Applying Entropy to Understand Drivers' Uncertainty during Car-following

Wei Lyu[1,2], Rafael C. Gonçalves[2], Fu Guo[1], Guilhermina A. Torrão[2], Vishnu Radhakrishnan[2], Pablo Puente Guillen[3], Tyron L. Louw[2] and Natasha Merat[2]

1 School of Business Administration, Northeastern University, Shenyang, China. lvweineu@gmail.com, fguo@mail.neu.edu.cn

2 Institute for Transport Studies, University of Leeds, Leeds, UK. tswl@leeds.ac.uk, tsrg@leeds.ac.uk, gtorrao@gmail.com, mn16vr@leeds.ac.uk, t.l.louw@leeds.ac.uk, n.merat@its.leeds.ac.uk

3 Toyota Motor Europe NV/SA, Hoge wei 33, 1930 Zaventem, BE. pablo.puenteg@gmail.com

Abstract

As one of the main processes in most microscopic simulation models and modern traffic flow theory, car-following has drawn huge academic attention from the engineering and physiological domains. However, given the inherently uncertain and unpredictable nature of human behaviour, car-following models have always faced challenges in capturing drivers' behaviour accurately and objectively. Therefore, to better capture drivers' uncertainty in car-following, this paper contrasts four different entropy algorithms (Shannon Entropy, Steering Wheel Entropy, Approximate Entropy and Sample Entropy) as a novel measure, based on time headway data during car following. Results showed that not all the entropy measures tested are suitable for the context of car-following, especially when it comes to measuring uncertainty in time headway data. Approximate and Sample entropy algorithms in a moving time window seem to be the most appropriate, as they consider drivers' prior time headway data as a factor in the perceived uncertainty. This paper contributes to the fields of microsimulation and human factors, as it demonstrates how entropy can be a precise and replicable measure of changes in behaviour, as well as anomalies in patterns of time headway data in car-following situations.

Introduction

The concept of car-following was first introduced by Pipes (1953, *p*. 275), and can be defined as '*the decision of the driver to follow the preceding vehicle efficiently and safely*'. Here, both efficiency and safety can be described in terms of time and distance between the preceding and following vehicle. Time headway (THW) is one way to characterise the safety margin in car-following, which is the extent to which the following vehicle is susceptible to unpredictable decelerations of the preceding vehicle (Boer, 1999). Car-following models aim to explain the interplay between phenomena at the microscopic level of individual driver behaviour and the macroscopic level of traffic flow. Over the past few decades, there have been numerous attempts to apply Newtonian-based models from the engineering domain, to approximate and interpret car-following (for a full review see Brackstone & McDonald, 1999). However, one of the issues with car-following models is that human behaviour is inherently random and unpredictable (Wilson, 1999), which is challenging to capture in classical mathematical models. In an attempt to further improve the accuracy of these models, there was a trend to incorporate psychological factors, such as motivation and attitude (Brackstone & McDonald (1999). However, models which included psychological variables tended to be no better at explaining drivers' car-following behaviour, partly due to the substantial intra-driver variability, or 'uncertainty', in terms of changes in driving behaviour or strategies in different driving stages or scenarios, and inter-driver differences in terms of risk-taking behaviour and demographics (Saifuzzaman & Zheng, 2014). Among these factors, the uncertainty of drivers' behaviour, as a ubiquitously inherent nature of drivers, is claimed to be taken into account in characterising car-following behaviour (Sheu & Wu, 2015). Therefore, there are clearly challenges with incorporating psychological metrics into these models. However, there is still a need to capture the 'uncertainty' element in drivers' behaviour. An alternative approach is to analyse the patterns of vehicle-based measures, such as time headway, to see how drivers' uncertainty has changed during the car-following process. Entropy theory is a possible candidate to illustrate 'uncertainty' in drivers' car-following behaviour.

