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RESEARCH ARTICLE

Modeling a Helical Fluid Inerter System with Time-Invariant Mem-Models

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Abstract

In this paper, experimental data from tests of a helical fluid inerter are used to model the observed hysteretic behavior. The novel idea is to test the feasibility of employing mem-models, which are time-invariant herein, to capture the observed phenomena by using physically meaningful state variables. Firstly we use a Masing model concept, identified with a multilayer feedforward neural network to capture the physical characteristics of the hysteresis functions. Following this, a more refined problem formulation based on the concept of a multi-element model including a mem-inerter is developed. This is compared with previous definitions in the literature and shown to be a more general model. Through-out this paper, numerical simulations are used to demonstrate the type of dynamic responses anticipated using the proposed time-invariant mem-models. Corresponding experimental measurements are processed to demonstrate and justify the new mem-modeling concepts. Focusing on identifying the unknown function forms in the proposed problem formulations, the results show that it is possible to formulate a unified model constructed using both the damper and inerter from the mem-model family. This model captures many of the more subtle features of the underlying physics, not captured by other forms of existing model.

KEYWORDS:

hysteresis; inerter; mem-models; dual input-output pairs; higher-order element; Masing model; mem-inerter

1 | INTRODUCTION

The inerter is a novel dynamic device that has been the subject of substantial research interest in both academia and industry. In particular, systems with inerters have been developed to improve passive control of dynamic systems, such as automotive suspensions, and more recently applications in civil engineering structures. This paper describes the development of a series of hysteresis models for an inerter by using new concepts recently introduced from other disciplines. The results are compared to experimental results from a fluid inerter system based on a fluid filled cylinder that induces flow in a helical pipe system.

The term inerter was first introduced by Smith (1) using the force-current analogy between mechanical and electrical networks. In this context, the inerter is considered to represent the equivalent of the capacitor. As a result it has the property that the force generated is proportional to the relative acceleration between its end points (or nodes). The constant of proportionality for the inerter is called inertance and is measured in kilograms.

Of course, in mechanical engineering the flywheel concept had been used for many centuries previously, but typically without relative accelerations. However in earthquake engineering, particularly in Japan since the late 1990s, several researchers have used inertial components to enhance the effectiveness of damping devices – see summary by (2) and references therein. There are several types of inerters: the rack and pinion inerter (1), the ball screw inerter (3), the fluid inerter (4, 5, 6, 7), and the electromagnetic inerter (8, 9). There are three main application areas, vehicle suspensions systems (3, 10, 11, 12, 13, 14, 15), train suspension systems (16, 17), and civil engineering systems (18, 2, 19, 20, 21, 22, 23, 24). The optimal performance of inerter-based vibration isolation systems has been considered by several authors, see for example the discussions in 25, 26, 27, 28. In general inertance is assumed to be fixed, but some recent devices have investigated the idea of variable inertance (29, 8, 5). The modeling in this study elucidates the natural variations in inertance that can occur in this type of fluid inerter.

Fluid inerters have the advantage of minimal moving parts, with the inertia effect being generated by the motion of the fluid. This inertial effect has two main ways of being realized. It is either due to the fluid driving a mechanical flywheel, which is usually called the hydraulic inerter (4), or the mass of the fluid itself moving in a helical pipe, which is defined as the helical fluid inerter, or just the helical inerter (5). In this paper we will describe the behavior of a helical inerter using memristive models, otherwise known as mem-models.

The models developed in this study, are based on two strategies: The first is based on the Masing model for hysteresis, which uses the experimental data to build the hysteresis curves. This model is cast into the memristive model framework defined in (30), specifically by using dual input-output pairs of variables. The second modeling approach assumes the system is modeled by a combination of mem-elements. Specifically in this case a mem-inerter and memristor. In this case the experimental data can be used to (i) identify the functional form of the mem-inerter function, and (ii) compare the performance of the mem-inerter model to an alternative nonlinear inerter model.

The rest of this paper is structured as follows. An overview of mem-modeling including the basic concepts and a suite of carefully designed numerical demonstrations named “idealized responses” will be given in Section 2. An overview of all experimental data sets, preprocessing, and the extraction of typical one-cycle loops of the steady-state responses are given in Appendix A. The Masing modeling approach and results are presented in Section 3, while the multi-element mem-modeling approach and results are given in Section 4. We conclude our study in Section 5.

2 | CONCEPTS AND NUMERICAL DEMONSTRATIONS

2.1 | Definitions for Mem-Models

Mem-models for dampers and springs are briefly introduced here, following the derivations of (31). The *general* mem-dashpot model, also called memristive system model, is defined as follows for a flow-controlled setting:

$$\text{state equations:} \quad \dot{\mathbf{y}}_d(t) = \mathbf{g}_d(\mathbf{y}_d(t), \dot{x}(t), t) \quad (1)$$

$$\text{input-output equation:} \quad r_d(t) = D(\mathbf{y}_d(t), \dot{x}(t), t) \dot{x}(t) \quad (2)$$

where $\mathbf{y}_d(t)$ is the state vector, x is relative displacement, and an overdot represents differentiation with respect to time t . Here in this constitutive relation, the input is \dot{x} while the output is the force r_d . D represents a generalized damping function of the states and velocity. When t is dropped from the right-hand side of both Eqs. (1) and (2), the definition becomes that of a time-invariant mem-dashpot. The exact functional forms for both \mathbf{g}_d and D are to be determined, which is a *key* challenge in applications and will be addressed in this study.

The *general* mem-spring model, also called memcapacitive system model, is defined as follows for a flow-controlled setting:

$$\text{state equations:} \quad \dot{\mathbf{y}}_s(t) = \mathbf{g}_s(\mathbf{y}_s(t), x(t), t) \quad (3)$$

$$\text{input-output equation:} \quad r_s(t) = S(\mathbf{y}_s(t), x(t), t) x(t) \quad (4)$$

where $\mathbf{y}_s(t)$ is the state vector. Here in this constitutive relation, the input is x while the output is the force r_s . S represents a generalized stiffness function of the states and the displacement. When t is dropped from the right-hand side of both Eqs. (3) and (4), the definition becomes that of a time-invariant mem-spring.

For hysteretic systems, the state variables displacement and velocity can be augmented with additional states to account for memory effects. For example, by including absement a and generalized momentum p , which are defined as the time integral of displacement x , and the time integral of the restoring force r , respectively. Note that r will be taken as either r_d , r_s , the inerter restoring force r_i (to be introduced later), or a combined element restoring force depending on the specific context.

The corresponding p will be p_d , p_s , and p_i , respectively. In general, absement and generalized momentum are given by the relationships

$$a(t) = \int_{-\infty}^t x(\tau) d\tau, \quad p(t) = \int_{-\infty}^t r(\tau) d\tau \quad (5)$$

An important property of mem-models is the so-called “zero-crossing” or “origin-crossing” property, which means that for mem-dashpots \dot{x} and r_d become zero simultaneously, and for mem-springs x and r_s become zero simultaneously. Mem-models have been developed to model nonlinear phenomena (32, 33, 34). Specifically, in engineering mechanics applications, mem-models are for modeling memory effects, i.e., hysteresis. For example, the use of a and p as state variables are introduced in 31, 30, 35 to capture history-/path-dependency.

We also would like to highlight the adjective “relative” for displacement to pave the road for the introduction of the mem-inerter. In fact, all kinematic quantities involved here including absement, displacement, velocity (and later, acceleration) are *relative* quantities being the difference of the two nodes that define the element. Mem-dampers and mem-springs are generalized dampers and springs, respectively. Relative velocity and relative displacement are specified for dampers and springs, e.g., in (36).

Consider the following mem-dashpot and mem-spring example defined using D and S , respectively, each as a simple time-invariant system model (31, 30, 35):

$$\text{mem-dashpot:} \quad D = -\text{sgn}(\dot{x}) \sin\left(\frac{\pi}{2} x\right) + 2 \quad (6)$$

$$\text{mem-spring:} \quad S = \text{sgn}(x) \cos\left(\frac{\pi}{2} a\right) + 2 \quad (7)$$

Subjecting the mem-dashpot and mem-spring to the defined sinusoidal relative displacement input $x(t)$ gives

$$\ddot{x}(t) = -A\omega^2 \sin(\omega t) \quad (8)$$

$$\dot{x}(t) = A\omega \cos(\omega t) \quad (9)$$

$$x(t) = A \sin(\omega t) \quad (10)$$

$$a(t) = \frac{A}{\omega} - \frac{A}{\omega} \cos(\omega t) \quad (11)$$

where $A = 1$ and $\omega = 1$ in this example. The results are shown in Figs. 1 and 2, respectively. Throughout this study, numerical

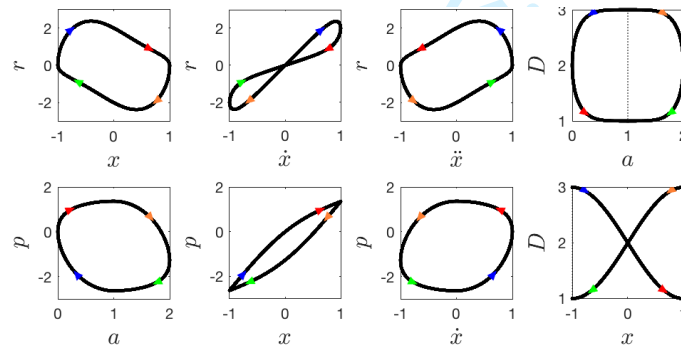


FIGURE 1 A particular mem-dashpot subject to the prescribed sinusoidal displacement input. For comparison using a linear dashpot, would change the behavior to: from the left to right columns, there would be ellipses, straight lines with a positive slope, ellipses, and constants of the value of the slope for D

differentiation with respect to time is carried out by using the central difference method. The MATLAB (37) code adopted here is `central_diff.m` (38) to ensure forward and backward differences at the left and right ends, respectively, and with the same second-order of accuracy as the central difference for the mid-portion. Numerical integration with respect to time is carried out by using the trapezoidal rule to obtain the integral counterpart for a specified input or solved output. Four colors, red, orange, green, and blue, highlight the responses to the four quarters of the sinusoidal input and are used in the figures to indicate the behavior of the hysteresis loops.

