The Measurement of Sequence of Returns Risk

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Abstract

We discuss the nature and importance of the concept of Sequence Risk, the risk that a bad return occurs at a particularly unfortunate time, such as around the point of maximum accumulation or the start of decumulation. This is especially relevant in the context of retirement savings, where the implications for withdrawal rates of a bad return can be particularly severe. We show how the popular 'glidepath' or target date savings' products are very exposed to such risk. Three different measures of Sequence Risk are proposed, each of which is intended to inform investors of the probability that a chosen investment strategy may not deliver desired withdrawal rates and hence these measures are intended to aid investment choices; conventional performance measures such as Sharpe or Sortino ratios are only indirectly related to this ability to achieve a given withdrawal experience. Finally, we note that, using US data, very simple portfolios comprising equities and bonds can achieve very low probabilities of failure to achieve popular desired withdrawal rates such as 5% p.a. providing the equity component is 'smoothed' by switching in and out of cash using a simple trend following rule.

Key words: Sequence Risk; perfect withdrawal rates; Monte Carlo; trend following.

1. Introduction

If poetry is the *best* possible words in the *best* possible *order*, (Samuel Taylor Coleridge, July 1827, in conversation), Sequence Risk is the *worst* possible investment returns in the *worst* possible *order*. Yet the *order* of returns is seldom considered in assessing the performance of an investment strategy. Probably the most widely used performance metric is the Sharpe ratio which compares an excess return over a risk-free return relative to volatility (usually standard deviation). The *order* of returns is of no consequence for this measure.

The order of returns becomes important when we introduce the notion of *time* into our discussion of investment performance, such as an accumulation or decumulation period, together with regular *additions* to, (e.g. savings such as pensions' contributions), or *withdrawals*, (such as pensions themselves) from the accumulated pot of savings.

Suppose one has an investment portfolio which loses a massive 40-50% either just before or indeed just after the end of one's accumulation period, i.e. 'at retirement'. The timing of this disaster could not be worse for an individual counting on this pot of money to last a lifetime with steady withdrawals. This is Sequence Risk in action: to lose such an amount at age 25 before 40 years of contributions together with investment growth have been accumulated is less painful than at the point of maximum wealth. A similar argument can be made for the impact of such a loss at, say, age 85, following 20 years of withdrawals, rather in year 1 of drawdown before any withdrawals have been made. Clearly, the *timing* of the large loss is crucial to the individual's decumulation experience, and this is Sequence Risk in action, as shown in Figure 1a.

Figure 1a
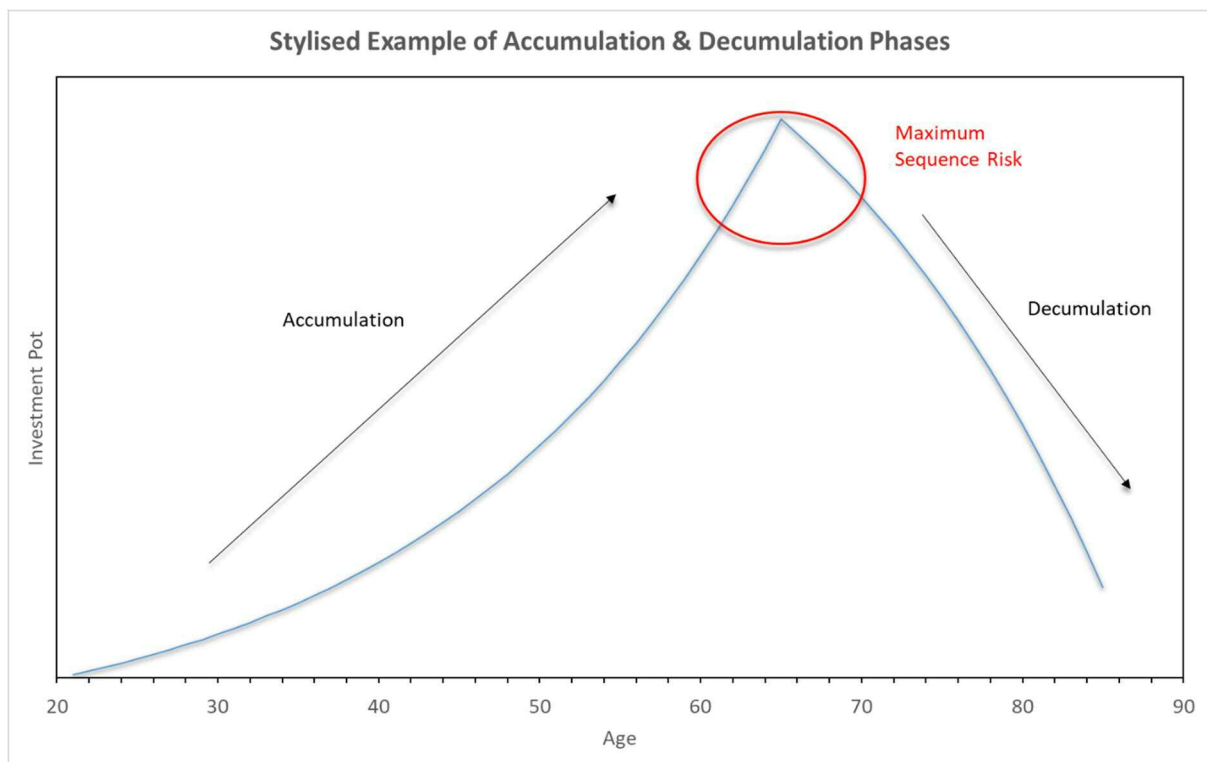
How can investors protect against such large drawdowns? Clearly market timing may be a solution if investment managers possess foresight, though this is indeed questionable in practice. An alternative is through the use of derivatives though this is expensive unless one can time the purchase of, say, puts ; Israelov (2017) shows that portfolios that are protected with put options have worse peak-to-trough drawdown characteristics per unit of expected return than portfolios that have instead simply reduced their equity exposure in order to reduce risk. This approach also echoes the views of Ilmanen (2016), who refers to index put buying as protection for equity portfolios as (looking at historical data) "roughly a minus one Sharpe strategy".

A third possibility and one with a growing following involves 'smoothing' returns by the use of trend following techniques which switch the portfolio between risky assets and the risk free using some simple systematic time series momentum rule, (see Israelov (2017)). This is explored further in a series of research papers by Clare et al (2017, 2019, 2020) where a concept directly related to withdrawal possibilities from an accumulated pot of savings (known as the Perfect Withdrawal Rate (PWR), Suarez et al, 2015) is strongly linked to the large drawdowns which some investment strategies experience (e.g. the 50/50 equity/bond portfolio of Bengen, 1994). If we cannot predict such events, then we must create investment strategies to avoid them. And a measure of success will be the ability to draw satisfactory incomes, i.e. a high PWR.

In terms of reducing Sequence Risk, Ge (2019) suggests adding low-volatility assets to the portfolio of equities and bonds, while Israelov (2017) emphasises the possible advantages of devoting a percentage of the portfolio to trend following assets which are robust to large equity drawdowns and are largely uncorrelated with conventional assets. They have in mind a CTA 'hedge' fund, for example, and possibly allocating to it up to 15% of wealth. This certainly improves the experience in reducing drawdowns (and hence Sequence Risk) but begs the question as to why stop at 15% - why not consider the advantages of trend adjustment for the whole portfolio?

