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# Analysis of Harmonic Amplification in AC Traction Systems Using LCL Filter with Active Damping

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## Abstract

An AC train traction drive generates harmonics which are injected into the traction supply lines and can be greatly amplified if they are near a resonant frequency of the supply network. The paper analyses the line resonant impedance. The most important are series resonances defined by admittance poles whose frequencies, and residues, vary according to the pantograph distance from the end of the traction supply line. An LCL filter, supplemented with a virtual notch filter for active damping, is designed for harmonic current suppression, and applied in traction drives using either a two interlaced H-bridge rectifier or a three-level NPC converter. Simulation results for a traction drive moving along a realistic overheadline are presented showing that the scheme is most effective for the NPC-based train drive.

## 1 Introduction

AC-powered traction has been widely adopted in recent high-speed railway development, due to the availability of power electronic converters with variable voltage and frequency. These are an enabling technology for using induction motors with attractive features of efficiency and robustness. A typical AC traction drive comprises a single phase PWM controlled active rectifier, a DC-bus, and a three-phase DC-AC inverter to drive the AC motors at the required torque and speed. These power converters mostly use high power and voltage IGBTs which can switch at high frequencies with low losses and can withstand high temperatures. However, the high switching frequency can generate harmonics with wider bandwidth and higher order than those generated by conventional thyristor rectifiers. These harmonics may resonate with traction power network natural resonant frequencies, causing over-current and over-voltage with potentially detrimental effects on electrified network components and in the external supply grid.

Harmonic penetration studies in AC traction systems of multiple parallel connected PWM rectifiers, with either two-level H-bridge or NPC-based topologies, have been reported [1]. Interleaving the switching waveforms of the parallel connected rectifiers increases the current ripple frequency, hence moving harmonic components further away from the line resonances. Using an LCL filter was also found effective in filtering out high frequency components [2]. Nevertheless the effect of the train position on the these harmonic current magnifications merits further analysis. A previous

study [3] showed that, with a fixed length of traction power supply line, its resonant frequency is determined by its line structure and impedance, and would not vary with the pantograph location. However, it was also found that the magnitudes of harmonic current, at the line resonant frequency, can be greatly amplified, depending on the train location. In general, the further away the train is from the supply substation, the greater is the magnification of current and voltage components near the resonance frequency.

This paper presents a rigorous analysis of harmonic current magnification as a function of pantograph distance from the end of the traction supply line. It then investigates the effects of suppression techniques using either two interlaced H-bridge rectifiers or a three-level NPC converter. An LCL filter with a notch filter as active damping is proposed for the rectifiers, and its effect in reducing current harmonic magnitudes is studied.

## 2. Overview of Traction Power Supply and Drive Systems

The traction network considered is supplied from the utility grid at about 220kV through a main transformer rated at 40MVA and set in the traction substation (SS). The primary side of the main transformer has two single-phase windings connected to the high voltage grid phase A and B, B and C. Each primary winding has two secondary windings with each rated at about 27.5 kV. These are connected in series, giving a total of 2x27.5kV. As shown in Fig 1, terminals of the two secondary windings are

connected at one end to the contact wire and at the other to the feeder line, and their junction point is connected to the rail. The two secondary windings corresponding to a different supply phase are also connected to the contact and feeder wires of a different track section which is isolated from the first. Treatment here assumes that the supply network uses the standard Auto-Transformer (AT) power supply mode. The traction network up and down lines are connected in parallel at the AT substations through the connection wire (CW). Locomotives running on the network are connected to feeder lines (F), and rails (R) of AT as shown in Fig 1(a). This forms a long loop between the train and the SS, and a short loop between the train and the AT substation; hence under the same power, the network voltage loss can be compensated and the current value greatly increased.