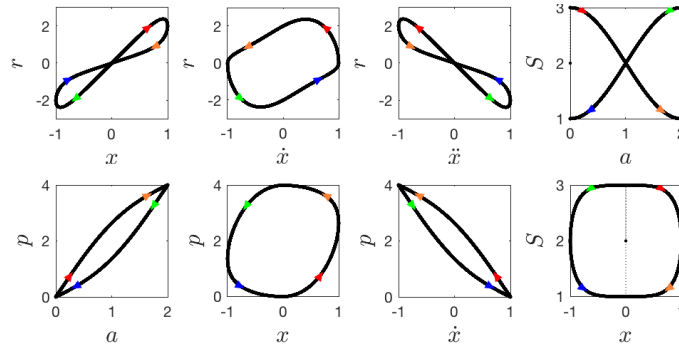


FIGURE 2 A particular mem-spring subject to the prescribed sinusoidal displacement input. For comparison a linear spring, would change the behavior to: from the left to right columns, there would be straight lines with a positive slope, ellipses, straight lines with the negative slope, and constants of the value of the positive slope for S

2.2 | Inerter Models

There are three sub-models of inerters. A linear inerter is defined as follows:

$$p_i = B\dot{x} \quad (12)$$

where B stands for inertance, with the unit of mass. The origin-crossing behavior is trivial, meaning that \dot{x} and p become zero simultaneously. This definition is consistent with the other one in (39), where a linear relationship between \ddot{x} and r is used. As mentioned previously, it should be emphasized that x is a relative motion. Referring to Fig. 2 for a linear inerter subject to the defined sinusoidal input, the response would be from the left to right columns: straight lines with a negative slope, ellipses, straight lines with the positive slope, and constants of the value of the positive slope for B .

A nonlinear inerter is defined as

$$\hat{p}_i = \hat{B}(\dot{x}) \quad (13)$$

where the origin-crossing behavior is assumed because it is physically meaningful. Notice that this definition differs from that in (39), where the linear relationship between \ddot{x} and r is generalized.

The mem-inerter, which is assumed to be a displacement-dependent inerter function, is defined as

$$p_i = B(x)\dot{x} \quad (14)$$

Or, equivalently, there is a one-to-one mapping as follows

$$\phi = H(x) \quad (15)$$

where the integrated generalized-momentum is denoted as:

$$\phi(t) = \int_{-\infty}^t p_i(\tau) d\tau \quad (16)$$

and the subscript i can be dropped for a more general definition of ϕ . The time derivative of $H(x)$ is $B(x)\dot{x}$ after applying the chain rule.

The *general* mem-inerter model *may* be defined as follows for a flow-controlled setting by exercising mathematical parallelism:

$$\text{state equations:} \quad \dot{\mathbf{y}}_i(t) = \mathbf{g}_i(\mathbf{y}_i(t), \dot{x}(t), t) \quad (17)$$

$$\text{input-output equation:} \quad p_i(t) = B(\mathbf{y}_i(t), \dot{x}(t), t) \dot{x}(t) \quad (18)$$

where $\mathbf{y}_i(t)$ is the state vector. Here in this constitutive relation, the input is \dot{x} while the output is p_i . B is a function that represents the generalized inertance, i.e. a function of the states and the velocity. When t is dropped from the right-hand side of both Eqs. (17) and (18), the definition becomes that of a time-invariant mem-inerter. For example, if the state variable is chosen as x , we *could* have a simple time-invariant mem-inerter model as follows:

$$p_i = B(x, \dot{x}) \dot{x} \quad (19)$$

Equations (17), (18), and (19) are the results of extending the mathematical parallelism in mem-models. In this study, we will demonstrate that these formulas are useful in modeling certain types of inerter, such as the helical inerter. However, as will be seen, the behavior of a physical inerter is complex, and it therefore unlikely that it could be modeled as a mem-inerter *alone*. It should also be noted that the signals recorded from an inerter test (discussed in detail in Section 3.1) could be interpreted using other nonlinear models. What we aim to show here is that the mem-model approach provides a useful framework for capturing some of the more subtle behaviors exhibited by these systems. To do this, we will first provide a set of numerical examples to demonstrate the modeling capability of Eq. (19) alone. Fig. 3 presents three gradually varying time-invariant mem-inerters in terms of Eq. (19) and subject to the defined sinusoidal input. These mem-inerters are defined using B as follows:

$$\text{mem-inerter:} \quad B_1 = \frac{\text{sgn}(\dot{x}) + 1}{2} \frac{0.2}{1 + e^{-10x}} + \frac{\text{sgn}(\dot{x}) - 1}{2} \frac{-0.2}{1 + e^{10x}} \quad (20)$$

$$B_2 = \frac{\text{sgn}(\dot{x}) + 1}{2} \frac{0.2}{1 + e^{-10x-3}} + \frac{\text{sgn}(\dot{x}) - 1}{2} \frac{-0.2}{1 + e^{10x+3}} \quad (21)$$

$$B_3 = \frac{\text{sgn}(\dot{x}) + 1}{2} \frac{0.2}{1 + e^{-10x-3}} + \frac{\text{sgn}(\dot{x}) - 1}{2} \left(\frac{-0.2}{1 + e^{10x+3}} - 0.1 \right) \quad (22)$$

Later, we will seek to reveal the inspiration of Eq. (19) and the adopted function forms in Eqs. (20) to (22) by using the experimental data of the helical inerter.

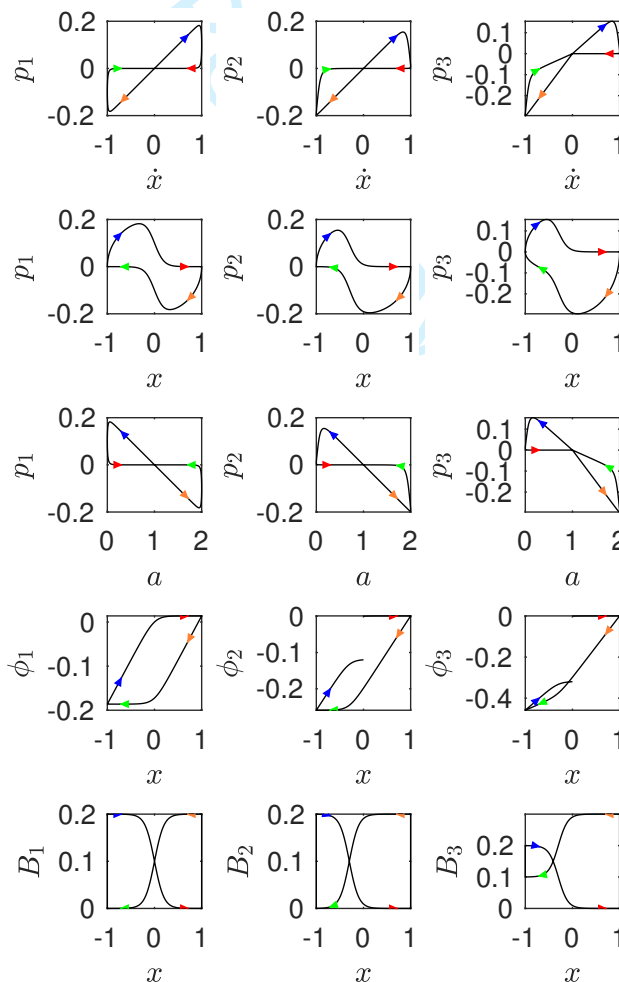


FIGURE 3 Three particular mem-inerters (following subscripts 1, 2, and 3) with each subject to the prescribed sinusoidal displacement input

The origin-crossing property for the mem-dashpot, mem-spring and mem-inerter is prominent in Figs. 1 to 3 showing r vs \dot{x} , r vs x , and p vs \dot{x} , respectively. The “figure of eight” structure of the response can be seen from the corresponding plots. Furthermore, the orientation of a loop can be observed following (31). For dashpots, r vs x rotates clockwise — it is symmetrical for a linear dashpot, while anti-symmetrical for a mem-dashpot. Similarly for springs, r vs \dot{x} rotates counter-clockwise, and likewise it is symmetrical for a linear spring, while anti-symmetrical for a mem-spring. Note also that the pairs of p vs x , and p vs a rotate clockwise for the dashpots, while p vs x rotates counter-clockwise for the springs. For p vs a , a mem-dashpot and mem-spring differ in terms of the shearing direction.

Column-wise in Fig. 3, three variations of a time-invariant mem-inerter model in the formulation of Eq. (19) are studied. While their different origin-crossing behaviors can be observed in the panels of p vs \dot{x} in the first row, the piecewise defined B vs x curves are plotted in last row with the equations given in Eqs. (20) to (22). Other rows in the middle reveal more inner-workings of the models. It can be seen that when B_1 vs x switches between the two curves mirroring about $x = 0$, p_1 vs \dot{x} is anti-symmetrical in the first and third quadrants. When the two curves on B_2 vs x do not mirror about $x = 0$, p_2 vs \dot{x} does not have the same loop in the first and third quadrants. When the two curves on B_3 vs x become even more complex, p_3 vs \dot{x} displays different loops in the first and third quadrants. Eq. (19) and the functional forms in Eqs. (20) to (22) demonstrate how the data-based models can be used to capture the complex and subtle physical behaviors exhibited by the helical fluid inerter. This observation appears to be robust to the fact that the quality of the available data does not always lead to good fitting for all data in this study.

Reference 40 proposes the concept of a mem-inerter and contrasts it with definitions of a linear and nonlinear inerter. The physical motivation seems to compare well with this study. However, the definition for the mem-inerter in Reference 40 is not as comprehensive as those for memristive and memcapacitive system models, such as those defined here.

2.3 | Dual Input-Output Pairs, Masing Model, and Higher-Order Elements

There are a wide ranging set of models for hysteresis — for recent treatments of the subject see for example (41, 42). Commonly used hysteresis models include Ramberg-Osgood (43), Bouc-Wen (44, 45, 41), bilinear hysteresis (46, 47), and classical Preisach models (48, 49). In this paper mem-models will be used based on the connection between mem- and classical models, as described by Reference 30. In the definition of the Masing model, as illustrated in Fig. 4, a virgin loading curve is critical because any other unloading and reloading curves are the scaled and shifted version of the virgin loading curve. However, in this specific application of the Masing model, we look into steady-state responses where we do not encounter the virgin loading curve anymore. Instead, we see only unloading and reloading curves, as will be described in the next Section.