The issue of the appropriate investment strategy for both accumulation and decumulation has been a relatively neglected area of study, labelled as being a 'known unknown' by Merton (2014). The justification for glidepath investing is that the fund is predominantly invested in higher return, higher risk asset classes while the individual is young, while this exposure gradually reduces as the individual ages in favour of ostensibly lower risk assets, primarily government bonds. The economic motivation for this glidepath, or lifecycle approach to investment strategy is that younger investors are better able to withstand equity risk because a large proportion of their total wealth is in the form of low-risk, human capital. This argument for the lifecycle pattern of equity holding was originally made by Merton (1971) developing on from the original Samuelson (1969) and Merton (1969) models which suggested that consumer/investors hold a constant share of the risky asset throughout their lifetime. The approach of Merton is developed in a more realistic form, allowing for the uncertain nature of labour income and costs of participation in equity markets, by Cocco et al (2005). This modelling has supported the development of Target Date Funds (TDFs) with their key feature

of an equity glide path, which reduces equity share with age[1]. See Figure 1b for major commercial providers' asset allocations for their 'Glidepath' strategies.

Figure 1b: Some Popular Commercial Glidepath Asset Allocations

Source: Journal of Performance Measurement

A particularly unfortunate and highly publicised example of Sequence Risk in action occurred in 2009-10 when the three biggest TDFs in the US lost around 30% of their value one year prior to the target date itself. Clearly, this was not supposed to happen given the funds' de-risking approach. However, around the target date, these strategies still have around 50% in equities which once every 10 years or so can experience severe drawdowns of 40-50%.

However, the Great Financial Crisis experience of such investing strategies mentioned above has led to concern that as the target date approaches, these funds may well be susceptible to large falls in value due to substantial and volatile equity components (see Dhillon, 2016). Ezra et al (2009) was the first to draw attention to the practical issues surrounding some important and large TDFs in the recent era. Long-only portfolios, whether passive or active, will inevitably suffer substantial drawdowns at regular (but unpredictable) intervals: even the famous and well diversified portfolios of the Harvard and Yale endowments suffered drawdowns of nearly 30% around 2008-9. The subsequent layoffs of staff and cancellation of capital projects demonstrated the very real impact of Sequence Risk for these institutions.

If Sequence Risk is so important why is there no metric comparable to, say, a Sharpe ratio to provide guidance to investors in choosing investment strategies?

This paper investigates the nature and potential measurement of Sequence Risk. We begin, in Section 2, with a look at the concept of withdrawal rates and how the sequence or order of returns is a key input in assessing the withdrawal possibilities for any strategy. By contrast, a

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[1] See a recent review of the impact of TDF's in the United States in Mitchell and Utkus (2020).

statistic for risk-adjusted returns, such as the Sharpe ratio, is agnostic about the *order* of the returns, yet in real world accumulation and decumulation, the (often-regular) contributions or withdrawals are crucial to the success or otherwise of the investors' actual experience.

In Section 3 we use the analytical framework of Perfect Withdrawal Rates (Suarez et al, 2015) and over 100 years of U.S. stock and bond data to examine the PWRs and the probability of failing to maintain required PWRs associated with varying portfolio asset allocations. We include in this analysis an asset allocation methodology (Clare et al, 2017) which systematically switches equity exposure to cash and *vice versa* according to a rules-based trend following methodology.

In Section 4 we note that the risk in decumulation for many is the risk of running out of money (or having to reduce PWRs to avoid running out of money) and we propose three different risk measurement approaches to better assess this risk, hence providing those entering decumulation with a framework with which to compare various asset class allocations and investment products. The findings develop the discussion in Section 3 regarding trend following strategies: the use of trend adjusted S&P 500 as the equity component (as opposed to long only equity) offers a powerful asset to substantially reduce the probability of *not* achieving a desired withdrawal rate: *a lower exposure to Sequence Risk implies a higher probability of sustaining a desired withdrawal rate.*

2. Sequence Risk and the Perfect Withdrawal Rate (PWR)

(i) Sequence Risk and Why it is Important

When comparing different investment strategies, the adviser and the individual naturally focus on expected returns, often extrapolated from past returns, and volatility, the latter loosely associate with 'risk' following Modern Portfolio Theory. In discussing financial planning there may also be an appetite for considering the maximum drawdown or maximum loss with which an individual may be comfortable. Indeed financial regulators have introduced both volatility of returns and maximum loss as parameters for calibrating how 'risky' a particular strategy is.

The primary focus on expected return and volatility in the form of the Sharpe ratio as a metric to compare alternative investment strategies ignores a very important aspect of risk: Sequence of Returns risk, or simply 'Sequence Risk'. Consider Figure 1a: as one approaches the year of maximum accumulation from contributions (often regular pension contributions) and investment and reinvestment income, then a sudden unexpected fall in investment values, e.g. equities, leads to a rapid drop in wealth as the individual is forced to both withdraw cash and watch the value of one's assets fall. The timing or sequence of the large negative return is most unfortunate: if the poor return had occurred at an early stage in life then it would not have been so impactful. The individual could respond by raising their savings for retirement by a modest amount over the large number of remaining years of work. This is not available to those in the last few years of their working life or after retirement. This is the effect of Sequence Risk in action and is not obviously related to volatility except indirectly. This concept for the example of US equity retirement portfolios since 1870 is explored in Clare et al (2020).

Ezra et al (2009, p 86) offer a simple example to illustrate how the sequential calendar timing of the poor return is crucial to the retirement experience of the individual. Given the assumption of an average 7.5% p.a. investment return, they show that a 20% fall in value at age 25 will have little impact on accumulated wealth at age 65, whereas such a fall the year before retirement will reduce the pot by about 25%. Can shifting asset allocation away from equity - so called 'glidepath' investing - remove or reduce the likelihood of such falls? Ezra et al (2009) and Dhillon et al (2016) would suggest not.

(ii) Perfect Withdrawal Rates

We wish to measure the impact of different sequences of returns on the withdrawal rates, which investors would experience for a variety of asset combinations in the context of retirement decumulation portfolios. To assess their relative performance, we use a metric called the Perfect Withdrawal Rate (PWR), see Suarez et al, (2015) and Clare et al, (2017). This concept measures the annual withdrawal rate (i.e. percentage of one's initial capital in real terms) one could withdraw each year for a pre-set number of years (e.g. 20 years), if one possessed perfect foresight of (future) returns.

The calculation of the PWR can be derived from the following expression for the constant annual withdrawal rate w which reduces the initial capital sum K_S at the start of decumulation to the final balance K_E after n periods.

$$w = [K_S \prod_{i=1}^{n}(1 + r_i) - K_E]/ \sum_{i=1}^{n}\prod_{j=i}^{n}(1 + r_j)$$

where the balance at the end of decumulation, K_E is set to zero if there are no bequests or similar targets and r_i is the rate of return in year i in annual percent. In our calculations, we assume no balance is left at the end of the decumulation, hence $K_E=0$, and set $K_S=1$. What remains is the perfect withdrawal rate:

$$PWR = \prod_{i=1}^{n}(1 + r_i)/ \sum_{i=1}^{n}\prod_{j=i}^{n}(1 + r_j)$$

This can be decomposed in two parts. The numerator is the sum of the product of all the returns during decumulation. We will refer to this as the Return Term. The order of returns during decumulation is irrelevant for this. For a constant denominator, the higher the Return Term, the higher will be the PWR. The denominator is referred to as the Sequence Risk Term. Hence, the order of returns over time can have substantial bearing on the PWR through this term.

