Influence of Spurious Modes on the Efficiency of Piezoelectric Transformers: a Sensitivity Analysis

Jack Forrester, Jonathan N. Davidson, Martin P. Foster and David A. Stone[1]

Abstract—An analysis of the efficiency degradation resulting from spurious modes in piezoelectric transformers (PTs) is presented. Circuit analysis is performed on PT equivalent circuit models with both a single resonant branch and with two resonant branches (spurious mode included), allowing the efficiency degradation from a spurious mode to be analysed. Multiple sensitivity analyses were then performed, highlighting frequency difference (between modes) and characteristic impedance of the spurious resonant circuit $\left(\sqrt{L_2/C_2}\right)$ as critical design parameters for minimising efficiency degradation from spurious modes. These two parameters are further analysed to determine optimum design conditions to ensure minimal efficiency degradation. Results of this analysis provide a method of estimating whether a spurious mode will degrade efficiency and consequently provides a method for improving PT designs.

***Index Terms*—Piezoelectric devices, sensitivity, converters, transformers**

I. INTRODUCTION

Piezoelectric transformers (PTs) are an alternative to traditional magnetic transformers. PTs are an assembly of one or more piezoelectric elements and make use of both the direct and converse piezoelectric effect to transform electrical energy. High power density, low EMI and the absence of magnetic fields are some of the advantages PTs offer over their magnetic counterparts. PTs are resonant devices and so are ideal for use in resonant converter and related power electronics applications [1]–[5]. One of the interesting properties of PTs is the large quality factor they exhibit—this typically leads to minimal losses and so, as a result, PT and PT-based converters can achieve high electrical efficiency [6].

PTs exhibit and consequently can be operated with a wide variety of different vibration shapes (modes), as shown in Fig. 1, with the longitudinal, planar and thickness modes being most common. Each PT can exhibit many of the possible vibration modes, each occurring at a different frequency. However, each PT topology has an optimum vibration mode that will allow optimum energy transfer through the device. The optimum vibration mode for a PT will typically be the mode that exhibits the highest electromechanical coupling and has the lowest losses, as can be observed by the 120kHz mode occurring in Fig. 1. Therefore, it is also the mode that exhibits the highest output power and efficiency. As a result, designers will often optimise a PT design exclusively for optimum mode performance.

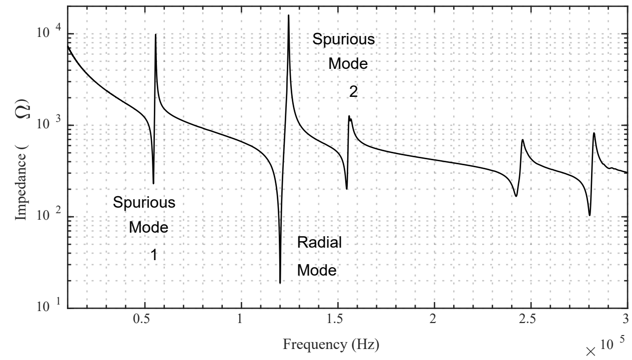

Fig. 1 - Impedance spectrum of the T1-13 radial mode Transoner PT, radial mode is exhibited at 120kHz

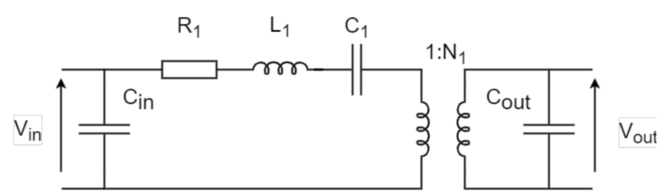

Fig. 2 - Mason equivalent circuit model of a PT

The Mason equivalent circuit model (Fig. 2) can be used to model the electrical behaviour of the optimum mode, of any topology or type of PT and, importantly, it allows the use of circuit analysis techniques on PTs. It should be noted that the model used here is a simplified Mason equivalent circuit; the full model contains two ideal transformers at the input and output. The full model is not necessary for the analysis in this paper. The individual components of the equivalent circuit are linked to the physical properties of the PT. Output and input capacitances model the parallel electrodes of the PT and the

[1] This work was supported by the Engineering and Physical Sciences Research Council (EP/P015859/1)

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ideal transformer models the transfer of energy from the electrical input to the mechanical vibration and, finally, to the electrical output. The RLC section models the resonant behaviour of the PT and, whereas the input and output capacitances exist electrically, the RLC section models purely mechanical behaviour.

Forrester *et al.,* [7] described a process for designing a radial mode transoner PT. Initially, the turns ratio is selected based on the input voltage and desired output voltage, giving the number of layers. Then, layer thicknesses are optimized to ensure zero voltage switching is achieved when used as part of a half-bridge-based inductor-less resonant converter [8]. Finally, the radius and total thickness are designed given the available disc sizes, desired resonant frequency and required output power [9]. The resulting transformer is then suitably designed for operation at the optimum mode.

However, the remaining non-optimum (spurious) vibration modes can interfere with the optimum mode of the PT, reducing the lifetime of the device and its energy transfer efficiency [10]. Therefore, one of the challenges PT designers must contend with is designing the PT in such a way as to limit the influence of these modes, whilst not affecting optimum mode performance.

Researchers take a variety of approaches to avoid these interactions. One of the most popular methods for avoiding interference from spurious modes is by careful selection of the piezoelectric material used. Ohnishi, *et al.*, [11] created a thickness mode PT using lead titanate. A thickness mode PT would typically have to contend with spurious longitudinal vibrations influencing the performance of the PT. One of the key features of lead titanate is that it exhibits a K_t coupling factor much greater than the K_{31} coupling factor. The coupling factor describes the ratio of stored mechanical energy to supplied electric energy [12]. Therefore, in a PT made from this material, spurious longitudinal vibrations (those in the 31 direction) transfer less energy compared to thickness (t-direction) vibrations. Similarly, Prieto, *et al.*, [13] also proposed using lead titanate for reducing the effect of spurious modes. However, they found that the resulting devices had lower electromechanical coupling in the optimum vibration mode than those made from traditional lead zirconium titanate (PZT) materials and therefore had reduced power density. To overcome the limitation, Prieto, *et al.*, proposed altering the shape of the PT, including adding a hole in the centre of the device to reduce unwanted vibrations for PZT-based devices; however, their method requires careful design and appropriate mounting to avoid damping the vibration. Sanz, *et al.*, [14] used interleaved electrodes to create a symmetrically-designed Rosen PT. Although this method shows some good results, it is only applicable to certain topologies and, even then, reduces the design space for those topologies.

While these efforts have reduced the impact of spurious modes, they require sacrificing device performance and, while they reduce the overall effect of spurious modes, these modes are not eliminated. Our approach in this paper, rather than attempting to eliminate spurious modes, is to design the PT in such a way that spurious modes have negligible effect on performance, without affecting optimum mode performance. The analysis will be aimed at giving a PT designer the information required, given some knowledge of the equivalent circuit of the PT, to determine whether a PT will exhibit minimal efficiency loss from spurious modes. The analysis will identify which design element or feature needs to change for the PT to achieve minimal interference from spurious modes.

For this analysis, the Mason equivalent circuit model of the PT is used, allowing the results of the analysis to be applicable to all PT topologies. This work will form part of a wider framework that explores the optimum geometric/material design of specific PT topologies for minimal spurious mode interaction, based on the results of the analysis presented here. Initially, the efficiency of the idealised single-mode PT is analysed and the elements of its equivalent circuit that influence this efficiency are determined. The equivalent circuit model of the PT is then extended to include another resonant element (i.e. a spurious mode) and the modified efficiency of this equivalent circuit is examined. This allows the efficiency degradation from the interaction between the optimum and spurious modes, when the PT is driven at the optimum (operating) mode, to be analysed through multiple sensitivity analyses. Using the results of this analysis, the key factors that affect the efficiency degradation are highlighted. Further, quantitative restrictions on the design space, in terms of equivalent circuit parameter values, are proposed. Observing this restriction will result in minimal degradation in efficiency for each specific PT. Finally, the results are verified by experimental measurements.