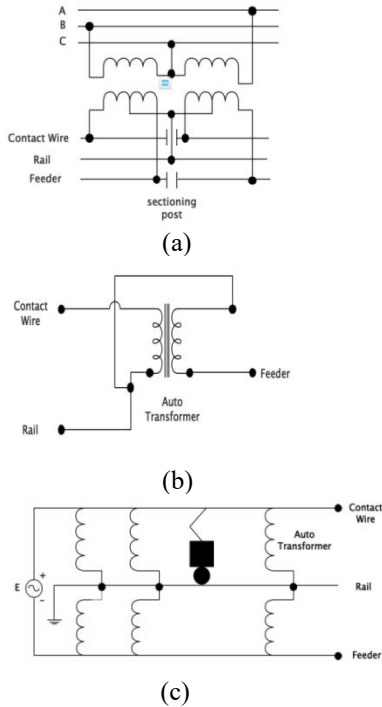


Fig.1(a) VX transformer in Substation(SS) (b) Auto-Transformer(AT) (c) Electric locomotive transmission system

A typical AC traction drive consists of an on-board transformer, a rectifier, intermediate DC-line and filters, DC-AC inverters and motors. Details of the inverter and motors are not required here, since the locomotive's rectifiers are the main source of harmonics, but two alternative rectifier configurations are now considered

### 2.1 Interlaced Two-level H-bridge-based rectifier

To improve waveforms and reduce harmonics on both lines, the traction rectifiers usually adopt multiple parallel structures to raise the equivalent switching frequency as seen from the AC supply side. Fig.2 shows the basic power unit based on two rectifiers connected in parallel on the DC-bus. The

conventional sine-triangle PWM scheme is assumed for both rectifiers, with their carrier wave phases separated by  $90^\circ$ . This interleaved configuration has twice the apparent switching frequency at the input AC terminals, and benefits the exterior circuit by pushing the harmonic frequencies to higher orders.

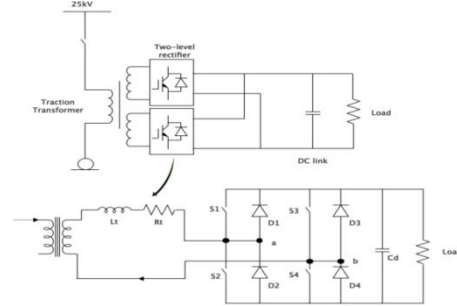


Fig.2 Interlaced rectifier with RL filter

### 2.2 Three-level NPC-based rectifier

The three-level NPC is as shown in Fig.3. Each phase limb has four switch-diode pairs and two clamping diodes, which gives the intermediate zero voltage level due to the diodes clamping the AC terminals to the neutral-point potential of the converter. The drawback of this topology is balancing of the DC-link capacitor voltages. When switching to the zero voltage level the current flows to charge and discharge the capacitors, causing the neutral point potential to fluctuate, and this may affect adversely the waveform performance.

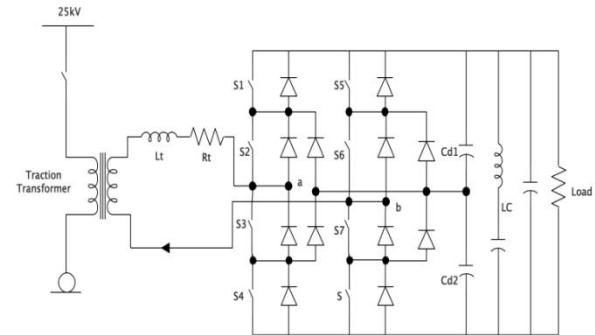


Fig.3. A three-level NPC converter

### 2.3 Analysis of traction network resonant current

The traction drives described in Section 2 draw current at 50Hz from the pantograph and become sources of harmonic currents injected at the same point and polluting the traction supply network. The harmonic current spectrum depends on the rectifier switching frequencies and wave forms. As is known a traction network, minus the train, has its own series of resonant frequencies determined by its line impedance, and if the harmonics injected by the train have frequencies close to a resonance frequency, the harmonic current fed into the grid may be greatly amplified, as now explained.

