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Non-Cooperative Game Based Congestion Control for Data Rate Optimization in Vehicular Ad Hoc Networks

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Abstract

The growth of connected vehicles in smart cities increases the number of information being communicated on the Internet of Vehicle networks. This causes wireless channel congestion problems, which degrades the network performance and reliability due to the low throughput, high average delay and the high packets loss. Therefore, this paper proposes a non-cooperative game approach to control congestion in the vehicular ad-hoc network channel where the nodes behave as selfish players requesting high data transmission rates. Moreover, the satisfaction of the Nash equilibrium condition for the optimum data transmission rate for each vehicle, is proven. A utility function is introduced based on data transmission rates, the priority of vehicles and contention delay in order to obtain the optimal rates. The performance of the proposed approach has been evaluated and validated in comparison with three others approaches over two testing scenarios for highway and urban traffic. The results show that the network performance and efficiency have been improved by an overall average of 35%, 30% and 37.17% in terms of packets loss, channel busy time and number of collision messages, respectively, as compared with the state-of-the-art-strategies for the highway testing scenario. Similar performance is achieved for the urban testing scenario.

Keywords: Vehicular ad hoc networks, Congestion control, Non-cooperative game approach, Data rate adaptation, IoV applications.

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1. Introduction

Intelligent Transportation Systems (ITSs) utilize Vehicular Ad hoc NETworks (VANETs) to disseminate information among vehicles in the communication networks. This improves traffic mobility and reduces the number of road accidents. VANETs have been employed to provide Vehicle to Vehicle (V2V) and Vehicle to Infrastructure (V2I) communication systems. The Wireless Access of Vehicular Environment (WAVE) [1] has been used by VANETs in order to support the communication among V2V and V2I communication systems. WAVE has been developed in the PHYsical layer (PHY) and the Medium Access Control (MAC) layer from the IEEE 609 and the IEEE 802.11p protocols [2]. This enables the V2V and V2I systems to disseminate traffic ifnormation covering short communication ranges.

VANETs have into two main types of applications: 1) safety applications in which two kinds of messages are sent over the Control CHannel (CCH) [3] - beacon messages and event-driven messages and 2) non-safety applications in which messages are sent through the Service CHannel (SCH) for congested road and parking availability notifications. The wireless channel congestion in VANETs is considered as a key challenge because it affects the transmitted traffic data and the network reliability. This problem occurs once a vehicle disseminates a large volume of data across the network or many vehicles send frequently multiple packets at the same time in a dense environment due to the limited capacity and buffer sizes of the channel. This causes communication overhead and decreases the data delivery ratio of the network. Therefore, the Quality of Service (QoS) is affected, especially the network throughput, delay and packet loss.

In order to control the transmitted data rates in VANETs, this paper proposes a non-cooperative game approach that can resolve the wireless channel congestion problem. A non-cooperative game approach is chosen in this paper due to its ability to provide an analytical model for the communication and decision-making problem in VANETs. Unlike a collaborative game approach, there is no requirement for the players in the game to communicate information relating to the optimization of the data rate. This is advantageous as such communication would further contribute to the solution of the congestion problem. In this paper, every vehicle is expressed as a greedy node and the data transmission rates are optimized. The proposed approach is called Non-Cooperative Game Approach for Congestion Control (NCGACC).

The main contributions of this work are the following:

1. A novel NCGACC data transmission approach is proposed. It can mitigate the channel congestion by adapting the vehicle data rate based on the vehicles' sending rate, maximum contention delay and priorities which are part of the the utility function for every vehicle to achieve the desired fairness. This approach differs from the initial results on a Game Theory Approach for Congestion Control (GTACC) [4], which does not consider any contention delay in the utility function. As illustrated in the performance evaluation provided in this paper, the inclusion of the contention delay in the utility function leads to improved QoS parameters.
2. The existence of a unique pure strategy Nash equilibrium has been proved for the VANETs congestion game.
3. The vehicle's utility function is formulated as a constrained non-linear optimization problem. In the initial work, [4], Lagrange multipliers were employed to determine the optimization problem. However, the addition of the contention delay in the final proposed utility function means this is no longer appropriate. Instead, it is proposed to find the optimal data transmission rates using the Newton-Raphson method for optimization.
4. An extensive performance evaluation is conducted for the proposed approach. This includes testing over both a highway and an urban-based scenario. Comparisons are also made with the following algorithms: GTACC, the Network Utility Maximization and Non-cooperative Beacon Rate and Awareness Control (NORAC) approach. The results show that the proposed approach is adapted to effectively optimize the data transmission rates to mitigate the congestion problem on the VANETs chanel.

The rest of the paper is structured as follows: Section 2 presents an overview of approaches providing solutions to the VANETs congestion control problem. Section 3 formulates the congestion avoidance problem as a non-cooperative game, and the proposed optimization approach is described in Section 4. Its performance evaluation is presented in Section 5. Finally, Section 6 summarizes the main results of the paper.

2. Related Work

The congestion control problem in VANETs has been widely investigated and many approaches have been proposed, such as the transmission power adaptation approaches, the frequency of traffic information adaptation approaches [5], messages scheduling and prioritizing strategies [6], strategies adapting the parameters of the Carrier-Sense Multiple Access with Collision Avoidance (CSMA/CA) [7] and heterogeneous approaches [8]. The focus is on the sending rate adaptation approaches due to their important role on the channel congestion and communication overhead. The adaptation of the sending rate leads to a decrease of the packet delivery ratio and affects the accuracy of the received traffic information.

Beacon adaptation approaches have been proposed in [9] and [10]. The beacon adjustment approach presented in [9] estimates the number of road lanes. Vehicles reduce their data transmission rates when they are driving on multiple lane roads. Nevertheless, this strategy does not consider the actual vehicle density on the road segment and it adjusts the information provision frequency depending on numbers of lanes of the road segment. Therefore, this reduces the accuracy of the traffic information in sparse data scenarios. In [10] another beacon adaptation strategy considers different parameters such as density, direction and status of vehicles in order to adjust the sending rate. This approach considers each of these parameters individually without examining their effects in a single cost function to achieve the ideal sending frequency. Moreover, the unbalanced decline of messages frequency affects the traffic data that should be disseminated between vehicles.