Information entropy, as proposed by Shannon (1948), provides a mathematical expression of the amount of uncertainty associated with a variable *X*, where the 'uncertainty' is the summation of all possible outcomes, where the outcomes are unknown. In the context of car-following, uncertainty can refer to the variability in the patterns of fluctuation in a driver's relative position to a lead vehicle. Mathematically, the measure of information entropy associated with each possible data value is the negative logarithm of the probability mass function for the value. According to Shannon's (1948) definition, events with high or low probability with p of approximately 0 or 1, will not contribute substantially to the final entropy value. By contrast, a uniform event where *p* equals approximately 0.5, will result in a much higher final entropy value. It is worth mentioning that the entropy *H* (expressed below) is a function of the probability distribution $\{p_1, p_2, \ldots\}$ rather than a function of values or statistical indicators of the original series $\{x_1, x_2, \ldots\}$. In this way, it can be concluded that the less regular the original series, the higher the uncertainty and the higher the entropy.

$$H(X) = -\sum_{x \in X} p(x) \log p(x)$$

Based on the entropy concept by Shannon, researchers have proposed various modifications of entropy and applied them to time series data in different fields. Nakayama et al. (1999), for example, introduced Steering Wheel Entropy based on prediction error, to quantify drivers' steering behaviour while performing non-driving-related-tasks (NDRT). Steering Wheel Entropy yielded significance where classical statistical methods failed (Nemoto, Yanagishima, Taguchi, & Wood, 2002). In addition, many researchers have adopted entropy-based methods to detect abnormal changes of drivers' physiological signals (such as the ones provided by electroencephalogram and electrocardiogram tests) in different driving experiments, especially drowsy driving (c.f. Huang, Pal, Chuang, & Lin, 2015), fatigue driving (c.f. Wang, Wang, & Fu, 2018), and distracted driving (c.f. Yu, Sun, & Zhang, 2011). These studies demonstrate the usefulness of using entropy in driving-related research to quantitatively measure randomness and uncertainty in time series data. Therefore, it may be helpful here in the analysis of vehicle metrics to understand underlying changes in drivers' car-following behaviour.

The aim of the paper is to investigate the suitability of different entropy algorithms to understand drivers' uncertainty in their car-following behaviour. To the best of the authors' knowledge, this is the first attempt to use entropy to understand drivers' uncertainty in a car-following context. This approach may provide a new perspective to measure and understand drivers' car-following behaviour.

Methods

In this section, four entropy algorithms will be introduced, including Shannon Entropy, Steering Wheel Entropy, Approximate Entropy and Sample Entropy. As the theoretical foundation of many other entropy algorithms, the concept proposed by Shannon is easy to understand and implement. Based on the framework of Shannon Entropy, Nakayama (1999) first introduced Steering Wheel Entropy in driving context to measure drivers' smoothness under different non-driving-related-tasks, and this entropy algorithm proved effective in detecting drivers' behavioural changes from the steering wheel angle. In addition, numerous authors have shown the usefulness of Approximate Entropy and Sample Entropy to show uncertainty and irregularity in time series data (Richman & Moorman, 2000; Xie, He, & Liu, 2008). The aim of this paper is to explore the use of entropy to capture drivers' behavioural uncertainty from time headway data series in the context of car-following, so we have chosen these four entropy algorithms as they were each designed to capture different situational-contexts.

Shannon Entropy. Shannon (1948) defined entropy as the negative logarithm of the probability mass function for a particular value. Once we obtain a time series (for example, time headway data), we can plot the histogram of the original data and compute the frequency of data points in each bin of the histogram. The frequency can serve as the probability in the entropy formula. Thus, we can calculate the entropy value of the original data series without having prior knowledge or other statistical characteristics of the raw data.

Steering Wheel Entropy. Steering Wheel Entropy was first proposed by Nakayama et al. (1999). It was based on the assumption that in free driving, a driver tends to control the steering wheel smoothly and predictably because of the anticipatory nature of preview control (i.e., a driver's continuous steering wheel corrections before entering a curve). In consideration of the validity of this entropy in quantitatively measuring drivers' lateral control of the vehicle, this paper uses it as a potential approach to assessing drivers' longitudinal control in car-following. Mathematically, this algorithm calculates the entropy value based on the prediction errors between prediction value and real value of the steering wheel angle time series. The second-order Taylor expansion was used to obtain the predicted steering angle. The prediction error $e(n)$ is defined as the difference between $\theta(n)$ and $\theta_p(n)$ (see Figure 1a). The 90th percentile value α is calculated from the frequency distribution of the computed prediction errors, which is then used to divide the distribution of $e(n)$ is into nine bins, as shown in Figure 1b. The proportion of prediction errors falling into each bin is computed and the steering entropy value Hp is calculated using the Shannon Entropy formula, while a log base is changed to 9 to assure the final entropy value falls between 0 and 1. As can be seen from Figure 1b, driving with NDRT results in a broader frequency distribution and a consequently higher Hp value.

