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Predictive Functional Control for Unstable First-Order Dynamic Systems

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Abstract. Predictive functional control (PFC) has emerged as a popular industrial choice owing to its simplicity and cost-effectiveness. Nevertheless, its efficacy diminishes when dealing with challenging dynamics because of prediction mismatch in such scenarios. This paper presents a proposal for reducing prediction mismatch and thus improving behaviour for simple unstable processes; a two-stage design methodology pre-stabilises predictions via proportional compensation before introducing the PFC component. It is demonstrated that pre-stabilisation reduces the dependency of the closed-loop pole on the coincidence point and also improves robustness to uncertainty. Simulation results verify the improved performance as compared to conventional PFC.

Keywords: PFC, coincidence horizon, pre-stabilisation, proportional compensation

1 Introduction

Predictive functional control (PFC) offers numerous beneficial attributes such as trivial coding, easy implementation and simple handling without needing sophisticated knowledge, software or specialised personnel. These qualities, along with systematic handling of constraints and dead-times compared to other conventional methods, say proportional-integral-derivative (PID) control, make PFC a popular alternative in industry, with numerous successful applications [1].

Conventional PFC [1–3] matches the plant output prediction to a desired first-order target trajectory at only one future point, the so-called coincidence point, by keeping the predicted input constant. One may ask if there exists a reliable criterion for selecting the desired target dynamics and coincidence point? Researchers have established generic guidelines for systems with relatively benign dynamics. For example, it is recommended [2] to use a one-step ahead model prediction for first-order plant as this guarantees target behaviour for first-order systems [4]. Alternatively, one recommendation for higher-order systems is to choose the point of inflection (where the gradient is maximum) on the step response curve as the coincidence point although it is arguable whether this would work well for systems with challenging dynamics. Moreover, for monotonically convergent higher-order systems, a coincidence point where the open-loop step response has risen to approximately 40-80% of the steady-state is often a better choice [4]. Nevertheless, matching underdamped, unstable and non-minimum phase dynamics with target first-order behaviour does not make sense and coincidence point selection for such systems is not straight-forward. Challenging dynamics demand a different parametrisation of the degrees-of-freedom [5], as the typical constant input assumption within the prediction horizon may be inappropriate. One recent attempt [6] parametrised the input with first-order Laguerre polynomial, which improves prediction consistency and convergence rate as compared to the original PFC for systems with simple dynamics; however, this approach is not really tailored to systems with difficult dynamics.

The main objective of this paper is to build on the ideas in [5, 7] and indeed conventional wisdom in PFC [2] which is to modify unstable dynamics before applying the PFC design. Accepted practice in the mainstream MPC community uses pre-stabilisation [8, 9], so this paper proposes a a two-stage PFC design methodology by integrating pre-stabilised dynamics with PFC decision making. Initially we restrict our study to first-order unstable plants focusing on the effects of a pre-stabilising structure on closed-loop performance, sensitivity and constraint handling. Specifically this paper analyses the relationship between the target pole, pre-stabilising gain and coincidence horizon and establishes guidelines for systematic and effective tuning. Generally a trade-off between closed-loop performance and sensitivity is observed, which signifies the importance of offline sensitivity analysis for proper selection of tuning parameters; something not in the conventional PFC literature. With pre-stabilisation, numerical simulations show improved closed-loop performance as compared to conventional PFC. Extensions for systems with higher-order dynamics constitutes future work.

The remainder of this paper is organised as follows: Section 2 succinctly formulates the control problem. Section 3 proposes the two-stage PFC and discusses sensitivity analysis, tuning procedures and constraint handling. Section 4 presents the numerical illustrations. Finally the paper concludes in Section 5.