A congestion control strategy called Utility-Based Packet Forwarding and Congestion Control (UBPFCC) [11] is able to adjust the data rate for non-safety applications. This strategy utilizes two parameters (size and cost of packets) to adapt the data frequency. The UBPFCC strategy estimates the average profit value of every vehicle based on the cost function of its transmitted messages and dynamically allocates the remaining data rate to the vehicles.

An Adaptation Beacon Rate (ABR) approach with fuzzy logic control has been proposed in [12] to reduce the congestion on the wireless channel by adapting the message frequency. This approach has utilizes the traffic flow information and the direction of vehicles belonging to the same road section as inputs to the fuzzy controller in order to get the output that is the ideal sending rate. Nevertheless, this strategy has only considered the beacon rate effects and neglects the effect of the event-driven messages on channel congestion once they are generated.

Another approach called LIMERIC [5] utilizes periodic messages as a linear control continuous feedback from the local neighbors. This approach estimates of the channel load and each vehicle updates its beacon frequency based on the variance in error within the calculated channel load and the actual value. LIMERIC assumes that all nodes contain the same channel load which can be considered unreliable due to the attenuation of a signal with various variables such as time, position and radio interference. The convergence of the vehicles to the same data rate is only determined when all the vehicles are in the same range and this cannot be employed for multi-hop scenarios.

Network Utility Maximization (FABRIC) [13] models have been the mathematical support for multiple allocation problems in communication networks especially for congestion control and adaptive routing, or for transmission power allocation in cellular networks, persistence probability optimization in Aloha-type MAC protocols and coordinated transmission in vehicular networks [14].

The FABRIC method [15] requires every vehicle to upload its current data rate and the estimated Lagrange multipliers to its beacon messages. Then vehicles utilize this uploaded data from their neighbors to update their own data transmission rates. However, the data rate has been adopted based only on one parameter and is limited to the vehicle density. This means the channels dynamics and data rate fluctuations have not been considered.

In FABRIC-P approach [16] each node can send its safety messages with a separate set of power levels. This approach estimates each data rate for every power transmission to measure the channel overload. The power transmission value is included in the beacon messages with other information such as data rate, Lagrange multipliers and traffic information. Then this information is sent to other neighbor vehicles to update the data rate. However, uploading such information to the beacon messages might be redundant and can increase the congestion on the wireless channel.

Recently, the NORAC non-cooperative game approach [17] has been introduced. It controls the congestion in the wireless channel based on two parameters: the beacon frequency and the channel busy ratio (CBR). Although the NORAC approach has shown a significant improvement in decreasing the CBR, it still has high packet loss in dense environments. This, in turn, affects the accuracy of the information which should be delivered to the driver in a timely manner.

The problem of the wireless channel congestion control can be solved through centralized or decentralized distributed approaches. The centralized approaches require that the vehicles exchange the data channel congestion information and the price that each vehicle needs to pay due to the channel congestion. The extra exchange of the information increases the channel overhead. On the other hand, the decentralized approaches can solve the problem by disintegrating the primary dilemma into pieces that are determined locally and an original problem such as primal decomposition and dual decomposition is used to reduce information exchanged among vehicles in the network [18]. However, by using these approaches, convergence to an optimal solution may require a long time and the solution in VANETs has to be fast. Therefore, this paper uses the Newton-Raphson method to fast convergence to an optimal solution.

Most of the above mentioned approaches have communication overhead generated due to the extra transferring of the channel information, an inequitable decline of the beacon frequency and the freezing MAC mechanism. All these shortcomings significantly affect the accuracy of the traffic data transmitted to each vehicle driving on the roads network. Therefore, this paper proposes NCGACC to mitigate the congestion problem by adapting the information frequency. In this approach, every node is behaving like a greedy player in the network.

3. The Proposed Approach

3.1. Formulation of the Non-Cooperative Game Approach

Each node in the VANETs transmits its traffic information to the adjacent vehicles or RSUs sharing the transmission range. Then these RSUs or vehicles disseminate the received information to the others nearby vehicles. Therefore, the channel congestion problem occurs in dense vehicular environments once the nodes start to broadcast information periodically at the same time or when the node transmits a high amount of traffic data over the network.

The channel congestion can be identified by using several measurement approaches such as estimating the channel usage levels, calculating messages number of the buffer size and determining the channel busy time [19]. This paper utilizes the channel usage level to discover the data congestion problem as in [20].

In this paper, a non-cooperative game theoretic approach is employed to adapt the data frequency and alleviate the congestion problem in VANETs based on the vehicle's sending rate, contention delay, and vehicle's priority. In the VANETs game, every vehicle is depicted as a greedy player. The optimal solution or the Nash equilibrium [21] is the information frequency or the sending rate for which each player can not enhance its profit by changing its sending frequency while other vehicles transmission frequencies remain constant.

In this game, we consider that each vehicle or RSU has a group of n players in its sending range $V = \{v_1, v_2, ..., v_i, ..., v_n\}$. These nodes contend with each other in order to maintain the channel and disseminate the packets at data rates (actions) $s = [r_1, ..., r_i, ..., r_n]$ to their nearby vehicles. The r_i represents the data frequency or rate of the vehicle v_i. This is given by [4]:

$$r_i = \begin{cases} r_b & \text{only beacon messages,} \\ \{w_1 r_e + w_2 r_b\} & \text{both beacon and emergancy messages,} \end{cases} \tag{1}$$

Here, r_b represents the beacon messages sending rate and the r_e depicts the emergency messages transmission rate. The optimal data rate of r_i^* is equal to the optimal r_b that is computed based on the Newton-Raphson method when no accident occurs on the road segments. However, if the accident occurs the optimal data rate is calculated by using the Newton-Raphson method for each of r_b and r_e individually. This means each application or generated messages rate is computed by using the Newton-Raphson method independently from each other. Then, due to the high priority of the emergency messages, we are using weights (w_1 and w_2) to give high chance (i.e. high data rate) for high priority applications (i.e. emergency messages). Thus, we set $w_2 = 0.7$ (emergency messages) and w_1=0.3 (beacon messages). This leads to a decrease in the load on the wireless channel that is created by sending two applications at the same time. w_1 and w_2 are represent the performance preference parameters and that have been chosen by the designer to meet the framework demands.

