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Extrapolative expectations and macroeconomic dynamics: Evidence from an estimated DSGE model

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Abstract
We outline a dynamic stochastic general equilibrium (DSGE) model with extrapolative expectations in asset pricing and fit the model to 50 years of quarterly U.S. macroeconomic time series data with Bayesian techniques. We conclude that extrapolative expectations in asset pricing are statistically significant, quantitatively relevant and result in a substantial improvement in the model's fit to the data. In particular, extrapolative expectations in asset pricing lead to more pronounced hump-shaped responses in the asset price and investment to shocks, and the model matches the degree of persistence observed in the asset price data significantly better than the alternative DSGE models considered here, which are the Smets and Wouters (2007; *American Economic Review*, *97*, 586–606) model, including a variant of the model with pre-determined investment expenditures, and the Gilchrist, Ortiz, and Zakrajsek (2009; *Credit risk and the macroeconomy: Evidence from an estimated DSGE model*. Mimeo) financial frictions model. Our findings are confirmed by numerous robustness exercises, including different prior assumptions, different sample periods and different time series variables, both excluding asset price data and the use of different asset price measures.

KEYWORDS
asset pricing, Bayesian techniques, business cycles, DSGE models, extrapolative expectations

1 | INTRODUCTION

A growing body of literature documents the prevalence of extrapolative expectations in financial decision making. Fuster, Laibson, and Mendel (2010, pp. 68, 70) noted that ‘studies in a wide variety of contexts suggest that actual people's forecasts place too much weight on recent changes, like the most recent quarterly growth rate in variables such as portfolio values or home prices’ and concluded that ‘while it is possible to explain any finding with a combination of rational expectations and a more- or less-elaborate surrounding story, introducing extrapolative features into models of expectation formation may provide a more parsimonious and general explanation for various empirical phenomena’.

For example, Barberis, Greenwood, Jin, and Shleifer (2015) calibrated an asset-pricing model with extrapolative expectations that is consistent with survey evidence of investor expectations. Greenwood and Shleifer (2014, p. 714) analysed investor expectations of stock market returns from several sources and concluded that ‘investor expectations tend to be extrapolative: they are positively correlated

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with past stock market returns, as well as with the level of the stock market'. Benartzi (2001) and Choi, Laibson, Madrian, and Metrick (2009) found that past stock returns affect savings in pension accounts. Chevalier and Ellison (1997), Greenwood and Nagel (2009), and Sirri and Tufano (1998) observed trend-chasing behaviour among mutual fund investors. Malmendier and Nagel (2011) found that past stock returns had long-term effects on people's financial risk taking, although more recent stock returns had stronger effects, and Vissing-Jorgensen (2003) observed that wealthy investors extrapolate their investment returns into the future.

However, although empirical research concerning asset pricing is sympathetic to the idea that asset prices may deviate from fundamentals due to extrapolative expectations, few macroeconomic models have incorporated asset mispricing or provided an empirical evaluation of its quantitative importance (see Hirshleifer, Li, & Yu, 2015, for an exception). In this article, we intend to fill this gap in the literature by modifying the dynamic stochastic general equilibrium (DSGE) model proposed by Smets and Wouters (2007) by allowing the asset price to be determined as a weighted average of the asset's fundamental value and its value according to trend extrapolation. The DSGE model is estimated with the same quarterly U. S. macroeconomic time series data as used by Smets and Wouters (2007); however, we added data on asset prices (measured as the market value of capital relative to its replacement cost, i.e., Tobin's q) and updated their data set such that it covers the period from 1966q1 to 2015q4.

To the best of our knowledge, the macroeconomic model reported in this article is the first DSGE model with extrapolative expectations in asset pricing that is fitted to data with Bayesian techniques. Several results should be emphasized. First, trend extrapolation in asset pricing is relevant. We find the (mean) weight attached to trend extrapolation to be 0.67 and the (mean) strength in trend extrapolation to be 0.60. At first glance, one could argue that the weight attached to assets' fundamental values in asset pricing is too small to be a reliable figure. However, in the model presented here, capital consists not only of capital traded in equity markets but also all physical capital in the economy.

Second, when evaluating the model's fit to the data, we obtain *very strong* evidence in favour of the DSGE model with extrapolative expectations in asset pricing over the DSGE model proposed by Smets and Wouters (2007), including both an alternative version of their model in which investment expenditures are pre-determined as in Bernanke, Gertler, and Gilchrist (1999), suggesting that 'trend extrapolation in asset pricing' is not merely a delayed response of investments to asset prices, and a model variant that includes financial frictions developed by Gilchrist, Ortiz, and Zakrajsek (2009). That the proposed model outperforms the Smets and Wouters (2007) model is an important result because this task is more challenging to accomplish than one may initially believe as Smets and Wouters (2007) already included most modelling features empirically relevant to business cycle dynamics. For example, the introduction of correlated disturbances (Cúrdia & Reis, 2010), the financial accelerator (Brzoza-Brzezina & Kolasa, 2013 and Gelain, Rodríguez Palenzuela, & Világi, 2009), and labour market search frictions (Gertler, Sala, & Trigari, 2008) did not result in a better fit to the data relative to the Smets and Wouters (2007) model.