2 Problem Statement

Consider an unstable first-order plant given by:

$$G_p(z) = \frac{b_p z^{-1-w}}{1 + a_p z^{-1}} \tag{1}$$

where a_p and b_p are the plant parameters, w is the system delay and $|a_p| \geq 1$ represents the open-loop unstable pole. The system eqn. (1) is subject to input, input rate and output constraints i.e.

$$u_{\min} \leq u(k) \leq u_{\max} \qquad \Delta u_{\min} \leq \Delta u(k) \leq \Delta u_{\max} \qquad y_{\min} \leq y(k) \leq y_{\max} \tag{2}$$

where $\Delta = 1 - z^{-1}$ is the difference operator. The objective is to design a PFC by first stabilising the prediction dynamics. Furthermore the controller is expected to show some degree of robustness against measurement noise, disturbances and multiplicative uncertainty.

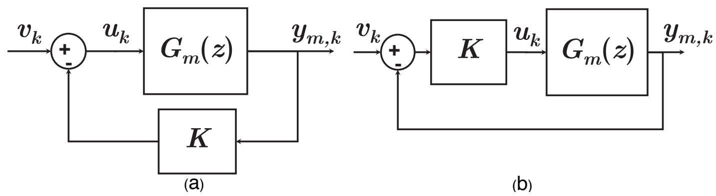

Fig. 1. Pre-stabilisation with proportional compensation

3 Two-Stage Predictive Functional Control

This section proposes a two-stage design approach to controlling the unstable system with PFC. In stage one, the prediction model is stabilised offline through proportional compensation before employing PFC. It should be noted that although open-loop PFC may stabilise unstable systems in an unconstrained environment, pre-stabilisation is necessary for accurate constraint handling. Denote the system model representing (1) as $G_m(z)$, ($a_m = a_p$ and $b_m = b_p$ if $G_m = G_p$):

$$G_m(z) = \frac{b_m z^{-1}}{1 + a_m z^{-1}} \tag{3}$$

The dead-time w is excluded from the prediction model and is added separately in the PFC control law. Next we discuss two alternatives to stabilise system eqn. (3).

3.1 Stage-1: Model Pre-stabilisation

The delay-free model (3) can be stabilised with proportional compensation either in the feedback path (Fig. 1(a)) or in the forward path (Fig. 1(b)). The closed-loop transfer function for both cases has the form:

$$T_m(z) = \frac{y_m(z)}{v(z)} = \frac{\beta z^{-1}}{1 + \alpha z^{-1}} \tag{4}$$

where $\beta = b_m$ and $\beta = K b_m$ for compensation in feedback and forward paths respectively and $\alpha = a_m + K b_m$. Evidently $T_m(z)$ is stable if $0 \leq |\alpha| < 1$. Moreover, the input u_k for feedback path compensation is parameterised as:

$$u_k = v_k - K y_{m,k} \tag{5}$$

and for forward path compensation as:

$$u_k = K(v_k - y_{m,k}) \tag{6}$$

The implementation of PFC with Fig. 1(a) for integral systems only was reported verbally in [7]. The current study generalises this concept for unstable dynamics and analyses the potential merits and demerits against the structure of Fig. 1(b). The expectation is to gain useful insights for generalising pre-conditioning with more advanced compensation for more complex plants.

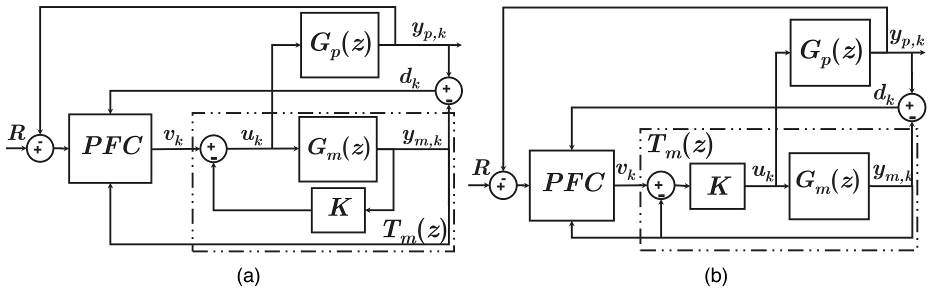

Fig. 2. PPFC structure—PFC on pre-stabilised model with proportional gain in (a) feedback path, and (b) forward path