Optimizing the transmission rates of vehicles and the RSUs is formulated as a non-cooperative game $G = (V, (S_i)_{i \in V}, (\Theta_i)_{i \in V})$ where the game has the following key components:

- *Players*: We consider V as a set of vehicles where n represents the number of nodes or vehicles that are communicated or participating in the communication range.

- *Strategies*: The actions or strategies act the feasible transmission frequency of traffic information of every player in the network. Every player or vehicle v_i can send at a highest and lowest sending frequency of r_i^{max} and zero, respectively. Therefore, $S_i = [0, r_i^{max}]$ is a group of feasible actions for the node or player i and the Cartesian product of action space for all players is $S = \prod_{i=1}^{n} S_i = [0, r_1^{max}] \times \cdots \times [0, r_i^{max}] \times \cdots \times [0, r_n^{max}]$.

- *Utility function*: The vehicle v_i utility function is specified by Θ_i and it has been utilized to increase the player profit. This can be obtained by optimizing the utility function with respect to r_i.

3.2. Utility Function Formulation

This paper formulates the utility function to consider three elements. The first element is that each vehicle's desire to transmit at as high a data rate as possible is considered in the payoff function. Then contention delay gives the delay generated when many vehicles are transmitting at the same time. Finally, there is also a priority attached to each vehicle determining which vehicles transmits data at a higher rate than the neighbor vehicles. The utility function has the following three main components:

- *Payoff function*: The sending data rate r_i of vehicle (player) v_i has been utilized to evaluate the payoff function $U_i(r_i)$ which is assumed to be a logarithmic function [22]. This is due to its distinctive characteristic of being strictly concave on its domain. Therefore, the cost or payoff function for vehicle v_i is given by [4]:

$$U_i(r_i) = \log(r_i + 1). \tag{2}$$

  Note, to avoid having values of payoff function equal to $-\infty$, + 1 has been added in (2). This is required as r_i can vary between 0 and r_i^{max}.

- *Contention delay*: The contention delay of vehicle (player) v_i has been denoted by $C_i(r_i; c_i)$. This function represents the number of vehicles affected by the data transmission contention along the road segment. The contention delay is calculated as follows:

$$C_i(r_i; c_i) = \frac{N}{r_{in}} - \frac{\tau}{B_o} \tag{3}$$

  where $r_{in} = \sum_{i=1}^{n} r_i$ is the aggregated throughput of all vehicles sharing the transmission range in packets per second, N represents the total number of vehicles are communicated on the road network and n is the number of vehicles that are sharing the communication range. The fixed values τ and B_o are the packet size and maximum allowed bit rates, respectively.

  In order to validate this equation, we assume that the value of τ= 800 bytes and $B_o = 3$ Mbps that is the bit rate. Therefore, the total transmission delay is 213 ms. Then, if the number of vehicles is N= 100 and the data rate $r_i = 20$ packet per second and according to the (3), the contention delay is equal to 4787 ms which is considered a very high value. Therefore, as N increases the contention delay increases and this means the road segment suffer from contention on the wireless channel.

- *Priority function*: The priority function of vehicle (player) v_i has been represented by $P_i(r_i; p_i)$. In order to identify high and low priority vehicles, a penalty has to be paid by each vehicle v_i, based on its transmission rate (r_i) and its priority (p_i). A player with a high p_i value has high priority. Thus, the vehicle v_i priority cost function is given by [4]:

  $$P_i(r_i; p_i) = \frac{r_i}{p_i}, \tag{4}$$

  where p_i is calculated as follows:

  $$p_i = \frac{D_{ij}}{R}. \tag{5}$$

  Here, D_{ij} denotes the gap between the transmitter and the recipient, where i denotes the sender node and j corresponds to the destination node. The sender node is either the RSU or the first vehicle sends the warning messages. This information is already included in the beacon messages and exchanged periodically amongst vehicles which help to avoid the communication overhead. The position of vehicles has been obtained by using a Global Position System (GPS) installed in each vehicle. The R variable represents the range of the communication of vehicle v_i or RSU. Hence, vehicles in the furthest area of the communication range have a larger opportunity to broadcast traffic information. On the other hand, vehicles closer to the original transmitter have a lower superiority, meaning they are less likely to transmit at a high sending rate.

Each player (vehicle) v_i utility function has been modelled as follows:

$$\Theta_i(r_i, r_{-i}) = \alpha_i \log(r_i + 1) - \beta_i[\frac{N}{r_{in}} - \frac{\tau}{B_o}] - \pi_i \frac{r_i}{p_i}. \tag{6}$$

Here, α_i, β_i and π_i are vehicle weight variables of utility functions $U_i(r_i)$, $C_i(r_i; c_i)$ and $P_i(r_i; p_i)$, respectively, where α_i, β_i and $\pi_i > 0$; $\forall i \in V$. The values of α_i, β_i and π_i are chosen by the author to meet the specifications and goals of the system. Note, r_{-i} is the data frequency of all other vehicles except v_i.

The initial work presented in [4] considered a simplified version of this utility function. The contention delay was not considered. As a result the utility function in [4] was given as follows:

$$\chi_i(r_i, r_{-i}) = \alpha_i \log(r_i + 1) - \frac{\pi_i r_i}{p_i}. \tag{7}$$

Proof for the presence of a unique Nash equilibrium for the utility function (6) is provided in Subsection 3.3. The same procedure can also be followed to prove the presence of a unique Nash equilibrium for the simplified utility function given in (7).

