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## Thermodynamic entropy as an indicator for urban sustainability?

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### Abstract

As foci of economic activity, resource consumption, and the production of material waste and pollution, cities represent both a major hurdle and yet also a source of great potential for achieving the goal of sustainability. Motivated by the desire to better understand and measure sustainability in quantitative terms we explore the applicability of thermodynamic entropy to urban systems as a tool for evaluating sustainability. Having comprehensively reviewed the application of thermodynamic entropy to urban systems we argue that the role it can hope to play in characterising sustainability is less general than has been suggested in the past. We show that thermodynamic entropy may be considered as a measure of energy efficiency, but must be complemented by other indices to form part of a broader measure of urban sustainability.

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### 1. Introduction

The notion of using thermodynamic concepts as a tool for better understanding the problems relating to “sustainability” is not a new one. Ayres and Kneese (1969) [1] are credited with popularising the use of physical conservation principles in economic thinking. Georgescu-Roegen was the first to consider the relationship between the second law of thermodynamics and the degradation of natural resources [2]. Despite the controversial nature of

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Georgescu-Roegen's work, the idea that the second law, in particular the concepts of entropy and exergy, can be successfully utilised to better understand sustainability remains prominent in the literature [3-8].

Cities are highly complex dynamic entities. They form thermodynamically open systems which facilitate huge flows of mass and energy both within the urban system itself, and across its boundaries. They are also responsible for around 75% of global resource consumption, 50% of global waste, and 60-80% of global greenhouse gas emissions [9]. Work by Wolman [10] and others in the study of urban metabolism has emphasised the similarities of cities to living organisms. This thinking naturally leads to the analogy of thermodynamic dissipative systems which maintain their structure through the constant dissipation of entropy [6, 11-13]. Thermodynamics if properly applied can thus have great potential to bring physical rigour and understanding to the impact our cities have on the environment within the upcoming paradigm of sustainability science. But can it be properly applied? Such a complex system as a city is after all very different from an ideal thermodynamic scenario.

In this work we present a review of the literature, identifying where and how thermodynamic entropy and the second law of thermodynamics have been applied to urban sustainability and related concepts. We believe that the relationship of the second law to urban sustainability has not yet been adequately addressed, something which we hope to achieve in this paper.

The term entropy outside of thermodynamics has long been applied in urban contexts, for example, the spatial entropies of Batty [14] and Wilson [15], as well as the Shannon-like entropy measure used as a recycling metric by Rechberger [16]. Whilst these methods present potential avenues and applications of entropy to urban sustainability, they remain broadly separate from the thermodynamic entropy we focus on in this paper.

## 2. Entropy and exergy

We first present a brief outline of the concepts of entropy and exergy, and their definitions within the thermodynamic literature.

### 2.1. Entropy

The concept of entropy is fundamentally rooted in thermodynamics, specifically the second law of thermodynamics, which Clausius stated in 1854 as: "heat cannot by itself pass from a colder to a warmer body" [17]. This law tells us that all real processes are *irreversible*, capturing what Eddington described as the "arrow of time", something hitherto absent from classical physics. The Second Law describes what we know from experience, but how do we quantify the notion of irreversibility?

For a thermodynamic process, the quantity of heat,  $Q$ , absorbed by a system depends on the path the process takes from the initial state of the system A to the final state B. Clausius showed however that division of  $Q$  by the temperature  $T$  at which the heat is supplied, produces a quantity which is path independent, depending only on the initial and final states, he called this quantity entropy,  $S$ . The change in entropy of the system being heated is thus given for a reversible process by:

$$\Delta S = S_B - S_A = \int_A^B \frac{dQ}{T}. \quad (1)$$

For a reversible process the total entropy change of the universe is zero, since the entropy gain of the system being heated is equal to the reduction in entropy of the system from which the heat is transferred. In an irreversible process, work is lost resulting in an additional production of entropy. Thus for an irreversible process, the total entropy change of the universe is always positive. This is a restatement of the second law, sometimes known as

Clausius' inequality,  $\Delta S \geq 0$ , and applies to any isolated system which cannot exchange energy with its surroundings.