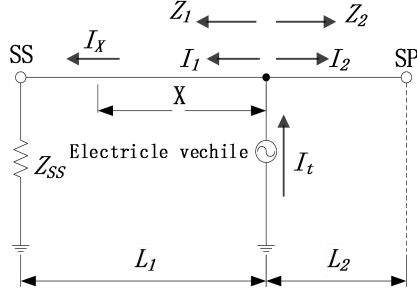


Fig.4 Simplified circuit model for traction supply system power supply

A simplified model for a train on the supply network is shown in Fig 4; on the left-hand-side is the network substation (SS) and on the right is the transmission line separation point (SP) which is approximated as an open circuit.

The pantograph and adjacent rail form a terminal pair at which the impedance seen by the harmonic source on the train can be defined, and for a given substation impedance  $Z_{SS}$  the whole external circuit forms a complete one-port network. This exhibits a series and parallel resonances, i.e. impedance minima and maxima. As discussed by Guillemin [4], impedance maxima occur at the natural frequencies of the system observed with open circuit port terminals and these (complex) frequencies should not depend on the port attachment points on the network. Simulations of the supply network indeed show parallel resonances occurring at frequencies that are independent of the pantograph position, provided  $Z_{SS}$  is fixed. Of more interest here for the current magnification effect are the series resonances defined by admittance poles whose frequencies, and residues, depend strongly on the train location.

Referring to Fig 4, the point SP is considered open circuited and the substation internal impedance  $Z_{SS}$  is connected at the input. Then standard transmission line equations give the impedance presented to the pantograph as:

$$Z = Z_1 \parallel Z_2 = \frac{Z_c \cosh \gamma(L-L_1) \cdot (Z_{SS} \cosh \gamma L_1 + Z_c \sinh \gamma L_1)}{Z_{SS} \sinh \gamma L + Z_c \cosh \gamma L} \quad (1)$$

Where  $L_1$ (km) is the distance from the train to the traction substation,  $L_2$  from the train ion to the SP, and  $L$  is the total line length. Since the harmonic current flowing to SS is:

$$I_1 = I_t \frac{Z_2}{Z_1 + Z_2} = I_t \frac{(Z_{SS} \sinh \gamma L_1 + Z_c \cosh \gamma L_1) \cosh \gamma L_2}{Z_{SS} \sinh \gamma L + Z_c \cosh \gamma L}, \quad (2)$$

the current flowing to SS when train at distance X is

$$I_X = I_t \frac{\cosh \gamma L_2 [Z_{SS} \sinh \gamma (L_1 - X) + Z_c \cosh \gamma (L_1 - X)]}{Z_{SS} \sinh \gamma L + Z_c \cosh \gamma L} \quad (3)$$

The amplification factor can be expressed as:

$$A_X = \frac{I_X}{I_t} = \frac{\cosh \gamma L_2 [Z_{SS} \sinh \gamma (L_1 - X) + Z_c \cosh \gamma (L_1 - X)]}{Z_{SS} \sinh \gamma L + Z_c \cosh \gamma L} \quad (4)$$

The largest amplification factor occurs with the train at the separation point, i.e.  $L=L_1=X$ ,  $L_2=0$ , giving:

$$A'_X = \frac{Z_c}{Z_{SS} \sinh \gamma L + Z_c \cosh \gamma L} \quad (5)$$

At a minimum of the denominator in equation (5), the system exhibits parallel resonance at a frequency:

$$f = \frac{1}{2\pi\sqrt{L_{SS}C}} \quad (6)$$

### 3. LCL filter with Voltage Oriented Current Control for Rectifier

The resonance problem can be addressed by reducing harmonic frequencies generated in the on-board rectifier and fed back to the pantograph terminal. The DC-bus voltage needs to be held constant by a voltage-oriented current control scheme which has an outer voltage feedback control plus an inner current control loop as shown in Fig.5. A phase locked loop (PLL) is used to keep the AC-side current in phase with the grid voltage and so maintain unity power factor.