II. Impact of PT Design on Efficiency Loss from Spurious Modes

First, increased frequency separation between optimum and spurious modes will be analysed for its impact on efficiency, using a COMSOL simulation. Then, analysis will be performed using AC circuit analysis techniques on simplified and extended Mason equivalent circuit models. Using the mathematical models generated for efficiency, an expression for the efficiency degradation from spurious modes can be found, allowing sensitivity analyses to be performed, highlighting key parameters for minimising efficiency degradation.

A. Spurious mode proximity to optimum mode

This section analyses the impact proximity of modes has on the overall efficiency of the PT. Maximising the frequency separation between operating and spurious modes is the most obvious method for minimizing interaction between modes. Spurious mode frequencies are generally influenced by changing the dimensions of a device; for example, controlling the radius/thickness ratio of a radial mode Transoner PT controls the spurious mode resonant frequencies. Intuitively, and correctly, the greater the separation in the frequency of the optimum and the spurious modes, the lesser the effect the spurious mode has on efficiency.

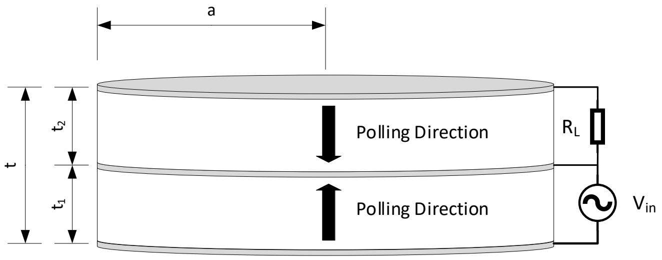


Fig. 3 – Two-layer radial mode Transoner PT

A COMSOL simulation was used to validate this assumption. A 2D axisymmetric, two-layer radial mode Transoner PT (Fig. 3) model is created in COMSOL. The radius of the PT is varied from 5mm-15mm and the thickness of the PT (t) is varied from 1mm-5mm. The two layers are equal in thickness ($t_1 = t_2$), the PTs are made from PZT4 and the Q-factor of the optimum mode is fixed at 150. For each PT studied the efficiency of the PT is determined at the optimum mode resonant frequency. A sinusoidal input voltage and a matched load are used. The resonant frequency of the closest spurious mode is also extracted. The resulting efficiencies are plotted against the percentage difference in frequency between the ideal and spurious modes. The results of this are shown in Fig. 4.

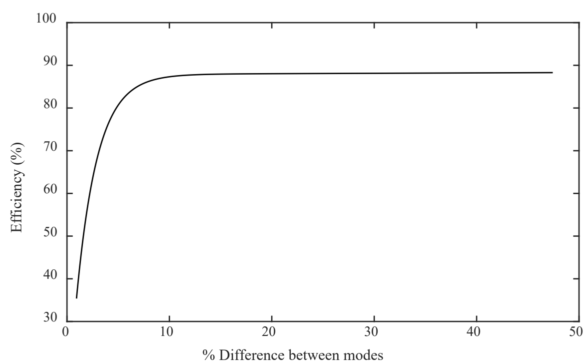


Fig. 4 – Exponential line of best fit for the simulated efficiency of multiple PT models from COMSOL against percentage difference between optimum and nearest spurious mode resonant frequencies

With increased frequency difference, higher efficiencies tend to be achieved (but there are exceptions). However, achieving a large frequency difference between optimum and spurious modes requires careful control of the PT dimensions and therefore limits the PT design space. As the frequency difference between the ideal and a spurious mode increases, the ideal mode might move to a frequency closer to that of another spurious mode because a PT can exhibit multiple spurious modes. Therefore, designing the PT to exhibit very large frequency difference between the optimum and one particular spurious mode is unlikely to result in an optimised design.

It is also likely that other design criteria will affect the frequency difference required to achieve a desired level of efficiency. Therefore, these other elements will be analysed.

B. Efficiency modelling

To analyse these design elements, the Mason equivalent circuit model will be used. First, the single branch Mason equivalent circuit model will be used to analyse the efficiency of an idealised single-mode PT (η_1)—this is an imagined PT with all spurious modes deleted. The Mason model will then be extended to include multiple resonant branches, allowing both operating and spurious modes to be modelled together. This allows the efficiency of a PT, driven at the optimum mode resonant frequency and including the effects of a spurious mode, to be analysed (η_2). Finally, a metric describing the loss in efficiency originating from the spurious mode ($\Delta\eta$) will be found.

1) Efficiency in the single branch model (η_1)

It is well-known that PTs can achieve high efficiency due to the high Q-factor exhibited by the hard piezoelectric materials employed in their construction. The efficiency ratio, η_1, of a device is defined in (1).

$$\eta_1(\omega) = \frac{P_{\mathrm{out}}(\omega)}{P_{\mathrm{in}}(\omega)} \tag{1}$$

By generating equations for the voltage and currents in the device, equations for the input and output power can be found. The input and output currents can be written, in the frequency domain, as

$$I_{\mathrm{in}}(\omega) = \frac{V_{\mathrm{in}} - \dfrac{V_{\mathrm{out}}(\omega)}{N_1}}{R_1 + \mathrm{j}X_1} + \mathrm{j}\omega C_{\mathrm{in}} V_{\mathrm{in}} \tag{2}$$

$$I_{\mathrm{out}}(\omega) = \frac{V_{\mathrm{out}}(\omega)}{R_{\mathrm{L}}} \tag{3}$$

where X_1 is the reactance of the LC branch

$$X_1 = \omega L_1 - \frac{1}{\omega C_1} \tag{4}$$

Which allows V_{out} to be calculated as

$$V_{\mathrm{out}}(\omega) = \frac{N_1 V_{\mathrm{in}}}{1 + (R_1 + \mathrm{j}X_1) N_1^2 \left(\dfrac{1}{R_{\mathrm{L}}} - \dfrac{\mathrm{j}}{X_{\mathrm{out}}}\right)} \tag{5}$$

where X_{out} is the reactance of capacitor C_{out}. Power in and out can then be calculated. Note that the input capacitor dissipates no power. Together with the output power, an expression for efficiency can be derived. Producing the full solution is trivial with computer algebra packages such as Maple but excessively long to present and is therefore omitted.

$$\begin{aligned} P_{\mathrm{in}}(\omega) &= \Re(V_{\mathrm{in}}(\omega) I_{\mathrm{in}}(\omega)^*) \\ &= \Re\left(\frac{V_{\mathrm{in}}^2 - \dfrac{V_{\mathrm{in}} V_{\mathrm{out}}(\omega)}{N_1}}{R_1 + \mathrm{j}X_1}\right) \end{aligned} \tag{6}$$

$$P_{\mathrm{out}}(\omega) = \frac{|V_{\mathrm{out}}(\omega)|^2}{R_L} \tag{7}$$

By combining all these equations, it can be shown that

$$\eta_1(\omega) = \frac{R_L}{R_1\left(\omega^2 {C_{\text{out}}}^2 {R_L}^2 + 1\right)N_1^2 + R_L} \quad (8)$$

Equation (8) suggests there are several equivalent circuit elements that ought to be carefully selected to achieve high efficiency. The damping resistance (R_1) has a significant effect on efficiency (from heat dissipation modelled as I^2R losses). However, R_1 is multiplied by $\omega^2 {C_{\text{out}}}^2 {R_L}^2$ and N_1^2, therefore higher efficiency operation is achieved for low values of all these parameters. However, C_{out} should be carefully designed to ensure zero-voltage switching (ZVS) can be achieved, as shown by Foster, *et al.*, and N will be chosen depending on the required application of the PT [8]. Due to the implications of minimising C_{out} and N, it is advantageous to ensure R_1 is minimised as this has no detrimental effects.

To ensure optimal efficiency, the load is often matched to the output capacitance as shown in (9) [15].

$$R_L = \frac{1}{\omega C_{\text{out}}} \quad (9)$$

And hence (8) can be simplified to

$$\eta_1(\omega) = \frac{1}{2N_1^2 \omega C_{out} R_1 + 1} \quad (10)$$

To analyse the effect of using an unmatched load on the efficiency of the PT, the efficiency of the single branch model can be simulated under various load conditions. Equation (8) was used with the equivalent circuit parameters for the optimum mode of the T1-13 Transoner PT defined in Table 1, and assuming the PT was driven at the series resonant frequency. The resulting efficiency against load curve is shown in Fig. 5.