For a system that can exchange matter and energy with its surroundings, i.e. one that is not isolated, the system's entropy change consists of the entropy flux due to the exchange of energy and matter with the surroundings  $\Delta S_E$ , and the internal entropy production due to irreversible processes  $\Delta S_I$ . Since the system can exchange energy with its surroundings, Clausius' inequality no longer applies, in this case the second law only means that the internal entropy production has to be greater than or equal to zero,  $\Delta S_I \geq 0$ .

In the 1870s Boltzmann developed statistical mechanics by considering entropy at the molecular level. Each molecule in a system has a set of discretised energy levels, the higher the temperature of the system, the greater the number of energy states that are available to each molecule. Each potential distribution of energy within the system is called a *microstate*. The relation between entropy and the number of microstates,  $W$ , accessible to the specified system at a given temperature and volume, the *macrostate*, is given by Boltzmann's equation:

$$S = k_b \ln W. \quad (2)$$

In this context, an increase in entropy means that the number of accessible microstates increases, and the probability that the system will be found in a certain energy state is reduced.

## 2.2. Exergy

The exergy of a system is defined as the maximum amount of work that may theoretically be obtained from it by bringing the system into equilibrium with an environmental reference state [18]. The concept of exergy has its roots in the early conceptions of classical thermodynamics, and is equivalent to what Gibbs in 1873 described as "available energy", but it wasn't until the 1960s and 1970s that the modern concept of exergy began to take off [19].

The development of exergy was driven by the inadequacies of energy accounting methods in capturing both the transformation of energy, and the concept of energy 'quality', or capacity to cause change [19]. For example 1kJ of electricity has a higher quality, or potential for performing work, than 1kJ of heat at ambient temperature. Exergy analysis is thus a common technique used within engineering to identify the thermodynamic imperfections of various systems by locating processes where exergy is "destroyed" [20, 21].

Kotas [20] distinguishes between four types of exergy relating to possible sources of energy within a system: kinetic, potential, physical, and chemical. Physical exergy relates to processes involving thermal interaction with the environment and is thus related to the system's pressure and temperature. Chemical exergy refers to the chemical properties of the substance in question, such as its chemical potential and concentration relative to the environment.

By definition, exergy requires a reference environment to be specified before any analysis can be performed. This requirement can be problematic, in part due to the lack of consensus on the definition of a reference environment [22, 23]. The difficulties are most prominent when it comes to chemical exergy; some authors prescribe the need for a universal reference environment [22, 24-26] based upon standard compositions of atmosphere, crust etc., however it seems as of yet whilst most methods are similar, no universal method has been adopted. Despite this, exergy remains an important and widely used tool in the application of thermodynamics to the analysis of real world systems.

## 2.3. Exergy vs entropy

The Gouy-Stodola relation [27] tells us that the destruction of exergy and production of entropy are two sides of the same coin:

$$Ex_{destroyed} = T_0 S_{produced} \quad (3)$$

This equivalence means that the two quantities may be used interchangeably given the factor of environmental temperature,  $T_0$ . The concepts of absolute exergy and entropy are however distinct, the exergy of a system can include kinetic and potential energy for example, this is still compatible with equation (3) because as soon as any of this potential energy is used, exergy is destroyed and entropy production occurs.

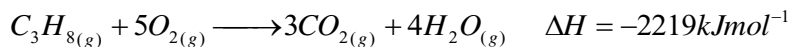
Both entropy and exergy require a reference state. The reference state for entropy is defined such that at absolute zero temperature the entropy of the system is zero, relating to the third law of thermodynamics. In studies of exergy destruction it is only the difference between input and output exergy that is relevant, thus the reference state chosen can be seen as irrelevant since it cancels out.