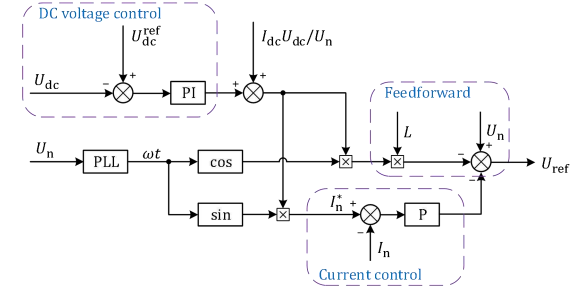


Fig.5 Voltage Control for Rectifier

PWM rectifier switching frequencies of for megawatt scale traction drives of MW power ratings are restricted to ~900 Hz, and the dominant harmonic with interleaving is about twice this, but harmonic filters are still required at the rectifier terminals. LCL filters have been found advantageous in providing better harmonic elimination with lower values of capacitance and inductance, and hence lower costs and volume [3]. Considering the LCL filter connected at the input terminals of the two-level rectifier as shown below in Fig.6,  $L_2=L+L_T$  represents the transformer secondary side leakage inductance summed with the inserted filter inductor.  $L_1$  is the converter side filter inductor,  $R_1$  and  $R_2$  are parasitic resistances of the inductors, while filter capacitor  $C$  may require an added series resistor  $R_d$  to damp the resonance.

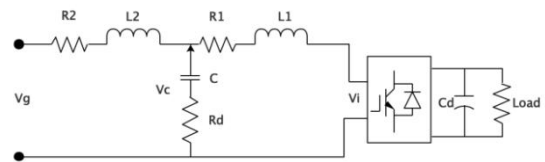


Fig.6 LCL filter applied to the ac input terminals of a rectifier

Neglecting  $R_1$  and  $R_2$ , the transfer function between the rectifier AC side voltage and load side current is readily found as:

$$G_d(s) = \frac{I_1(s)}{V_{ab}(s)} = \frac{1 + CR_d s}{L_1 L_2 C s(s^2 + \omega_{res}^2 CR_d s + \omega_{res}^2)} \quad (7)$$

where  $\omega_{res}$  is the resonant frequency equal to  $\sqrt{(L_1+L_2)/(L_1L_2C)}$ . On the DC-bus side,  $C_d$  is the DC side support capacitor.

The converter-side inductor  $L_1$  is set to constrain the rectifier DC-bus ripple current amplitude to about 10% of the maximum rectifier current,  $I_m$ , whence inductor  $L_1$  can be evaluated as:

$$L_1 = \frac{V_{dc}}{0.1n f_{sw} I_m} \quad (8)$$

Where  $f_{sw}$  is the switching frequency and  $n$  corresponds to the number of converter voltage levels; for a two level converter with unipolar PWM, or an NPC converter,  $n=3$ .  $L_2$  relates to resonant frequency, and is evaluated [6] as

$$L_2 = \sqrt{\frac{1}{k_a^2} + 1} \quad (9)$$

where  $k_a$  defines the desired attenuation ratio which is about 20%.

Selection of capacitor  $C$  is based on the reactive power constraint, with a maximum percentage power factor variation  $\lambda$  in the grid.

Hence:  $\lambda = \frac{2\pi f_0 E_n^2 C}{P_n}$  and is usually less than 5%.

$E_n$  is the rated voltage at the rectifier's AC side,  $P_n$  the rated power for the rectifier, and  $f_0$  is the grid frequency. Substituting the above parameters to evaluate the resonant frequency gives  $f_{res} = 609.3\text{Hz}$  which satisfies the condition on the cut-off frequency:  $10f_0 < f_{res} < 0.5f_{sw}$ . Note LCL filters have very low impedance, determined by the load impedance, at their resonant frequencies and hence require effective damping. A damping resistor  $R_d$  calculated as  $R_d = 1/(3\omega_{res}C)$  may be connected in series with the capacitor, to attenuate the output magnitude at the resonant frequency. In this case it varies from zero to  $3.2\Omega$ , increasing with the attenuation required at the resonant frequency. Table 3.1 lists the specimen design parameters.