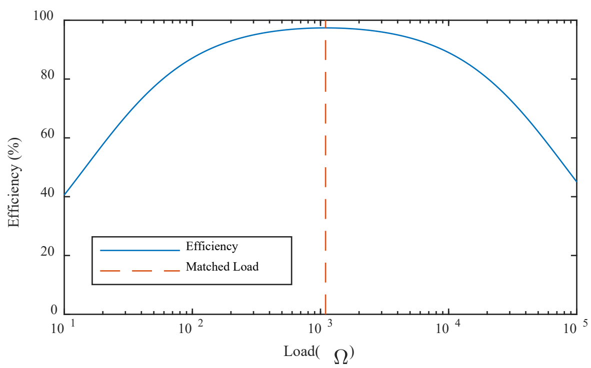


Fig. 5 – Efficiency of the single branch equivalent circuit model of the T1-13 radial mode PT against load

Table 1 – T1-13 Radial mode PT equivalent circuit parameters, including optimal and closest occurring spurious mode

R_1	R_2	L_1	L_2	C_1
18.9 Ω	233.7 Ω	12.7 mH	95.2 mH	138.7 pF
C_2	N_1	N_2	C_{in}	C_{out}
89.8 pF	0.88	1.32	1.90 nF	1.21 nF

As can be seen in Fig. 5, η_1 peaks when the load is equal to the matched load. Very small or very large loads should be avoided as this gives low efficiency. When R_L is small, the output voltage is low and therefore the voltage across the resonant circuit is maximised, increasing I^2R losses. When R_L is large, the output current is much lower than the current through the resonant circuit, again lowering the efficiency. Also from (10), it can be seen that the matched load is dependent on frequency, which adds to the complexity of designing and controlling PT-based power converters.

2) Multiple Branch Model

The model shown in Fig. 2 is a simplification of the full equivalent circuit and, as a result, is only applicable when the PT is driven at the resonant frequency of the optimum mode and when all other vibration modes have negligible effect. The full equivalent circuit model of a PT contains many RLC branches which model each of the different vibration modes. Lin [16] showed that each of these RLC branches should be connected to the output node of the PT through an ideal transformer to correctly model the impedance and performance of a PT, as shown in Fig. 6. Although the PT contains many vibration modes, most of these have high impedance at the operating frequency and pass negligible current, and thus have negligible influence on the performance of the PT. They can therefore be omitted from the model. However, even when considering a small number of spurious modes, the analysis on the efficiency of the PT is difficult. As a result, the investigation in this paper will consider only two modes interacting: the optimum mode and a single spurious mode.

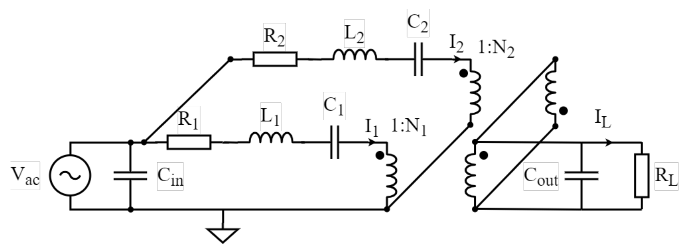


Fig. 6 - Extended Mason equivalent circuit model of a PT

a) Efficiency in the multiple branch model (η_2)

An understanding of the efficiency loss mechanism in the multiple branch model is required before analysing the impact of a spurious mode on efficiency. To begin, a frequency domain analysis was performed using the equivalent circuit model in Fig. 6 and the equivalent circuit parameters of a T1-13 radial mode Transoner PT (Table 1). The efficiency suggested by the model was examined and shown in Fig. 7.

As can be observed in Fig. 7, while the efficiency is usually high around the optimum mode, there is a region of significant inefficiency near the spurious mode resonant frequency. This effect is due to the interaction between the two resonant modes. To understand the reason for this interaction, the current through the resonant branches, when the PT is operated at the minimum efficiency frequency (~43kHz in Fig. 7), is analysed and is shown in Fig. 8, where I_1, I_2 and I_L are the currents through the optimum mode branch, spurious mode branch and the load respectively.

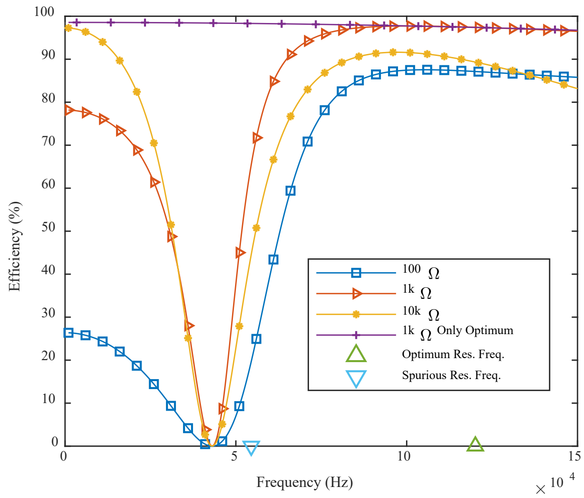


Fig. 7 - Efficiency of a two-branch equivalent circuit model of the T1-13 PT against operating frequency under various load conditions, single branch efficiency included for 1kΩ load for comparison

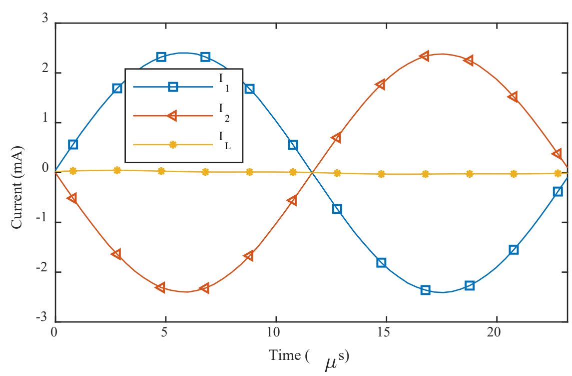


Fig. 8 - Current flowing through the optimum mode, spurious mode and load at 43kHz

The resonant branch currents are near-equal in magnitude and in antiphase and when combined through the transformer, this results in negligible output current. Energy loss is incurred through damping: there is still notable input power. There is a large region of frequencies where this interaction occurs and degrades the efficiency. In this region, the currents suffer destructive interference. It should be noted that other factors, such as the load, have an impact on the range of frequencies that are affected by the efficiency loss as demonstrated by the different curve shapes shown in Fig. 7.

To minimise the effect of spurious modes on the efficiency of a PT, the condition where there is significant interference should be avoided or should occur at frequencies away from the optimum mode, where the power transfer is already negligible. As shown in Fig. 4, increasing the frequency difference between modes helps to ensure high efficiency because the minimum efficiency frequency would then occur away from the optimum mode. Also, the range of frequencies at which destructive interference occurs should be minimised.

b) Efficiency modelling for two modes

To determine the best method of minimising spurious mode influence on efficiency, it is first necessary to derive an expression of the efficiency of a PT containing an ideal and a spurious vibration mode.