Table 1 shows three sets of returns. Each of them has an annual compound return of 5%. In Scenario A, every annual return is 5%, i.e. there is zero volatility, and thus the Return Term is $(1 + 0.05)^{10} = 1.63$. No matter how these returns are ordered, the Sequence Risk Term remains the same at 13.21. One can consider the zero-volatility set of returns as the base case. If the Sequence Risk Term for another 10-year set of 5% annual compound returns exceeds 13.21 then volatility has worked against the retiree, if it is less then volatility has been to their benefit.

Scenarios B & C have the same set of returns as Scenario A except two of the 5% returns have been replaced with a big gain, +120%, and a big loss, -50%. The compound annual return remains 5%, however, and the Return Term (i.e. the numerator) is the same in all cases. The Sequence Risk term for this (constant) series is 13.21 with a PWR of 12.33% pa. It is clear, however, that the Sequence Risk terms have remained anything but the same between the three orderings. In Scenario B, the large loss arrives early, at a time when the retirement pot is substantial, whilst the big gain arrives at the end when decumulation has whittled the pot down to a small amount. The early loss taken here significantly affects the ability to withdraw money in the future years and the PWR is just 6.28%. Scenario C reverses the return order, with the large gain arriving first and the big loss last up. The Sequence Risk term is now just 7.14 compared to 25.88 in Scenario B resulting in a much higher PWR of 22.77%.[2]

Year	Table 1		
	Perfect Withdrawal Rate and Sequence Risk Example		
	Returns (%)		
	A	B	C
1	5	-50	120
2	5	5	5
3	5	5	5
4	5	5	5
5	5	5	5
6	5	5	5
7	5	5	5
8	5	5	5
9	5	5	5
10	5	120	-50
Return Term	1.63	1.63	1.63
Sequence Risk Term	13.21	25.88	7.14
PWR (%)	12.33	6.28	22.77

The point of the example above is to illustrate that even if one had perfect foresight in predicting the compound return of a portfolio over a long period, it is still possible to get wildly different retirement outcomes, unless one also has the unworldly ability to know the sequence that the annual returns will arrive in. For many people in retirement, working another couple of years to fund any deficit in pension savings is not an option. Therefore, for those in retirement, it seems reasonable to assume that one wishes to avoid particularly adverse outcomes. Taking on substantial volatility is, therefore, less appealing in case one encounters an unfortunate sequence of returns. However, also if one adopts an ultra-defensive portfolio, the level of return might be so low, equivalent to a small Return Term in our example, that the PWR will be commensurately very low despite having mitigated the chance of experiencing a substantial negative sequencing risk event. In the rest of this paper, we look at a variety of

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[2] A similar example is presented by Ge (2019).

portfolios and the risk-reward trade off in a retirement context. We then suggest some explicit measures of Sequence Risk.

3. PWRs, Failure Rates and Portfolio Allocation: US evidence 1926-2018

If I have a particular target withdrawal rate in retirement, how likely is it that I will not be successful in achieving that target given a particular asset allocation? We call this the *failure rate*.

Table 2 shows the summary statistics for various combinations of stock and bond portfolios (all values are from 1926-2018, real USD, stock returns data from the Shiller database[3], bond returns data from updates of Welch and Goyal (2008) in the Goyal database[4]). The real annual compound return and volatility are those experienced from 1927 onwards (we use the first year of data for some other calculations later in the paper). For each portfolio combination, we then calculate a 20-year PWR based on randomly drawing annual returns with replacement from the set of data available. We run 20,000 Monte Carlo simulations for each portfolio allocation.

Table 2
Summary 20-Year Perfect Withdrawal Rate Statistics for Stock and Bond Portfolios using 20,000 Monte Carlo Simulations

	100% Stock	80% Sk 20% Bd	60% Sk 40% Bd	40% Sk 60% Bd	20% Sk 80% Bd	100% Bond
Annualized Real Return (%)	6.81	6.19	5.44	4.56	3.56	2.44
Annualized Real Volatility (%)	15.52	12.56	9.99	8.19	7.73	8.82
Mean PWR (%)	8.91	8.49	7.98	7.44	6.89	6.31
Median PWR (%)	8.65	8.28	7.89	7.35	6.77	6.15
10th PWR Percentile	4.85	5.27	5.42	5.44	5.14	4.53
5th PWR Percentile	4.04	4.55	4.82	4.98	4.74	4.17
1st PWR Percentile	2.77	3.39	3.85	4.16	4.06	3.51
Prob (PWR <3%)	1.43	0.50	0.19	0.03	0.00	0.14
Prob (PWR <5%)	11.14	7.92	6.28	5.21	8.17	19.07
Prob (PWR <7.5%)	36.35	38.01	42.58	53.83	69.06	80.19
Prob (PWR <10%)	65.32	73.01	83.77	93.40	97.59	98.14
Prob (PWR <12%)	82.47	90.03	96.77	99.43	99.80	99.85

As one might reasonably expect, the greater the proportion of stocks in the portfolios, the higher the compound return. Volatility decreases as more bonds are added, but only up to a point. The portfolio with 20% stocks has a lower volatility than that comprised entirely of bonds. We next consider some alternative measures of PWR risk by studying the distribution of outcomes from the simulations.

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[3] Stock returns are the continuously compounded returns on the S&P 500 index, including dividends from Robert Shiller's website, see http://www.econ.yale.edu/~shiller/data.htm.
[4] Bond returns are from the Goyal database, see http://www.hec.unil.ch/agoyal/docs/PredictorData2017.xlsx.

The percentiles shown in Table 2 can be considered as Value-At-Risk (VaR)-type measure of risk. For instance, the 10th percentile of the 100% stock portfolio is 4.85%, i.e. there is a 10% risk of achieving a value less than this although this says nothing about how much less than 4.85% this might be.

Whilst we find that the median value for the 100% stock portfolio is highest at 8.65%, this is not the case by the time one reaches the 10th percentile. Both the 60% stocks - 40% bonds and 40% stocks - 60% bonds allocations have a PWR value of 5.4%. Indeed, only the 100% bond portfolio has a lower value than 100% stocks at 4.53%. Once one reaches the fifth and first percentiles, the 40% stocks - 60% bonds portfolio clearly has the highest PWR. The inference is that the high volatility of stocks is sufficient to generate some poor outcomes at this point despite the high average return whilst the low returns of bonds relatively outweigh their low variance.

An alternative way of looking at the retirement problem would be to assume someone has say $1m in their retirement pot and they want a minimum of $50,000 to withdraw from it over 20 years, i.e. they require a 20-year PWR that is greater than or equal to 5%. Anything less than 5% could be considered a shortfall or failure. Figure 3 shows the cumulative frequency of the stock-bond portfolios whilst selected values are shown in Table 2.