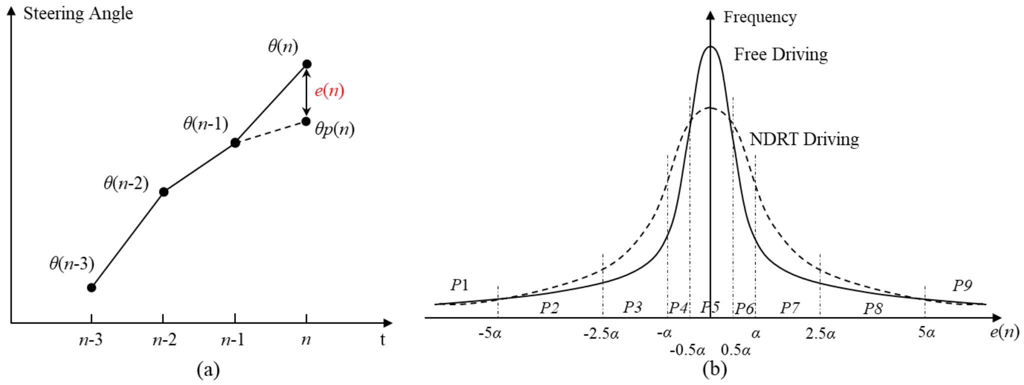

Figure 1 Steering Wheel Entropy (Nakayama et al., 1999), (a) diagram of the prediction error $e(n)$, (b) frequency distribution of the prediction error in driving with/without NDRT.

Approximate entropy (ApEn). ApEn measures the logarithmic probability that nearby pattern runs remain close in the next incremental comparison (Delgado-Bonal & Marshak, 2019). If there is a greater likelihood of the data remaining close and similar to the next incremental comparison, which would indicate regularity, it will yield lower ApEn values and vice versa. In the ApEn algorithm, a pair of parameters (m, r) are set as input. More specifically, m is the length of the template (length of the window of the different vector comparisons), and r is a noise filter (superposition of noise much smaller in magnitude than r barely affects the calculation). ApEn (m, r, N) measures the logarithmic frequency with which blocks of longitude m that are close together stay together for the next position, if possible, with the same number of observations N due to the bias mentioned below. Delgado-Bonal & Marshak (2019) provide a comprehensive tutorial on how ApEn is defined, calculated and applied.

Sample Entropy (SampEn). In the calculation of ApEn, there are two important limitations due to its bias. The first is that the ApEn result with different r may be different, and the second is that ApEn is influenced by the length of the data series. To eliminate the potential bias of ApEn, Richman and Moorman (2000) proposed SampEn as an updated version which does not involve self-counting. In other words, in the ApEn algorithm, the comparison vector $x(i)$ counts itself to avoid the presence of log0, which will present when there are no similar patterns. SampEn (m,r,N) is the negative value of the logarithm of the conditional probability that two similar sequences of m points remain similar at the next point m+1, counting each vector over all the other vectors except on itself. The calculation of the SampEn is highly similar to ApEn, and the main differences lie in whether the sum of all template vectors is inside or outside other logarithms, as can be seen from the below formula. For the detailed calculation and illustration, Delgado-Bonal & Marshak (2019) provided a comprehensive tutorial for both ApEn and SampEn.

$$ApEn(m,r,N) \approx -\frac{1}{N-m}\sum_{i=1}^{N-m}\log\frac{\sum_{j=1}^{N-m}\left[number\ of\ times\ that\ d\left[\left|x_{m+1}(j)-x_{m+1}(i)\right|\right]<r\right]}{\sum_{j=1}^{N-m}\left[number\ of\ times\ that\ d\left[\left|x_{m}(j)-x_{m}(i)\right|\right]<r\right]}$$

$$SampEn(m,r,N) = \text{-}\log\frac{\sum_{i=1}^{N-m}\sum_{j=1,j\neq i}^{N-m}\left[number\ of\ times\ that\ d\left[\left|x_{m+1}(j)-x_{m+1}(i)\right|\right]<r\right]}{\sum_{i=1}^{N-m}\sum_{j=1,j\neq i}^{N-m}\left[number\ of\ times\ that\ d\left[\left|x_{m}(j)-x_{m}(i)\right|\right]<r\right]}$$