3.3. Nash Equilibrium Proof and Existence

In this game dilemma of the channel congestion control $G = (V, (S_i)_{i\in V}, (\Theta_i)_{i\in V})$, a Nash equilibrium (an action profile (sending frequency) $s^* \in S$, where $s^* = [r_1^*, \ldots, r_i^*, \ldots, r_n^*]$) exits if and only if no node (vehicle) has the motivation to ameliorate its profit by adjusting its action, while the actions of all other players stay constant. In this game, the pure-strategy Nash equilibrium is a V-tuple $\{r_i^*\}_{i\in V}$ which satisfies:

$$\Theta(r_i^*, r_{-i}^*) \geq \Theta(r_i, r_{-i}^*)$$

$\forall r_i^*, r_i \in S_i, r_i^* \neq r_i, \forall i \in V$.

To prove that a singular pure-strategy Nash equilibrium exists for the problem under consideration the following theorems are shown to be applicable. Such an approach has been accepted as sufficient for proof of a unique a pure-strategy Nash equilibrium, for example see [21].

Theorem 3.1. *The formulated non-cooperative VANETs congestion game G has at least one pure-strategy Nash equilibrium, if every strategy set S_i is compact and convex and the $\Theta_i(r_i, r_{-i})$ is strictly concave and continuous in S_i.*

It is clear that the strategy vector S_i is compact for all $i \in V$. This is because the strategy vector $S_i = [0, r_i^{max}]$ for all vehicles (player), is closed and bounded.

The set S_i is convex if and only if for any $a, b \in S_i$ and any $\theta = [0, 1]$,

$$0 \leq \theta a + (1-\theta)b \leq r_i^{max}$$

Here, the point $\theta a + (1-\theta)b \in S_i$. Therefore, the set S_i is convex; $\forall i \in V$.

Definition: A twice continuously differentiable utility function Θ_i is strictly concave if and only if $\dfrac{\partial^2 \Theta_i}{\partial r_i \partial r_j} \leq 0 \; \forall i, j \in V$, and the Hessian matrix is Negative Definite (ND) for all $s \in S$.

To prove that the Θ_i is strictly concave, let's define the Hessian matrix of $\Theta_i(s)$, and $s_i = \{r_i, r_{-i}\}_{i \in V}$, as follows:

$$H(s) = \begin{bmatrix} f''_{11} & f''_{12} & \cdots & f''_{1n} \\ f''_{21} & f''_{22} & \cdots & f''_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f''_{n1} & f''_{n2} & \cdots & f''_{nn} \end{bmatrix}, \tag{8}$$

where $f''_{ij} = \dfrac{\partial^2 \Theta_i}{\partial r_i \partial r_j} \; \forall i, j \in V$.

Hence, for all r_i such that $\alpha_i, \beta_i, \pi_i > 0; \forall i \in V$, this is then given by (see Appendix A):

$$f''_{i,j} = \begin{cases} -\dfrac{\alpha_i}{(r_i+1)^2} - \dfrac{2N\beta_i r_i}{(r_{in})^4} & \text{if } i = j; \forall i, j \in V \\ -\dfrac{2N\beta_i r_j}{(r_{in})^4} & \text{if } i \neq j; \forall i, j \in V \end{cases}. \tag{9}$$

Preposition: A matrix is negative definite if and only if k leading principle minors alternate in sign with the odd order ones being < 0 and the even order ones being > 0 [23]. Therefore, it is clear that the leading principal minors [23] and [24] of $H(s)$ are Negative Definite. Thus, the $\Theta_i(r_i, r_{-i})$ is strictly concave in S_i; $\forall i \in V$.

According to [25], the above conditions (in *Theorem* 3.1) are enough to prove that the game G has at least one Nash equilibrium.

Theorem 3.2. *Given a game* $G = (V, (S_i)_{i \in V}, (\Theta_i)_{i \in V})$ *of VANETs channel congestion control, where every action set* S_i *is compact and convex,* $\Theta_i(r_i, r_{-i})$ *is strictly concave and continuous in* S_i*. Let* $\boldsymbol{q} = [q_1, q_2, ..., q_n]$ *be a random vector of constant positive variables and if the Diagonal Strict Concavity (DSC) property is satisfied. Then the formulated game G of VANETs channel congestion problem has a unique pure-strategy Nash equilibrium[21].*

Let the weighted positive sum of the utility functions $\Theta_i(r_i, r_{-i})$; $\forall i \in V$ be given by

$$\psi(r_i, r_{-i}; s) = \sum_{i=1}^{n} q_i \Theta_i(r_i, r_{-i}). \tag{10}$$

Then the pseudo-gradient of $\psi(r_i, r_{-i}; r)$ is estimated by:

$$g(r_i, r_{-i}; q) = \begin{bmatrix} q_1 \nabla\Theta_1(r_1, r_{-1}) \\ q_2 \nabla\Theta_2(r_2, r_{-2}) \\ \vdots \\ q_n \nabla\Theta_n(r_n, r_{-n}) \end{bmatrix} \tag{11}$$

where $\nabla\Theta_i(r_i, r_{-i}) = \dfrac{\alpha_i}{r_i + 1} + \dfrac{N\beta_i}{(r_{in})^2} - \dfrac{\pi_i}{p_i}$, $\forall i \in V$. See Appendix A for derivation of the first partial derivative of the utility function.

Then, the Jacobian matrix $(G(r_i, r_{-i}; q))$ with respect to r_i of $g(r_i, r_{-i}; q)$ is given as follows:

$$G(r_i, r_{-i}; q) = \begin{bmatrix} E_{11} & E_{12} & \dots & E_{1n} \\ E_{21} & E_{22} & \dots & E_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ E_{n1} & E_{n2} & \dots & E_{nn} \end{bmatrix} \tag{12}$$

where $E_{i,j} = q_i f''_{i,j}$; $\forall i, j \in V$.