### 3. Applications of thermodynamic entropy to urban systems

#### 3.1. Degradation of resources

Georgescu-Roegen's 1971 magnum opus "The Entropy Law and the Economic Process" linked the second law of thermodynamics to the function of the economy, which he argued is primarily the transformation of "low entropy" into "high entropy" [2]. This eventually led Georgescu-Roegen to introduce a "Fourth law of thermodynamics" which posits that the complete recycling of matter is theoretically impossible due to the irreversible degradation of resources and an increase of "material entropy" [28]. The idea is that low entropy natural resources enter the economy where they are systematically degraded, and irreversibly transformed into high entropy wastes thereby placing fundamental limits on resource scarcity [29, 30].

This is an attractive idea, since the systems approach of inputs and outputs would allow for direct application to urban systems. We consider Georgescu-Roegen's hypothesis in further detail by analysing a hydrocarbon fuel, a source of high quality energy that is ultimately emitted from the urban system as low grade energy in the form of waste heat. We start with the equation for the combustion of propane [31]:



The enthalpy change of the reaction,  $\Delta H$ , tells us how much energy stored within the propane fuel is released as heat, this release of heat necessitates a change in entropy of the global system. The total entropy change of the global system is given by

$$\Delta S_{system} = (H_r - H_p)/T_0 - (S_r - S_p), \quad (4)$$

where  $H_r$ ,  $H_p$ ,  $S_r$ , and  $S_p$  are respectively the standard molar entropies and enthalpies of combustion. Inserting numbers, and assuming standard conditions, reveals a large increase in entropy [32]:

$$\Delta S_{system} = (7446 - 101)JK^{-1}mol^{-1}. \quad (5)$$

The entropy change of the system contains two contributions: the first accounts for the net heat released by the breaking and reforming of chemical bonds, the second term accounts for the change in standard molar entropy of the molecules due to a change in molecular configurations.

Therefore, it is clear that the economic activity of fuel combustion leads to an increase in entropy of the universe. What is less clear however, is what exactly is meant by description of a transformation of a "low entropy" fuel to a

“high entropy” waste. Firstly the terms low and high are arbitrary without respect to a reference, the material outputs actually have a lower standard molar entropy than the material inputs, hence the negative second term in equation (5). It is only because of the heat released that the total entropy of the system increases. It is therefore incorrect to attribute the entropy produced during the transformation as a property of the combustion products. It would be more appropriate to emphasise the *transformation* as “entropy producing”, rather than describing entropy as a property of the inputs and outputs. Alternatively we can say that the fuels have a “high exergy content”, and combustion is the process by which this can be released.

The concept of degradation of material flows in relation to entropy is the crux of the problems with Georgescu-Roegen’s hypothesis [33, 34]. At first the analogy seems obvious, a city “consumes” material inputs ultimately transforming and degrading them to unusable waste products, this is apparent for fuels, even if entropy is not considered a property of the material inputs. For other material flows where the utility of the material is unrelated to its exergy content or suitability as a fuel, the problems become more visible. Whilst it can be argued that the utility of the material flow is degraded in the human system, the link to thermodynamic entropy is merely an analogy. Thermodynamic entropy cannot account for the loosely defined human concept of ‘utility’, it can only hope to measure the potential for use as a fuel.

The conflation of entropy with “disorder” is to blame for the mistaken belief that thermodynamic entropy production can be observed in relation to macroscopic material flows [35]. A bottle smashed into a thousand pieces for example represents a negligible increase in entropy, simply because on a molecular level, the number of chemical bonds that have been destroyed is insignificant.

To some extent, it can be argued that entropy production can serve as a proxy measure of utility loss [23], but this glosses over the issue first raised by Georgescu-Roegen and subsequently highlighted by many others that entropy categorically fails in accounting for qualitative aspects [35, 36]. Perhaps the clearest example is to point out that the entropy change due to releasing a kilogram of hydrogen cyanide into the atmosphere is roughly equivalent to releasing the same amount of carbon dioxide. Despite these issues, Georgescu-Roegen’s pioneering work on the study of energy and matter within economics has been hugely influential in linking sustainability to thermodynamics [7, 23, 34, 37].