3.2 Stage-2: PFC design

The pre-stabilised PFC (PPFC) structure employing a PFC loop on the stabilised model is shown in Fig. 2. In PFC, the output prediction, $y_{p,k}$ is required to follow target first-order dynamics such that:

$$y_{p,k+i} = R - (R - y_{p,k})\rho^i \tag{7}$$

where R is the steady-state set-point value and ρ is the target closed-loop pole. The PFC control law matches the output prediction $y_{p,k+i}$ and target output $R - (R - y_{p,k})\rho^i$ at a single point in future, known as the coincidence point h, while assuming a constant predicted input, i.e. $v_k = v_{k+i|k}$, $\forall i > 0$. Hence, after recursion on model (4), an i-step ahead model prediction is obtained [1, 3]:

$$y_{m,k+i} = (-\alpha)^i y_{m,k} + [(-\alpha)^{i-1}\beta + (-\alpha)^{i-2}\beta + \cdots + \beta]v_k \tag{8}$$

The prediction equation (8), requires correction from bias due to uncertainties with the offset term d_k where $d_k = y_{p,k} - y_{m,k}$. Thus PFC is defined from:

$$y_{p,k+i} = y_{m,k+i} + d_k = R - (R - y_{p,k})\rho^i \tag{9}$$

Substituting from (8), the solution to (9), or PPFC law, is given as:

$$v_k = \frac{R - (R - y_{p,k})\rho^h - (-\alpha)^h y_{m,k} - d_k}{\sum_{j=1}^{h}(-\alpha)^{h-j}\beta} \tag{10}$$

Theorem 1. *For a given ρ and h either pre-stabilisation technique results in the same control law provided equal proportional gain is used.*

Proof. First using eqn. (10) in eqn. (5) with $\beta = b_m$, gives:

$$u_k^{fback} = \frac{R - (R - y_{p,k})\rho^h - (-\alpha)^h y_{m,k} - d_k}{\sum_{j=1}^{h}(-\alpha)^{h-j} b_m} - K y_{m,k} \tag{11}$$

Now using eqn. (10) in eqn. (6) with $\beta = Kb_m$:

$$u_k^{forward} = K\left[\frac{R-(R-y_{p,k})\rho^h-(-\alpha)^h y_{m,k}-d_k}{K\sum_{j=1}^{h}(-\alpha)^{h-j}b_m} - y_{m,k}\right] = u_k^{fback}$$

Thus same control law results irrespective of the pre-stabilisation technique. □

Remark 1. Theorem 1 shows there is no obvious advantage of either pre-stabilisation method. Thus for complex systems, pre-conditioning in the feedback path is expected to give same performance as in the forward path.

Remark 2. System delays can be easily incorporated into PFC control law [3] by noting that $E(y_{p,k+w}) = y_{p,k} + y_{m,k} - y_{m,k-w}$. Therefore eqn. (10) becomes:

$$v_k = \frac{R-[R-E(y_{p,k+w})]\rho^h-(-\alpha)^h y_{m,k}-d_k}{\sum_{j=1}^{h}(-\alpha)^{h-j}\beta} \tag{12}$$

where $d_k = y_{p,k} - y_{m,k-w}$. When $w = 0$, eqn. (10) and eqn. (12) are no different.