Finally, extrapolative expectations in asset pricing clearly affect the dynamic responses of macroeconomic variables in the DSGE model. In particular, extrapolative expectations lead to more pronounced hump-shaped responses in the asset price and investment to shocks, and the model matches the degree of persistence observed in asset price data significantly better than alternative models, including the Smets and Wouters (2007) model. Taken together, these results support the hypothesis that fluctuations in main U.S. macroeconomic variables are affected by deviations in asset prices from their fundamental values as defined by a present-value model. The results also indicate that asset price misalignments should be an important ingredient in DSGE models aiming to understand business cycle dynamics. Notably, our findings are confirmed by numerous robustness exercises, including different prior assumptions, different sample periods and different time series variables, both excluding asset price data and the use of different asset price measures.

The remainder of this article is organized as follows. In Section 2, we examine why misaligned asset prices may be relevant for a better understanding of business cycle dynamics. A DSGE model with extrapolative expectations in asset pricing is presented in Section 3, and a quantitative analysis of this model and alternative DSGE models is performed in Section 4 along with robustness checks. The conclusions are presented in Section 5.

2 | ASSET PRICES AND BUSINESS CYCLE DYNAMICS

Asset prices in DSGE models are defined by the price of capital. Therefore, we first calculate Tobin's q to generate a time series observable that matches the concept of the price of capital.[1] The log of Tobin's q (lQ_t) is displayed in Figure 1.

Two observations are notable in Figure 1. First, the deviations from the steady-state value of the log of the price of capital, which should be zero, can be large and persist for long durations (the value of the first-order

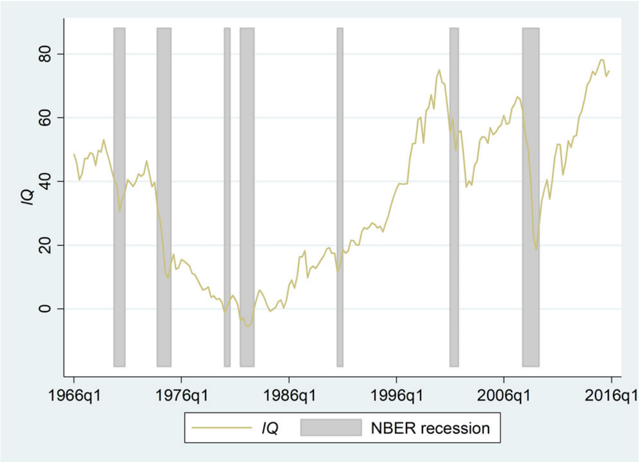

FIGURE 1 Time series of the log of Tobin's q [Colour figure can be viewed at wileyonlinelibrary.com]

autocorrelation is 0.98), suggesting that factors other than fundamentals likely influence the dynamics of the price of capital. Second, the deviations from the steady-state value appear to have the following relevant cyclical component: Tobin's q increases during economic booms and falls during economic recessions. This pattern is also apparent in other measures of asset prices relative to fundamentals (Cochrane, 2005, pp. 356–359; Goliński, Madeira, & Rambaccussing, 2015).[2]

Indeed, the value (0.46) of the contemporaneous correlation of the cyclical component of the log of Tobin's q ($l\hat{Q}_t$) with the cyclical component of the log of real gross domestic product ($lG\hat{D}P_t$) is substantial.[3] In fact, Tobin's q actually falls prior to the National Bureau of Economic Research's recession dates, suggesting that it leads the business cycle. To assess this aspect more rigorously, we performed the following linear regression:

$$lG\hat{D}P_t = \beta_0 + \beta_1 lG\hat{D}P_{t-1} + \beta_2 l\hat{Q}_{t-1} + \varepsilon_t = 0.005 + 0.777 \cdot lG\hat{D}P_{t-1} + 0.051 \cdot l\hat{Q}_{t-1}, \tag{1}$$

confirming that Tobin's q leads the business cycle because β_2 is positive and highly statistically significant (the p-value is .000).

In summary, (a) the time series of asset prices as measured by Tobin's q significantly deviates from its theoretical long-run value and is highly persistent, indicating that asset prices are misaligned relative to fundamentals; and (b) Tobin's q has an important cyclical dimension since it is procyclical and leads output. These findings suggest that an improved modelling of asset prices is relevant for a better understanding of business cycle dynamics.

3 | DSGE MODEL WITH EXTRAPOLATIVE EXPECTATIONS IN ASSET PRICING

Our choice of the Smets and Wouters (2007) model as the reference model is motivated by its inclusion of a wide variety of real and nominal frictions, its good fit with U.S. macroeconomic time series data and, in a slightly different version, its good fit with Eurozone data (Smets & Wouters, 2003).[4] Cúrdia and Reis (2010, p. 24) noted that 'central banks around the world have adopted variants of this model', which also influenced our choice of this model as our reference model.