### 3.2. Degradation of the environment

Other authors, particularly within the environmental sciences present entropy production as a form of pollution which is damaging to the environment. Cities are analogous to dissipative structures, a term coined by Prigogine in the 1960s [38] to describe systems which are able to exist in organised states far from thermodynamic equilibrium, in part through the dissipation of energy to their environment. Common examples include hurricanes and living organisms [11]. The implication of this is that cities produce a large amount of entropy which is dumped into the surrounding environs. This is mostly true, cities use large amounts of energy so a large production of entropy is inevitable, what is less clear however is the consequences of this for the surrounding environment itself.

As we have discussed in the previous section, entropy production is almost solely related to flows of energy, it cannot capture material wastes or pollution, and thus the entropy ‘dumped’ into the surroundings is really only waste heat. Whilst the pollution, greenhouse gases, and material waste produced in our cities pose major problems for sustainability, these have little to do with entropy. The Earth itself is a dissipative system, meaning any anthropogenic entropy produced will ultimately leave the Earth system, being radiated into space in the same manner as the natural entropy production which occurs on our planet. Of course it is wrong to disregard urban heat production, indeed the ill-effects of the urban heat island on human and ecosystem health, water quality, and local climate are well documented [39].

The Earth balances a huge net influx of solar energy with an equally large outflow of long wave radiation energy, in order to maintain its temperature and sustain life. With this energy balance comes a massive production of entropy, since solar energy is emitted from the sun at  $T_{sun}$  and then reemitted from the earth at a much lower temperature  $T_E$ , this entropy production is equivalent to  $S_I = Q(1/T_E - 1/T_{sun})$ , where  $Q$  is the solar energy flux incident on the Earth. Piexoto et al (1991) [40] estimate this figure to be roughly  $589\text{mWm}^{-2}\text{K}^{-1}$ . Globally this greatly dwarfs any extra anthropogenic entropy production caused, for example, by the liberation of energy hitherto stored in the Earth's crust for millennia from the combustion of fossil fuels; Weiss (1994) [41] estimates the anthropogenic contribution to be equivalent to roughly 0.03% of natural entropy production.

Despite this, the Earth's energy balance is delicate, so the Earth's anthropogenic heat flux cannot be considered to be negligible. Zhang and Caldeira (2015) [42] show however that the contribution to climate forcing due to anthropogenic heat is small compared to the release of  $\text{CO}_2$  into the atmosphere, representing approximately 1.7% of the radiative forcing attributable to current accumulation of  $\text{CO}_2$ . They also argue that ultimately the forcing due to  $\text{CO}_2$  exceeds that of the anthropogenic heat by a factor of approximately 100,000, since its lifetime in the atmosphere greatly exceeds that of the anthropogenic heat. Thus, whilst the anthropogenic heat flux can present problems at local and regional scales, the production of  $\text{CO}_2$  poses a much more pressing problem in relation to climate change on a global scale.

It is important to emphasise that whilst the anthropogenic entropy production can provide a measure of the anthropogenic heat flux, its focus is perhaps the wrong one. The division by temperature of the heat flux muddies the waters, with hotter cities producing less entropy per Joule of heat relative to cooler cities, the physical implications of this are unclear. Thus rather than focusing on the entropy produced by urban systems, it is more meaningful to focus on the urban outputs that have negative implications for sustainability whether that is anthropogenic heat or pollution, material waste, and greenhouse gases.