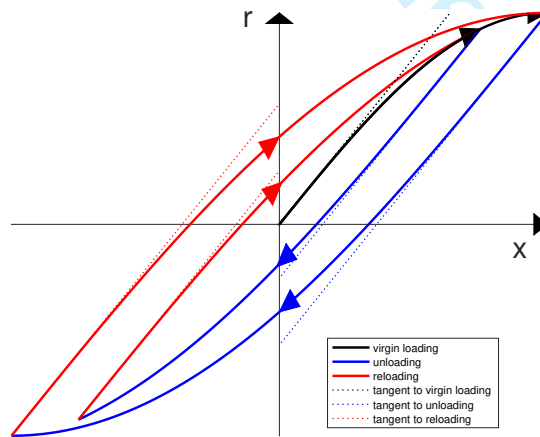


FIGURE 4 A sketch of Masing model highlighting the identical initial slopes as in Fig. 8 and later utilized in Fig. B7 for Groups 1 and 2 modeling, respectively

The key idea in (30) is that the commonly seen hysteresis models are treated in the plane of r vs x where there are no origin-crossing properties due to permanent deformations involved, however all these models can be treated on the plane of \dot{r} vs \dot{x} instead, where there are origin-crossing properties given the piecewise-defined restoring forces. That is, both \dot{x} and \dot{r} become

zero simultaneously. For these models, r vs x form an input-output pair called an integral pair, where \dot{r} vs \dot{x} form an input-output pair called a differential pair. Apart from the origin-crossing properties, these two pairs are inherently connected, for example, tangent quantities for an integral pair are secant quantities for a differential pair. Available models for integral pairs can be transformed to models for differential pairs.

In this study, we will investigate an alternative set of dual input-output pairs: the integral pair of p vs x and differential pair of r vs \dot{x} . Initially, we will create a model for the inerter based on the pair of p vs x , from which we will obtain the other quantities needed to define the pair of r vs \dot{x} . The model for the p vs x is inspired by the Masing model that is typically used for modeling hysteretic behavior in the r vs x domain. See (50, 51, 52) for the Masing model for r vs x or equivalently, stress vs strain. This modeling approach is new, and we show that it is successful in using one model (i.e., the Masing model on p vs x plane) to capture some dominant features for multiple test data sets. The effectiveness of this simplified modeling strategies, one of the two main approaches in this study, will be demonstrated in Section 3. The results using this strategy appear to be more accurate and insightful than the linear modeling approach in (53).

This initial success does not come as a surprise if we think broadly in terms of higher-order element (HOE) in the multi-physics domain. Among other papers on this subject, (54) introduced the concept of (α, β) elements, or higher-order elements for two-terminal electronic devices. Also, (55) further generalized the concept to other multiple physical domains. For mechanics, HOE promotes examining the derivatives and integrals of kinematic-kinetic pairs that we normally examine in order to define constitutive relations. Mem-models and inerters are just special cases of these type of HOEs.

3 | THE MASING MODEL

In this Section we will use experimental data from tests on the inerter system in order to create a Masing type model.

3.1 | Experimental data overview

The experimental inerter test set-up has been previously described by (5), and for completeness a brief description is included in Appendix A. The inerter was subject to a sinusoidal excitation with a prescribed frequency and amplitude. Only the displacement (x) and restoring force (r) time histories were recorded from the experiment. All the other time histories to be shown were obtained by post-processing the recorded results based on standard signal processing methods to obtain close approximations to the quantities defined in Section 2. The integrated quantities, absement a , generalized-momentum p and its integral ϕ were seen to have drift and/or offset. This is primarily caused by numerical drift that is well known to occur in integrated quantities (56, 57). In order to mitigate these effects the MATLAB `detrend` function was applied to the recorded restoring force data to limit the effects on generalized momentum p and its integral ϕ . The resulting signals of a specific sample dataset are shown and discussed in Appendix A.

3.2 | Integral Input-Output Pair using Masing Model

In seeking to create an effective model of the inerter, we utilize the concept of higher-order elements in conjunction with the Masing model. The initial step is to form two groups using the majority of the available test data sets. See Table 1. Group 1 consists of the data sets which clearly exhibit memristive type behavior. Group 2 consists of the data sets that show a mixture of memristive and other behavior. Within each group, the analysis is the same. It is important to emphasize that this division criterion is arbitrary. However, it allowed us to carry out the subsequent analysis in a consistent manner.

Figures 5 and 6 contrast Groups 1 and 2 in terms of r vs \dot{x} origin-crossing behavior. Recalling the panel of r vs \dot{x} in Fig. 1, it can be seen that Group 1 display features of memristive system models given both the origin crossing and the switching behaviors from the unloading in the first quadrant (in red arrow) to the loading in the third quadrant (in orange arrow). Group 2, on the other hand, does not so clearly display the origin-crossing behavior (although some are close) and thus we assume that these would not necessarily be approximated well with memristive system models.

An important question is whether it is possible to approximate all test results within Group 1 using one model, and those within Group 2 using another model. This calls for an examination of the features of all test results within each group in a collective manner. To do so, we extract five consecutive loops from one test, and collect all test results within Group 1 that contains a

TABLE 1 Two groups of typical results selected from test data to demonstrate the usefulness of integral input-output pair using Masing model. Note that the data sets are labeled in terms of the input amplitude and frequency of the sinusoidal excitation

Group 1: memristive system	Group 2: mixed behavior
20mm 2Hz	30mm 2Hz
15mm 2Hz	5mm 2Hz
10mm 2Hz	15mm 3Hz
10mm 3Hz	2.5mm 7Hz
5mm 4Hz	2.5mm 10Hz
	3mm 7Hz
	2mm 10Hz

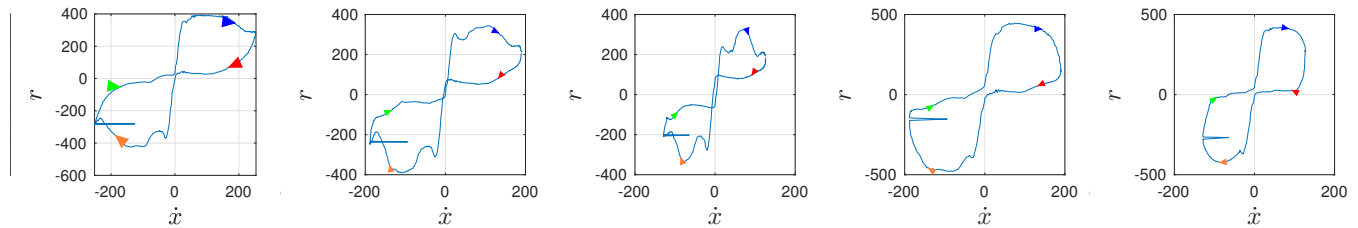


FIGURE 5 Useful loops of a typical one-cycle steady-state inerter response showing the origin-crossing behavior on the r vs \dot{x} plane in an approximate sense: Group 1 in Table 1. From left to right, they are: 20mm 2Hz, 15mm 2Hz, 10mm 2Hz, 10mm 3Hz, and 5mm 4Hz

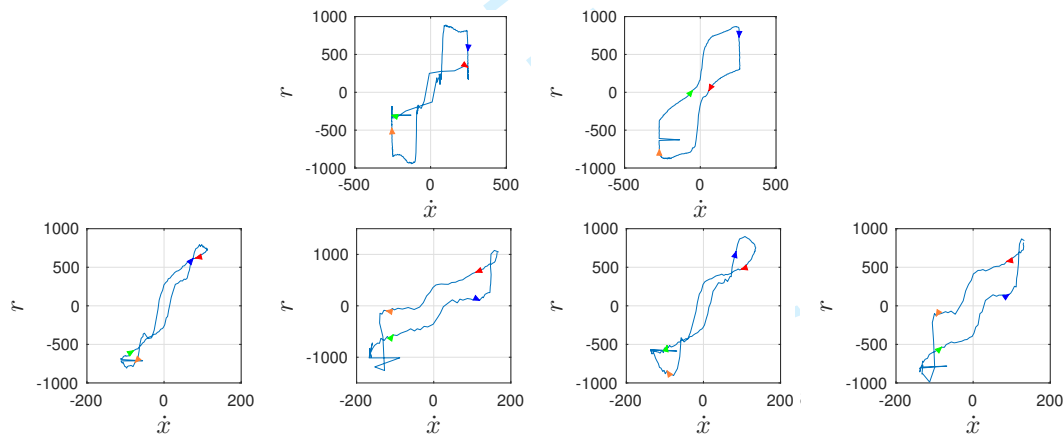


FIGURE 6 Useful loops of a typical one-cycle steady-state inerter response *not* showing the origin-crossing on the r vs \dot{x} plane in an approximate sense: Group 2 in Table 1. From top to bottom and left to right, they are: 30mm 2Hz, 5mm 2Hz, 15mm 3Hz, 2.5mm 7Hz, 2.5mm 10Hz, 3mm 7Hz, and 2mm 10Hz

total of 25 cycles. For Group 1, Fig. 7 helps examine possible underlying structure in terms of higher-order input-output pairs including p vs \dot{x} , p vs x , p vs a , and ϕ vs x , respectively.

In order to make sense of these figures, we first focus on the p vs x plot in Fig. 7. Then we shift each loop in the p vs x so that they are all positioned with the same upper right vertex. This is the so-called shifted p vs x plot in Fig. 8. It indicates that all the data sets conform to one Masing model because each data set only corresponds to a different range of the same Masing model.