The symmetric matrix $[G(r_i, r_{-i}; q) + G^T(r_i, r_{-i}; q)]$ is ND for all $r_i, r_{-i} \in S$. Then, the function $\psi(r_i, r_{-i}; q)$ meets the DSC property. Hence, based on Rosen's Theorem [21], the VANETs congestion game G has a unique pure strategy Nash equilibrium.

4. Solution of the VANETs Game

This section describes two solution to the formulated above VANETs game. The first solution relies on an optimization using Lagrangian multipliers as in [4]. The second solution is based on the Newton-Raphson optimization method which is required due to the extra complexity caused by adding the contention delay to the final utility function proposed in this paper.

4.1. GTACC Approach Implementation

In an initial work [4], an approach that is called GTACC has been introduced to alleviate the channel congestion problem in VANETs. The GTACC approach is based on optimizing the data rates using the utility function in (7). This leads to the following maximization problem:

$$\begin{aligned} \underset{r_i \in S_i}{\text{maximize}} \quad & \chi_i(r_i, r_{-i}) \\ \text{subject to} \quad & \sum_{i=1}^{n} r_i \leq C \\ & 0 \leq r_i \leq r_i^{max},\ \forall i \in V, \end{aligned} \tag{13}$$

where, the Maximum Data Load (MDL) is represented by C, which is introduced to avoid congestion of the wireless channel of the network. Here, we assume that the $\mathcal{L}_i(r_i, \lambda_i, \xi_i)$ represents the Lagrangian function of the node (vehicle) i in order to solve (13) by optimizing:

$$\mathcal{L}_i = \chi_i(r_i, r_{-i}) + \lambda_i(C - \sum_{i=1}^{n} r_i) + \xi_i(r_i^{max} - r_i), \tag{14}$$

where λ_i and ξ_i represent Lagrange multipliers. The Karush Kuhn Tucker (KKT) conditions of a node (vehicle) v_i to find the global maximum are as follows [4]:

$$\begin{gathered} \lambda_i, \xi_i \geq 0 \\ r_i \geq 0 \\ r_i^{max} - r_i \geq 0 \\ \nabla_{r_i}\chi_i(r_i, r_{-i}) + \lambda_i \nabla_{r_i}(C - \sum_{i=1}^{n} r_i) + \xi_i \nabla_{r_i}(r_i^{max} - r_i) = 0 \\ \lambda_i(C - \sum_{i=1}^{n} r_i), \xi_i(r_i^{max} - r_i) = 0. \end{gathered}$$

Therefore, a global maximum or the optimal sending frequency solution (r_i^*) for a vehicle v_i; $\forall i \in V$

$$r_i^* = \begin{cases} 0 & \text{if condition 1} \\ r_i^{max} & \text{if condition 2} \\ \frac{\alpha_i p_i}{\pi_i} - 1 & \text{otherwise} \end{cases} \tag{15}$$

where condition 1 and condition 2 are as follows:

$$\frac{\pi_i}{p_i} \geq \alpha_i. \tag{16}$$

$$\frac{\pi_i}{p_i} \leq \frac{\alpha_i}{r_i^{max} + 1} \tag{17}$$

See Appendix B for the derivation of the GTACC optimal solution.

4.2. NCGACC Approach Implementation

This work proposes to solve the considered optimization problem

$$\begin{aligned} \underset{r_i \in S_i}{\text{maximize}} \quad & \Theta_i(r_i, r_{-i}) \\ \text{subject to} \quad & \sum_{i=1}^{n} r_i \leq C \\ & 0 \leq r_i \leq r_i^{max}, \; \forall i \in V, \end{aligned} \tag{18}$$

using the Newton-Raphson method [26] since the formulated utility function cannot be optimized via the Lagrange multipliers and KKT conditions. Under the condition that the utility function is differentiable, the method finds its derivative and sets it to zero. Then, if the function satisfies the assumptions made in the derivation of the utility function and the initial assumption is close, then a better approximation $r_{i,k+1}$ is achieved. The main characteristics of the Newton-Raphson method are fast, root quadratic convergence and easy conversion to multiple dimensions. Algorithm 1 describes the procedure of finding the vehicles optimal data rate r_i^* in VANETs.

Algorithm 1 Newton-Raphson method.

```
*Initialization*:
    Set variables $\alpha_i$, $\beta_i$ and $\pi_i$
    Set $r^{max}$
    Set $r_0$
$k = 0$
Find the value $r_i^*$ of $r_i$ that maximizes $\Theta_i(r_{i,k}, r_{-i,k})$
**while** $\Theta_i'(r_{i,k}, r_{-i,k}) >$ tolerance
    **do**
    $r_{i,k+1} \leftarrow r_{i,k} - \frac{\Theta_i(r_{i,k},r_{-i,k})}{\Theta_i'(r_{i,k},r_{-i,k})}$
    $r_{i,k+1}^* \leftarrow r_{i,k+1}$
    $k \leftarrow k + 1$
**Return** $r_{i,k+1}^*$
```

In VANETs systems, vehicles or RSUs broadcast their data to the nearest vehicles or RSUs that are sharing the range of the communication. However, once a vehicle broadcasts a large amount of traffic data or several nodes begin to disseminate their information simultaneously without considering the channel capacity and the traffic flow conditions, this leads to a data congestion problem in the channel of the VANETs. Here, every vehicle or RSU will check the congestion conditions periodically by estimating the level of the channel usage and compare it with a predefined threshold as in [20]. Similarly, in this paper, the channel usage level threshold is also assumed 70%. Thus, if channel usage level exceeds the threshold, it is assumed that the communication channels face to congestion. After congestion detection, congestion control is carried out by second component of the proposed strategy. When the congestion is identified, the vehicles adapt their data rate as in Algorithm 1. Figure 1 summarizes the steps of the proposed NCGACC approach.