#### 4. Efficiency and sustainability

Entropy production and exergy destruction are really just measures of how *efficiently* energy is utilised. This should be evident from our discussion above of "lost work". Indeed historically the main purpose of exergy has been to analyse the inefficiencies of industrial processes [19-21], and studies exist using entropy production in a similar manner, most prominently the work of Bejan [27, 43]. The application of exergy analysis to assess national, regional, and urban energy efficiency also exists in the literature [44-50]. This presents the possibility of bridging the link between entropy and sustainability by employing the production of entropy as a measure of urban energy efficiency.

##### 4.1. Thermodynamic efficiency

It is important to highlight that the use of the term "efficiency" in our context has a very specific definition, applying only to processes which use or result in the production of heat. There are several different types of efficiency in thermodynamics which can lead to confusion, perhaps the most common conception of efficiency is given by:

$$\eta_I \equiv \frac{\text{useful energy output}}{\text{energy input}}. \quad (8)$$

This is often known as the *first law efficiency* [51] since the conservation of energy, as prescribed by the first law of thermodynamics, limits its range between 0 and 1. The second law of thermodynamics however places a fundamental limit on the efficiency of all thermodynamic processes, as it states that the complete conversion of heat to work is impossible. The *second law efficiency* [51, 52] thus gives the ratio between the actual first law efficiency and the 'ideal' first law efficiency that would be obtained from an equivalent perfect reversible process:



$$\eta_{II} \equiv \frac{\eta_I}{\eta_{reversible}}. \quad (9)$$

This is equivalent to the ratio of the actual work output to the ideal reversible work output. In this way the second law efficiency can be linked to lost work, and thus it can be seen that entropy production and exergy destruction provide measures of the second law efficiency. The link to lost work allows the second law efficiency to account for energy quality, and for example can identify how much work is lost in waste heat, something the first law efficiency fails to do.

#### 4.2. Applications to urban systems

The application of exergy analysis to societal systems was pioneered by Reistad (1975) [44] who analysed the efficiency of the US energy system. Subsequent studies have extended this application to analysing the energy efficiencies of other nations and regions [45-48]. Hammond and Stapleton (2001) [46] for example presented an exergy analysis of the UK's energy system by examining both the main energy sources, and a breakdown of energy use by different sectors. An exergetic improvement potential is produced which identifies the combustion and heat transfer processes associated with power generation, transport, and space heating account for most exergy losses within the system.

The application of these same methods to cities presents opportunity to identify and target the major sources of inefficient energy use at the urban scale. Of course the reliance on data at this scale becomes more crucial, since often energy use data at reasonable spatial and sectoral aggregation is either difficult to obtain or non-existent. Where adequate data exists however, an urban exergy analysis can be performed. Balocco et al (2004) [49] analyse Castelnuovo Berardenga a municipality in Italy using exergy methods, building on a previous work considering entropy generation [50]. Using an extended exergy accounting method (EEA) [53], which considers a life cycle approach accounting for "invested exergy", the residential housing stock is analysed for energy efficiency.