Except for allowing the asset price to be determined as a weighted average of the asset's fundamental value, defined by the same present-value model as in Smets and Wouters (2007), and its value according to trend extrapolation, the DSGE model presented here is identical to the DSGE model presented by Smets and Wouters (2007). Hence, to conserve space, we only show how the asset price is determined in our version of their model and refer to Smets and Wouters (2007) for the remaining equations.[5]

Specifically, we assume that the asset price, q_t, which is the price of capital, is determined as a weighted average of the asset's value according to trend extrapolation, q_t^c, and the asset's fundamental value, q_t^f. Therefore, the aggregate price index equation for assets, or capital, is as follows:

$$q_t = \omega q_t^c + (1-\omega) q_t^f, \tag{2}$$

where $0 \leq \omega \leq 1$ is the weight attached to trend extrapolation in asset pricing.[6] This equation is similar to the aggregate price index equation for goods (and also wages if the model includes wage rigidity) in typical New Keynesian models (see, e.g., (20)–(23) in Galí & Gertler, 1999).

The asset's value according to trend extrapolation is equal to the previous asset price plus the previous change in the asset price multiplied by a strength parameter in trend extrapolation, ϑ, as follows:

$$q_t^c = q_{t-1} + \vartheta(q_{t-1} - q_{t-2}). \tag{3}$$

We are of two minds regarding the microeconomic foundations underlying trend extrapolative behaviour in financial decision making. Although not ideal, this ambiguity is by no means uncommon in contemporary macroeconomic research (e.g., backward-looking price and wage setting have become standard specifications in DSGE models and are already included in the model proposed by Smets & Wouters, 2007).

The asset's fundamental value is determined by the same present-value model as in Smets and Wouters (2007). According to this model, the current asset price depends positively on its expected future price and the expected future real rental rate on capital and depends negatively on the (ex ante) real interest rate and a risk premium shock as follows:

$$q_t^f = q_1 E_t q_{t+1}^f + (1-q_1) E_t r_{t+1}^k - \left(r_t - E_t \pi_{t+1} + \varepsilon_t^b\right), \quad (4)$$

where r_t^k is the real rental rate on capital, r_t is the nominal interest rate controlled by the central bank, π_t is the inflation rate, and ε_t^b is the risk premium shock that represents a wedge between the interest rate and the return on assets held by households as follows:

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b, \quad (5)$$

where η_t^b is a Gaussian white-noise process with a zero mean and a standard deviation of σ_b. Moreover, $q_1 = \beta\gamma^{-\sigma_c}(1-\delta)$, where β is the discount factor applied by households, γ is the steady-state growth rate of the economy, σ_c is the elasticity of intertemporal substitution, and δ is the depreciation rate of the capital stock.

If $\omega = 0$, the asset price is determined solely by its fundamental value, and the DSGE model considered here becomes identical to the DSGE model proposed by Smets and Wouters (2007). If $0 < \omega < 1$, the asset price is only partially determined by its fundamental value because past asset prices also affect the current asset price. Thus, the asset price is misaligned in this case because $q_t \neq q_t^f$ in general.

4 | QUANTITATIVE ANALYSIS

We fit the DSGE model with extrapolative expectations in asset pricing and the DSGE model proposed by Smets and Wouters (2007) to quarterly U.S. macroeconomic time series data. This approach allows us to study the implications of asset mispricing for business cycle dynamics. Alternative specifications of the Smets and Wouters (2007) model, including a model in which investment expenditures are pre-determined as in Bernanke et al. (1999) and a model variant involving financial frictions developed by Gilchrist et al. (2009), are also examined, and various robustness checks are performed. The data set is described in Section 4.1, and the models are estimated and simulated in Sections 4.2 and 4.3, respectively.

4.1 | Data set

The data set consists of the same quarterly U.S. macroeconomic time series data as used by Smets and Wouters (2007); however, we added data on asset prices and updated their data set such that it covers the period from 1966q1 to 2015q4. The following time series data are included in the data set: (a) the log-difference of Tobin's q; (b) the log-difference of the real gross domestic product (GDP); (c) the log-difference of real consumption; (d) the log-difference of real investment; (e) the log-difference of the real wage; (f) the log of hours worked; (g) the log-difference of the GDP deflator; and (h) the federal funds rate.

The following measurement equations are employed:

$$\begin{bmatrix} dlQ_t \\ dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{bmatrix} = \begin{bmatrix} \Delta\bar{q} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} \Delta q_t + \varepsilon_t^q \\ \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ \Delta w_t \\ l_t \\ \pi_t \\ r_t \end{bmatrix}, \quad (6)$$

where l and dl denote the log and log-difference, respectively; $\Delta\bar{q}$ is the time series average of the quarterly trend growth rate in Tobin's q; ε_t^q is a Gaussian white-noise process with a zero mean and a standard deviation of σ_q; $\bar{\gamma} = 100\cdot(\gamma-1)$ is the time series average of the common quarterly trend growth rate in real GDP, real consumption, real investment, and real wage; $\bar{l}$ is the time series average of hours worked normalized to zero; $\bar{\pi} = 100\cdot(\Pi_* - 1)$ is the time series average of the inflation rate, where Π_* is the steady-state inflation rate; and $\bar{r} = 100\cdot\left(\frac{\Pi_*\gamma^{\sigma_c}}{\beta} - 1\right)$ is the time series average of the nominal interest rate.