To determine the efficiency of the circuit in Fig. 6, when operated at the optimum mode resonant frequency, an equation for the input and output power is derived in terms of the equivalent circuit component values. The power and efficiency can be derived by considering the current in each resonant branch and the output current in terms of input and output voltages.

$$I_1(\omega) = \frac{V_{\text{in}} - \dfrac{V_{\text{out}}(\omega)}{N_1}}{R_1 + \mathrm{j}X_1} \tag{11}$$

$$I_2(\omega) = \frac{V_{\text{in}} + \dfrac{V_{\text{out}}(\omega)}{N_2}}{R_2 + \mathrm{j}X_2} \tag{12}$$

$$I_{\text{in}}(\omega) = I_1 + I_2 + j\omega C_{\text{in}} V_{\text{in}} \tag{13}$$

$$I_{\text{out}}(\omega) = \frac{V_{\text{out}}(\omega)}{R_{\text{L}}} \tag{14}$$

where X_1 and X_2 are the reactances of the LC combinations of branch 1 and 2, respectively. That is to say

$$X_n = \omega L_n - \frac{1}{\omega C_n} \tag{15}$$

Assuming only these two branches are significant, (11), (12), (14) can be re-arranged to give an expression for V_{out}

$$V_{\text{out}}(\omega) = \frac{\dfrac{V_{\text{in}}}{(R_1 + \mathrm{j}X_1)N_1} + \dfrac{V_{\text{in}}}{(R_2 + \mathrm{j}X_2)N_2}}{\dfrac{1}{N_1^2(R_1 + \mathrm{j}X_1)} + \dfrac{1}{N_2^2(R_2 + \mathrm{j}X_2)} + \dfrac{1}{R_L} - \dfrac{\mathrm{j}}{X_{\text{out}}}} \tag{16}$$

$$P_{\text{out}}(\omega) = \frac{|V_{\text{out}}(\omega)|^2}{R_L} \tag{17}$$

$$\eta_2(\omega) = \frac{P_{\text{out}}(\omega)}{P_{\text{in}}(\omega)} = \frac{P_{\text{out}}(\omega)}{\Re(V_{\text{in}}(\omega) I_{\text{in}}(\omega)^*)} \tag{18}$$

A closed-form expression for η_2 can be easily derived by solving (11), (12), (13), (16), (17), (18), but it is too long to print here. Instead, we will write η_2 in function format.

$$\eta_2 = f(L_1, L_2, C_1, C_2, R_1, R_2, N_1, N_2, C_{\text{out}}, \omega, R_L) \tag{19}$$

3) Efficiency degradation (Δη)

It is helpful to introduce the concept of efficiency degradation, $\Delta\eta$. This parameter describes the proportional loss in efficiency due to a proximate spurious mode. It neglects the reduction of efficiency that can occur from losses in the optimum resonant mode regardless of spurious-mode interactions. We define it thus

$$\Delta\eta = 1 - \frac{\eta_2}{\eta_1} \quad (20)$$

where η_1 is the efficiency calculated from the single-branch model, and η_2 is the efficiency calculated from the two-branch model. It is worth noting that $\Delta\eta$, like η_2, is a function of L_1, L_2, C_1, C_2, R_1, R_2, N_1, N_2, C_{out}, ω and R_L. It is useful to perform a series of substitutions so that efficiency is written in terms of resonant frequencies.

$$\omega = \omega_1;\ \omega_1 = \frac{1}{\sqrt{L_1 C_1}};\ \omega_2 = \frac{1}{\sqrt{L_2 C_2}}; \\ \zeta_1 = \sqrt{\frac{L_1}{C_1}};\ \zeta_2 = \sqrt{\frac{L_2}{C_2}} \quad (21)$$

where ω_1 and ω_2 are the resonant frequencies of the optimum and spurious mode, respectively; and ζ_1 and ζ_2 are the magnitude of the reactance of L_1 and L_2 (or C_1 and C_2), respectively, at the relevant resonant frequency. These substitutions eliminate L_1, L_2, C_1 and C_2 from (20) and also eliminates, ζ_1, therefore simplifying the (still long) expression.

C. Sensitivity analysis

Owing to the complexity of the efficiency degradation mechanism in a PT, it is not easy to determine which equivalent circuit parameters have a significant effect. We therefore perform a sensitivity analysis to isolate the relevant parameters.

1) One factor at a time

The 'one-factor-at-a-time' is the simplest type of sensitivity analysis to perform and to understand. The results of this analysis will help to show the general trends that each of the circuit parameters have on efficiency degradation. Whilst the results of this analysis are focused around one specific PT (T1-13 in this case), similar results would be achieved for other PT topologies, noting the similarity between equivalent parameters.

The analysis is performed with the exemplar PT parameter set in Table 2. However, the T1-13 PT has low efficiency degradation owing to the significant frequency difference between its optimum (radial) mode and any spurious mode. However, as described in [7] spurious modes are typically not considered in the design process and therefore, an optimised frequency difference is not likely to be true of initial designs. Subsequent sensitivity analyses will therefore be performed on a modified set of parameters, based on the T1-13, which brings ω_2 closer to ω_1 whist maintaining all other parameters. This PT model, which we will term T1*, has the parameters given in Table 2. The new design improves the viewability of the sensitivity analysis and represents a realistic early PT design. At matched load and optimum resonant frequency, this modification increases the efficiency degradation from -0.04% to 1.46% (for single-branch efficiency, η_1, of 97.4%).

To perform the sensitivity analysis, a range of parameters to study is required. Each of the parameters will be varied through the range $x_{\text{n}}/100 \le x \le 100x_{\text{n}}$ where x is the parameter and x_{n} is its nominal value from Table 2. This allows the analysis to cover a wide range of potential devices. The only exception is the turns ratios (N_1, N_2) which will only be varied in the range $N_{\text{n}}/10 \le N \le 10N_{\text{n}}$ because the turns ratio typically correlates linearly to the geometry, which is physically constrained.

a) Initial study on exemplar PT, T1-13

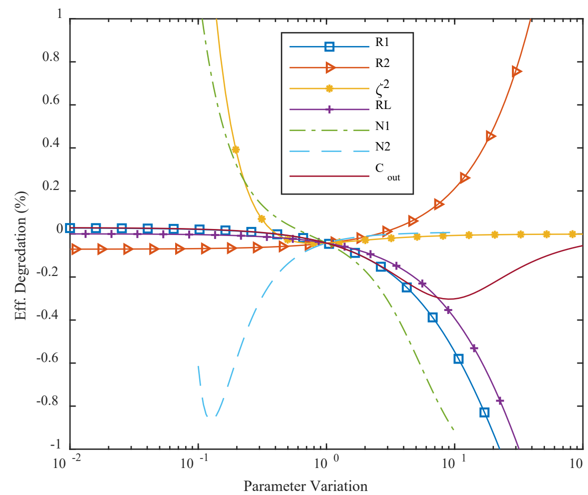


Fig. 9 - Sensitivity analysis on the influence of parameter value on the efficiency degradation of a nominal T1-13 PT

*b) Study on modified PT, T1**

Each curve in Fig. 9 demonstrates the effect of varying a single parameter while keeping the other parameters at their nominal value. As shown in Fig. 9, ζ_2 has the greatest effect on efficiency degradation. As ζ_2 decreases from its nominal value, the efficiency degradation rapidly increases. Each of the other parameters have minimal effect on the efficiency degradation across its full range of variation. However, as discussed above, the overall low efficiency degradation is unrealistic for early designs (such as those generated using [7]), due to the large frequency difference between modes. Nevertheless, from the initial analysis, we can see that ζ_2 is a vital design parameter in ensuring low efficiency degradation.

Table 2 – Equivalent circuit parameters for the T1-13 PT and the adjusted T1-13 PT, termed the T1*

	R_1	R_2	$\omega_1/2\pi$	$\omega_2/2\pi$	ζ_1	ζ_2	N_1	N_2	C_{in}	C_{out}
T1-13	18.9 Ω	233.7 Ω	120.1 kHz	54.4 kHz	9556.3 Ω	32568 Ω	0.88	1.32	1.90 nF	1.21 nF
T1*	18.9 Ω	233.7 Ω	120.1 kHz	108.3 kHz	9556.3 Ω	32568 Ω	0.88	1.32	1.90 nF	1.21 nF

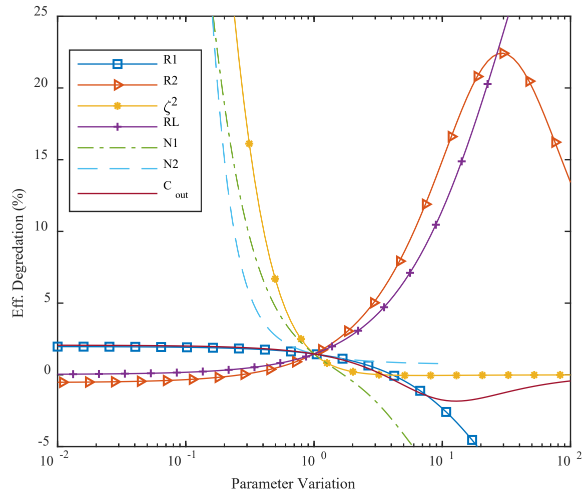


Fig. 10 - Sensitivity analysis on the influence of parameter value on the efficiency degradation of the T1* PT

Fig. 10 shows the same analysis performed on the modified model, T1*. Comparing Fig. 10 to Fig. 9 shows that each of the parameters has an increased effect on the efficiency degradation when the spurious mode is closer to the optimum mode. ζ_2 remains the parameter with the largest effect on efficiency degradation, again highlighting its importance in design. N_2, R_L N_1 and R_2 all have significant effects on the efficiency degradation, with proximity of the modes determining how impactful the parameter is. For this case study, R_1 and C_{out} have negligible effect on efficiency degradation compared to the other parameters. It should be noted that negative efficiency degradation, means an efficiency increase caused by the spurious mode. However, using (8) it can be concluded that increasing these parameters decreases optimum mode efficiency and so should typically be avoided.