3.3 Sensitivity Analysis

The ability of a feedback loop to reject unwanted perturbations in the form of noise, disturbance and multiplicative uncertainty can be assessed with frequency domain sensitivity analysis [10]. Control law (11) can be re-arranged as:

$$u_k = F(z)R - M(z)y_{p,k} - N(z)y_{m,k} \tag{13}$$

where $F(z)$, $M(z)$ and $N(z)$ are appropriate polynomials. Note further:

$$\{y_{m,k} = G_m(z)u_k, \;\; (13)\} \;\;\Rightarrow\;\; D(z)u_k = F(z)R - M(z)y_{p,k} \tag{14}$$

with $D(z) = 1 + N(z)G_m(z)$. Eqn. (14) is represented in the block diagram of Fig. 3 where disturbance $d_{y,k}$ and measurement noise n_k are also shown; the effective control law is $C(z) = M(z)D^{-1}(z)$. Consequently, $P_C(z) = 1 + C(z)G_p(z) = D(z)A(z) + M(z)B(z)$ is the closed-loop pole polynomial. From Fig. 3, sensitivity of the plant input to noise is found to be:

$$S_{un}(z) = C(z)[1 + C(z)G_p(z)]^{-1} = M(z)P_C^{-1}(z)A(z) \tag{15}$$

whereas sensitivity of the plant output to disturbance is:

$$S_{yd}(z) = [1 + C(z)G_p(z)]^{-1} = A(z)P_C^{-1}(z)D(z) \tag{16}$$

Sensitivity $S_\delta(z)$ of the closed-loop pole to multiplicative uncertainty uses:

$$\begin{aligned} P_C(z) &= 1 + C(z)[G_p(z) + \delta G_p(z)] \\ &= [1 + C(z)G_p(z)]\left(1 + \delta C(z)G_p(z)[1 + C(z)G_p(z)]^{-1}\right) \end{aligned}$$

where δ is possibly a frequency dependent scalar. Thus:

$$S_\delta(z) = C(z)G_p(z)[1 + C(z)G_p(z)]^{-1} = M(z)P_C^{-1}(z)B(z) \tag{17}$$

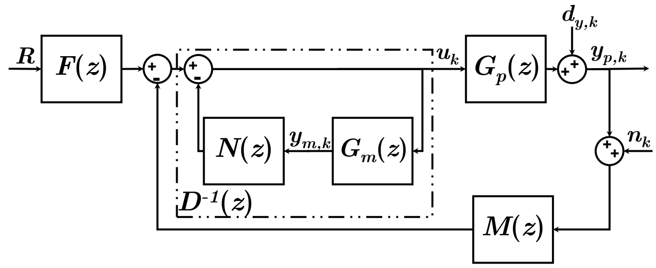

Fig. 3. PPFC block diagram for sensitivity analysis

3.4 Tuning

There are two tuning parameters for a given target pole ρ: the pre-stabilising gain K and the coincidence point h. K determines the position of the pole $|\alpha|$ in z-plane which logically should be restricted between ρ and 1. Therefore K can be tuned within a range where K_U and K_L are upper and lower limits:

$$K_L = -\left(\frac{1+a_m}{b_m}\right) < K \leq -\left(\frac{\rho+a_m}{b_m}\right) = K_U \tag{18}$$

Theorem 2. *Closed-loop pole $z_{CL} = \rho$ is guaranteed if either*
(i) proportional gain $K = K_U$ irrespective of h, or
(ii) coincidence point $h = 1$ irrespective of K.

Proof. We know from [4] that:

$$z_{CL} = -\alpha + \frac{\rho^h - (-\alpha)^h}{\sum_{j=1}^{h}(-\alpha)^{h-j}}$$

(i) selecting $K = -(\rho + a_m)/b_m$ implies:

$$\alpha = a_m - \left(\frac{\rho+a_m}{b_m}\right) b_m = -\rho$$

Consequently $z_{CL} = \rho$ is guaranteed irrespective of h.
(ii) selecting $h = 1$ implies:

$$z_{CL} = -\alpha + \frac{\rho^1 - (-\alpha)^1}{1} = \rho$$

Hence $z_{CL} = \rho$ is guaranteed irrespective of K. □

Corollary 1. *If $K = K_U$ then S_{un} and S_δ are independent of h.*

Proof. $A(z) = 1 + a_p z^{-1}$ and $B(z) = b_p z^{-1}$ do not involve h, plus Theorem 2 proves $P_C(z)$ does not depend on h either. This leaves only $M(z)$ to check. First note that for $K = -(\rho + a_m)/b_m$,

$$\sum_{j=1}^{h}(-\alpha)^{h-j} b_m = \frac{b_m[1-(-\alpha)^h]}{1+\alpha} = \frac{b_m(1-\rho^h)}{1-\rho} \quad \Rightarrow \quad M(z) = \frac{1-\rho^h}{\frac{b_m(1-\rho^h)}{1-\rho}} = \frac{1-\rho}{b_m}$$

which makes $M(z)$ free from h. Thus both S_{un} and S_δ are independent of h. □

Algorithm 1. Select K mid way in its range i.e. $K = (K_U + K_L)/2$. This implies $\rho \leq z_{CL} < (1+\rho)/2$ for $1 \leq h < \infty$ and also keeps h relevant for tuning offline sensitivity functions.

3.5 Constraint Handling

Another important aspect is the proper handling of constraints. Since pre-stabilisation changes the PFC control variable to v_k, this implies a transfer of constraints from u_k to v_k is necessary, for example via a process of back calculation [2]. For pre-stabilisation with proportional gain in the feedback path:

$$\begin{aligned} u_{min} + K y_{m,k} \leq & v_k \leq u_{max} + K y_{m,k} \\ \Delta u_{min} + K \Delta y_{m,k} \leq & \Delta v_k \leq \Delta u_{max} + K \Delta y_{m,k} \end{aligned} \tag{19}$$

and if proportional gain is placed in the forward path:

$$\begin{aligned} \frac{u_{min}}{K} + y_{m,k} \leq & v_k \leq \frac{u_{max}}{K} + y_{m,k} \\ \frac{\Delta u_{min}}{K} + \Delta y_{m,k} \leq & \Delta v_k \leq \frac{\Delta u_{max}}{K} + \Delta y_{m,k} \end{aligned} \tag{20}$$

Output constraints on the other hand are incorporated through predictions (8). At each time sample k, output constraints have to be satisfied throughout and beyond the coincidence horizon, that is until the predictions have settled [9]. From eqns. (8)-(9), the predictions for constraint horizon n_c are:

$$y_{p,k+j} = P_j y_{m,k} + H_j v_k + L_j; \quad j = 1, 2, \ldots, n_c, \quad n_c \gg h \tag{21}$$

Therefore the output constraints $y_{min} \leq y_{p,k} \leq y_{max}$ are transferred to:

$$y_{min} \leq P_j y_{m,k} + H_j v_k + L_j \leq y_{max}; \quad j = 1, 2, \ldots, n_c \tag{22}$$

One can utilise a simple loop to test each constraint in turn and select the v_k closest to the nominal value from (10) which satisfies all the constraints [6].

Theorem 3. *Given that v_{k-1} is feasible by assumption and one is able to select $v_k = v_{k-1}$, constraints can always be satisfied in the nominal case as long as n_c is large enough. Proof equivalent to that in [6].*

4 Simulation Results & Discussion

This section examines the performance of the proposed PPFC controller and compares it with conventional PFC. The unstable plant and constraints are:

$$G_1 = \frac{0.2361 z^{-6}}{1 - 1.118 z^{-1}}; \quad -0.4 \leq u(k) \leq 0.3, \quad -0.1 \leq \Delta u(k) \leq 0.1; \quad 0 \leq y(k) \leq 0.9$$