5. Performance Evaluation

5.1. NCGACC Approach Parameters Selection

This section shows the effect of selecting different values of (α_i, β_i and π_i) on the beacon rate and CBR. The α_i represents the preference parameter of sending rate payoff function, the higher

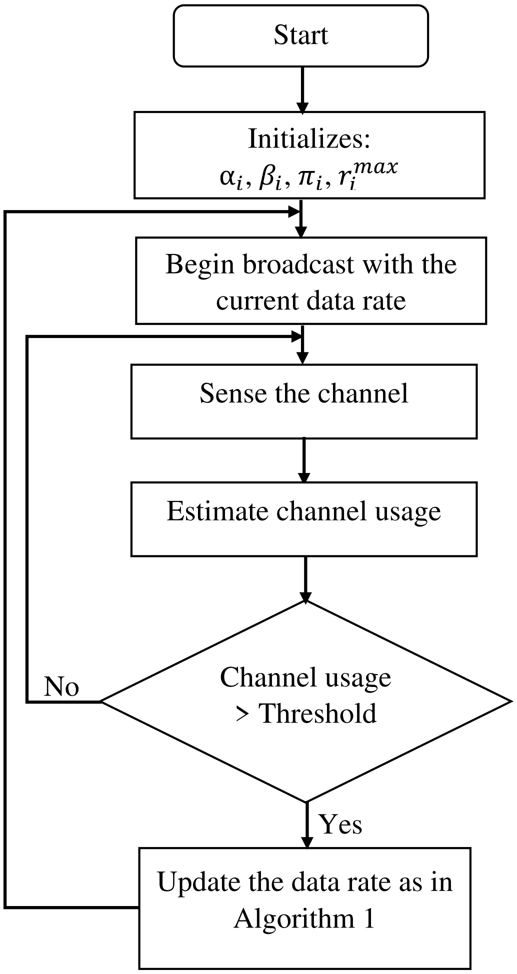

Figure 1: The flow chart of congestion control in VANETs

the α_i value the higher the data rate transmitted by the vehicles. The β_i represents the preference parameter of the cost function in the utility function. This parameter value reflects how much the cost will be comparing to the transmitted information. The π_i represents the weight parameter that plays an important role in identifying of a penalty has to be paid by each vehicle based on its transmission rate. CBR is a parameter that reflects how regularly the wireless MAC channel is busy. Here, a high way scenario with 150 vehicles has been tested and evaluated in order to select the desired parameters that satisfy the system requirements.

Figures 2 and 3 show the effect of changing weights in the cost function on beacon rate and CBR. For example, when β_i and π_i are constant and equal to 5.0 and 2.0 respectively, and α_i has

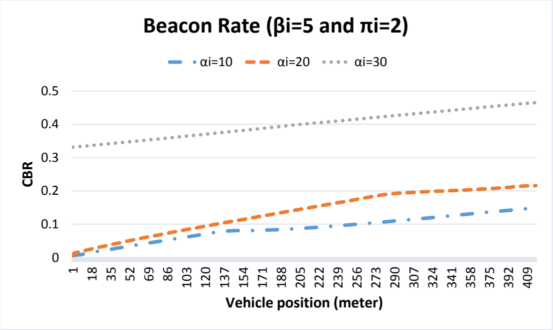

Figure 2: α_i vs CBR

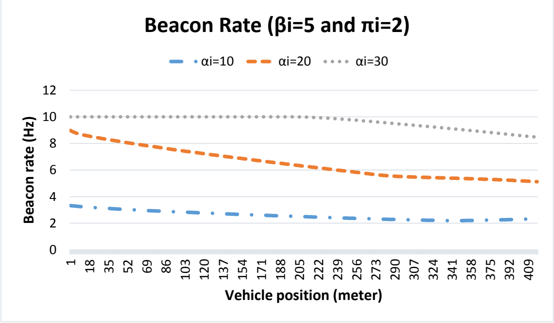

Figure 3: α_i vs data rate

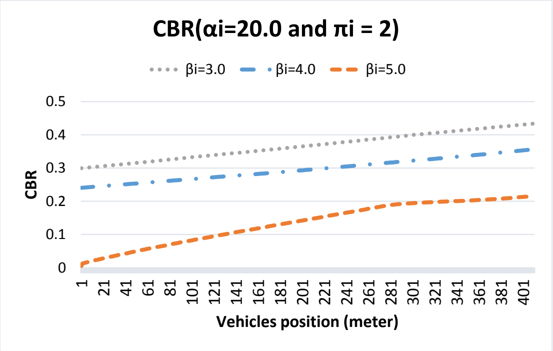

Figure 4: β_i vs CBR

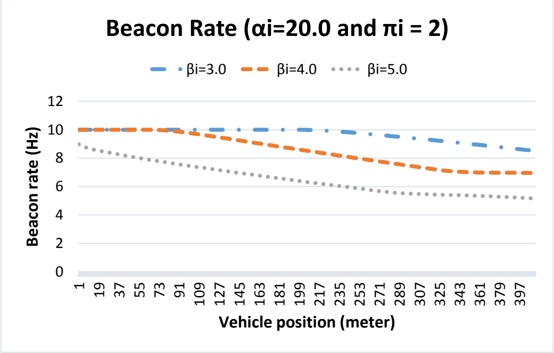

Figure 5: β_i vs data rate

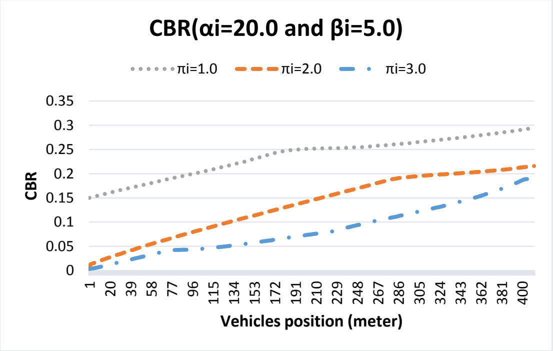

Figure 6: π_i vs CBR

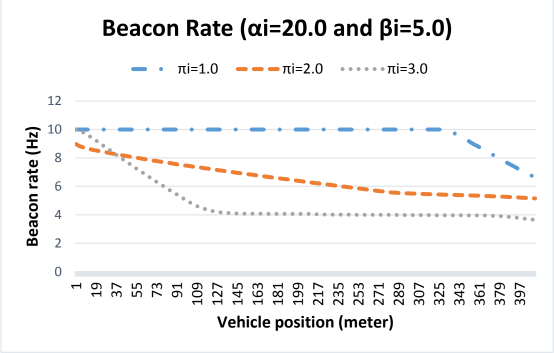

Figure 7: π_i vs data rate

different values of (10.0, 20.0 and 30.0), respectively. It is clear from the Figures 2 and 3 that increasing the values of α_i will increase the vehicle data rate and that will be at the expense of using high bandwidth.