Application & Discussion

In this section, the four entropy algorithms will be applied to the time headway data in a car-following driving scenario. Data used here is from an experiment which aimed to assess changes in driver behaviour after exposure to automation in car-following situations. In this experiment, the participant was instructed to resume control of the vehicle from an automated driving system, and then follow the preceding vehicle until the end of the trail. The lead vehicle maintained a constant speed of 40 mph (64.4 km/h). In this paper, the time headway data of one participant is selected to serve as an example for the application of the four algorithms discussed in the previous section. The data used here include 200 s of time headway data, starting at the point the participant resumes manual control from the automated driving systems. The sample rate of driving data collection is 60 frames per second, which resulted in 12 000 time headway data points.

Entropy in a fixed time window

If one wants to measure how the variation of time headway over time affects the progression of the level of entropy in a given stream of data, it is feasible to assume that using a fixed accumulated time window might be a possible option. The advantages of this approach is that by comparing the difference of the accumulated entropy values from the beginning of the series to a certain point in the time series (e.g. from 0 to 10s) and its previous iterations (from 0 to 8; from 0 to 6, etc.) it is possible to see how the entropy in time headway progresses over time. As shown in Figure 2, all the four entropy algorithms are sensitive to the changes in time headway data at the beginning when the computed data series is relatively short. However, after 90 seconds after take-over, values from all the entropy algorithms tend to be flat, ignoring the change of time headway data. These entropy algorithms are somewhat dependent on the length of the original data. It appears that when the computed time headway data series is long enough (for example, more than 90 seconds in this case), a small incremental time window (2 seconds in this case) will bring little additional information and influence to the existing data. This may explain the absence of fluctuations in all four entropy measures. One possible explanation for this phenomenon is that all entropy algorithms are based on previous observations in the same dataset. That being said, the larger is the dataset, the less likely a new incremental piece of data would affect the overall distribution.