4.2 | Estimation of the DSGE models

4.2.1 | Methodology

The DSGE models are fitted to the data with Bayesian techniques. We use Dynare to estimate and simulate the models. First, the mode and standard deviation of the posterior distribution are estimated by maximizing the log-posterior function that combines prior information regarding the parameters with the likelihood of the data set. Second, the Metropolis–Hastings algorithm is used to obtain a more complete picture of the posterior distribution and evaluate the marginal likelihood of a model. As described by Smets and Wouters (2007), a sample of 250,000 draws is created for each model, neglecting the first 50,000 draws, and Markov chain Monte Carlo univariate and multivariate diagnostics are used to determine convergence and stability in the parameter moments.

Next, we describe the priors used to estimate the DSGE model with extrapolative expectations in asset pricing. The prior of $\Delta\bar{q}$ is assumed to follow a normal distribution with a mean of 0.12 (i.e., the time series mean of the quarterly growth rate of Tobin's q between 1966q1 and 2015q4) and a standard deviation of 0.1. The prior of σ_q is assumed to follow an inverse gamma distribution with a mean of 0.1 and a standard deviation of 2. When choosing the priors of the parameters characterizing trend extrapolation in asset pricing, we adopt the principle of indifference (i.e., we assign equal probabilities to all possibilities). Therefore, we use a uniform distribution as the prior for the weight attached to trend extrapolation, $\omega \in [0,1]$, and the strength in trend extrapolation, $\vartheta \in [-3,3]$. Thus, the latter interval includes non-positive values, although extrapolative behaviour implies a positive value for this parameter (which also proves to be the case in the estimations, see Sections 4.2.2 and 4.2.5).

The priors of the remaining parameters in the DSGE model coincide with those in the Smets and Wouters (2007) model. Thus, we refer to their paper for a discussion of the priors. In addition, a few parameters, which are the same as those in the Smets and Wouters (2007) model, are fixed in the estimations at the same values as in their model. Finally, the Gilchrist et al. (2009) model includes parameters that are not included in the Smets and Wouters (2007) model. For these parameters, we use the same priors as described in the Gilchrist et al. (2009) model.

4.2.2 | Is trend extrapolation in asset pricing relevant?

The prior and posterior distributions of the structural parameters and shock processes related to trend extrapolation in asset pricing are shown in Table 1.

Both parameters that distinguish the DSGE model with extrapolative expectations in asset pricing from the DSGE model used by Smets and Wouters (2007), ω and ϑ, are quantitatively large. The estimated mode of the weight attached to trend extrapolation (ω) is 0.70, the estimated mean is 0.67, and the 5th and 95th percentiles of the posterior distribution are 0.52 and 0.80, respectively. Furthermore, the estimated mode of the strength in trend extrapolation (ϑ) is 0.56, the estimated mean is 0.60, and the 5th and 95th percentiles of the posterior distribution are 0.19 and 1.00, respectively.[7] Thus, our findings suggest that asset price misalignments are important for a better understanding of business cycle dynamics.

4.2.3 | Other findings

The estimates of most parameters in the DSGE model with extrapolative expectations in asset pricing are close to those obtained by Smets and Wouters (2007). Thus, we focus only on the parameter values that are significantly altered by the introduction of extrapolative expectations. The prior and posterior distributions of the relevant structural parameters and shock processes are shown in Tables 1 and 2, where the DSGE models in Table 2 are the Smets and Wouters (2007) model, an alternative specification of their model in which investment expenditures are pre-determined as in Bernanke et al. (1999), and a model variant involving financial frictions developed by Gilchrist et al. (2009).[8]

Of the structural parameters, the steady-state elasticity of the cost of adjusting capital (φ) significantly differs among the models. In the model with extrapolative expectations in asset pricing, the estimated mode and mean are 3.42 and 3.65, respectively, whereas the estimated mode and mean in the Smets and Wouters (2007) model are 3.97 and 4.28, respectively. In the model with pre-determined investment expenditures, the estimated mode and mean are 5.46 and 5.87, respectively. Thus, investment is more responsive to changes in the asset price in the model in this article than in these other models because investment inversely depends on the steady-state elasticity of the cost of adjusting capital. Compared with the Gilchrist et al. (2009) model, investment is less responsive to changes in the asset price because the estimated mode and mean are 2.62 and 2.93, respectively.