Note that the Masing model (50, 51, 52) has been used to model restoring force r vs displacement x , or stress σ vs strain ϵ relations. Here we use the Masing model to model generalized momentum p vs displacement x relation instead. After the approximation, we will simply perform time derivatives on both generalized momentum p and displacement x to obtain a model

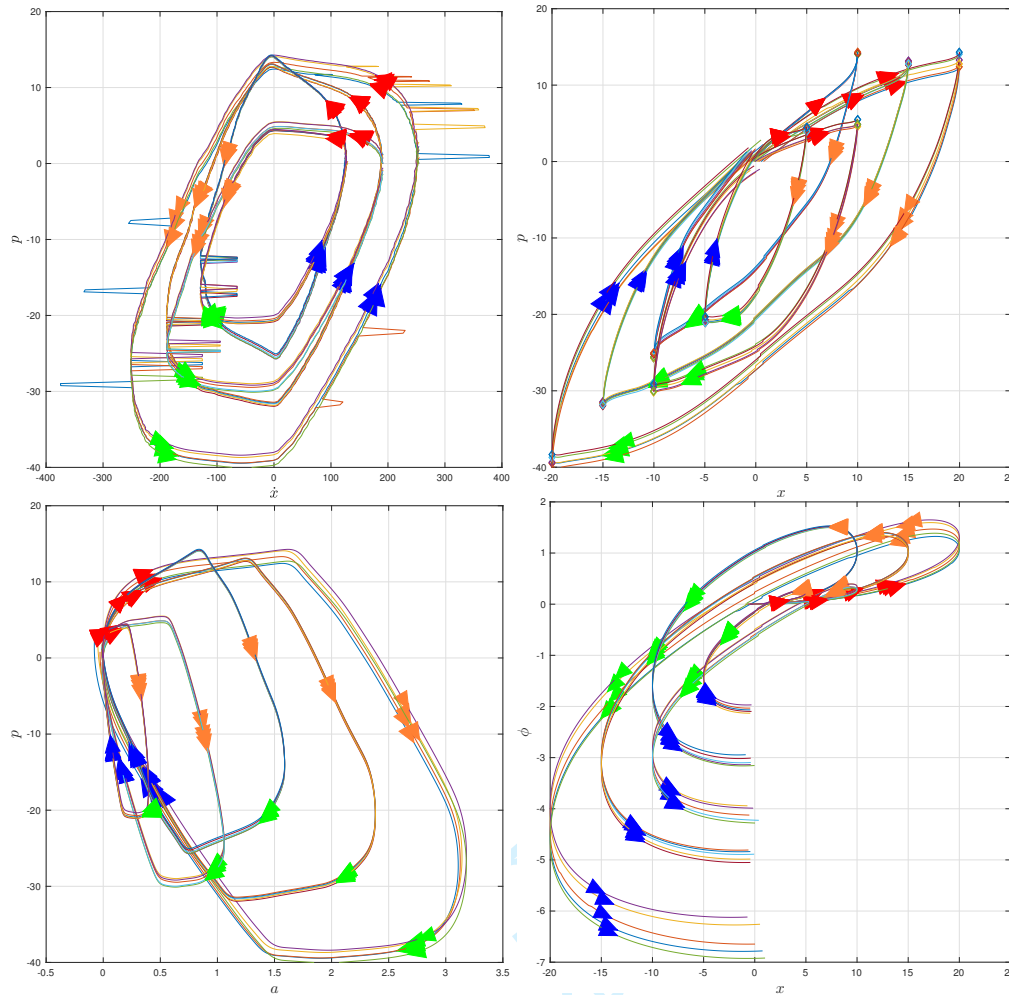


FIGURE 7 p vs \dot{x} , p vs x , p vs a , and ϕ vs x plots of all data sets with a somewhat ambiguous underlying structure: Group 1 in Table 1

for restoring force r vs velocity \dot{x} , which is our true interest for characterizing the inerter behavior. This modeling strategy including leveraging and taking time derivatives follows the approach described in (30).

In summary, p vs x pairs from the steady-state in different tests form different loops, the so-called hysteresis loops as in Fig. 7. These loops can be shifted and form a set in Fig. 8, which can be considered from the same Masing model in terms of p vs x but with different turning points. Notice that shifting in an integral pair p vs x will not affect the differential pair r vs \dot{x} based on calculus.

Looking at Fig. 8, we decide to assume that all five tests are following the same unloading curve. Since the 20mm 2Hz test covers the largest range, we fit its unloading curve with a multilayer feedforward neural network (FFNN), a universal approximator. Five hidden nodes were chosen to produce the best approximation result. An alternative would be to collect the unloading curves of all tests for fitting, say, using an FFNN. Figure 9(a) presents the experimental unloading curve and trained unloading curve. In terms of the reloading curve, the Masing rule was used given the approximated unloading curve. The corresponding time histories are consistent with those in Fig. A4 if following the colors in the order of orange, green, blue and red.

To predict the r vs \dot{x} plots for *all* other tests in Group 1, we will have to predict their corresponding p vs x plots first. We used linear interpolation applied to a given time history x with respect to that of 20mm 2Hz to obtain a portion of the 20mm 2Hz's time history p . This is not a perfect choice because not all time histories of x fall 100% to the path of that for 20mm 2Hz, and so approximation errors are thus anticipated. We then numerically differentiate the given time history x and predicted time history p to obtained time histories of \dot{x} and r , respectively, before combining them. Fig. 10 presents all predictions for the rest of the tests in Group 1.

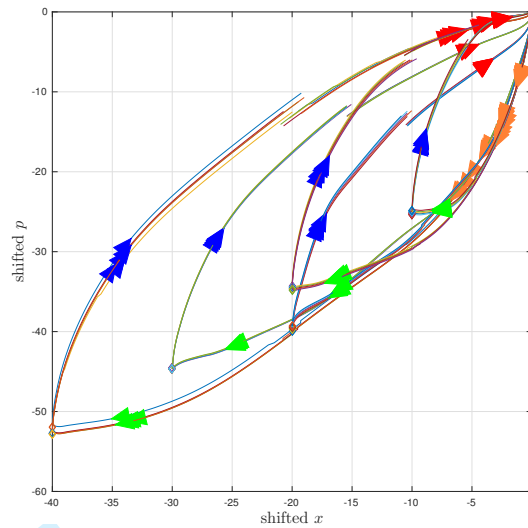


FIGURE 8 p vs x plots with shifted x and p of all data sets indicating a clearer structure hinting an application of the Masing model: Group 1 in Table 1

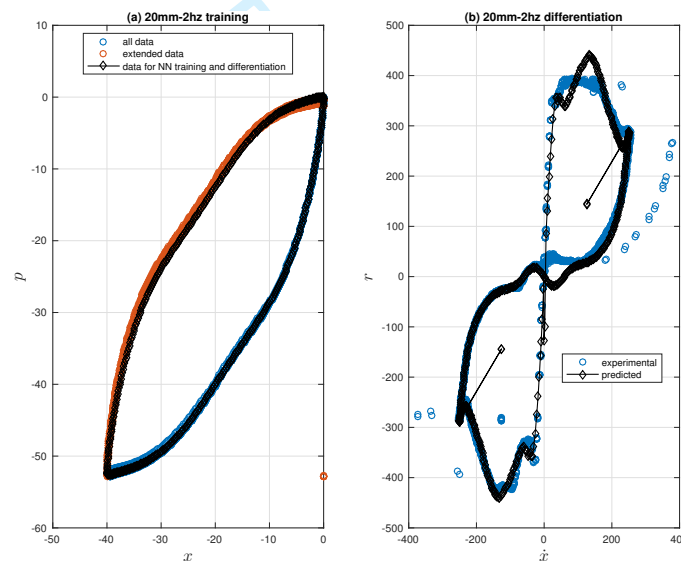


FIGURE 9 The experimental and approximated dual input-output pairs for the 20mm 2Hz test in terms of (a) p vs x , and (b) r vs \dot{x} plots. In detail, the FFNN was used to approximate the lower branch of p vs x , while the upper branch was placed by following the Masing's rule. The r vs \dot{x} was simply obtained by differentiating the time histories of p and x before combining them

The capability of the proposed modeling method has thus been demonstrated. Even though some of the more subtle behaviors are not entirely captured, the overall features and maximum r values are represented to a reasonable degree using a unified model, especially when comparing with linear modeling where there isn't a unified model. Recall that we only need one data set for modeling, where the excitation frequency is relatively low (thus the range of x is relatively high), to predict all of the rest of the responses given known displacement inputs. This is similar to some other techniques, when we intend to predict dynamic responses (with high frequencies) by using static or pseudo-static tests (with low frequencies).

The same analysis and data processing are carried out to the Group 2 test results. See Figs. B6 to B9 in Appendix B for Group 2. The performance is not as good as that for Group 1, although there are some similar features.

It deserves serious thoughts on why the p vs x pair could be selected for fitting the Masing model by entirely following the experimental data. p vs x is an integral input-output pair for a mem-dashpot. A non-one-to-one but a looping p vs x as in a Masing

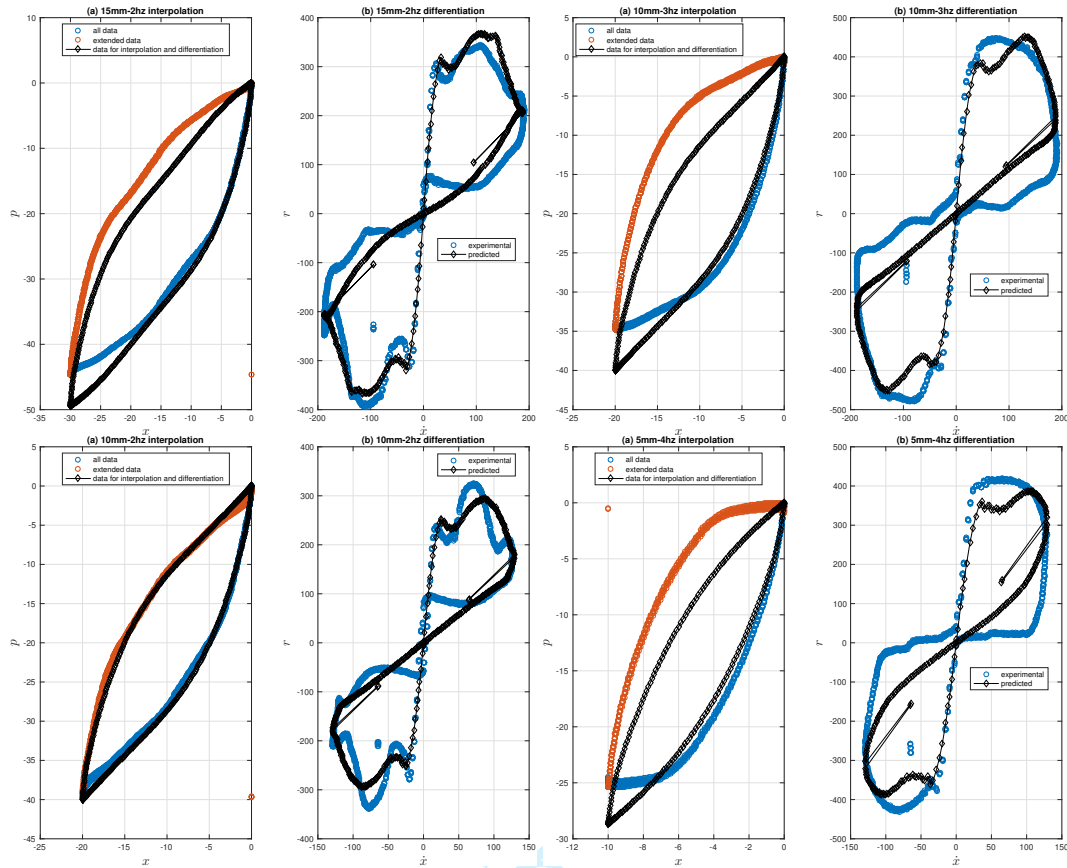


FIGURE 10 The experimental and predicted dual input-output pairs for the 15mm 2Hz, 10mm 3Hz, 10mm 2Hz, and 5mm 4Hz test in terms of (a) p vs x , and (b) r vs \dot{x} plots. In detail, linear interpolation was used to obtain the lower branch of p vs x , while the upper branch was placed by following the Masing's rule. The r vs \dot{x} was simply obtained by differentiating the time histories of p and x before combining them

model corresponds to a memristive system model as was studied in (30). This means that we chose the first approximation of the inerter by using a memristive system model (see both Eqs. (1) and (2) for the most comprehensive mathematical definition for a flow-controlled memristive system model). We do not claim that this modeling is completely accurate, however the feasibility should not come as a surprise. At least, both a helical fluid inerter and a mem-dashpot are variable energy-dissipative devices. Finally we observe that the mem-models fit best with input that are low frequency and large amplitude. This applies to most data sets in Group 1 and few in Group 2. This occurs because the time and amplitude range for the history-dependent effects to develop were more significant, and as a result we believe this is why the mem-modeling approach works better for this situation.