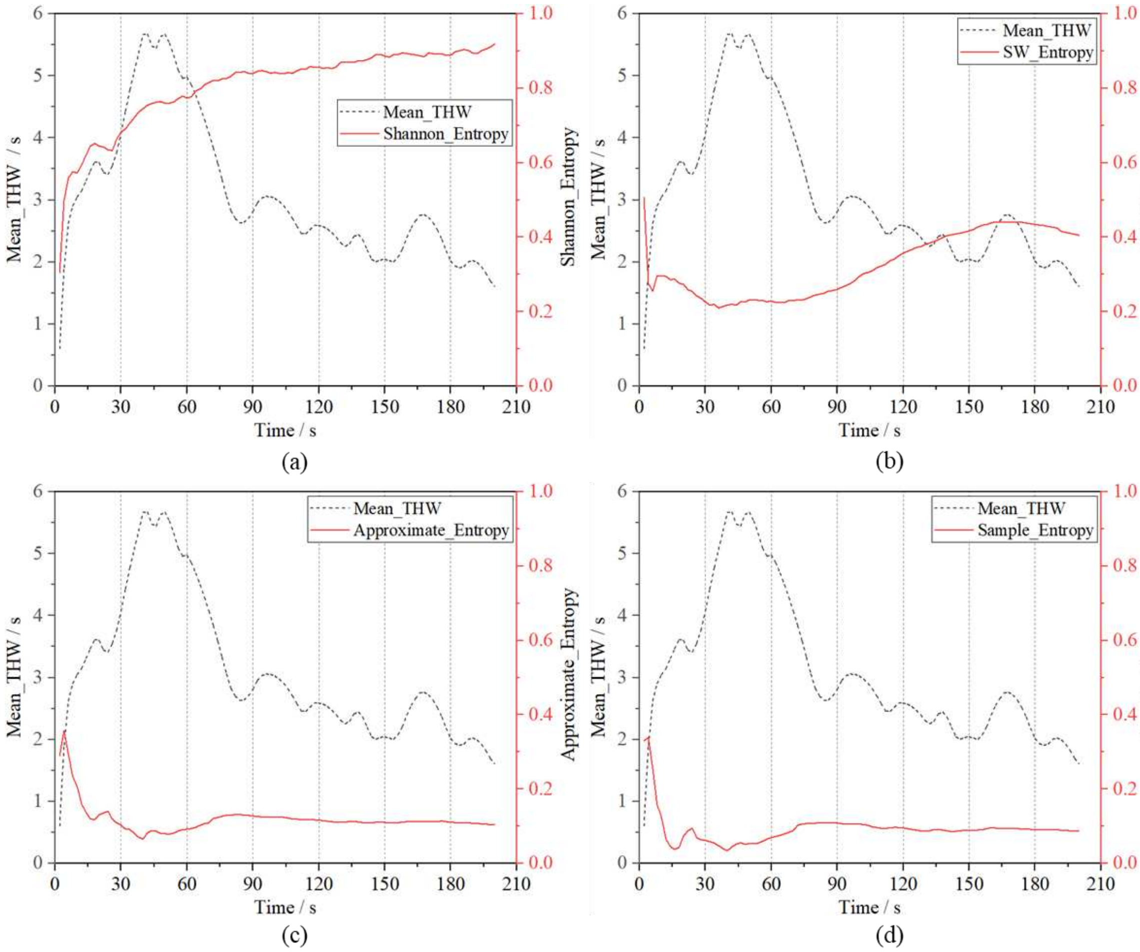

Figure 2 Mean time headway and corresponding entropy of different algorithms in accumulated fixed time window, (a) Shannon Entropy, (b) Steering Wheel Entropy, (c) Approximate Entropy, (d) Sample Entropy. Note: To improve readability, the values of ApEn and SampEn are multiplied by 40.

Entropy in a moving time window

Instead of a fixed time window, a moving window strategy can also be used to detect drivers' behaviour in a comparable length of each segmentation. As this paper focuses on the measurement of drivers' uncertainty in car-following, the fixed time window strategy would assume that the uncertainty or changes in driver behaviour are based on their previous actions during the whole period of car-following, as the calculation involves all the driver's prior time headway data. This approach is unlikely to explain drivers' real behaviour, as control of a vehicle on a 'manoeuvring level' is mainly based on constantly-updated information in a short period (seconds), and not based on long-term information (Michon, 1985). That being said, shorter and moving time windows would theoretically be more suitable, as it accounts just for the relevant information to the driver when it comes to changes in their behaviour. Figure 3 presents the mean time headway and its corresponding entropy of different algorithms with a moving time window of 30 s and a step size of 1 second. Based on the results shown in Figure 3, overall, the moving window strategy seems more appropriate to distinguishing the entropy of the time headway data, as compared to the fixed window strategy.

As a direct implementation of the information entropy, Shannon's Entropy is easy to understand and implement, and it can reveal the randomness to a certain degree. However, according to its definition, Shannon entropy is highly dependent on the distribution of the time series and ignores the order of the values within the series, which makes it barely possible to detect patterns of variation. This can be seen in Figure 3a, where the above-mentioned differences in time headway before and after stabilisation are not reflected in the entropy values. As a comparison, Steering Wheel Entropy (SW entropy) inverts the logic, since it does not handle the raw data directly but tries to calculate the predicted values from the raw data. From Figure 3b, it can be seen that immediately after regaining control, the time headway increases and then decreases linearly, making the pattern less uncertain and easier to predict, which, therefore, results in a low entropy value. Conversely, if the driver's time headway fluctuates more during car-following, the entropy value increases. However, the SW entropy measure does not seem to be sensitive enough to detect small changes in the time headway data, which is not entirely consistent with the result from Nakayama et al. (1999). This entropy algorithm was initially devised for assessing drivers' lateral control of the vehicle, more specifically, the high-frequency steering corrections (Nakayama et al., 1999). As there are more adjustments and fluctuations in steering behaviour, the algorithm will yield higher entropy value, compared with the time headway data in our car-following experiment, which describes the longitudinal accelerating or decelerating behaviour with less uncertainty.