Using the earlier example, one is assuming that the aim is to minimise the probability of a PWR outcome being less than 5% through a choice of asset allocation. The extent that any portfolio solution exceeds the 5% target is a bonus but of no consideration in this analysis. Focus is solely aimed at avoiding anything less than the target outcome (here, 5%). In terms of Figure 3 this would involve selecting the line which has the lowest cumulative frequency at the point where PWR equals 5%. From Table 2 it would be selecting the lowest value from the probability (PWR <5%) row. In this case, an asset allocation of 40% stocks - 60% bonds would be selected and the probability of failure would be just over 5%. Choosing 100% stocks or 100% bonds would lead to double-digit failure rates due to the high volatility of the former and insufficient return of the latter.

It becomes clear, from looking at a range of PWR targets, that there is no value at which a 100% bond portfolio is desirable. In every case, there is a smaller chance of failure by adopting a 20% stocks - 80% bond portfolio. As one increases the target PWR, so the probability of failure logically increases and one has to take on increasing proportions of equity in an effort to meet the desired withdrawal rate. From Figure 3, it appears that somewhere around a PWR of 6.5% is the point at which one has to switch to a portfolio comprised entirely of equity in order to have the best chance of meeting one's target. This is a somewhat crude measurement, though, since one is transitioning from an 80 - 20 portfolio to a 100 - 0 allocation, with no subdivisions in between.
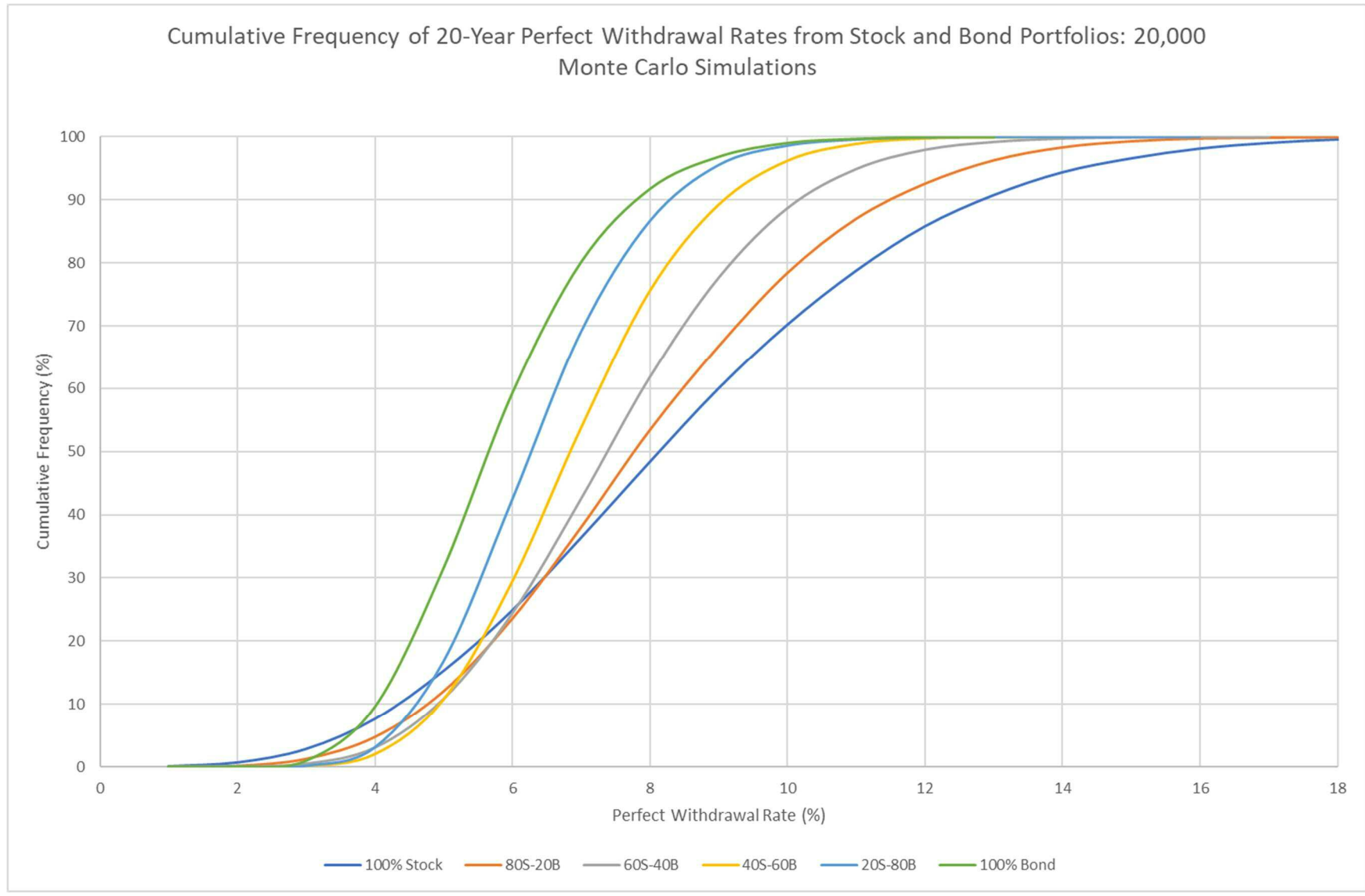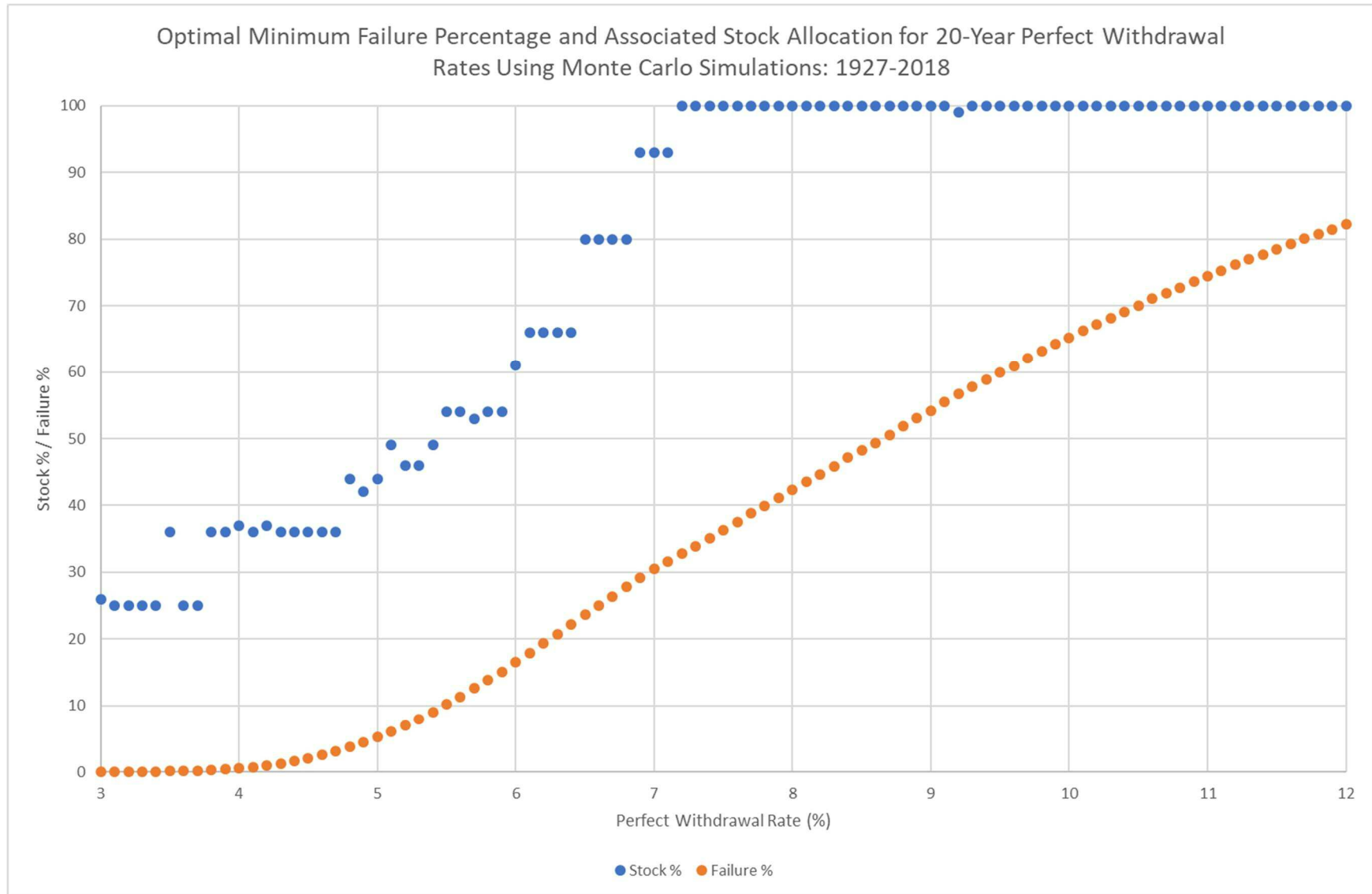
Figure 3