Table3.1 Parameters of traction drive rectifier and corresponding filter values

Parameter	Value
$E_n$	2944V
$f_0$	50Hz
$f_{sw}$	1250Hz
$L_1$	3.93mH
$L_2$	4.66mH
$R_d$	3.2 $\Omega$
$C$	32.16 $\mu\text{f}$
$f_{res}$	609.3Hz

Passive damping, by connecting resistors in either the capacitor or inductor filter branches as above, reduces the overall efficiency. Active damping techniques are preferable; one uses a notch filter in the rectifier current feedback control loop; when properly tuned it can produce a high attenuation at

the resonant frequency, hence effectively neutralizing the LCL filter resonance. This method of active damping is considered a good option since it does not affect filter performance in the high frequency range, nor does it require extra sensors as applies when implementing a virtual resistance.

The transfer function of a notch filter is given as

$$G_{notch}(s) = \frac{s^2 + 2\zeta_1\omega_0s + \omega_0^2}{s^2 + 2\zeta_2\omega_0s + \omega_0^2} \quad (10)$$

where  $\zeta_2, \zeta_1$  (with  $\zeta_2 > \zeta_1$ ) are the damping ratios of the zeros and poles respectively. At the given resonant frequency, these values set the level of attenuation and can be tuned to provide the desired damping. The zeros cancel out the filter poles, while an extra pair of complex poles is introduced further inside the stable region of the root locus, thereby increasing the range of stability.

## 4. Simulation Studies and Results

Traction drives, using either an interlaced two-level rectifier or a three-level NPC converter with the same power rating, and supplying DC power to the machine drive, have been simulated. Since the main source of high frequency harmonics is the front end rectifier, the machine drive operating on the DC-bus is represented by a 7.4  $\Omega$  resistor. Both rectifiers are fitted with an LCL filter plus a notch filter-based active damper designed according to the described procedures. Their AC supply side voltage and current waveforms, and their harmonic contents are compared.

Table4.1 Parameters used in the Simulated Train Model

Parameters	Value
DC side voltage	3600V
AC side inductor	5.5mH
DC side resonant circuit inductor	0.84mH
DC side resonant circuit capacitor	3mF
DC side support capacitor	9.12mF
Switch frequency	900 Hz
Load	7.4 $\Omega$

A typical practical double-track network has supply arms comprising seven significant conductor pairs, namely contact wires, bearing cables, positive feed lines, rails, return lines, protection lines and reinforcing ground lines. It is very desirable to simplify this by omitting the last three line pairs, combining the contact wires and bearing cables (at nearly equal voltages) as one, and merging all the rails. The resulting equivalent circuit has five mutually coupled conductors. Line parameters were found using Carson's theory [7] and the 5<sup>th</sup> order  $\mathbf{Z}$  and  $\mathbf{Y}$  matrices of the model were simplified by a matrix order reduction algorithm [8]. The parameters and matrices are listed in [3].

### 4.1 Simulation Results

The impedance magnitude vs frequency of the simulated 20 km traction line, and spectra, are shown at two example points(4km and 8km from AT) in Figs.7(a) and (b).