The effect R_2 has on the efficiency degradation is interesting. Initially, as one would expect, increasing R_2 increases the efficiency degradation due to higher I^2R losses. However, beyond a certain value (30 times nominal), the efficiency degradation peaks, thereafter falling with increased R_2. This effect is due to reduced power flow in the spurious mode, therefore, if $R_2 = \infty$, there is no power through the spurious mode and so no efficiency degradation. However, as damping in PTs is largely due to mechanical losses, common to all modes, higher damping in the spurious mode typically results in higher damping in the optimum mode [17]. Therefore, while increasing the damping (i.e. R_2) in the PT may minimise efficiency degradation, it will typically cause a decrease in the optimum mode efficiency of the PT (due to increases in R_1) and thus degrade performance. However, if a designer could increase the damping in a specific mode without affecting the optimum mode, this would be a good option for improving the efficiency degradation in the device.

2) Parametric Sweep

While studying variations of a single parameter gives useful insight into how the efficiency degradation of a specific PT is affected by each parameter in isolation, the results of a 'one-factor-at-a-time' analysis are dependent on the device studied. To study multiple parameters simultaneously, a parametric sweep-based sensitivity analysis is performed. For this analysis, all parameters are varied in a 7-dimentional space with each dimension having the same range of variation as the previous analysis, which was derived from the T1-13 PT (Table 2).

To visualise the results, each parameter (dimension) is varied in turn and the 6-dimensional space extracted. We then define a good device, as one which achieves an efficiency degradation of less than 5%. The proportion of good devices in this space is calculated. Graphical plots are then produced showing the trend in proportion of good devices with the parameter at hand.

To further describe this type of analysis, a simplified scenario is shown with only 2 input variables, ζ_2 and R_2. The efficiency degradation will be analysed for each variation of both parameters. The T1* PT will be used for the remaining parameters, as given in Table 2. Firstly, this data can be visualised on a 3D graph, as only 2 parameters are considered. This is shown in Fig. 11.

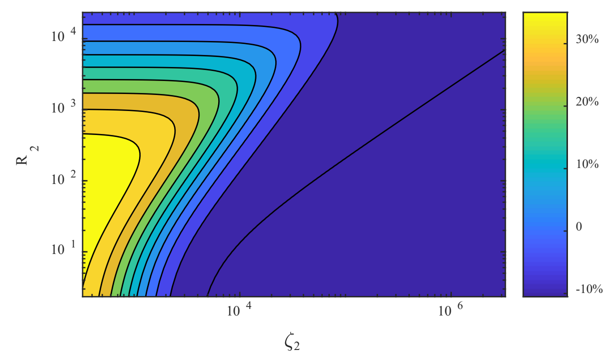


Fig. 11 - Contour plot of efficiency degradation with changes in ζ_2 and R_2

Although, with greater than 2 input variables, such as in our analysis, it is impractical to visualise the data on a single graph.

To overcome this, first, for each variation of a parameter's value a probability density plot can be produced, showing the likelihood of achieving various levels of efficiency degradation. An example plot is shown for $R_2 = 5.4\Omega$.

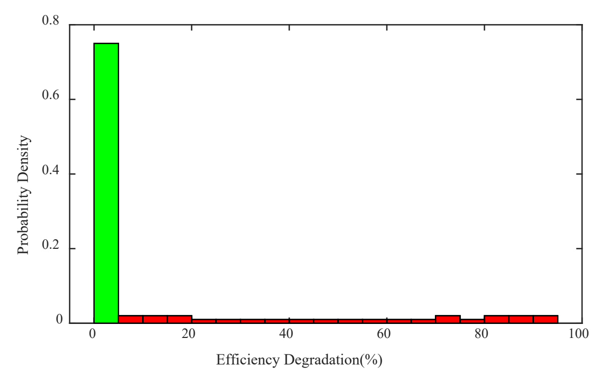


Fig. 12 – Probability density plot for $R_2 = 5.4\Omega$, the green bar highlighting the desired (<5% $\Delta\eta$) specification

From Fig. 12, we calculate that 75% of devices are within the specification (<5% $\Delta\eta$) when $R_2 = 5.4\Omega$. This process is then repeated for all values of said parameter, and a graph of percentage of devices in specification against the variation in the parameter value can be produced, as shown in Fig. 13. This process is then repeated for the remaining parameter(s), as also shown in Fig. 13.

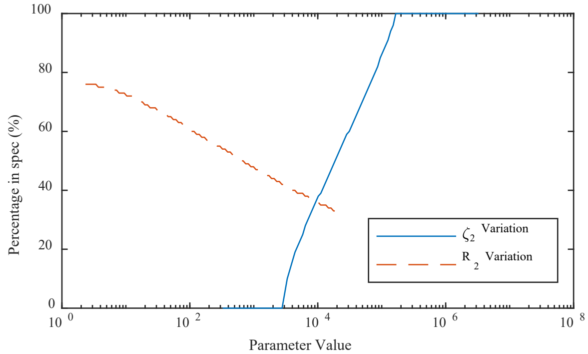


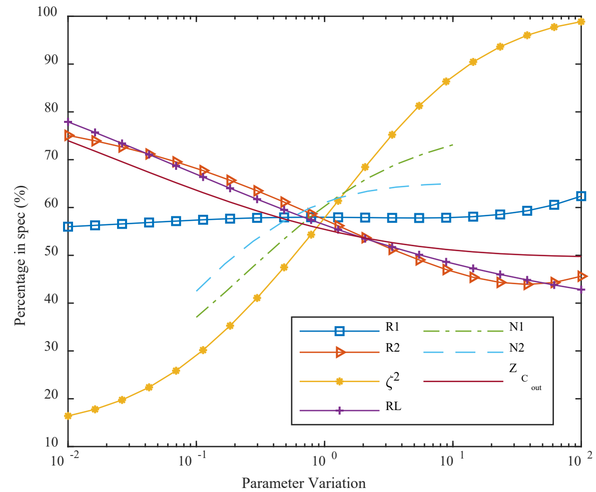
Fig. 13 - Percentage of datapoints in specification against variation in each parameter's values

From Fig. 13, we can see a similar general trend as was visable in the 3D plot in Fig. 11, with a positive and a negative correlation observed for ζ_2 and R_2 respectively. It should be noted that more subtle trends are lost in this analysis, such as $\Delta\eta$ improving with very large R_2 values.

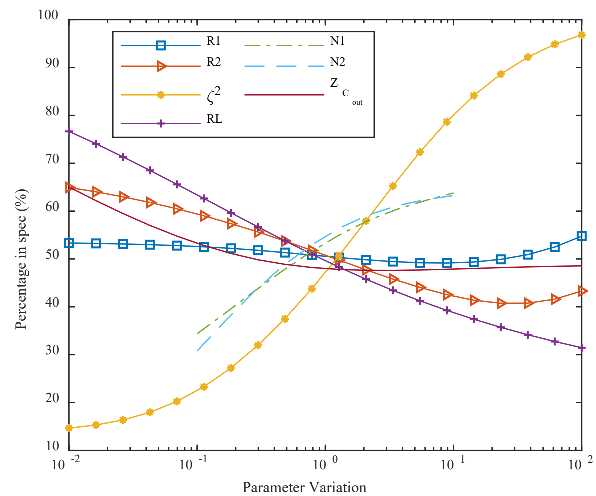
To perform this analysis on a PT, several changes need to be made from the 'one-factor-at-a-time' analysis, although, the range of parameter variation will be kept the same. Firstly, as there is no longer a nominal device, it is important that all results are indepedent of the chosen optimum mode resonant frequency. Therefore, C_{out} will be considered as an impedance $\left(X_{C_{\text{out}}} = -1/\omega C_{\text{out}}\right)$, to ensure matched load conditions don't affect the results. In this case, $X_{C_{\text{out}}}$ will be varied from 13.2 Ω to 132 kΩ. Secondly, the spurious mode resonant frequency is set as $\omega_2 = 0.9\omega_1$ and $\omega_2 = 1.1\omega_1$. The close proximity of the modes to the optimum mode, will accentuate the affect each parameter has and, using multiple spurious resonant frequencies, highlights the differences between modes occurring at frequencies either side of the optimum mode resonance. The results of the parametric sweep sensitivity analysis are presented in Fig. 14.