4 | MEM-INERTER MODELING

We now assume the inerter to be modeled is a parallel combination of mem-elements, dominated by the mem-inerter and a memristor. Initially we pose the problem by assuming that we are unsure of the exact type of elements in the model, and want to use the experimental data to identify this. We also investigate whether the definition for the mem-inerter in (40) is adequate in this context.

With these uncertainties regarding the sub-models in mind, we start with the following overall equations that are general and flexible enough to capture the points mentioned above. Thus we define our possible models as

$$p = \begin{cases} p_i + p_d = \underbrace{B(x)\dot{x}}_{\text{mem-inerter}} + \underbrace{G(x)}_{\text{memristor}}, \text{ or} \\ \hat{p}_i + p_d = \underbrace{\hat{p}(\dot{x})}_{\text{nonlinear inerter}} + \underbrace{G(x)}_{\text{memristor}} \end{cases} \quad (23)$$

where the first terms on the righthand-side (RHS) account for the contribution from the inerter, while the second terms on RHS is from a damping element, assumed in the first instance to be a memristor, the simplest form of a mem-dashpot. Capturing both variable inertance and variable damping follows well with the physics observed in the experiments. If the definition for the mem-inerter in Reference (40) is inadequate, then it would show up in the term of $B(x)$. We treat this as a model selection problem: Based on our identification, we choose the model that fits the data better.

First, when $\dot{x} = 0$ in either Eqs. (23) or (24), we can extract all points to model $p_d = G(x)$ for the memristor alone. If the identified function form is linear (without memory), then it means that there is only a linear dashpot. If the identified function is a one-to-one nonlinear function, then it means that there is a mem-dashpot. Two sets of these points are presented in Figs. 11 for Groups 1 and 2. An FFNN with three hidden nodes was used to fit each sets of the points; the fitted results are presented in the same figures. We highlight that when $x = 0$, $p = 0$, and the first and third quadrant behaviors are not symmetrical.

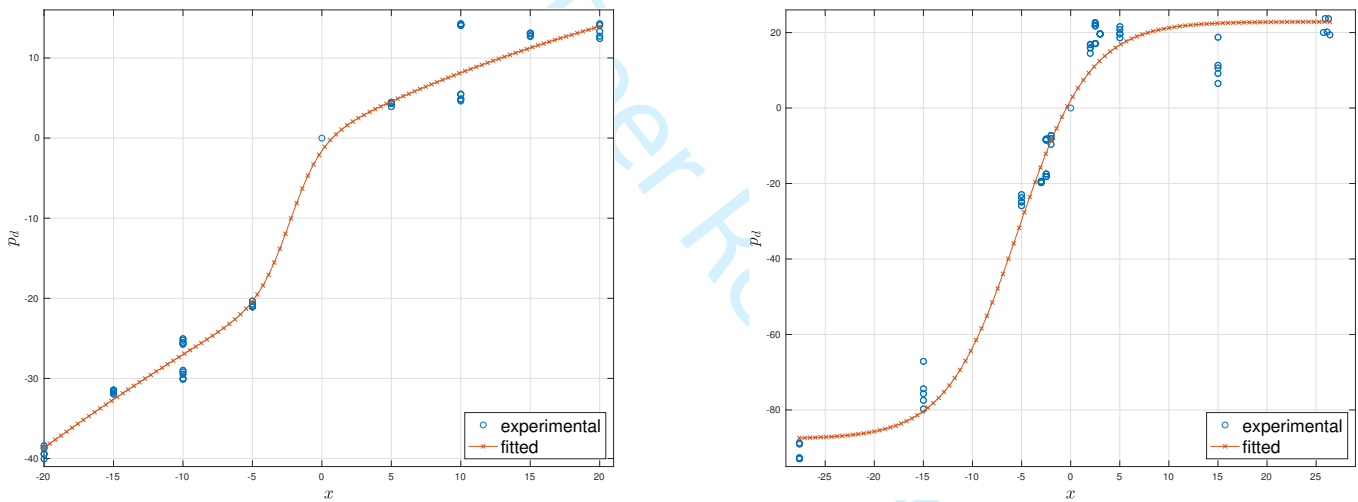


FIGURE 11 The experimental and approximated $p_d = G(x)$ for the mem-dashpot component of all experiments in Table 1 using a FFNN with three hidden nodes: Left plot Group 1, right plot Group 2.

To appreciate what the fitted $p_d = G(x)$ curves mean, Fig. 12 shows the r vs \dot{x} plots belonging to the memristor portions for all Group 1 tests. Since the fitted $p_d = G(x)$ functions are the integral pairs of the memristors, we used the experimental input x as the domain of each $p_d = G(x)$, and obtained p_d by simulating the trained FFNN. Then we numerically differentiated the time histories of both x and p_d before combining them for a r_d vs \dot{x} loop. The origin-crossing is anticipated for r_d vs \dot{x} with $r_d = D(x)\dot{x}$, where $D = \frac{dp_d}{dx}$. These loops do not have a switching feature because they are for memristors not memristive system models.

The generalized momentum for the memristor portion, p_d , as just obtained was removed from the total generalized momentum to obtain the generalized momentum for the mem-inerter portion, p_i . Fig. 13 presents all plots belonging to the mem-inerter portions for Group 1 tests. These plots include ϕ vs x , p_i vs \dot{x} , and B vs x , where both numerical differentiation and numerical integration were used, and $B = \frac{p_i}{\dot{x}}$.

Four out of five p_i vs \dot{x} plots in Fig. 13 for Group 1 display the origin-crossing behavior, which is the signature of an inerter. Nonetheless, negative B values on B vs x are not meaningful. It is not difficult to match $B < 0$ portion to those branches in the

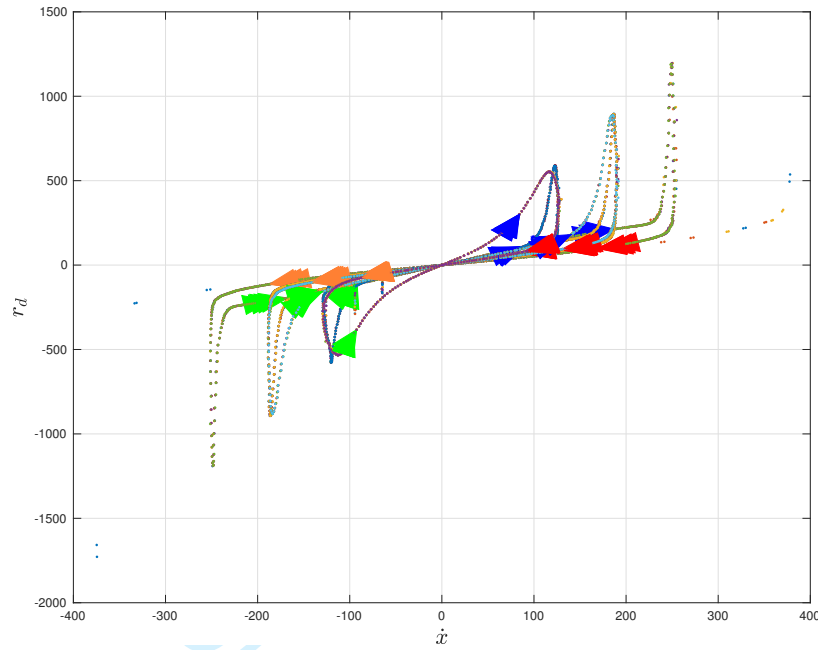


FIGURE 12 The approximated $r_d = D(x)\dot{x}$ for the mem-dashpot component of all experiments in Group 1 in Table 1 using a FFNN with three hidden nodes

second and fourth quadrants on p_i vs \dot{x} , very similar to disallowing $D < 0$ for the mem-dashpot in terms of r_d vs \dot{x} . We can trim those physically meaningless portions.

Four out of five B vs x plots in Fig. 13 show some consistency, indicating the possibility of using *one* model for all tests in Group 1. It seems that there is *no* one-to-one mapping between x and B . Instead, it seems that B switches between two functions of x according to \dot{x} . With this said, we can redefine the functional form of the model to be

$$p = p_i + p_d = \underbrace{B(x, \dot{x})\dot{x}}_{\text{mem-inerter}} + \underbrace{G(x)}_{\text{memristor}} \quad (25)$$

meaning that indeed, the behavior is better modeled by a mem-inerter, not a nonlinear inerter. In addition, this mem-inerter is more complicated than the definition in (40). This mem-inerter is a system-level model as proposed previously in Eq. (19). Similar to the system-level mem-models studied in (31), \dot{x} switches between two functions of $B(x)$.

Extracting physically meaningful portion of the data from (the four out of five cases in) Fig. 13 and overlaying them to identify a unified function form, e.g., in terms of B vs. x , would not lead to a good result given the quality of the data. Nonetheless we can qualitatively compare these five cases with those three numerical examples given previously in Fig. 3. In addition to finding similarities in terms of p vs \dot{x} and B vs x , ϕ vs x is now qualitatively comparable. In fact, these three numerical examples were inspired by the experimental results discussed here. As a result, we believe it is possible that Eqs. (20) to (22) could thus inspire more refined models using future experimental data.