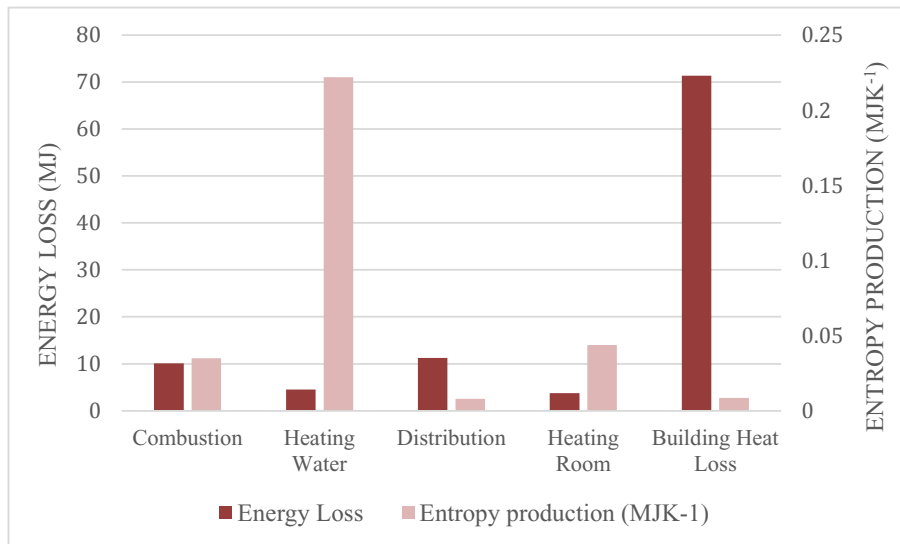
Analysis of additional sectors of energy use would be crucial for a full analysis, studies analysing the efficiency of transport and industrial sectors exist, and indeed these are included in many national exergy accounts (e.g. Hammond and Stapleton). The main issues of obtaining similar analysis for an urban system again rests on data requirements, possible solutions where available data is inadequate depend on what data is available, extrapolation can be made based on previous studies and weighting towards the composition of the city in question. Even if a full urban exergy analysis is not possible, exergy still remains a valuable tool for analysing competing scenarios, particularly for large urban projects such as district heating systems or largescale efficiency installations such as the roll-out of solar PV or insulation schemes.

Whilst the literature tends to focus on exergy analysis, the application of entropy analysis is equally valid even if it is not as common. Since entropy production and exergy destruction provide equivalent measures of efficiency they can be used interchangeably given a fixed value of the environmental temperature. In fact, entropy can be seen as a more fundamental thermodynamic quantity than exergy, and for scenarios where the environmental temperature is difficult to determine, e.g. during short and long wave radiation exchanges, entropy production can be the more applicable of the two. Bejan [27, 43] presents a general method of entropy generation minimisation as a method for targeting and improving on inefficiencies.

We can present a brief example of the benefits of an entropy based efficiency approach by considering a basic steady state space heating model with a conventional gas boiler and hot water distribution system. Fig. 1 presents the breakdown of the energy loss and entropy production that occurs during each energy transfer process that takes place whilst heating a small home, from the combustion of gas in the boiler, to the ultimate loss of heat into the outside environment.



Whilst all the energy is ultimately emitted from the house as heat loss, the production of entropy, exergy destruction, and loss of energy quality does not occur in correlation to the amount of heat lost during each process. Whilst most of the energy loss occurs due to heat loss from the building itself, this process produces little entropy since the temperature gradient between the building's exterior and interior is relatively small. Conversely, most entropy production occurs within the boiler due to the large temperature gradient between the burner gas and the water being heated, despite little energy loss occurring here. This analysis reveals that the greatest cause of irreversibilities and thermodynamic inefficiency is the process of heating water from the combustion flame before being distributed throughout the home. This process is inherent to a gas boiler heating system and thus whilst we can



act to minimise heat losses, we cannot avoid this large proportion of entropy production (although minimising losses does act to reduce the overall heat demand). What this analysis ultimately reveals is that the combustion of fuel for the primary purpose of space heating is fundamentally an inefficient use of energy. Potential methods to circumvent this on an urban scale include the use of district heating networks for space heating which utilise waste heat from industrial processes for example [54].

Fig. 1. Energy loss and entropy production of each process within the heating of a home by a conventional gas boiler, following a simple steady state analysis. An interior temperature of 293K was assumed, and an exterior temperature of 283K. Efficiency data and working temperatures were adapted from Yildiz and Güngör (2009) [55], analysis based on 1 day with average UK energy use for space heating per home [56].

#### 4.3. A link to sustainability?

The link between efficiency and sustainability is often taken for granted and passes unquestioned. Of course a more thermodynamically efficient system implies lower energy use which is clearly more sustainable if it leads to a reduction in fossil fuel consumption. The benefits of a reduction in fossil fuel consumption are multi-faceted, the reduction in the depletion of a finite resource, the reduction in the production of greenhouse gases, and the reduction in pollution released from combustion, all promote sustainability. These factors only apply to a reduction in the combustion of fossil fuels however, the link between the efficiency of renewable energy use and sustainability is less clear. Of course there are financial aspects to consider, especially in regards to renewable energy sources being competitive with fossil fuels, but at face value the efficiency of renewable energy use is not clearly related to sustainability.