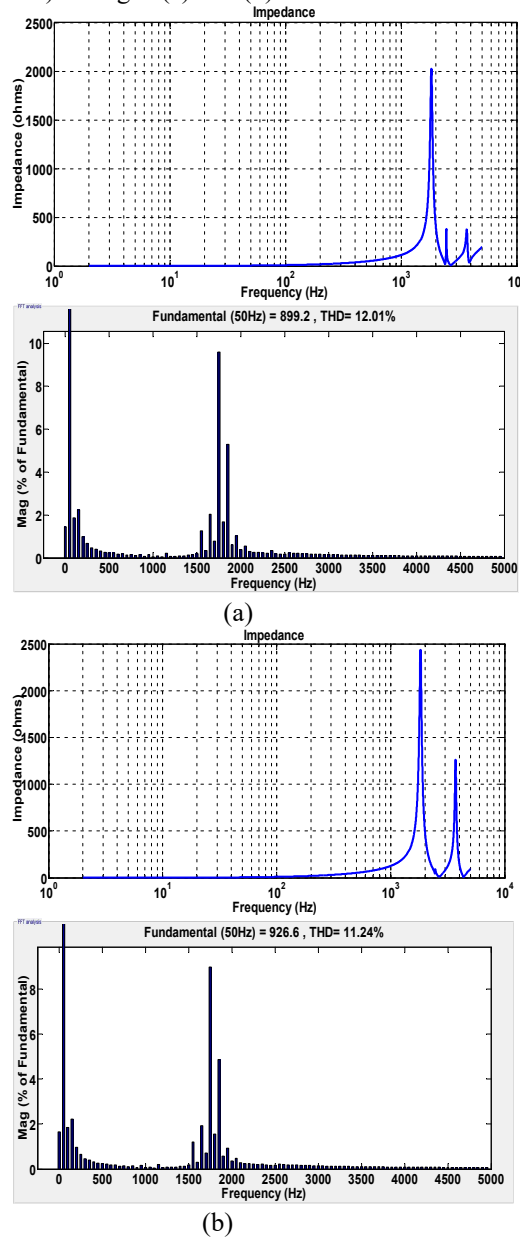


Fig.7 AC supply side harmonic current analysis for a traction unit running on a 20km line. (a) External impedance magnitude seen by pantographs vs Frequency, and current spectrum, at point 4km from AT.(b) Corresponding plots at point 8km from AT. As can be seen the impedance pole resonant frequency is at about 1750Hz and, as expected, this does not change at different points of the line. However when a traction drive with a single PWM rectifier is connected to the line at different points, since it injects harmonic element which coincides with a line series resonant frequency, the harmonic current at 1750Hz is magnified as large as ten times of its original value when at the end of the section and varies with the locomotive location as shown in Table 4.2.

Table 4.2 Harmonic Current Amplifications corresponding to train position measured from the end of the 20km transmission line

Train position (distance from SP km)	Current Amplification (linear scale) at 35 <sup>th</sup> harmonic	Current Amplification (linear scale) at 37 <sup>th</sup> harmonic
0	10.02	5.55
4	9.61	5.29
8	8.98	4.87
12	7.34	3.78
16	5.48	2.55s

In Fig.8(a) and (b), the harmonic analysis for a traction drive using an interlaced rectifier without and with an LCL filter fitted at the SS transformer input terminals is shown. (a) shows AC current spectra when the LCL filter is not used; the harmonics at twice the switching frequency and its side bands are the highest. In Fig 8(b), with a properly tuned LCL filter, these elements are minimized. However this rectifier introduces some lower order harmonics which can be seen clearly in both spectra.

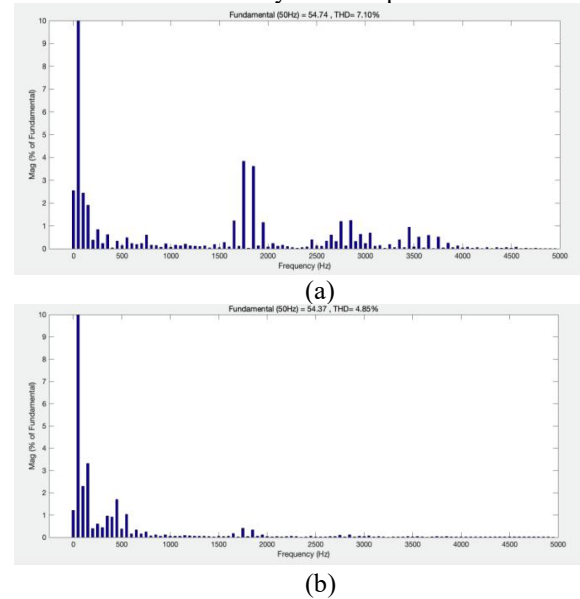


Fig.8 Current Spectrum analysis. (a) Interlaced rectifier with no filter, (b) Interlaced rectifier with LCL filter