Fig. 14 shows that all parameters have some influence on the proportion of good devices and hence the efficiency degradation. However, ζ_2 has the greatest effect on the number of good devices, with a change of up to 82.5 percentage points (from worst to best case ζ_2 value) in the percentage of devices exhibiting <5% efficiency degradation. This analysis also highlights the impact the load resistor, R_L, has on the proportion of acceptable devices. A change of up to 45.2 percentage points is observed across the parameter range.

In a similar way to R_L, increases in R_2 and C_{out} cause a decrease in the proportion of good devices. Conversely, N_1 and N_2 have a similar impact on the proportion of good devices, each with an increase of up to 36 percentage points across the parameter range. R_1 has a negligible effect when compared to the other parameters.



a)



b)

Fig. 14 – Parametric sweep-based sensitivity analysis. (a) $\omega_2 = 0.9\omega_1$ (b) $\omega_2 = 1.1\omega_1$

III. PT DESIGN FOR MINIMAL EFFICIENCY DEGRADATION

From the previous analyses, a few parameters have been highlighted as vital for low efficiency degradation. Most notably these include, frequency difference and ζ_2. Previous analyses have also shown that both variables have a negative correlation with efficiency degradation. From the parametric sweep sensitivity analysis, it was also found that for ζ_2 values greater than ~18MΩ, all PTs have minimal efficiency degradation, irrespective of the other parameter values (given they are in the range used for this analysis). This is useful from a design perspective, as efficiency loss from spurious modes can be negated by careful design of only a single parameter, especially as ζ_2 has no impact on optimum mode performance. However, the critical value of ζ_2 (that required 100% of devices tested to exhibit <5% $\Delta\eta$) from the parametric sweep sensitivity analysis is at least two orders of magnitude greater than that exhibited by the T1-13 PT, and such a large value may be unachievable.

A. Typical ζ_2 values achieved by PTs

The following section will analyse the typical ζ_2 values that are typically achieved in various PTs topologies. This will help to give some context to ζ_2 values that are required to avoid minimal spurious mode interaction for all PTs.

To determine typically achievable values of ζ_2, a COMSOL simulation study was performed on various different PT topologies [18]–[23]. For each PT examined, an eigenfrequency study was performed, allowing the resonant frequencies to be extracted. The impedance of each PT at frequencies surrounding the two nearest spurious modes to the optimum mode were simulated, and ζ_2 extracted using a method presented by Forrester *et al* [24]. The results of this analysis are presented in Fig. 15.

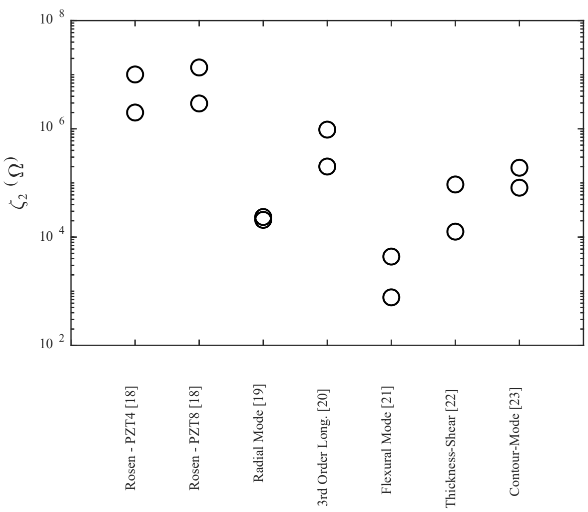


Fig. 15 - Typical ζ_2 values exhibited by several different PT topologies

As shown in Fig. 15, there is significant variation in ζ_2 exhibited across the range of PT topologies. The bar shaped PTs (Rosen and 3rd order longitudinal) exhibit very large ζ_2, so large that these PTs should have minimal efficiency degradation irrespective of the other equivalent parameters. However, the other PT topologies exhibit significantly smaller ζ_2 values. Fig. 15 also shows that using a harder PZT, such as PZT8, leads to larger ζ_2 values than the softer PZT4. Importantly, this analysis has also shown that most PTs, even if made from PZT8, will not achieve a ζ_2 greater than the ~18MΩ that is required to ensure minimal $\Delta\eta$.

However, Fig. 14 shows that changing the frequency of the spurious mode, changes the percentage of devices exhibiting less than 5% efficiency degradation for the same ζ_2 value. This is further confirmed by the 'one-factor-at-a-time' analyses, as for the same ζ_2 value, different levels of efficiency degradation were achieved at different $\Delta\omega$ values. Therefore, the critical value of ζ_2, at which all PTs analysed had <5% efficiency degradation, will change with proximity of the spurious mode to the optimum mode. Furthermore, with greater frequency difference, the critical value of ζ_2 will be smaller and more achievable.

B. Analysing the influence of frequency separation and ζ_2 on the percentage of good devices

Previous analyses have proven that mode proximity and a large ζ_2 value are key factors in ensuring low efficiency degradation. Therefore, the following analysis will focus on analysing the combined impact these two parameters have on the efficiency degradation. This will be extended by analysing various scenarios which constrain the other equivalent parameters. This will provide insight into the optimum frequency separation for a given ζ_2, informing PT design decisions.

The relationship between frequency separation ($\Delta\omega = (\omega_2 - \omega_1)/\omega_1$), ζ_2 and the percentage of devices achieving <5% efficiency degradation will be analysed. Similar to the parametric sweep sensitivity analysis, all parameters will be independently varied in a 7-dimentional space. However, in this analysis, the spurious mode resonant frequency will also be included in the variation, with load being matched to the output capacitance, and therefore not independently varied. The PT will be driven at the optimum mode resonant frequency and all other parameters varied in the same way as the previous parametric sweep analysis. The percentage of good devices (<5% $\Delta\eta$) for variations in ζ_2 and $\Delta\omega$ will be extracted and plotted, using the same method as in the parametric sweep analysis. The range of variation for all parameters is kept the same as in previous sensitivity analyses, with $\Delta\omega$ varied between $-0.75 < \Delta\omega < 0.75$ by changing ω_2, with ω_1 fixed at 100kHz (although, choice has no impact on the results). A contour plot of $\Delta\omega$, ζ_2 and percentage of good (exhibiting <5% $\Delta\eta$) devices is shown in Fig.16 below.

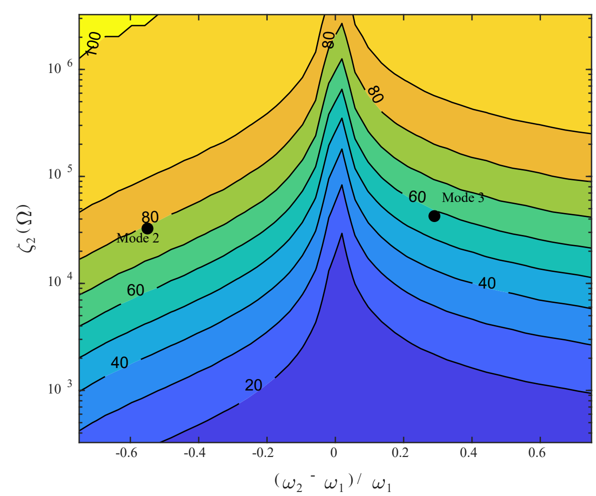


Fig. 16 – Contour plot of the percentage of good devices, against ζ_2 and Δω. Modes 2 and 3, corresponding to the 1st and 2nd spurious modes respectively, of the T1-13 PT.