In terms of mem-inerter modeling, when $x = 0$ in either Eqs. (23) or (24) and when we also assume $G(0) = 0$, we can extract all points belonging to the inerter component meaning the B and \dot{x} terms, and the corresponding difference between the p and G terms. This is done but not presented here due to space considerations. It is uncertain how these points should be fitted making an origin-crossing curve. The differential pair is always more challenging to fit than its integral pair counterpart.

Finally, differentiating Eqs. (23) and (24), we have

$$r = \begin{cases} r_i + r_d = \underbrace{\dot{B}(x)\dot{x}^2 + B(x)\ddot{x}}_{\text{mem-inerter}} + \underbrace{D(x)\dot{x}}_{\text{memristor}}, & \text{or} \\ \hat{r}_i + r_d = \underbrace{\dot{p}(\dot{x})\dot{x} + \hat{p}(\dot{x})\ddot{x}}_{\text{nonlinear inerter}} + \underbrace{D(x)\dot{x}}_{\text{memristor}} \end{cases} \quad (26)$$

$$(27)$$

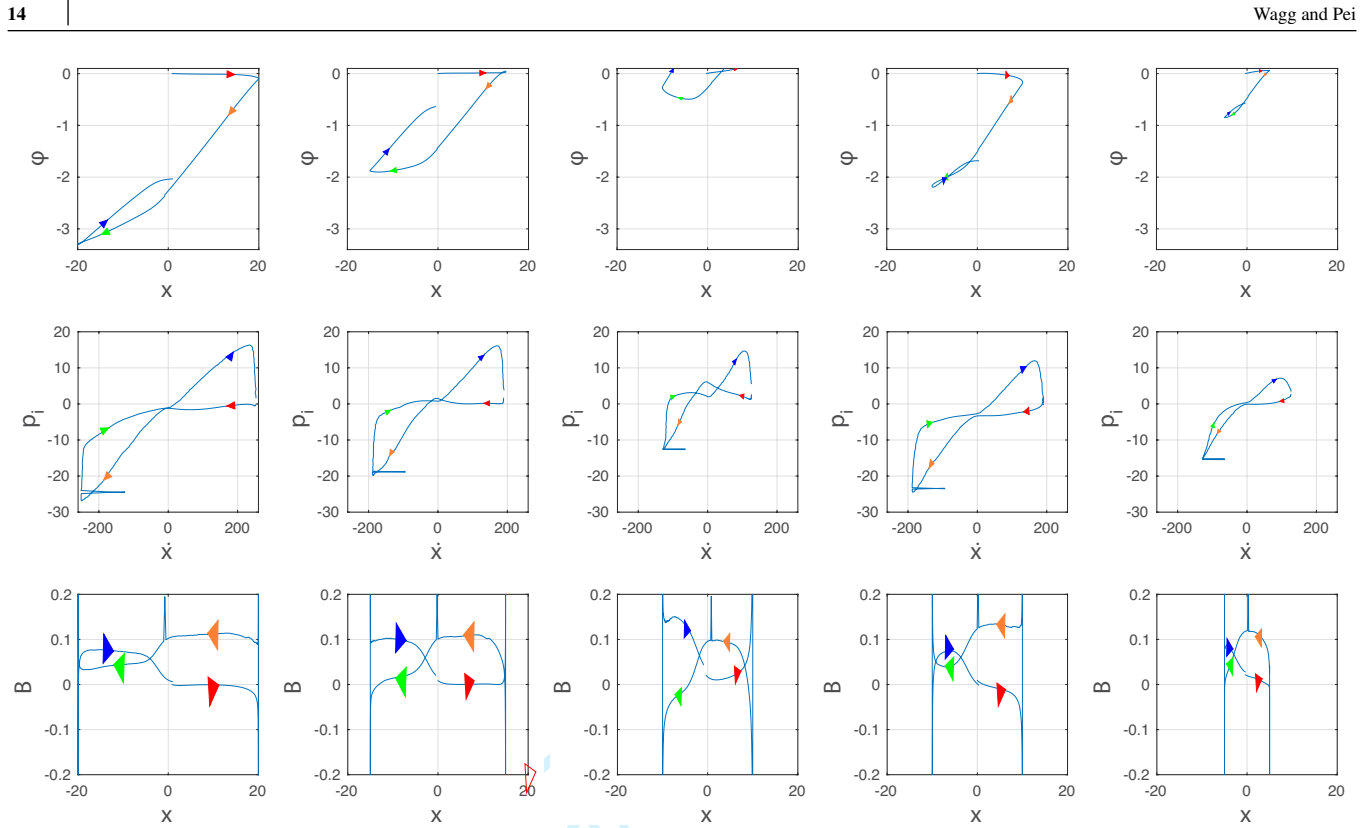


FIGURE 13 The approximated $p_i = B(x)\dot{x}$ for the mem-inerter component for a typical one cycle: Group 1 in Table 1. From left to right, they are: 20mm 2Hz, 15mm 2Hz, 10mm 2Hz, 10mm 3Hz, and 5mm 4Hz. Notice: There are physically meaningless portions

When $\dot{x} = 0$, we can extract all corresponding \ddot{x} so as to verify the existence of $B(x)$. If this does not go well for a specific set of data, it indicates that perhaps the inerter is a nonlinear inerter rather than a mem-inerter. Note that Eq. (26) differs from Eq. (11) in Reference (40) because the latter is differentiated from Eq. (14), a less general version than Eq. (23).

5 | CONCLUSION

The helical fluid inerter is a new device with important properties for passive structural control. Specifically it can provide both inertance and viscous damping simultaneously in a device with minimal moving components. However, its inherent hysteretic characteristics are quite challenging in terms of creating an effective model for the device. Our exploration in this study shows that some of the physics exhibited by the helical fluid inerter can be accurately captured using a newly developed theory called mem-models for hysteresis. More specifically, the functional relationships beyond the commonly-seen combinations of displacement, velocity, and restoring force have been demonstrated. These relationships depend on the integral quantities including absement, generalized momentum, and the integral of generalized momentum, which represent the natural memory effects that occur in the device.

By following the origin-crossing formulation of mem-modeling, which comes from the underlying instantaneous switching of the governing quantities, we have been able to demonstrate the effectiveness of the mem-modeling approach for the helical fluid inerter. In particular the results reported in this paper included both the Masing model (see Figs. 9 and B8 for training results and Figs. 10 and B9 for validation) and a mem-elements model that revealed the functional form of the mem elements (see both Figs. 11 and 13). These models indicate the potential usefulness that mem-modeling can offer. In future work, the current data could be potentially augmented by using different excitation. For example, creep or relaxation-like tests as suggested in (35) or quasi-static force tests to estimate $p_d = G(x)$ for mem-models.

We have considered how a combination of mem-elements, such as mem-inerter and memristor may be used to capture more fully the physical behavior of the helical fluid inerter device. This also included a discussion of the subtle difference between the

mem-inerter and a nonlinear inerter model – an important distinction when modeling practical devices. The results demonstrate that mem-modeling can enable engineers to build models of this type of device that more closely match the experimental data, and therefore give better validation results. Ultimately, we believe this will enable engineers to design helical fluid inerters with increased accuracy and therefore increased design confidence.

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APPENDIX

A EXPERIMENTAL SETUP

Details of the experimental testing can be found in (5). For completeness a brief description of the system is included here. A schematic diagram of the fluid inerter system considered in this paper is shown in Fig. A1. The inerter system is designed with a central fluid filled cylinder, radius r_2 , which is attached to a helical coil on the outside of the cylinder. The helical coil has an internal radius of r_3 , and the helix radius is r_4 . The fluid is driven through the cylinder using a plunger of radius r_1 .

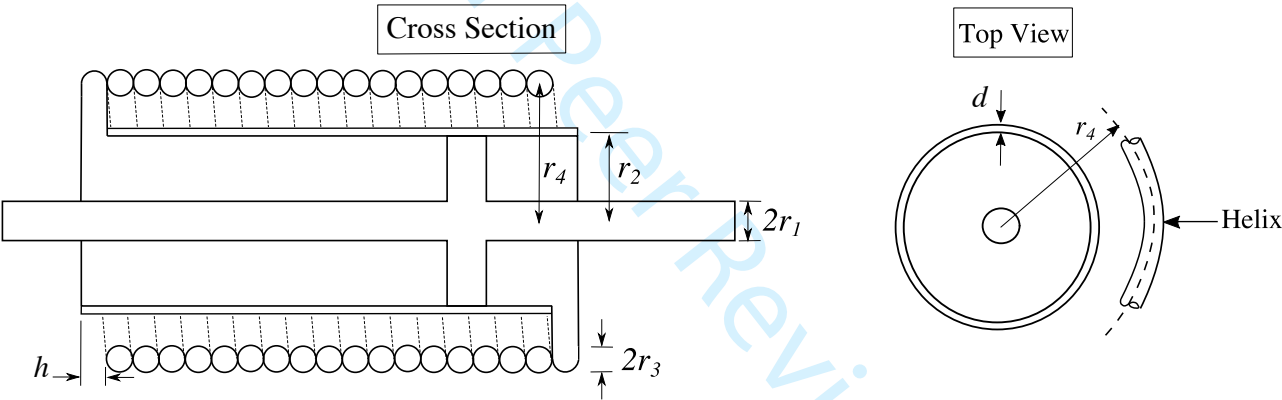


FIGURE A1 Schematic diagram of the inerter design showing the longitudinal cross section, and the top view of the system

Having a larger helix radius will lead to a higher inertance being generated by the fluid inerter system. In this paper the radius was set at its maximum possible value during testing of 120mm.

For the purpose of testing the inerter was installed in a test rig as shown in Fig. A2. The test rig consists of a hydraulic test machine that can be used to give a dynamic displacement signal to the upper input of the inerter. The lower input of the inerter is constrained not to move, and connected to a force transducer. The upper input is attached to a hydraulic actuator, the position of which is measured with a linear variable differential transformer (LVDT). The actuator is controlled using bespoke computer software associated with the test machine. Finally, a laser thermometer was used to monitor the inerter temperature to ensure a consistent operating temperature. A close up photograph of the inerter system is shown in Fig. A2, where the helical pipe system can be clearly seen. The helical pipe was made of flexible tubing held in place by pipe clips. The central chamber was made of steel.

Details of the specific set-up parameters for each test phase are given in Table A1.