Figure 4

Optimal Minimum Failure Percentage and Associated Stock Allocation for 20-Year Perfect Withdrawal Rates Using Monte Carlo Simulations: 1927-2018

We attempt to obtain greater refinement of this in Figure 4. Now 20,000 Monte Carlo simulations are carried out for each 1% portfolio allocation increment in stocks and bonds. Each blue dot on the chart represents the percentage invested in stocks (with the remainder in bonds) that provides the lowest chance of failing to meet the associated PWR target on the x-axis. For each PWR target, the orange dot shows the chance of failure using the optimal stock allocation, where in this context 'optimal' refers to the stock allocation for a given a selected PWR target that provides the least-worst possible experience.

As an example, at the 4% PWR target the optimal allocation was approximately 37% stocks - 63% bonds, with a failure probability of 0.55%. Some degree of interpolation has to be allowed for due to the nature of the simulations, but the general pattern becomes clear. For a 6% PWR target, the optimal allocation is 61% stocks and the failure rate is 16.6%. At the point one has chosen a PWR target of a little over 7%, the optimal stock allocation is 100% in stocks.

This allocation is essentially the same for all higher PWR targets but the failure rate naturally rises, the greater the selected withdrawal rate. It should be noted at this point that these failure rates are based on looking backwards. With historically low bond yields and above-average stock valuations, one can legitimately question if these targets can be met at the same failure rates, with similar asset allocations, looking forward. A combination may be required of greater investment pots at the start of decumulation, retirements commencing later in life and an acceptance of lower PWR targets. The quest essentially remains the same, however; to find an asset allocation which offers the smallest chance of failure, for the selected target PWR.
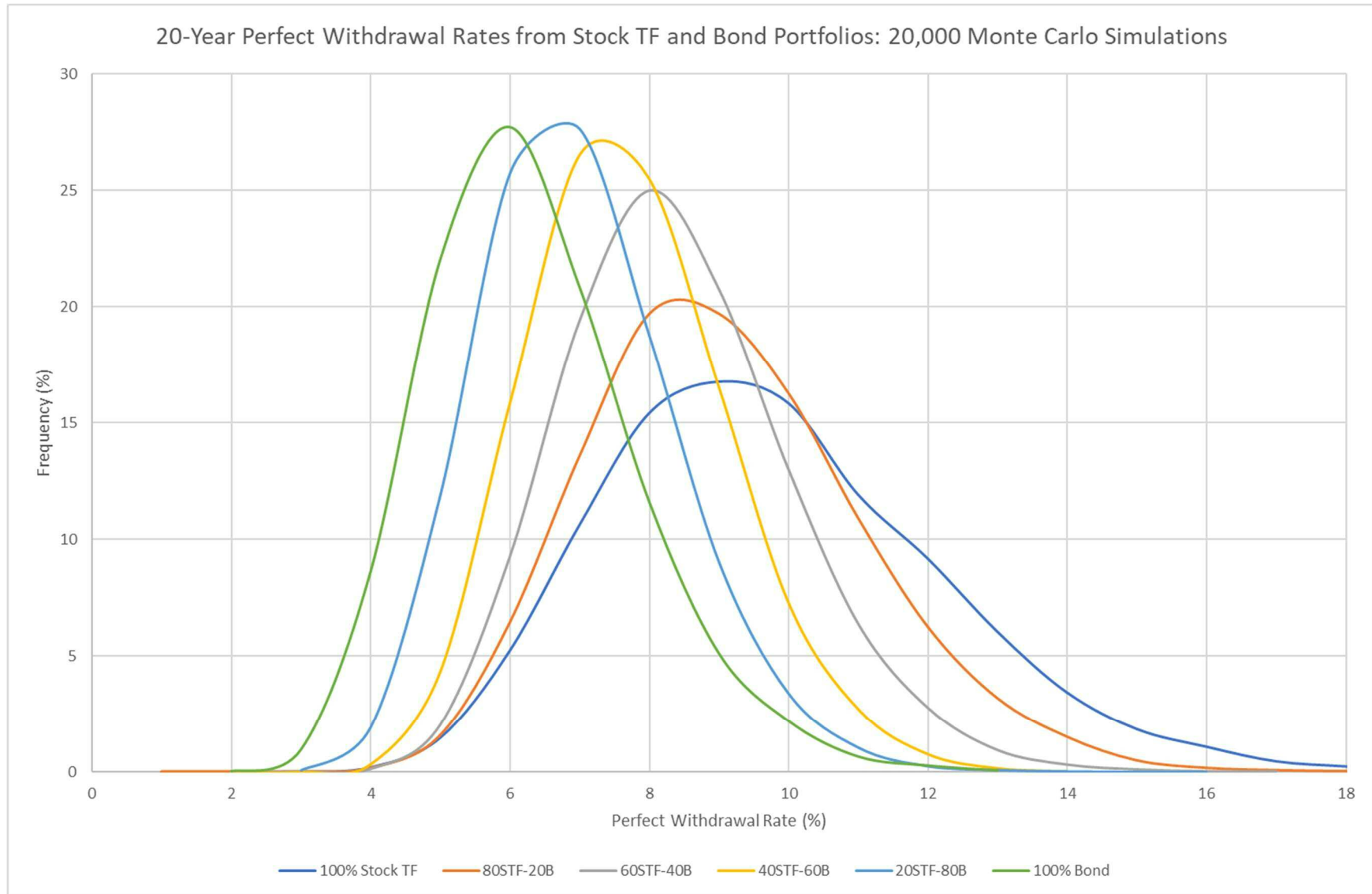
Clare et al (2017) suggest that a useful method for historically reducing the volatility of stock investments[5] for retirement portfolios without sacrificing return is to apply a trend following rule. They use a simple filter which says that if the price of the stock index is above the 10-month average then a long position in stocks is taken otherwise the allocation is placed in T-Bills. We adopt this methodology here and replace the stocks component in earlier Tables and Figures with Stocks TF (STF) i.e. trend adjusted.