Additionally, the habit parameter in consumption (λ) significantly differs among the models. In the model with extrapolative expectations in asset pricing, the estimated mode and mean are 0.72 and 0.67, respectively, whereas the estimated mode and mean in the Smets and Wouters (2007) model are 0.52 and 0.53, respectively. In the model with pre-determined investment expenditures, the estimated mode and mean are 0.45 and 0.47, respectively. Thus, compared with these other models, the habit formation in consumption is stronger in the model with extrapolative expectations. Compared with the Gilchrist et al. (2009) model, the habit formation in consumption

TABLE 1 Estimation results of the DSGE model with extrapolative expectations in asset pricing

	Prior distribution			Posterior distribution				
Parameter	Type of distribution	Mean	*SD*	Mode	*SD*	Mean	5%	95%
ω	Uniform [0,1]	-	-	0.70	0.08	0.67	0.52	0.80
ϑ	Uniform [−3,3]	-	-	0.56	0.24	0.60	0.19	1.00
φ	Normal	4.00	1.50	3.42	0.76	3.65	2.18	5.08
$\Delta\bar{q}$	Normal	0.12	0.10	0.12	0.09	0.12	−0.03	0.27
ρ_b	Beta	0.50	0.20	0.37	0.10	0.53	0.30	0.78
ρ_i	Beta	0.50	0.20	0.91	0.04	0.90	0.82	0.98
σ_q	Inv. Gamma	0.10	2.00	3.88	0.19	3.92	3.59	4.24

Note: Marginal likelihood of the model: −1,732.42.
Abbreviation: DSGE, dynamic stochastic general equilibrium.

TABLE 2 Estimation results of the alternative DSGE models

	Smets and Wouters' (2007) model			Smets and Wouters' (2007) model with pre-determined investment expenditures			Gilchrist et al.'s (2009) model		
Parameter	Mode	*SD*	Mean	Mode	*SD*	Mean	Mode	*SD*	Mean
ω	–	–	–	–	–	–	–	–	–
ϑ	–	–	–	–	–	–	–	–	–
φ	3.97	0.92	4.28	5.46	1.08	5.87	2.62	0.75	2.93
$\Delta\bar{q}$	0.12	0.09	0.13	0.12	0.09	0.12	0.12	0.09	0.12
ρ_b	0.84	0.04	0.83	0.82	0.04	0.79	0.29	0.11	0.37
ρ_i	0.76	0.07	0.76	0.76	0.05	0.76	–	–	–
σ_q	4.01	0.20	4.05	4.14	0.21	4.18	3.91	0.19	3.95

Note: Marginal likelihood of the Smets and Wouters (2007) model: −1,745.76. Marginal likelihood of the Smets and Wouters (2007) model with pre-determined investment expenditures: −1,772.40. Marginal likelihood of the Gilchrist et al. (2009) model: −1,758.99.
Abbreviation: DSGE, dynamic stochastic general equilibrium.

is weaker because the estimated mode and mean are 0.76 and 0.72, respectively.

The shock processes with the estimates that differ the most among the models include the risk premium shock and the investment-specific technology shock. The risk premium shock in the model with extrapolative expectations in asset pricing and the Gilchrist et al. (2009) model is less persistent than in the Smets and Wouters (2007) model and the model with pre-determined investment expenditures. The mean ρ_b is 0.53 (and the mode is 0.37) in the extrapolative expectations model, and the mean is 0.37 (and the mode is 0.29) in the Gilchrist et al. (2009) model, whereas the mean is 0.83 (and the mode is 0.84) in the Smets and Wouters (2007) model and the mean is 0.79 (and the mode is 0.82) in the pre-determined investment expenditures model.

The first-order autocorrelation coefficient (ρ_i) in the equation that governs the investment-specific technology shock (see Smets & Wouters, 2007, p. 589) is 0.90 in the model with extrapolative expectations in asset pricing (and the mode is 0.91), whereas the mean (and the mode) is 0.76 in the Smets and Wouters (2007) model and the model with pre-determined investment expenditures. Thus, the investment-specific technology shock is more persistent in the extrapolative expectations model. In the Gilchrist et al. (2009) model, there is no investment-specific technology shock because, according to Justiniano, Primiceri, and Tambalotti (2010), such a shock may be viewed as a disturbance to the financial sector and is, therefore, replaced by shocks associated with the financial accelerator in their model.

We also note that the volatility of the error term in the measurement equation for Tobin's q is slightly lower in the model with extrapolative expectations in asset pricing. The mean σ_q is 3.92 (and the mode is 3.88), the mean is 4.05 (and the mode is 4.01) in the Smets and Wouters (2007) model, the mean is 4.18 (and the mode is 4.14) in the alternative model with pre-determined investment expenditures, and the mean is 3.95 (and the mode is 3.91) in the Gilchrist et al. (2009) model. The high values indicate