Additionally, whilst an increase in efficiency tends to imply a reduction in use, this relationship is often not observed. Jevons' paradox refers to the effect of an increase in efficiency actually leading to an increase in consumption [57]. Generally increase in the efficiency of a resource use means a reduction in the price of the

resource which can lead to an increase in demand, when the increase in demand exceeds the efficiency savings Jevons' paradox is observed. The existence of this effect is somewhat controversial in regards to energy efficiency, however some ecological economists argue that policy must be utilised to counter the effect, with increases in efficiency being accompanied by drives to cut consumption [58, 59].

The analysis presented above suggests that the direct transformation of high quality energy sources to heat is fundamentally wasteful. Possible solutions may include a move towards technologies that minimise the inefficiencies of this transformation such as heat pumps, or systems that utilise waste heat produced elsewhere such as district heating networks. What this also tells us however is that work is not the only form of energy that is deemed a useful output in our cities, heat output is a fundamental requirement in some form. Thus whilst it may be largely desirable to minimise entropy production in our cities, one should bear in mind that a generation of heat and thus entropy can also be desirable, Patterson argues that this is a fundamental oversight of indicators of thermodynamic efficiency [52].

Additionally, retrospective exergy analysis whilst informative is often not as useful in practice as if applied during the design process since most major losses require a superior design, and are thus non-trivial to implement. Walsh and Thornley (2012) [60], through consultation with academic and industrial stakeholders, identified key barriers to improving thermodynamic efficiency, with an emphasis on reclaiming lost heat. The most significant barriers were found to be those related to cost, location, and the availability of infrastructure, with the risks such as halt in production and use of untested technology perceived to far outweigh the benefits. One must therefore bear practical considerations in mind if performing a retrospective analysis.

## **5. Conclusions**

We have shown that the realisation of utilising entropy and the second law of thermodynamics to describe the sustainability of urban systems is marred with problems. The major difficulty of this realisation is the inability of entropy to quantify material flows and degradation in a meaningful way. The move from a linear urban metabolism of separate inputs and outputs, to a more circular metabolism where outputs can be processed and recycled to feed back into the system, is perhaps a key path to achieving resource sustainability. Unfortunately this is not something that entropy can hope to shed light upon.

The failure of thermodynamics to fully capture the human degradation of resources as envisioned by Georgescu-Roegen is in part due to its lack of the notion of 'usefulness' or utility. This is not surprising, as the concept of utility itself is a fuzzy one. Thermodynamics is of course the best tool to tell us the thermodynamic utility of a source of energy, but this is as far as it goes. Likewise entropy fails as a measure to quantify the impact of human waste into the environment. The Earth is open to energy flows, and hence entropy cannot accumulate in the environment and act to degrade it since it is simply radiated into space like all natural heat outputs.

Entropy production, like the destruction of exergy can however be utilised as a measure of thermodynamic efficiency, and it is here that it can provide the most benefit to analyses of urban sustainability. Entropy applied directly in an entropy generation minimisation analysis, or indirectly in an exergy analysis, can identify the major sources of lost work in urban systems, thereby highlighting key opportunities to cut down energy use. Data requirements may mean that in some cases a comprehensive analysis of the whole urban system is difficult to realise, but it may be used on a smaller scale to analyse the thermodynamic efficiency of several competing scenarios in an urban planning phase. Whilst the employment of efficiency measures by itself cannot be considered equivalent to sustainability, their use in tandem with drives to cut consumption, and suitable transitions to renewable energy sources can present a major opportunity to realise a path to energy sustainability.

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