As theorised, the results in Fig. 16 show that the values of ζ_2 required to ensure all devices have <5% $\Delta\eta$ decreases with spurious mode proximity to the optimum mode. However, even with a frequency difference of -0.75, ζ_2 values in excess of 1MΩ are required for 100% of devices to be classed as good, with only 1.25% of combinations (of ζ_2 and $\Delta\omega$) leading to 100% of devices being good. The asymmetry in the results should also be noted. As a result, for the same value of ζ_2, a spurious resonance would require a greater frequency difference if it occurs at a frequency greater than the optimum mode, compared to a mode occurring below ω_1.

C. Parameter range effect on required ζ_2 and $\Delta\omega$

Several different stipulations will be applied to the range of parameter variation in order to provide insight into realistic requirements for ζ_2 and $\Delta\omega$.

1) Reducing parameter range

The wide range of parameter variation used throughout the previous analyses, ensures that most potential PT designs are included. However, typically PT designs will have parameters similar to the nominal T1-13 PT. Therefore, removing some of the more extreme parameter variations will give results that are more relevant to typical PT designs. A similar analysis to that in Fig. 16 will be performed, however, the range of parameter variation will be $x_n/10 \le x \le 10x_n$ where x is the parameter and x_n is its nominal value from Table 2 and $N_n/5 \le N \le 5N_n$ for the turns ratios (N_1 and N_2). Again, $\Delta\omega$ is varied, by changing ω_2, between $-0.75 < \Delta\omega < 0.75$, ω_1 is again assumed to be 100kHz. The results of this analysis are shown in Fig. 17.

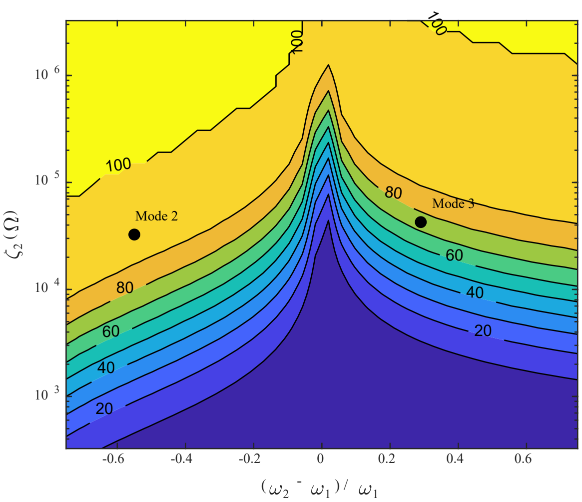


Fig. 17 - Contour plot of the percentage of good devices against ζ_2 and Δω, with the parameter space of the other parameters reduced. Modes 2 and 3, corresponding to the 1st and 2nd spurious modes respectively, of the T1-13 PT

Fig. 17 shows that reducing the range of parameter variation greatly reduces the required ζ_2 values for 100% of devices to exhibit <5% $\Delta\eta$, with 16.3% of combinations (of ζ_2 and $\Delta\omega$) leading to 100% of devices being good. However, this graph also shows that for small $|\Delta\omega|$ and ζ_2 values, a smaller percentage of devices have <5% $\Delta\eta$. This highlights that while the extreme parameter variations in some cases greatly increase $\Delta\eta$, in other cases these extreme variations can allow a PT to be have <5% $\Delta\eta$, even when exhibiting small $\Delta\omega$ and ζ_2. However, $\Delta\omega$ and ζ_2 values required for 80%+ of devices to be good, has greatly reduced.

2) Lower damping

Typically, a PT will be designed for high optimum mode efficiency. As described earlier (III-B-1), minimising R_1 is vital for high optimum mode efficiency. R_1 values of <50Ω are typical in PZT based devices and as a result, 90%+ efficiency can be easily achieved irrespective of the other equivalent circuit parameters, when operated at the matched load. Also discussed previously, spurious mode damping is linked to optimum mode damping and so, if a PT is designed for high optimum mode efficiency (low R_1), then R_2 is also typically, relatively small. Again, these stipulations will be analysed. The range of parameter variation will be the same as in Fig. 16. However, devices exhibiting $R_1 > 50\Omega$ and/or $R_2 > 500\Omega$ will be excluded from the analysis. It should be noted that 50Ω was chosen by assuming 95% or better efficiency is desired in the optimum mode (if it is assumed $C_{\text{out}} = 0$, R_L = 1kΩ and $N_1 = 1$) and 500Ω was chosen noting the ratio of R_1/R_2 in the T1-13 PT. It should be noted that $X_{C_{\text{out}}}$, R_L and N_1 are all still varied through the same range as was used in Fig. 16, the assumptions above were used exclusively to determine the desired range of R_1 and R_2. The results of this analysis are shown in Fig. 18.

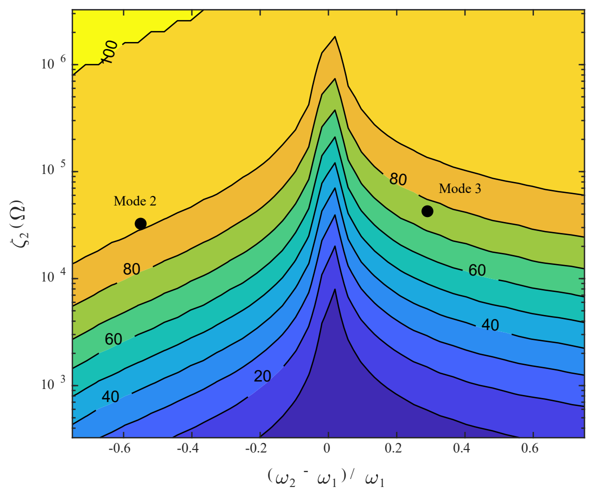


Fig. 18 - Contour plot of the percentage of good devices against ζ_2 and Δω, with the damping resistances, R_1 and R_2, less than 50Ω and 500Ω respectively. Modes 2 and 3, corresponding to the 1st and 2nd spurious modes respectively, of the T1-13 PT

The results in Fig. 18 are similar those in Fig. 17, with lower ζ_2 values leading to a higher percentage of devices achieving <5% $\Delta\eta$ for the same $\Delta\omega$, compared to Fig. 16. However, again for small $|\Delta\omega|$ and small ζ_2 values, a slightly lower percentage of good devices is achieved, compared to Fig. 16. Fig. 18 also shows that reducing R_1 and R_2 makes achieving <5% $\Delta\eta$ easier.

3) Design case study – 5V, 1W output

A potential application for PTs is in low power, resonant converters, such as those used for charging mobile phones, tablets or laptops. In these applications typically a 5V output is required, providing 1W to the load, leading to a load resistance of 25Ω. Ideally, the transformer should have high efficiency, as losses across the switching and rectification elements will also reduce the overall system efficiency, which is beyond the scope of this analysis. Therefore, in this analysis, the PT will be designed to achieve an optimum mode efficiency greater than 80%. Given these stipulations, an analysis can be performed to determine desired ζ_2 and $\Delta\omega$ values for minimal $\Delta\eta$.

For this analysis, all parameters will be varied through the same range as in Fig. 16 with the load fixed at 25Ω. All combinations of parameters which lead to an optimum mode efficiency of <80% will be removed from the analysis. Noting the ratio of R_1/R_2 in the T1-13 PT, devices exhibiting R_2 values > $15R_1$ were also removed from the analysis. It should be noted that, as the input voltage is not given in this scenario, the desired value of N_1 to achieve 5V output is unknown. Its value is therefore varied through the full range as was used in Fig. 16. The results of this analysis are presented in Fig. 19.