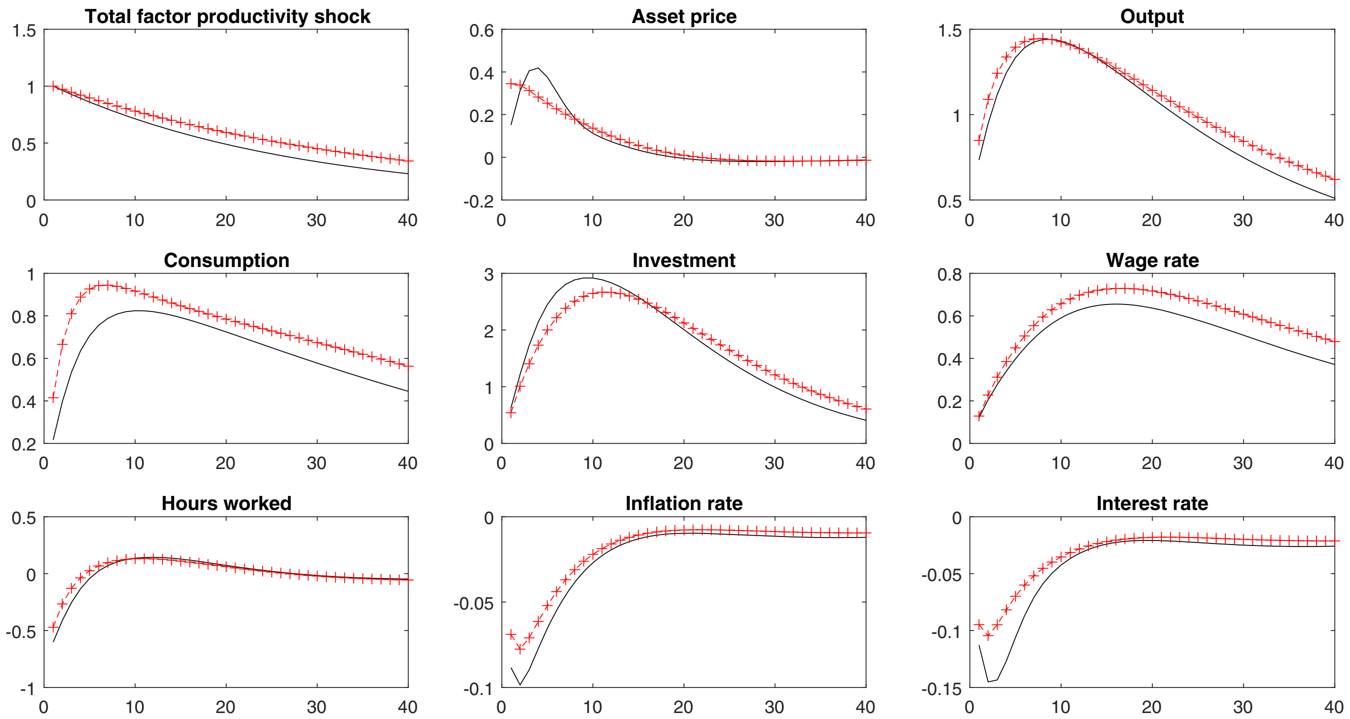

FIGURE 2 Impulse response functions in the context of a total factor productivity shock. Note: – refers to the DSGE model with extrapolative expectations in asset pricing, and +– refers to the DSGE model used by Smets and Wouters (2007). Abbreviation: DSGE, dynamic stochastic general equilibrium [Colour figure can be viewed at wileyonlinelibrary.com]

that all DSGE models show difficulty in generating the observed fluctuations in asset prices.

4.2.4 | How well does the model with extrapolative expectations in asset pricing fit the data?

Because the marginal likelihood of a model provides an indication of the model's out-of-sample prediction performance, it represents a benchmark for comparing different models (see An & Schorfheide, 2007, for an overview of Bayesian techniques for model comparison). Therefore, we compute the marginal likelihood by modified harmonic mean estimation for all four models: the model with extrapolative expectations in asset pricing, the Smets and Wouters (2007) model, an alternative specification of their model with pre-determined investment expenditures, and the Gilchrist et al. (2009) model that involves financial frictions. As a robustness check, we also compute the marginal likelihood by Laplace approximation. However, because those values are nearly identical to the values obtained by the modified harmonic mean estimation, we report only the latter estimates.

The marginal likelihood of the model with extrapolative expectations in asset pricing is −1,732.42, whereas the marginal likelihood of the Smets and Wouters (2007) model is −1,745.76. One might argue that the reason that the model in this article is the preferred model is that 'trend extrapolation in asset pricing' captures the delayed response of investment to the asset price. However, this argument does not appear to hold because the marginal likelihood of the model with pre-determined investment expenditures is −1,772.40, which is the lowest value among all four models. The marginal likelihood of the Gilchrist et al. (2009) model is −1,758.99. These findings suggest that the incorporation of extrapolative expectations in asset pricing into the Smets and Wouters (2007) model improves the fit of the model to the data.

How substantial is this improvement? To answer this question, we compute the Bayes factor (BF) of the model with extrapolative expectations in asset pricing against the three competing models. The motivation for using this approach is that richer models are not necessarily preferred because such models 'have many more hyperparameters and the Bayes factor discriminates against these'. Therefore, BF 'embodies a strong preference for parsimonious modeling' (Fernández-Villaverde & Rubio-Ramírez, 2004, p. 176). Kass and Raftery (1995) proposed that 2logBF values above 10 can be considered very strong evidence in favour of the tested model. Values between 6 and 10 represent strong evidence, values between 2 and 6 represent positive evidence, and values below 2 are 'not worth more than a bare mention' (Kass & Raftery, 1995, p. 777). We refer to this statistic as the KR statistic.