Figure 3c and 3d show the time headway and Approximate Entropy and its updated version, Sample Entropy. Due to their construction, these two entropy algorithms are more generic and independent of the nature of the dataset, ignoring the distribution of the original data and focusing more on the patterns of the series. Additionally, the value of ApEn and SampEn is non-negative and finite for deterministic processes with noise, as the parameter r serves as the noise filter as well. It can be seen from the plot that, using the sliding window strategy, the entropy value changes correspondingly. More specifically, the entropy value increases when there are more new patterns in the computed window, while the entropy value decreases if the period is more predictable and regular. They are sensitive enough to tell the changes in series and can be used to detect variance when drivers change their behaviour. Considering the theoretical definition of uncertainty (Boer, 1999), as unpredictable or unaccounted changes on human behaviour when it comes to their adopted time headway to a lead vehicle, both Approximate and Sample entropy can highlight the moments where variations in the fluctuation pattern happen, represented by sudden spikes in the entropy value. In other words, it is possible to assume that those two algorithms are a good surrogate metric for the degree of uncertainty in drivers' behaviour, with the advantage of being directly quantifiable and replicable for comparison of different experimental datasets, adding scientific value for a previously subjective concept.

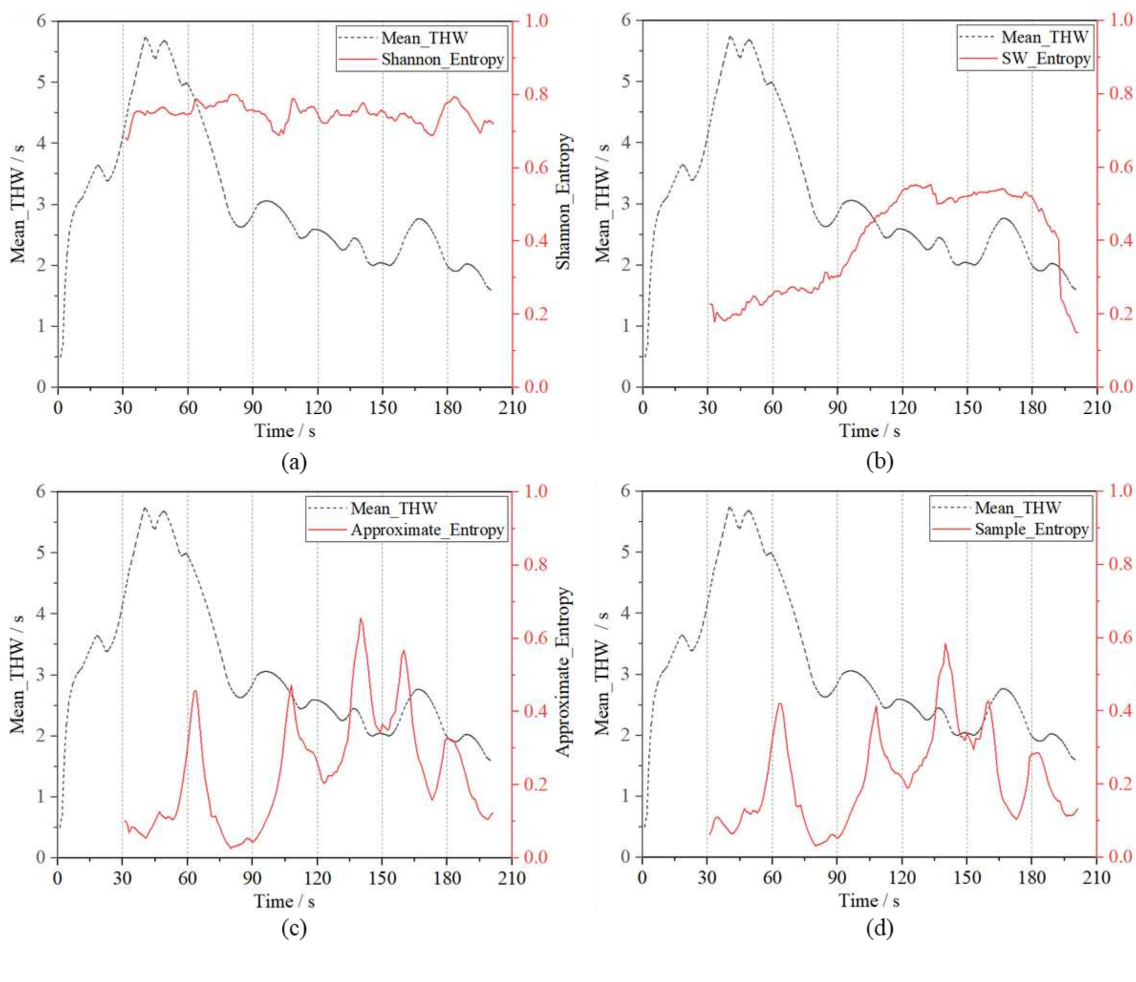

Figure 3 Mean time headway and corresponding entropy of different algorithms in moving time window (window=30s), (a) Shannon Entropy, (b) Steering Wheel Entropy, (c) Approximate Entropy, (d) Sample Entropy. Note: To improve readability, the values of ApEn and SampEn are multiplied by 40.