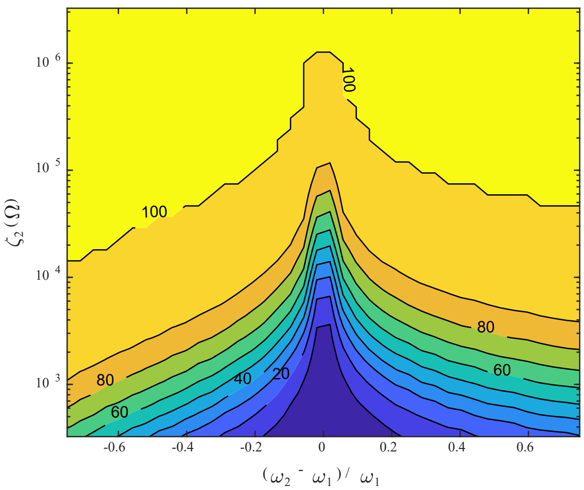


Fig. 19- Contour plot of the percentage of good devices against ζ_2 and Δω, with optimum mode efficiency η_1 greater than 80%, load fixed at 25Ω and the damping resistance, R_2, no larger than $15R_1$

As is shown in Fig. 19, the required ζ_2 value to achieve <5% $\Delta\eta$ in 100% of devices for a given $\Delta\omega$ is greatly reduced in this scenario. As in previous figures, for small ζ_2 and $|\Delta\omega|$ values, fewer devices achieve <5% $\Delta\eta$, than in Fig. 16. However, observing the ζ_2 values presented in Fig. 15, in most cases a frequency difference of -0.37> $\Delta\omega$ and/or $\Delta\omega$ > 0.56 will ensure that, so long as the other equivalent circuit values are within the relevant ranges, the PT should exhibit minimal efficiency degradation.

However, as with all of the results published here, if a PT exbibits ζ_2 and $\Delta\omega$ values which lead to <100% of devices achieving <5% $\Delta\eta$, then it is still possible that a specific PT will have minimal $\Delta\eta$. Therefore, the results presented should be used as a guideline for design and if accurate estimations of the efficiency degradation in the PT are required, then they can be generated using the equations presented earlier.

IV. Experimental Validation

To validate the findings, the T1-13 PT will have its efficiency measured at the optimum frequency. The input impedance spectrum for this device is shown in Fig. 1. The efficiency is compared to predictions made using the equivalent circuit parameters presented in Table 2 and the various contour plots (Fig. 16-18). It is important to note that it is not possible to directly measure efficiency degradation (as the modes are inseparable). The single-branch efficiency (η_1) was therefore estimated from the equivalent circuit parameters. A sinusoidal voltage was supplied to the PT and the input power, output power and efficiency were measured using a Yokogawa PX-8000 power oscilloscope. Using the estimated single-branch efficiency and measured efficiency, an estimate of efficiency degradation can be made. For this analysis, a load of 1kΩ (approx. matched) was used. Also, in this PT there are two offending spurious modes, these will be analysed separately.

First the equivalent circuit parameters of the third most significant mode (second spurious mode in Fig. 1), are extracted and shown in Table 3. Efficiency degradation from this mode can be found by using R_3, ω_3, ζ_3 and N_3 in place of R_2, ω_2, ζ_2 and N_2.

Table 3 - Equivalent circuit components for mode occurring at a frequency above the optimum in the T1-13 PT

	R_3	$\omega_3/2\pi$	ζ_3	N_3
T1-13	234.9 Ω	154.7 kHz	42,637 Ω	1.31

The ζ and $\Delta\omega$ values for the 1st and 2nd spurious modes (Fig. 1) are shown in Table 4 below.

Table 4 - ζ and Δω values for the two spurious modes occurring closest (in terms of frequency) to the optimum mode

	Mode 2	Mode 3
ζ	32.6 kΩ	42.6 kΩ
$\Delta\omega$	-0.55	0.29

In the contour plots (Fig. 16-18), the percentage of good devices (<5% $\Delta\eta$) with the ζ_2 and $\Delta\omega$ values from Table 4, can be found. They are labelled on each figure and summarised in Table 5.

Table 5 – Estimated percentage chance of achieving <5% $\Delta\eta$ for both modes from the contour plots previously presented in the respective figures

	Mode 2	Mode 3
Fig. 16	80.8 %	59.4 %
Fig. 17	97.9 %	77.0 %
Fig. 18	92.2 %	77.8 %

As Table 5 shows, both modes exhibit high percentage chances of achieving <5% $\Delta\eta$, with a maximum of 97.9% and 77.8% for 1st and 2nd spurious modes respectively. This suggests that both 1st and 2nd spurious modes are not guaranteed to have no influence on optimum mode performance. However, with these modes achieving a high percentage chance of <5% $\Delta\eta$ on Fig. 16-18, it is highly likely they would have no influence on optimum mode performance, especially considering they exhibit a high turns ratio and low damping compared to the maximum 500Ω used in the analysis shown in Fig. 18.

For validation, the efficiency degradation of the PT is measured. The optimum mode efficiency was estimated using the equivalent circuit values in Table 2 and using equation (8). Using this value and the experimentally measured efficiency, the efficiency degradation is found.

For this PT and a 1kΩ load, the optimum mode efficiency is calculated at 97.3%. The efficiency is then experimentally measured as 96.9%. Using (20), the efficiency degradation is

0.4%. Therefore, confirming the PT does exhibit <5% efficiency degradation from both spurious modes.

V. Discussion

The work presented here has highlighted the importance of optimising certain parameters at the design stage. As a result, designing a PT to exhibit large ζ_2 is vital, as this is the most influential parameter for minimising efficiency degradation, whilst having negligible effect on optimum mode performance. Secondly, decreasing damping in both the optimal and spurious modes is important for high efficiency. Both are typically determined by the piezoelectric material chosen, the electrode material chosen and the construction of the PT. Finally, increasing the spurious mode turns ratio also has a beneficial effect on $\Delta\eta$, whilst not affecting optimum mode performance. However, designing this parameter is difficult, as it requires knowledge of the type of spurious mode and how geometry or material changes affects its parameters.

It can be seen that efficiency loss from spurious modes can be minimised though careful design of the spurious mode resonant frequencies and ζ_2. It has also been shown that the required ζ_2 can be reduced through careful design of the other equivalent circuit values. Therefore, the analysis in this paper provides designers with a method of appraising designs and, if required, provides information regarding potential design improvements.

It should also be noted that, while the analysis here looks at the required components and design to achieve less than 5% efficiency degradation, it does not analyse how far below or above 5% efficiency degradation represents a good or bad design. Readers will be able to reproduce the graphs for any desired efficiency degradation or nominal PT parameters using the presented equations.

The presence of spurious modes also causes an unwanted reduction in the output voltage. Therefore, even with appropriate optimum mode design, a PT may not supply the intended output voltage. However, improvements to efficiency degradation by adjusting spurious mode parameters (ω_2, R_2, N_2 and ζ_2), minimises the output voltage reduction, further demonstrating the importance of appropriate design for these parameters. R_1 and C_{out} have a complicated relationship with voltage degradation and so decreasing their value (to decrease $\Delta\eta$) can increase or decrease voltage degradation depending on other parameters. Increasing the optimum mode turns ratio (N_1) to decrease $\Delta\eta$, causes voltage degradation to increase and therefore should be avoided. However, in T1-13 PT the voltage degradation was <1%, and so in most cases, voltage degradation is unlikely to be an issue.

Due to the number of variables that influence the efficiency degradation and the wide range of potential variation in these parameters, there will be cases where a parameter is outside the presented range of variation. The results of this analysis should be used as a guideline for appropriate design with a view to producing an exact specification following circuit and finite element analysis. The parameters of a PT design should be carefully analysed using the equations presented in this paper to calculate the efficiency degradation and the required ζ_2.

Spurious modes will typically occur at frequencies both above and below the optimum mode frequency. However, the analysis in this paper has primarily considered interaction with a single spurious mode. We assume that each spurious mode can be considered and analysed separately, which is likely in practice.

VI. Conclusion

An analysis of the impact of spurious modes on the efficiency of piezoelectric transformers was presented. Circuit analysis was performed on PT models with both a single resonant branch and with two resonant branches to generate equations for the efficiency of each model. Multiple sensitivity analyses were then performed and ζ_2 was highlighted as a critical design consideration. The value of ζ_2 required for minimal efficiency degradation in all devices was found. Furthermore, analysis of the combined effect of ζ_2 and $\Delta\omega$ was performed. This analysis was repeated with the parameter space of all other materials restricted in various scenarios. Results of this analysis provide a method of estimating the extent to which a spurious mode will degrade efficiency and consequently, through the results of the sensitivity analyses, provide a method for improving PT designs.

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