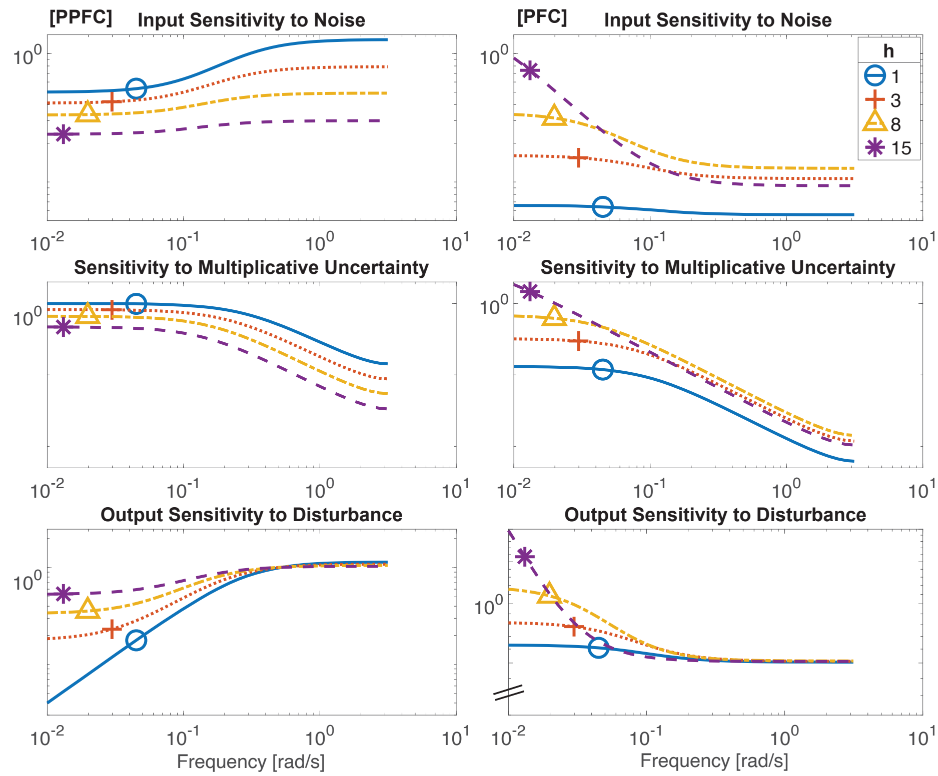

Fig. 4. Comparison of sensitivity plots for G_1 as function of h between PPFC with $K = 1.03$ and conventional PFC (vertical scales for bottom two figures are not equal)

A disturbance $d_{y,k} = 0.5$ is introduced at the 35th sample and white sensor noise $n_k \epsilon [-0.1, 0.1]$ after the 55th sample; the multiplicative uncertainty is $\delta = 0.5$. The target dynamics are governed by $\rho = 0.75$ and $R = 1$ resulting in $0.4998 < K \leq 1.5587$. We choose the middle value of gain according to Algorithm 1, thus $K = 1.03$ guarantees $0.75 \leq z_{CL} < 0.875$. Note that the output upper limit is intentionally kept below set point to analyse the efficacy of PPFC constraint handling.

4.1 Sensitivity Analysis

A sensitivity analysis is used to select the coincidence horizon h. Fig. 4 shows a comparison of sensitivity functions between PPFC and conventional PFC for different h. An worsening trend in sensitivities can be observed for PFC with higher h whereas for PPFC this trend is reversed apart from disturbance rejection that deteriorates slightly. This is expected because with PFC a larger h means the control law is being based on an increasingly large/divergent open-loop prediction and thus is unreliable. The core point is that PPFC clearly outperforms