On the other hand, increasing the value of β_i will increase the price of contention which will

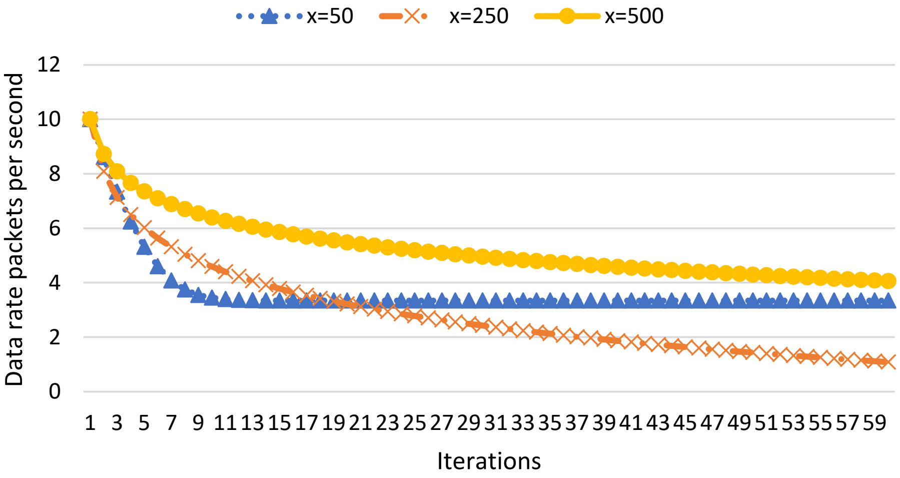

Figure 8: Data rate convergence for different vehicle positions.

decrease the CBR due to use lower data rate as shown in Figures 4 and 5. Finally, Figure 6 and 7 show the effect of changing π_i parameter on the data rate and the CBR when α_i and β_i are constant. It is clear that by increasing value of π_i the data rate and the CBR will decrease and vice versa. In this paper, these values have been chosen in order to reach trade-off among weights and satisfy the congestion requirements.

Figure 8 shows the data rate convergence in every iteration of the proposed algorithm when each vehicle send data at rate of 10 packet per second at the begin of the simulation time. This figure shows the data rate convergence for different vehicle positions at $x = 50$, $x = 250$ and $x = 500$ for same values of α_i, β_i and π_i. It is clear that once the vehicle approach congested area the wireless channel congestion increases and more number of iterations needed to convergence. However,this adaptation approach was sufficient to address requirements of the safety application.

5.2. NCGACC Simulation Results

The vehicular network simulator Veins [27] has been utilized to evaluate and test the proposed approach. This simulator has combined the Simulator for Urban MObility (SUMO) [28] which is responsible for controlling the flow of vehicles on the roadmap with the network simulator OMNeT++ [29] that provides the communication tools for the V2V and V2I systems. Two scenarios have been used to validate the introduced approach(one direction 4-lanes a highway road and two intersections an urban road). The implemented approach has been employed for various vehicle numbers, with sending traffic data frequency being improved in each scenario. All the simulations have been replicated 10 times with different seeds.

Six various measurements have been analysed in this simulation estimation:

- **Average number of sent packets**: It represents the average number of transmitted messages by all the senders in the network simulation.
- **Average number of received packets**: It represents the average number of received messages at all the receivers in the network simulation.
- **Average number of packets loss (Number of packets)**:It represents the average total numbers are lost either in the MAC buffer or in the wireless channel.
- **Average number of collision**: Indicates the average number of collision in the wireless channel during sending of messages.
- **Average channel busy time**: Indicates the wireless channel busy time within a given interval.

The behaviour and performance of the proposed NCGACC approach have been tested over two test scenarios (highway and urban traffic) and compared to the initial results reported in [4], FABRIC and NORAC approaches which are implemented as in [16] and [17], respectively.

5.3. A Highway Scenario

In this scenario, a high way road that includes four lanes with the one direction traffic flow has been performed in SUMO to test the performance evaluation of the proposed approach as displayed in Figure 9.

Table 1 presents the simulation parameters that have been employed over the implemented scenarios, where the vehicle speeds have been determined by the authors based on background with similar dilemma instances and utilizing the guide of the U.K. road laws. The values of α_i, β_i and π_i in Table 1 are selected to provide a proper trade-off among the optimization criteria regarded in the utility function. In essence, they decide the relative importance of the terms in the utility function. For example, if the value of π_i is increased it means the weighting of priority function is increased, meaning it is given more importance in the optimization. Note, from experience changing these values has an impact on the values of the utility function. However, the position of the optimal value of the utility function remains constant for a range of parameter values.

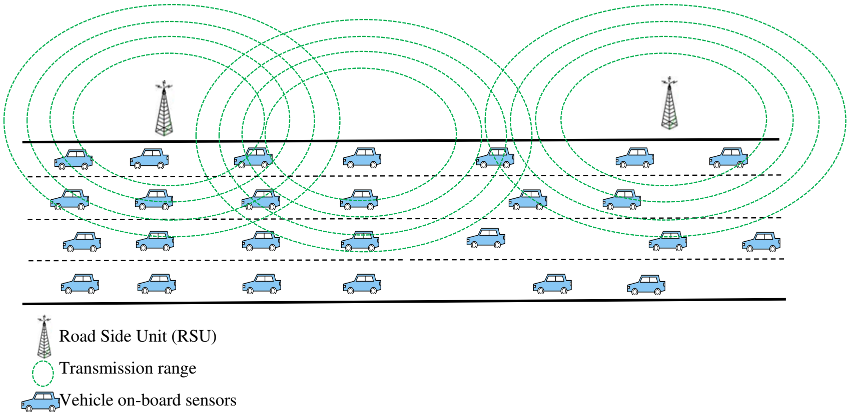

Figure 9: A highway scenario considered in SUMO.