In Fig. A3, the restoring force r has been detrended before any other numerical manipulation. Figure A3 also shows approximations to the quantities $D = \frac{r}{\ddot{x}}$, $S = \frac{r}{\dot{x}}$, and $B = \frac{r}{x}$. It is important to clarify these computations are only meaningful for a mem-dashpot, mem-spring, and mem-inerter, respectively, and it is therefore to be expected that not all values in these time histories are physically meaningful, e.g., there are values of $D < 0$, $S < 0$, and $B < 0$. This supports the assertion that this type of inerter cannot be modeled as a single mem-element *alone*. The analysis of the recorded responses will enable us to develop both a simplified model and a more sophisticated model.



FIGURE A2 Photograph of the inerter mounted vertically in the test machine showing the external helix piping

TABLE A1 System parameters

Property	Value	Units
Helix pitch, h	30	mm
Wall thickness, d	5	mm
r_1	14	mm
r_2	25	mm
r_3	6	mm
r_4	120	mm
Stroke of cylinder	150	mm
Kinematic viscosity at 40°	2.1	cSt
Oil density	802	kg/m ³

In order to understand the physical behavior of the inerter system in more detail, the data from Fig. A3, was processed into a representative set of data for a single period of oscillation. The results are shown in Fig. A4. Here absement and generalized-momentum are numerically integrated from zero for each cycle.

In addition to time series representations, the data was plotted in the form of cyclic loops for a single steady-state oscillation, and the results are shown in Fig. A5. Four colored arrows, red, orange, green, and blue, again, highlight the responses to the four quarters of the sinusoidal input. The top row contains r vs \ddot{x} , r vs \dot{x} , r vs x , and \dot{x} vs x plots, which are commonly used in analysis of cyclic signals. The bottom row contains p vs \dot{x} , p vs x , p vs a , and ϕ vs x plots, which will be examined closely in the rest of the paper following the concept of HOE. The key idea is that it may be challenging to create a model by using the commonly seen (top row) plots, however we may be able to seek insights and even solutions from those less commonly seen (bottom row) plots.

B RESULTS OF GROUP 2 IN SIMPLIFIED MODELING

Figures B6 to B9 are results of Group 2 in Section 3.

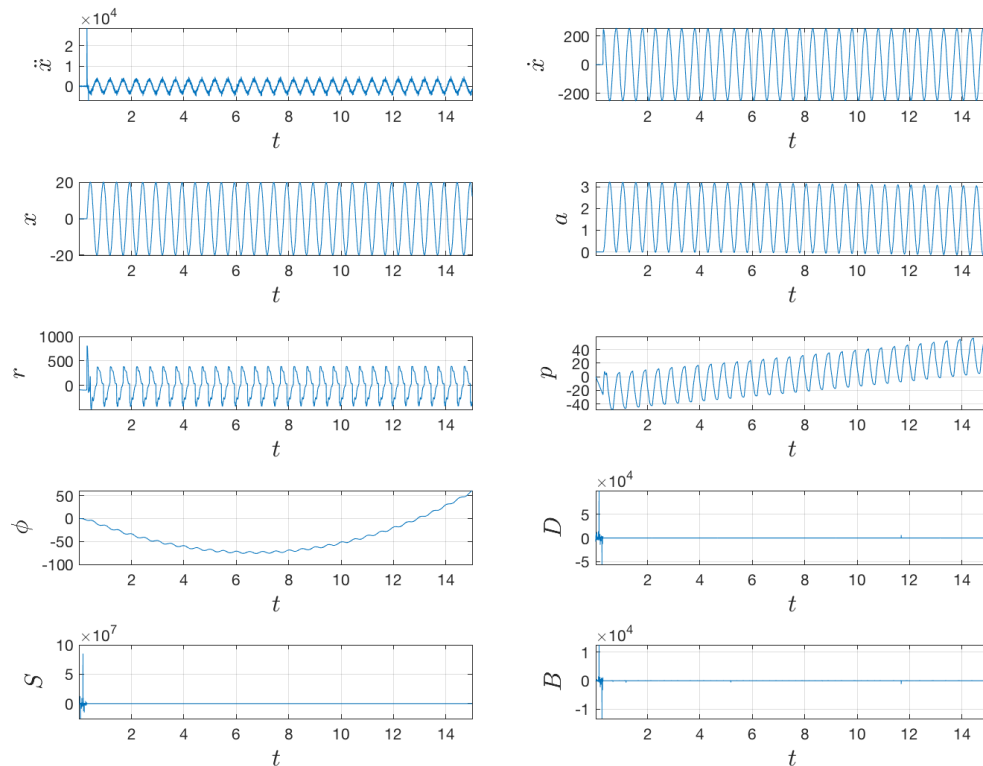


FIGURE A3 Detrended time histories of inerter response from the onset of a test: 20mm 2Hz

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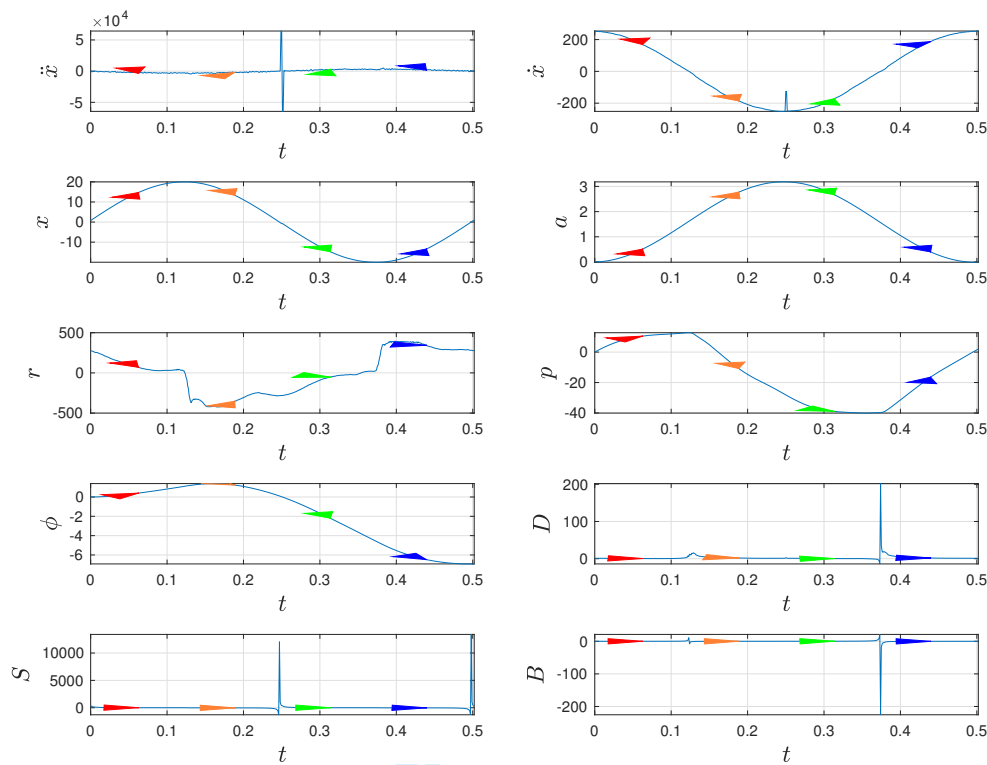


FIGURE A4 Time histories of a typical one-cycle steady-state inerter response: 20mm 2Hz. Four colored arrows, red, orange, green, and blue, highlight the responses to the four quarters of the sinusoidal input

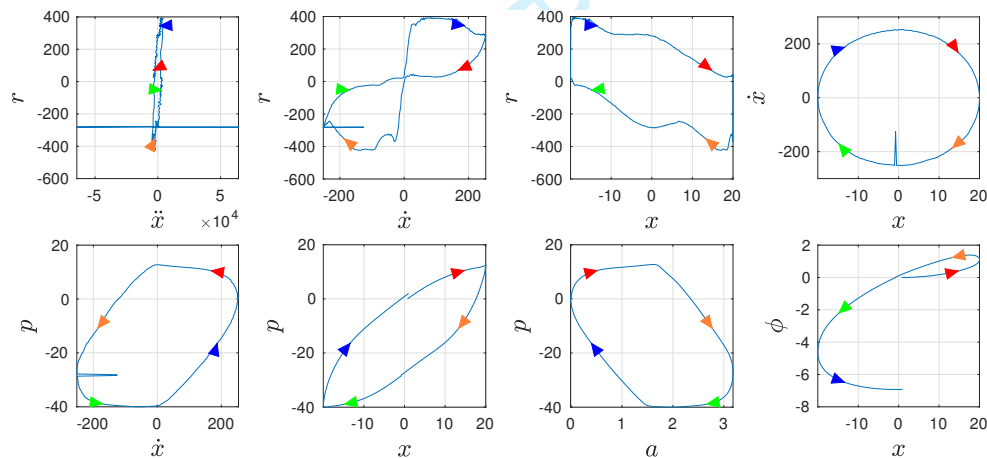


FIGURE A5 Useful loops of a typical one-cycle steady-state inerter response: 20mm 2Hz