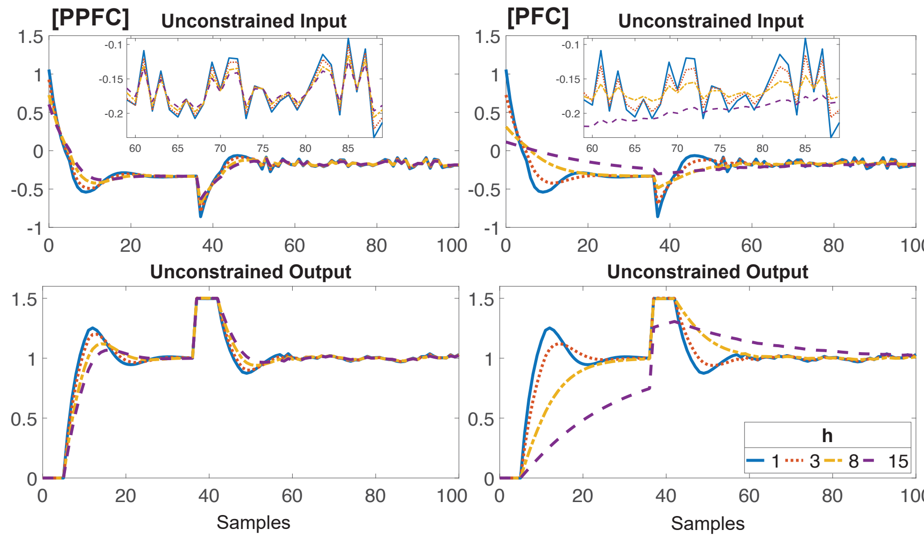

Fig. 5. Comparison of unconstrained input and output for G_1 as function of h between PPFC with $K = 1.03$ and conventional PFC

PFC in terms of sensitivity and one can choose h to get some trade off between the different sensitivity functions.

4.2 Closed-loop Behaviour

The unconstrained time-domain performance shown in Fig. 5 agrees with the results of sensitivity analysis, although nominal performance is affected somewhat by the parameter uncertainty. Again PPFC clearly outperforms PFC and has much more consistent behaviour as h changes. It is particularly notable that PFC begins to fail for large h which is the opposite observation one gets with stable open-loop processes.

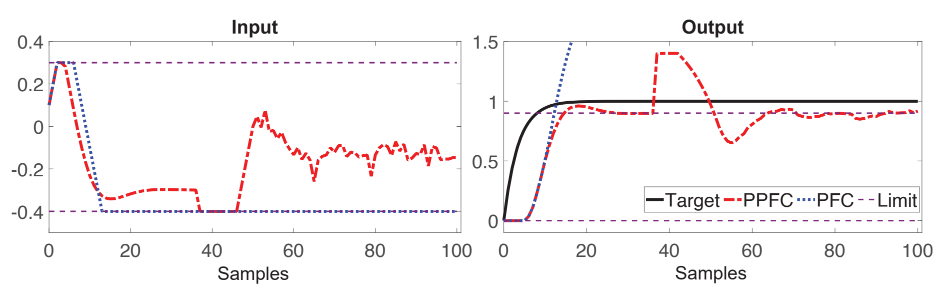

Fig. 6. Comparison of constrained input and output for G_1 between PPFC with $K = 1.03$, $h = 15$ and conventional PFC with $h = 1$

When constraints are introduced the advantages of PPFC are even more pronounced as seen in Fig. 6. Notably PPFC performs well notwithstanding the unstable pole and retains feasibility, whereas PFC fails and has an unstable closed-loop.

5 Conclusions

This paper proposes a two-stage design approach to controlling unstable first-order plants with PFC. It has been shown that pre-stabilisation with a simple proportional gain improves performance, both with and without constraints. The paper establishes systematic guidelines for selection of both the proportional gain and other tuning parameters and proposes some offline analysis to consider their impact on overall performance. The theoretical aspects of this study have been validated through numerical simulations which demonstrate superior closed-loop control with the proposed scheme.

In future the authors plan to extend this study to more challenging unstable and/or higher-order dynamics. We expect a similar approach to pre-condition oscillatory and non-minimum phase behaviour with PD loops could be exploited which otherwise are difficult to control with conventional PFC alone. It is noted that complex pre-conditioning loops within the PFC framework might involve a slightly more demanding constraint handling procedure, but given modern computing capacity this is not likely to be a problem.

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