Figure 4 shows the Approximate Entropy and Sample Entropy in different moving time windows (40 s or 20 s). The motivation for the experimentation with different time windows is that entropy is affected by the patterns seen in the dataset. As car-following datasets are characterised by a constant fluctuation on time headway values, including a larger amount of data might allow the algorithms to identify patterns which could not be seen in smaller samples, as a tradeoff for sensitivity (i.e. the larger the dataset, less likely it is for small changes to make a difference to the entropy values). Based on this observation, it is worthy to note that the size of the time window used for the calculation of entropy must be in line with the experimental design. In datasets with slower fluctuations, a longer time window should be required in order to reveal a pattern in driver's car-following. Conversely, in datasets with more constant fluctuations, a larger time window would affect the variable's sensitivity to highlight uncertainty on the observed pattern.

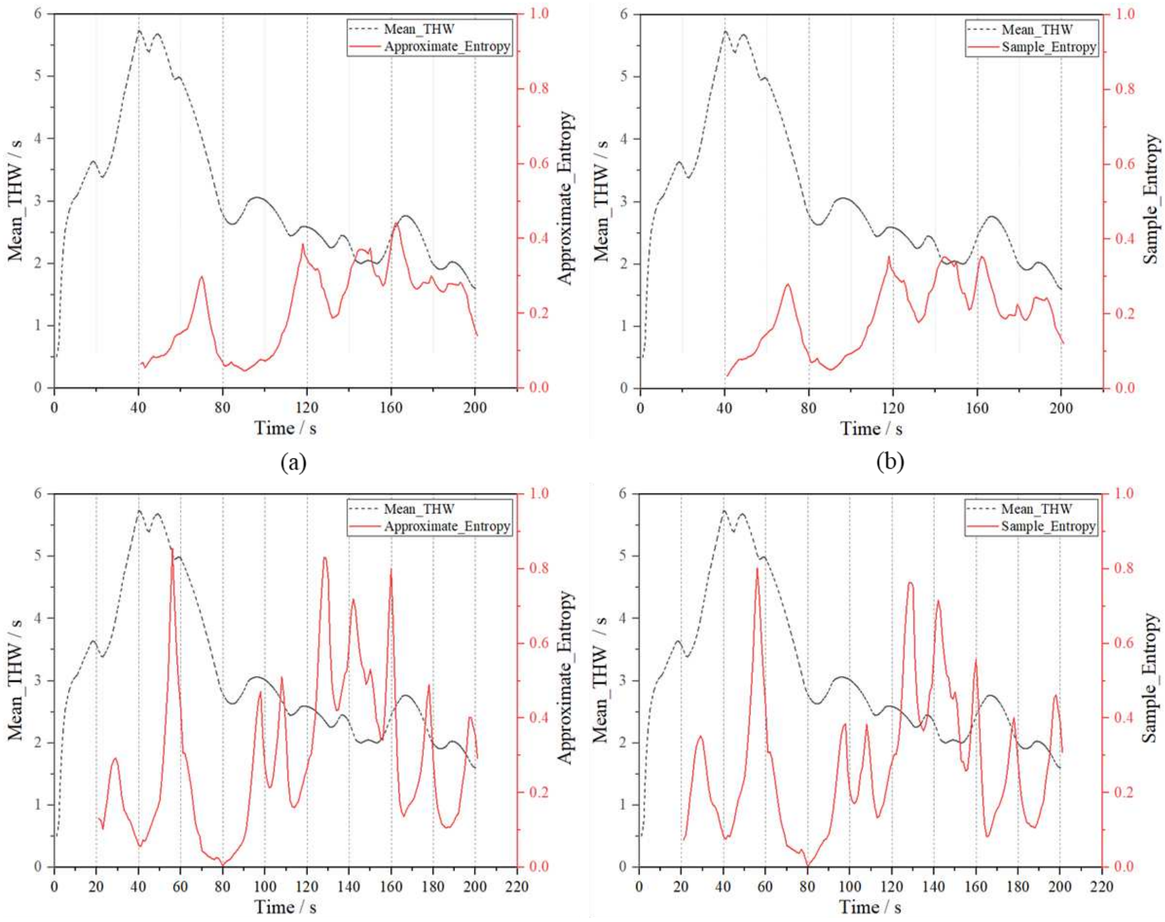

Figure 4 Mean time headway and corresponding entropy of different algorithms in moving time window, (a) Approximate Entropy (window=40s), (b) Sample Entropy (window=40s); (c) Approximate Entropy (window=20s), (d) Sample Entropy (window=20s). Note: To improve readability, the values of ApEn and SampEn are multiplied by 40.

