the prediction was good, showing correlations considerably above 0.71 ($p < .05$, $N = 8$) with an average correlation of 0.86. For the other ten, however, the correlations between predicted and observed probabilities were unreliable. Note that the predicted probabilities are considerably flatter for color associations than emotion associations, which corresponds to observed distributions in the original data: color association is in many instances ambiguous.
The final analysis examines whether prediction of colour association can be further improved when perceived emotion is taken as a basis rather than predicted from (predicted values of) emotional valence/arousal and strength.

5 Relating colour to emotion

One of the main hypotheses of the original study was whether the color choices for the music could be predicted from the emotion associations with the music and the emotion associations with the colors. To test this, the ratings of emotion to the music in Experiment 3 were multiplied by the ratings of emotions to colors in Experiment 2 and the results were compared to the choices of colors to the music in Experiment 1. Here, we will redo this analysis using a probabilistic approach.
As above the ratings data from Experiments 2 and 3 were transformed into probabilities, using an exponential transform of the data followed by a normalization to get probabilities. Secondly, the probability of a color j, given a musical piece was estimated using (4):

$$p\left(color_j \middle| piece\right) = \sum_{i=1}^{n} p\left(color_j \middle| emotion_i\right) * p(emotion_i | piece) \quad (4)$$

The probability of a color given a piece was estimated to be equal to the sum for all emotions of the product of the probability of a color given an emotion and the probability of the emotion in the context of this piece. In other words, the probability of a color in the context of a piece depends on the probability of an emotion in the context of that piece and the probability of the color in the context of that emotion. Probabilities of colors related to different emotions are weighted according to the conditional probability of the emotion and summed.
The probabilities of color given emotions were calculated on the basis of the ratings data from Experiment 2 [5], and the probabilities of emotions given musical excerpts were calculated from the ratings data of Experiment 3 [5].
Table VIII shows the predicted probabilities and the correlation between these predicted probabilities of colour and the observed proportion with which each colour was chosen in the context of a piece (Experiment 1 [5]). The median correlation was 0.84: for 18 out of 24 preludes, the prediction was good, showing correlations above 0.71 ($p < .05$, $N = 8$) with an average correlation of 0.87. For the other six, the correlations between predicted and observed probabilities were unreliable, and in two instances

negative. The preludes with negative correlations were identified as outliers in the original study in terms of colour association. It can be seen in Table VIII that, as in the previous analysis, Yellow, Blue and Red were frequently reliably identified.

TABLE VIII: PREDICTED PROBABILITIES OF PERCEIVED COLOR PER PRELUDE FROM EMOTION RATINGS TRANSFORMED TO PROBABILITIES. PROBABILITIES ARE HIGHLIGHTED IN BOLD THAT WERE PREDICTED AND OBSERVED TO BE HIGHEST. CORRELATION BETWEEN PREDICTED AND OBSERVED PROBABILITIES IS GIVEN IN THE RIGHT COLUMN.

Prelude	Red	Orange	Yellow	Chartreuse	Green	Cyan	Blue	Purple	Correlation
A Maj	0.04	0.132	0.344	0.056	0.183	0.107	0.078	0.061	0.41
A Min	**0.371**	0.113	0.231	0.04	0.108	0.025	0.021	0.091	0.786
Ab Maj	0.049	0.14	**0.366**	0.052	0.186	0.105	0.044	0.058	0.917
B Maj	0.056	0.154	**0.384**	0.059	0.183	0.061	0.031	0.072	0.717
B Min	0.107	0.037	0.026	0.086	0.062	0.087	**0.387**	0.208	0.985
Bb Maj	0.051	0.155	**0.387**	0.06	0.185	0.057	0.03	0.074	0.82
Bb Min	0.081	0.034	0.021	0.08	0.063	0.087	**0.412**	0.223	0.971
C Maj	0.018	0.062	0.178	0.038	0.154	0.205	0.298	0.045	0.739
C Min	**0.712**	0.057	0.054	0.016	0.034	0.012	0.017	0.098	0.949
C# Maj	0.045	0.15	**0.379**	0.058	0.184	0.075	0.041	0.068	0.711
C# Min	0.063	0.024	0.025	0.061	0.08	0.181	**0.444**	0.122	0.828
D Maj	0.055	0.158	**0.389**	0.06	0.183	0.052	0.028	0.075	0.921
D Min	0.187	0.068	0.142	0.059	0.105	0.129	0.23	0.08	-0.266
E Maj	0.013	0.034	0.1	0.035	0.131	0.244	0.403	0.04	0.482
E Min	0.075	0.033	0.027	0.078	0.069	0.115	**0.415**	0.19	0.993
Eb Maj	0.114	0.089	0.239	0.043	0.164	0.137	0.154	0.06	-0.428
Eb Min	0.08	0.033	0.023	0.083	0.065	0.104	**0.411**	0.201	0.978
F Maj	0.072	0.154	**0.379**	0.058	0.179	0.055	0.029	0.075	0.857
F Min	0.062	0.028	0.02	0.07	0.071	0.135	**0.438**	0.176	0.939
F# Maj	0.032	0.148	0.372	0.055	0.176	0.102	0.056	0.058	0.609
F# Min	**0.369**	0.112	0.231	0.04	0.109	0.027	0.021	0.091	0.847
G Maj	0.091	0.155	**0.375**	0.059	0.173	0.042	0.025	0.079	0.678
G Min	0.141	0.046	0.061	0.067	0.078	0.115	**0.344**	0.148	0.918
G# Min	0.064	0.024	0.021	0.059	0.077	0.174	**0.449**	0.133	0.959

6 Discussion and conclusion

I argued for the power of a probabilistic, and in particular a Bayesian approach to model emotion and color associations with music. Two applications of Bayes' formula were explored to predict emotion on the basis of three musical cues, and to predict color on the basis of these musical cues in the context of 24 preludes from Bach's Well-Tempered Clavier, Book I. To relate color to music, color was interpreted in terms of valence, arousal and strength and associations between these dimensions and musical features were estimated. The predictions showed promising results, yielding high correlations for a majority of the preludes between predicted and observed probabilities for both emotion and color. The prediction of color could however be further improved when basing it directly on perceived emotion.

There are several ways in which the details of the models used in the explorations can be further fine-tuned and improved. For example, the probabilities for the relationship between cues and emotions and between cues and colors were set by hand. These settings can be fine-tuned on the basis of independently collected empirical data or corpus analyses. Furthermore, categorization of the cues facilitated the estimation of the probabilities in this instance, but this is not necessary, nor likely to be the best solution. In reality, features such as tempo and intensity and probabilities related to these may change in a continuous rather than discrete manner. Similarly, a dimensional approach was used to define emotions and colors. This meant needing to combine the conditional probabilities of cues for valence and arousal (and strength) to estimate the conditionality probability of the cues in the context of an emotion. Product was used to combine probabilities, assuming independence between them, which was a simplification that requires further investigation and verification. Future research may compare this approach to working directly with emotion and color categories in relation to musical cues.

The broader promise of a Bayesian approach is to include context-sensitive estimates of prior probabilities of emotions and features: depending on the musical culture, genre, and piece (e.g. its lyrics), certain emotions may be more readily perceived in music than others, and certain features may be more distinctive and as such more informative than others. Whilst Bayesian probabilities are central to many commonly used statistical and data mining processes, its cognitive plausibility is worth exploring [17].

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