Table 3 shows that the average return for 100% stocks TF is more than 1.5% per annum higher than the comparable value in Table 2 whilst volatility is around one-third lower. There is an associated uplift in all the portfolios that contain some proportion of trend adjusted stocks, STF, also. What becomes apparent from looking at the 10th, 5th and 1st percentiles is that the 100% trend adjusted stock, STF, portfolio has the highest value in each case, i.e. there is now no need to take on any bonds. A similar picture emerges when one looks at the shortfall values in the bottom of Table 3. It is only for a PWR target of 5%, where the probability of failure is one-tenth of one percent less by going to 80% STF - 20% bonds. In both cases, the values are less than 1% so it is a relatively minor detail. These conclusions can be confirmed visually from studying Figures 5 and 6.

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[5] Clare et al (2019) find that applying trend following to lower volatility bonds leads to only minimal differences in outcome. For the ease of simplicity, we thus stick to applying trend following to just stocks in this paper.

Figure 5

20-Year Perfect Withdrawal Rates from Stock TF and Bond Portfolios: 20,000 Monte Carlo Simulations

Figure 6

Cumulative Frequency of 20-Year Perfect Withdrawal Rates from Stock TF and Bond Portfolios: 20,000 Monte Carlo Simulations
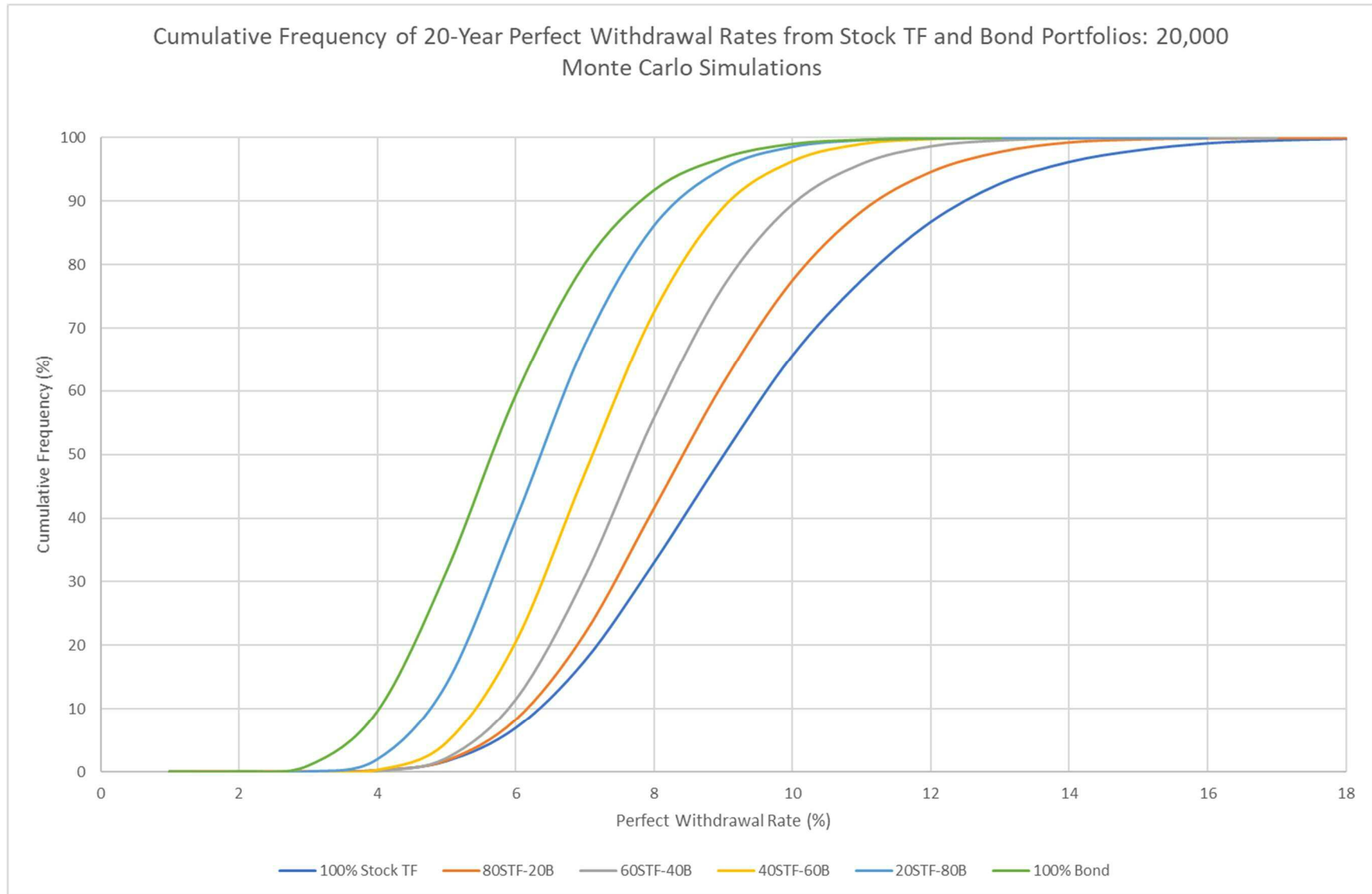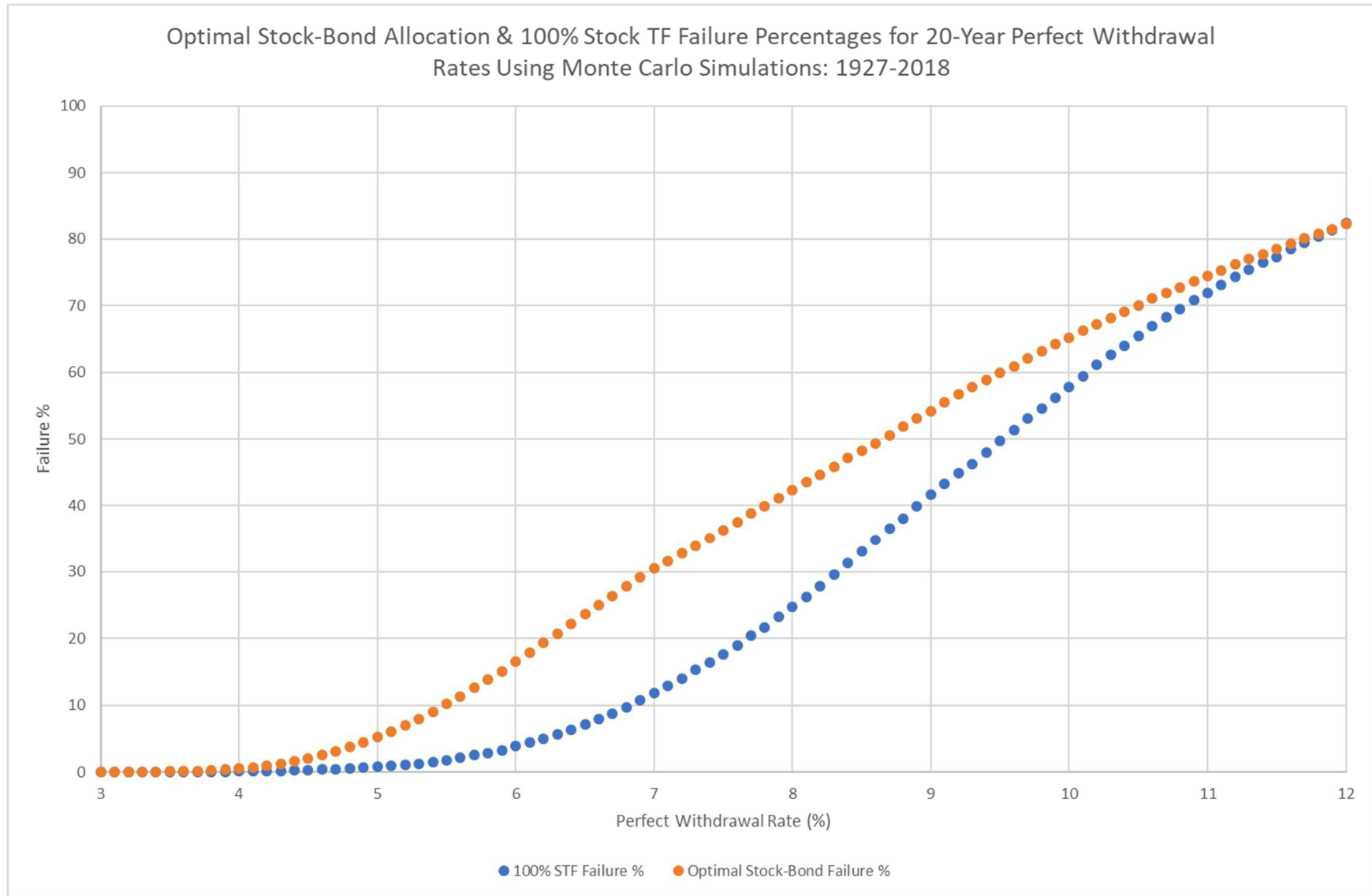
Figure 7