Conclusion

The aim of this paper was to contrast different entropy algorithms as a measure of drivers' uncertainty in car-following. Four different entropy algorithms were applied to a time headway data series, and their results and suitability were contrasted and discussed. Our analysis showed that not all the entropy measures tested were suitable in the context of car-following, especially when it comes to measuring uncertainty in time headway data. Sample and Approximate entropy seem to be the most appropriate, especially when it comes to the moving time windows, as they consider the drivers' prior time headway data as a factor in the perceived uncertainty, and avoid a flattening of the entropy progression as the sample size increases. This paper contributes to the field of human factors and automation, as it demonstrates how entropy can be a precise and replicable measure of changes in behaviour, as well as anomalies in patterns of time headway data. The entropy algorithms used here can be used in future data analysis of time headway datasets, as a proxy to directly access drivers' level of uncertainty during the car-following. However, it is important to note that entropy measures are affected not only by the size of the entire data set but also the size of the time window sample used in the calculation of entropy.

References

Boer, E. R. (1999). Car following from the driver's perspective. *Transportation Research Part F: Traffic Psychology and Behaviour*, 2(4), 201–206.

Delgado-Bonal, A., & Marshak, A. (2019). Approximate entropy and sample entropy: A comprehensive tutorial. *Entropy*, 21(6), 541.

Huang, C. S., Pal, N. R., Chuang, C. H., & Lin, C. T. (2015). Identifying changes in EEG information transfer during drowsy driving by transfer entropy. *Frontiers in Human Neuroscience*, 9(OCTOBER), 1–12.

Michon, J. A. (1985). A critical view of driver behavior models: what do we know, what should we do?. In *Human behavior and traffic safety* (pp. 485-524). Springer, Boston, MA.

Nakayama, O., Futami, T., Nakamura, T., & Boer, E. R. (1999). Development of a steering entropy method for evaluating driver workload. *SAE transactions*, 1686-1695.

Nemoto, H., Yanagishima, T., Taguchi, M., & Wood, J. M. (2002). Driving workload comparison between older and younger drivers using the steering entropy method. *SAE Transactions*, 2040–2047.

Pipes, L. A. (1953). An operational analysis of traffic dynamics. *Journal of Applied Physics*, 24(3), 274–281.

Richman, J. S., & Moorman, J. R. (2000). Physiological time-series analysis using approximate entropy and sample entropy maturity in premature infants Physiological time-series analysis using approximate entropy and sample entropy. *Americal Journal of Physiology Heart and Circulatory Physiology*, 278, H2039–H2049.

Saifuzzaman, M., & Zheng, Z. (2014). Incorporating human-factors in car-following models: A review of recent developments and research needs. *Transportation Research Part C: Emerging Technologies*, 48, 379–403.

Sheu, J. B., & Wu, H. J. (2015). Driver perception uncertainty in perceived relative speed and reaction time in car following - A quantum optical flow perspective. *Transportation Research Part B: Methodological*, 80, 257–274.

Wang, F., Wang, H., & Fu, R. (2018). Real-Time ECG-based detection of fatigue driving using sample entropy. *Entropy*, 20(3), 196.

Wilson, T. D. (1999). Models in information behaviour research. *Journal of Documentation*, 55(3), 249–270.

Xie, H. B., He, W. X., & Liu, H. (2008). Measuring time series regularity using nonlinear similarity-based sample entropy. *Physics Letters, Section A: General, Atomic and Solid State Physics*, 372(48), 7140–7146.

Yu, L., Sun, X., & Zhang, K. (2011, July). Driving distraction analysis by ECG signals: an entropy analysis. In *International Conference on Internationalization, Design and Global Development* (pp. 258-264). Springer, Berlin, Heidelberg.