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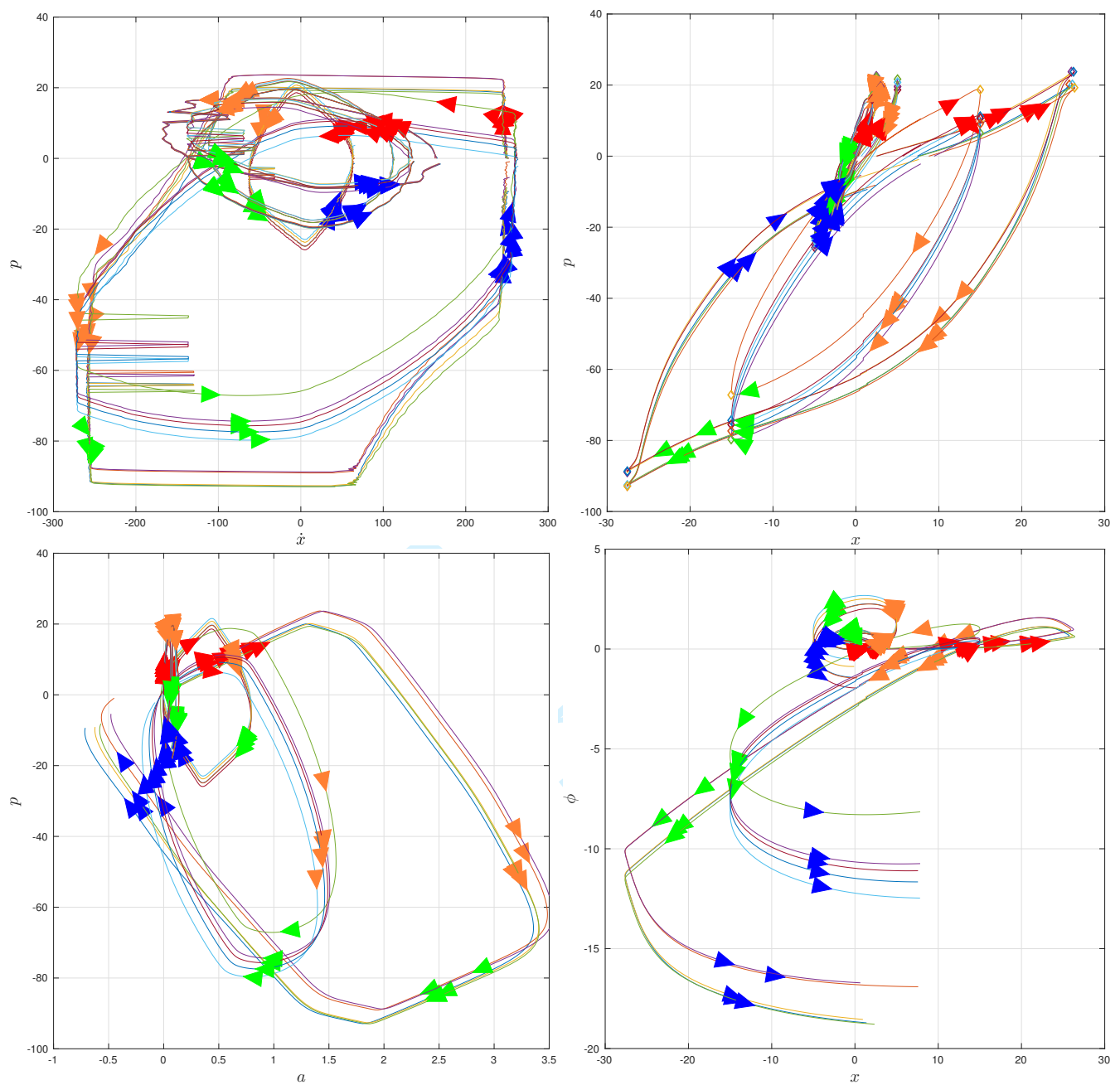


FIGURE B6 p vs \dot{x} , p vs x , p vs a , and ϕ vs x plots of all data sets without a strong indication of underlying structure: Group 2 in Table 1

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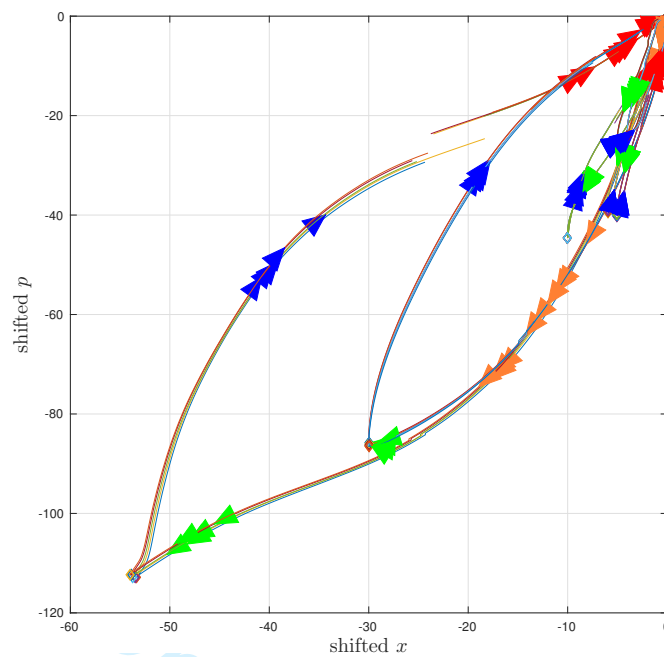


FIGURE B7 p vs x plots with shifted x and p of all data sets indicating a clearer structure hinting an application of the Masing model: Group 2 in Table 1

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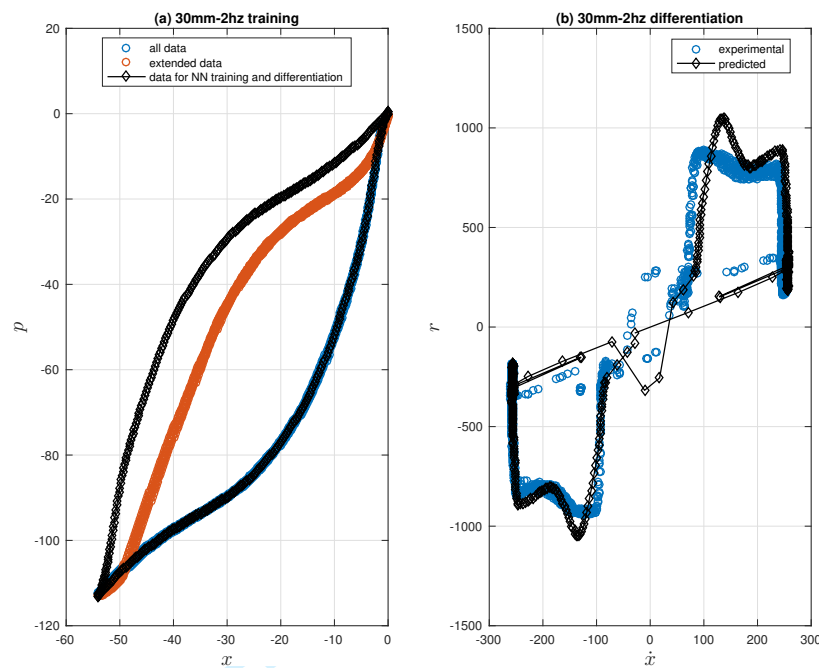


FIGURE B8 The experimental and approximated dual input-output pairs for the 30mm 2Hz test in terms of (a) p vs x , and (b) r vs \dot{x} plots. In detail, the FFNN was used to approximate the lower branch of p vs x , while the upper branch was placed by following the Masing's rule. The r vs \dot{x} was simply obtained by differentiating the time histories of p and x before combining them

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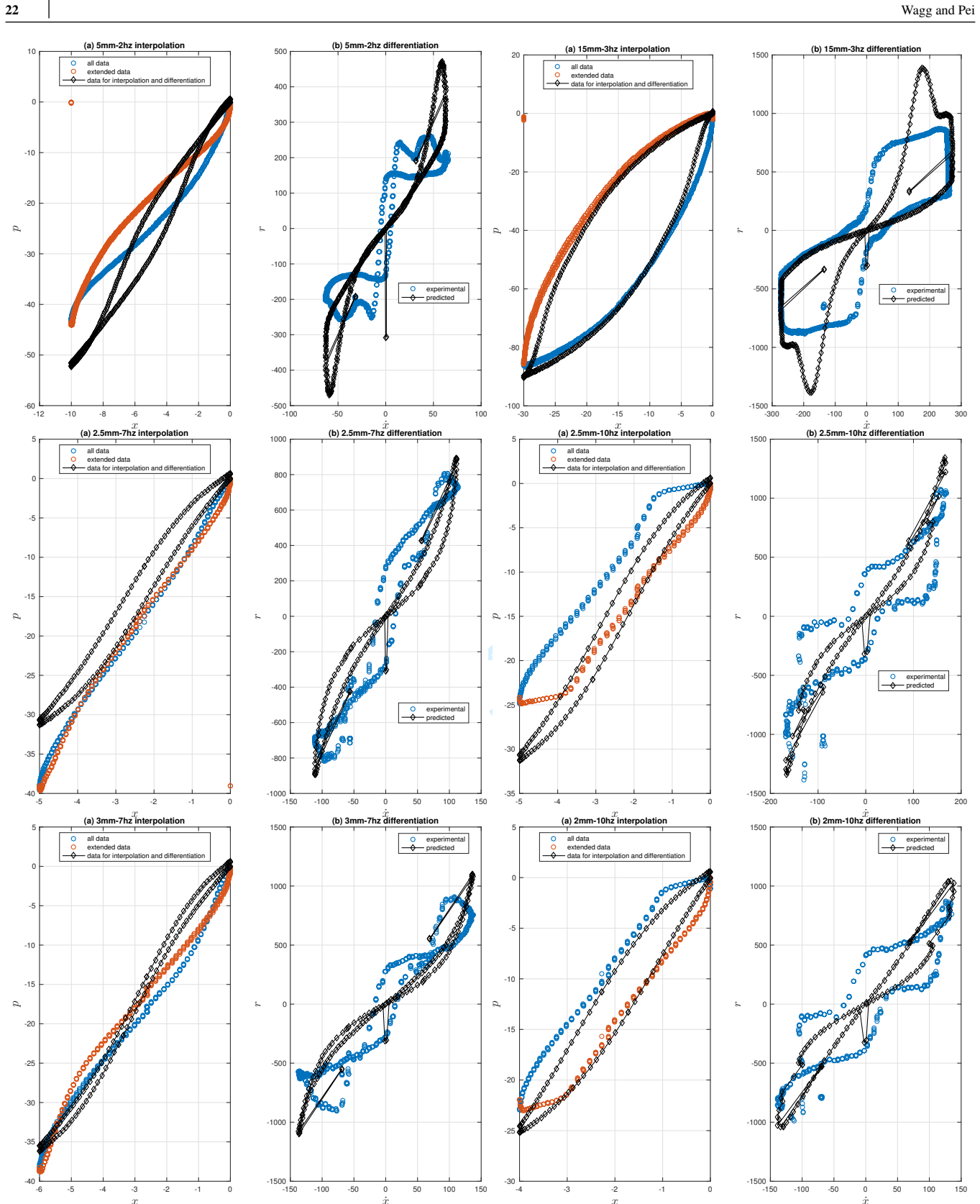


FIGURE B9 The experimental and predicted dual input-output pairs for the 5mm 2Hz, 15mm 3Hz, 2.5mm 7Hz, 2.5mm 10Hz, 3mm 7Hz, and 2mm 10Hz test in terms of (a) p vs x , and (b) r vs \dot{x} plots. In detail, linear interpolation was used to obtain the lower branch of p vs x , while the upper branch was placed by following the Masing's rule. The r vs \dot{x} was simply obtained by differentiating the time histories of p and x before combining them

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