When we consider the model with extrapolative expectations in asset pricing against the Smets and Wouters (2007) model, we obtain 26.68 as the value of the KR statistic. Moreover, when we compare the model with the alternative specification of the Smets and Wouters (2007) model in which investment expenditures are pre-determined, the value of the KR statistic is 79.96. Finally, when we consider the model against the Gilchrist et al. (2009) model with financial frictions, we obtain 53.14 as the value of the KR statistic. Thus, these results support the hypothesis that fluctuations in main U.S. macroeconomic variables are affected by deviations in asset prices from their fundamental values.

4.2.5 | Robustness checks

Our results are confirmed by several robustness checks.[9] First, the adoption of a uniform distribution with only non-negative values for the prior of the strength parameter in asset pricing, $\vartheta \in [0,3]$, does not significantly alter the results. The estimated mean of the weight attached to trend extrapolation (ω) is 0.66, and the estimated mean of the strength in trend extrapolation (ϑ) is 0.60. In fact, the marginal likelihood of the model is slightly better than the model in which the prior for the strength parameter includes negative values (−1,731.83).

Second, following Galí, Smets, and Wouters (2011), we estimate the models for the period from 1966q1 to 2007q4 due to concerns that the nonlinearities induced by the zero lower bound of the federal funds rate could distort the estimates. In this case, the estimated mean weight attached to trend extrapolation (ω) is 0.76, and the estimated mean strength in trend extrapolation (ϑ) is 0.67. When we consider the model against the Smets and Wouters (2007) model for the same period, we obtain 44.82 as the value of the KR statistic. Thus, we obtain even stronger evidence in favour of the model with extrapolative expectations in asset pricing over their model. Moreover, when we compare the extrapolative expectations model with the Smets and Wouters (2007) model with pre-determined investment expenditures and the Gilchrist et al. (2009) model, the value of the KR statistic is 69.78 and 44.60, respectively.

Third, we estimate the models using only the same seven quarterly U.S. macroeconomic time series data as used by Smets and Wouters (2007). Thus, we do not include data regarding the log-difference of Tobin's q. The estimated means of the weight attached to trend extrapolation (ω) and the strength in trend extrapolation (ϑ) are 0.64 and 0.71, respectively. When we consider the model against the Smets and Wouters (2007) model, we obtain 10.44 as the value of the KR statistic. Although the value of this statistic is lower than that obtained when including data regarding asset prices in the estimations, this result represents very strong evidence in favour of the model with extrapolative expectations in asset pricing over their model. Moreover, when we compare the extrapolative expectations model with the Smets and Wouters (2007) model with pre-determined investment expenditures and the Gilchrist et al. (2009) model, the value of the KR statistic is 44.78 and 50.48, respectively.

Finally, we estimate the models using asset price data from the Dow Jones Wilshire 5000 index rather than using Tobin's q. The estimated means of the weight attached to trend extrapolation (ω) and the strength in trend extrapolation (ϑ) are 0.38 and 0.92, respectively. Thus, these estimates differ from the estimates in the baseline model. Nonetheless, we find strong evidence in favour of the model with extrapolative expectations in asset pricing against the Smets and Wouters (2007) model. The value of the KR statistic is 6.70. Moreover, we find very strong evidence in favour of the extrapolative expectations model over the Smets and Wouters (2007) model with pre-determined investment expenditures and the Gilchrist et al. (2009) model as the value of the KR statistic is 40.52 and 62.12, respectively.

4.3 | Simulation of the DSGE models

To more deeply scrutinize the properties of the Smets and Wouters (2007) model and our modification of their model, we simulate the responses to various shocks under the respective means of the posterior distributions. We concentrate on these models because they outperform the alternative variants of the Smets and Wouters (2007) model. The resulting impulse response functions of the following variables were examined: asset price (q_t), output (y_t), consumption (c_t), investment (i_t), real wage (w_t), hours worked (l_t), inflation rate (π_t), and nominal interest rate (r_t). The shocks in the models are a risk premium shock, a fiscal shock, an investment-specific technology shock, a total factor productivity shock, a price mark-up shock, a wage mark-up shock, and a monetary policy shock. Figure 2 describes the impulse response functions in the context of a total factor productivity shock.[10]

In the model with extrapolative expectations in asset pricing, the asset price and investment exhibit more pronounced hump-shaped responses to the shocks. Thus, the impulse response functions suggest that the improved fit to the data as a result of introducing extrapolative expectations in asset pricing is caused by the increased persistence of the aggregate variables. This suggestion is consistent with the observation that the risk premium shock had higher estimated volatility in the extrapolative expectations model.