When one compares the failure rates of the 100% STF portfolio with the optimal asset allocations from Table 2, we observe that around a 3% PWR target the failure rates are both very close to zero. At a PWR target of 12% they also very close, both 100% stocks and 100% trend adjusted stocks, STF, have a failure rate of approximately 82.5%. This is clearly a level that should not be targeted using any method if one has a reasonable hope of success. For PWR targets between 3% and 12%, however, we find substantial differences in the failure rates.

Table 3

Summary 20-Year Perfect Withdrawal Rate Statistics for Stocks with Trend Following and Bond Portfolios (no Trend Following adjustment) using 20,000 Monte Carlo Simulations

	100% STF	80% STF 20% Bnd	60% STF 40% Bnd	40% STF 60% Bnd	20% STF 80% Bnd	100% Bond
Annualized Real Return (%)	8.42	7.36	6.22	5.02	3.76	2.44
Annualized Real Volatility (%)	10.36	8.54	7.26	6.84	7.43	8.82
Mean PWR (%)	9.76	9.09	8.39	7.71	6.98	6.31
Median PWR (%)	9.52	8.93	8.26	7.61	6.85	6.15
10th PWR Percentile	6.84	6.67	6.40	5.94	5.27	4.53
5th PWR Percentile	6.22	6.15	5.96	5.53	4.91	4.17
1st PWR Percentile	5.23	5.21	5.18	4.88	4.25	3.51
Prob (PWR <3%)	0.00	0.00	0.00	0.00	0.01	0.14
Prob (PWR <5%)	0.66	0.56	0.66	1.50	6.10	19.07
Prob (PWR <7.5%)	17.55	21.77	30.81	47.06	67.34	80.19
Prob (PWR <10%)	57.85	69.83	84.12	93.37	97.30	98.14
Prob (PWR <12%)	82.57	91.96	97.56	99.52	99.79	99.85

Figure 7 plots the failure rate for 100% STF on the same chart as the failure rate for the optimal stock-bond allocation shown in Figure 4. We observe that failure rates for conventional portfolios begin to rise around the 4% PWR target level. By the point one reaches a PWR of 5% they show a failure rate of around 5% whereas it is less than 1% for the 100% trend adjusted stock portfolio, STF. The gap continues to widen until around the 7% PWR level when the optimal conventional portfolio failure rate is approximately 30% and 100% STF is 12%. From here upwards, the lines begin to converge. The 50% failure rate equates to a PWR target of 8.7% for optimal stocks and bonds and about 9.5% for 100% STF. As discussed earlier, the lines converge again at a PWR of approximately 12%.

We thus conclude that adding trend following to stock allocations can be a powerful tool for portfolios where one is concerned for trying to protect against outcomes that fall below certain targets. In this case, the lower volatility of trend following in stocks reduces the likelihood of sequences of large negative returns at inopportune times that can have a big impact on withdrawal rates as evidenced by our simple example in Table 1.

4. Exploring Measures of Sequence Risk

How can an investor entering drawdown compare different investment strategies to protect their investment from Sequence Risk? This will involve developing a new 'risk rating' methodology for decumulation. The financial services industry has developed many types of risk ratings for investment products, using various measures usually based on return and volatility statistics over differing time periods, to help match a product to the presumed risk tolerance of an investor. Regulators worldwide are keen that investors are not sold either a product that turns out to have been too volatile (risky) or alternatively one with too little risk with a return that therefore fails to meet their investment objectives.

As consumers, we generally know whether we want a fast sports car, or a safe family pick up before going to the dealer. Consumer awareness is very less acute when it comes to buying an investment product, so help is required. However, all of the current risk ratings are primarily intended for use in the accumulation stage of life when it is assumed that there is still time left to recover from any market downturn or losses. There are currently no risk rating methodologies specifically designed to rank investment products suitable for those in decumulation or pension drawdown. Industry advice today is typically to adjust a portfolio increasingly towards exposure to bonds and cash and reduce exposure to stocks, as is implicit in target date funds or glidepath strategies. However, as we have seen from the preceding study of stock and bond returns over nearly 100 years of data, this is a sub-optimal solution due to the importance of Sequence Risk.

(a) Three Approaches to Measuring Sequence Risk
(i) Characterising the Distribution of PWRs

We consider the distribution of PWRs for a given decumulation period, say 20 years, as in Figures 2 and 5. Essentially, we wish to focus on certain aspects of the distribution relating to the left tail in the belief that it is low PWRs which investors wish to avoid. Of course we could mimic the Sharpe ratio by dividing a measure of central tendency, say the mean PWR, by a measure of dispersion, say the interquartile range, or similar. However, this ignores the behaviour of PWRs in the left tail.

A simple measure in this spirit could be as follows (see Table 4):

$$20-r \qquad R \quad R \quad (20 \quad RR) = \frac{Mean\ Historic\ -\ r\quad PWR}{Probability\ of\ Not\ Obtaining\ a\ 5\%\ PWR}$$

The higher the 20 DRR ratio, the more appropriate is the asset mix for decumulation. Adoption of the trend following filter as proposed by Clare et al (2017) clearly is optimal in all cases and particularly at the 80% Stock-20% Bond asset mix (16.23) while a 100% bond allocation is the least desirable (0.33) when considering a 20-year decumulation timeframe. This analysis can be repeated for shorter time frames and so provide the basis for the construction of an optimal glidepath.