Corresponding results for the NPC-based traction drive, without and with LCL filter, are shown in Figs 9(a)-(c); without the filter the NPC harmonic current amplitudes at around twice the switching frequency are significantly lower than for the interlaced. There are some lower order harmonics but their amplitudes are also much lower.

With the filter installed, all higher order harmonics are almost completely eliminated as seen in Figs 9 (b)-(c). Comparing these three spectra shows that the use of active damping by using notch filter is less important, since harmonics near the LCL filter resonant frequency are already small.



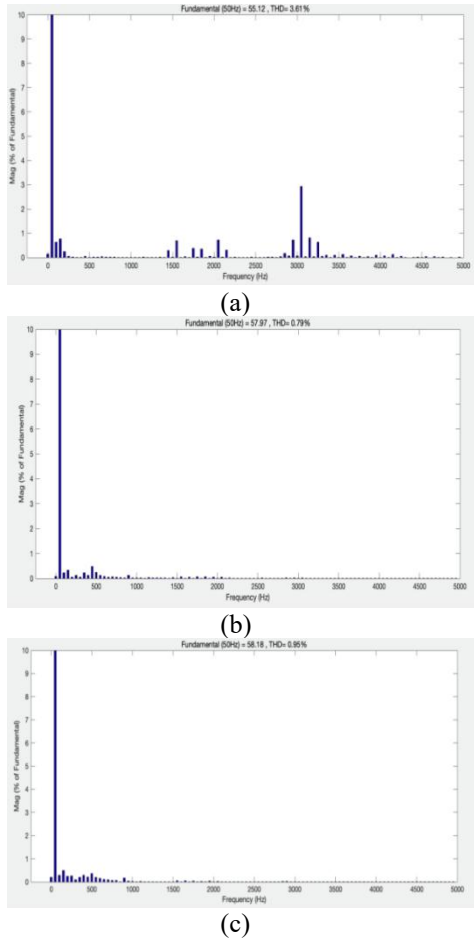


Fig.10 Current Spectrum analysis for (a) NPC without filter (b) NPC with LCL (c) LCL with Notch filter.

Table 4.3 Current Amplifications at 35<sup>th</sup> order, versus train location, for interlaced and NPC rectifiers

Train Position (distance to the substation)	Interlaced rectifier with no filter	Interlaced rectifier with LCL filter
4km	2.71	0.31
8km	3.04	0.35
12km	3.67	0.39
16km	3.95	0.41
20km	4.11	0.43

Table 4.3 contrasts all results obtained, i.e. the harmonic current amplification factors varying with train position changes for different rectifier drives and without and with LCL filter used. Compared to that listed in Table 4.2, using an interlaced rectifier the amplification factor of harmonic current at different train positions are significantly reduced; the maximum scale factor at the end of the line is 4.11, and the minimum at 4km is 2.71. With the

LCL filter the amplification scales are reduced further down to 0.43 the maximum. This shows the effectiveness of the LCL filter in suppressing the current harmonics fed into the overhead line. The most effective topology is the NPC rectifier with the resultant harmonic amplification seen in the list, without LCL filter. With LCL filter the harmonic current at the line resonant frequency is suppressed and changes little with the train location.

## 5. Conclusions

A rigorous analysis of the overhead line resonance impedance has been performed, leading to the derivation of the current amplification factor. It was shown how this varies with the train location. The paper also discussed the design of an LCL filter, with a virtual notch filter as active damping, which can greatly reduce the harmonic current at the traction supply side, and effectively overcomes the resonance problem.

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