This result is important because Chari, Kehoe, and McGrattan (2000) found that the standard New Keynesian model exhibits difficulty in accounting for the persistence of aggregate economic variables, such as the inflation rate and output. Here, the persistence in the time series of the log of Tobin's q is quite high; the first-order autocorrelation coefficient is 0.98. The model with extrapolative expectations in asset pricing matches this aspect of the data rather closely; the first-order autocorrelation coefficient is 0.96. The same coefficient in the Smets and Wouters (2007) model is 0.82. Moreover, as shown in the data, the log-difference of Tobin's q has a positive first-order autocorrelation coefficient in the extrapolative expectations model (0.04, compared with 0.17 in the data), whereas the first-order autocorrelation coefficient is negative in the Smets and Wouters (2007) model (−0.01).

5 | CONCLUSIONS

An important topic of current debate is the role of extrapolative expectations in asset pricing in business cycle fluctuations. In their study of eight centuries of economic crises, Reinhart and Rogoff (2008b) found that a run-up in asset prices is common in most crises. The mispricing of assets also appears to have played a large role during the Great Recession that began in the United States in December 2007 and thereafter spread worldwide with devastating effects (Reinhart & Rogoff, 2008a). Therefore, both historical evidence and recent events highlight the need to better understand how asset mispricing affects aggregate economic variables, including an empirical evaluation of its importance. To address this issue, we modified the reference DSGE model proposed by Smets and Wouters (2007) by allowing the asset price in this model to be determined as a weighted average of the asset's fundamental value and its value according to trend extrapolation. Thus, we introduced extrapolative expectations to their model.

First, we found that extrapolative expectations in asset pricing are statistically significant and quantitatively large. Second, we found that extrapolative expectations resulted in a substantial improvement in the DSGE model's fit to quarterly U.S. macroeconomic time series data compared with the reference DSGE model used by Smets and Wouters (2007), an alternative specification of the Smets and Wouters (2007) model in which investment expenditures are pre-determined as in Bernanke et al. (1999), and a model variant involving financial frictions developed by Gilchrist et al. (2009). This finding is promising because this task is more challenging to accomplish than one may initially believe. Third, we found that extrapolative expectations lead to more pronounced hump-shaped responses in the asset price and investment to shocks. In conclusion, extrapolative expectations in asset pricing should be included as an important component of DSGE models aiming to understand business cycle dynamics.

This article is intended to be informative to researchers working on the issue of how monetary policy should respond to asset price misalignments. We provided empirical support for the idea that these misalignments are relevant to business cycle fluctuations. Thus, our parameter estimates could be informative when calibrating new models within this area of research. Researchers who may also benefit from our findings include those employing the agent-based approach in an attempt to overcome the difficulties of asset pricing theories in explaining asset price movements (see the handbook by Hommes & LeBaron, 2018, for a thorough survey of the theoretical, computational and empirical literature on heterogeneous agents models, including agent-based models).

ENDNOTES

[1]Following Laitner and Stolyarov (2003), we compute Tobin's q as the market value of U.S. businesses divided by the replacement cost of the capital stock and obtain a time series of the stock of reproducible capital by adding business inventories and private non-residential fixed assets. The time series of private non-residential fixed assets is the only time series not available at a quarterly frequency. Therefore, we convert the annual time series into a quarterly time series by means of linear interpolation. The data are obtained from the Z.1 Release of the Flow of Funds Accounts of the United States provided by the Federal Reserve.

[2]Adam, Marcet, and Beutel (2017) present a model that not only generates boom-bust cycles in stock prices but also delinks stock prices from fundamentals. The key ingredient in their model is the incorporation of subjective beliefs that display excessive optimism (pessimism) at market peaks (troughs) in an otherwise standard asset-pricing model with rational agents. The model captures many features of actual stock prices.

[3]The cyclical component is obtained by means of the Hodrick-Prescott filter (Cornea-Madeira, 2017).

[4]The few empirically relevant rigidities not included in the model include firm-specific capital (Madeira, 2015 and Woodford, 2005) and firm-specific employment (Madeira, 2014).

[5]Complete descriptions of the DSGE models examined here are provided in the Appendix of this paper, which is available upon request.

[6]Notable examples of macroeconomic models with heterogeneous expectations and trend extrapolative behaviour include those proposed by Branch and McGough (2010), Cornea-Madeira, Hommes, and Massaro (2019), De Grauwe (2012), and Lines and Westerhoff (2012).

The research conducted by Brock and Hommes (1997, 1998) served as the theoretical foundation for this research.

[7]These estimates are consistent with the results reported by Hommes, Sonnemans, Tuinstra, and van de Velden (2005), who found that most traders follow a trend extrapolation strategy of the same type as that considered here and obtained values for the strength in trend extrapolation between 0.36 and 1.17.

[8]The complete estimation results, including the results of various robustness checks, are provided in the Appendix of this paper, which is available upon request.

[9]See the Appendix of this paper, which is available upon request.

[10]To save space, the impulse response functions associated with the other six shocks in the models are provided only in the Appendix of this paper, which is available upon request. We choose to display the impulse response function to the total factor productivity shock because it plays an important role in explaining cyclical fluctuations among both Real Business Cycle and New Keynesian proponents.

DATA AVAILABILITY STATEMENT

The data set is available upon request from the authors.

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