A new industry benchmark index for retirement (and associated default tracker fund) would be helpful for all retirees, financial advisors and product providers. It would provide a helpful

benchmark against which to measure all other products offered for decumulation. It is proposed that the index be based on the optimal asset allocation as determined by the DRR ratio, re-analysed annually. Currently the 20-year index would therefore be based on the return series of an 80% equity - 20% bond portfolio with a 10-month trend following overlay as shown in Table 4. As a possible practical implementation, the underlying components of the 80/20 index portfolio could perhaps be 80% MSCI World index and 20% FTSE World Government Bond Index.

Table 4
Decumulation Risk Ratio (DRR) for Various Stock (with and without TF) and Bond Portfolios

Stock/Bond Portfolio	Mean PWR (%)		Prob. of Obtaining <5% PWR (%)		20 DRR	
	No TF	With TF	No TF	With TF	No TF	With TF
100% Bond	6.31	6.31	19.07	19.07	0.33	0.33
20% Stock / 80% Bond	6.89	6.98	8.17	6.10	0.84	1.14
40% Stock / 60% Bond	7.44	7.71	5.21	1.50	1.42	5.14
60% Stock / 40% Bond	7.98	8.39	6.28	0.66	1.27	12.71
80% Stock / 20% Bond	8.49	9.09	7.92	0.56	1.07	16.23
100% Stock	8.91	9.76	11.14	0.66	0.79	14.78

(ii) Further investigation of the Left Tail

Would it be possible to investigate further the distribution in the left tail? Indeed, if the most important thing remains the downside, then we can construct a metric which considers the probability of successfully achieving this minimum, and, *if failure occurs*, how bad it is. We can characterise the latter concept by looking at the distribution in more detail in the left tail, possibly calculating the mean conditional on being in, say, the bottom 5% of outcomes.

For instance, suppose one desires a PWR of 5%. For a 100% Stock portfolio this has a probability of success (based on the Monte Carlo simulations) of 88.87%. In those cases where the PWR came in *below* 5%, the mean PWR was 3.98%, or 79.6%. If we multiply the probability of success (88.87%) by the average PWR failure outcome (79.6%) we get a score of 70.7% (the higher the score, the better - if there is no chance of failure then success probability equals 100% and average PWR failure outcome also equals 100%).

Tables 5a and 5b report the probabilities of success using 20% portfolio increments. As one would expect, the scores are higher when one uses TF since the left tail is more 'to the right'. However, for the TF portfolios the minimum failure rate occurred using 80STF-20B but the score is higher for both 60STF-40B and 40STF-60B. The assumption is thus that when these lower risk portfolios fail to meet the specified PWR they do so by a smaller amount than the higher risk portfolios.

Table 5a						
Percentage Chance of Success for Various Stock & Bond Combinations						
	100% Stock	80% Stock 20% Bond	60% Stock 40% Bond	40% Stock 60% Bond	20% Stock 80% Bond	100% Bond
PWR (5%)	70.68	77.01	81.81	85.50	83.38	71.29
Table 5b						
Percentage Chance of Success for Various Stock (with Trend Following) & Bond Combinations						
	100% Stock TF	80% Stock TF 20% Bond	60% Stock TF 40% Bond	40% Stock TF 60% Bond	20% Stock TF 80% Bond	100% Bond
PWR (5%)	91.21	92.19	92.70	92.32	86.13	71.29

(iii) A more intuitive approach?

An intuitive approach to comparing investment products for decumulation which a retiree may readily appreciate and find relevant to their analysis is to rank investment options simply by their historic failure rate or the risk of running out of money in decumulation (without having to reduce PWRs). This approach is also dependent on the time left in decumulation (in this case 20 years) and a minimum PWR required (in this case 5% p.a.).

Taking the 5% PWR failure risk of mixed asset portfolios without trend following from Table 2 and those with the inclusion of trend following from Table 3, we find the following risk categorisation, shown in Table 6. This analysis provides some stark conclusions given the underlying constraints for someone entering decumulation with a 20-year time horizon. An 80% stock / 20% bond portfolio with trend following applied, has historically returned a 5% PWR with very low risk of failure, a rating of 1. Without trend following, a 60% stock 40% bond portfolio is the least risky with a rating of 4, while 100% stock and 100% bond are found to be at high or very high risk of failure with ratings of 6 and 7 respectively. Table 6 clearly shows the benefit of reduced failure rates if one constructs decumulation portfolios comprising smoothed asset returns. And note the high probability of failure for the 100% bond portfolio, often associated in conventional thinking with 'safe' investing.

Of course, when thinking of applying this to specific funds we have to acknowledge that most funds will not have sufficient data to even manage one independent 20 year period. Results from simulations could be very misleading if one only has say 5 years data to work with. We would have to express retirement solutions in terms of asset classes that have more extensive histories, say like the UK ones, which go back to 1971, to assess PWR probabilities (e.g. see Clare et al, 2020). These would then have to be populated with ETFs and/or funds that reflect the chosen asset class mix. Assumptions would have to be made about whether historic returns are likely to be repeated from current valuation levels.

Table 6

20 Year Decumulation Portfolio Risk Rating Schedule for Various Stock and Bond allocations, With and Without Trend Following for Stocks

Rating scale	1	2	3	4	5	6	7
Probability of PWR < 5%	0 to 1%	1 to 2.5%	2.5 to 5%	5 to 7.5%	7.5 to 10%	10 to 15%	15% +
Risk Portfolios	Very Low	Low	Low to Medium	Medium	Medium to High	High	Very High
With Trend Following	100% STF, 80% STF 20% Bond, 60% STF 40% Bond	40% STF 60% Bond		20% STF 80% Bond			
Without Trend Following				40% STK 60% Bond, 60% STK 40% Bond	80% STK 20% Bond, 20% STK 80% Bond	100% STK	100% Bond

5. Conclusions

The successful uptake of defined contribution schemes in the U.S. after pension reforms in 2006, led to the rapid growth of assets allocated to Target Date Funds. Unfortunately, those that had selected the 2010 TDFs suffered a significant loss of capital, on average 34%, after the market downturn in 2007/2008, a dramatic example of Sequence Risk in action. In the UK, recent pension reforms make it likely that TDFs will also grow in popularity, exposing a growing cohort of pensions savers to Sequence Risk also.

In this paper we show how the timing or sequence of large negative market returns, particularly around the point of greatest asset (usually) pension accumulation can result in a most unfortunate outcome. To measure the impact of different sequence of returns on withdrawal rates, we restate the metric of PWRs, noting that Sequence Risk is not obviously related to market volatility except indirectly, making the use of volatility-based risk rating methodologies potentially ineffective when considering risk in decumulation

We use the PWR metric to analyse a long period of U.S. stock and bond data to find that asset allocation which offers the smallest chance of failing to achieve a target PWR. We find that employing a trend following rule as proposed by Clare et al (2017) is a powerful tool to reduce the likelihood of sequence of large negative returns at inopportune times and hence raise target PWRs given a tolerance for a certain failure rate.

Finally, we suggest three ways of risk rating portfolios using the PWR metric, which all incorporate a measure of Sequence Risk. These risk rating measure provide a new way of analysing various asset allocations, which is particularly important when selecting TDFs or other investment products for decumulation.

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