Table 1: Configuration parameters for the implemented examples

Simulation parameters	**Value**
Map dimension	1000 m highway scenario, and 650 m × 1000 m urban scenario
Vehicles speed	22-34 m/s highway scenario, and 13-27 m/s urban scenario
Number of vehicles	50, 70, 90, 110, 130, 150
Simulation time	200 s
MAC/PHY	IEEE 802.11p
Transmission range	300-1000 m
Transmission rate	3-27 Mbps
Bite rate	6 Mbps
Message size	600 Bytes
Safety messages data rate	10 packet/s
Maximum iterations	60
α_i	20
β_i	5
π_i	2
w_1	0.7
w_2	0.3

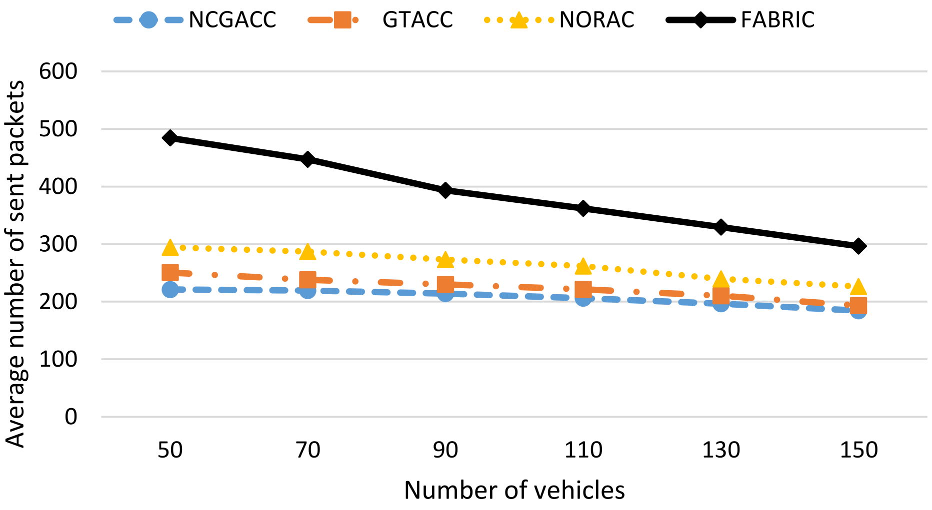

Figure 10: Total average sent packets in the highway scenario

5.3.1. Average Number of Sent Packets

Figure 10 presents the obtained results of total average sent packets recorded by the four tested approaches. Figure 10 shows that with an increment in the number of vehicles the average sent messages decreases. It is obvious that the average sent of NCGACC approach is the most stable one as compared to the GTACC, FABRIC, and NORAC. Using NCGACC, GTACC, NORAC, and FABRIC approaches, the total sent packets for 150 of vehicles is 129.13, 151.42, 191.94 and 261.67 packets, respectively. The recorded results depict that the developed approach better than the other approaches and it is able to achieve a better performance in VANETs. This is because the NCGACC adjusts the transmission frequency of messages by choosing the optimal value as well as regards the contention delay and priorities of vehicles once the channel congestion is detected. On the other hand, the FABRIC, GTACC, and NORAC do not consider the contention delay in their optimization in order to alleviate the channel congestion of the VANETs. This generates many messages being ready for transmission within the network, especially during peak transmission times.

5.3.2. Average Number of Received Packets

Figure 11 describes the contrast of the number of vehicles with the recorded average received packets. It is obvious that received packets has a proportional relationship with the number of sent

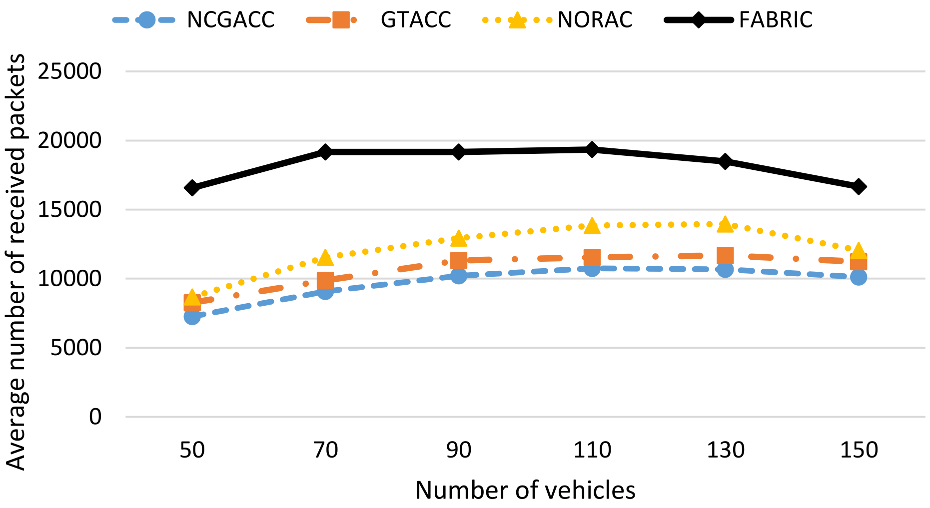

Figure 11: Total average received packets in the highway scenario

packets in the network. This is because increasing number of sent packets will increase the number of received at the destination nodes. However, many vehicles will start to compete among each other in order to access the wireless channel and send data at a maximum frequency which causes a long waiting time and delay in the received information at the destination nodes. The results reveal that the received packets in the NCGACC approach is smaller than the GTACC, NORAC and FABRIC approaches but without a significant impact on the accuracy of the received information. Additionally, it is clear when there is an increase in the number of vehicles, the NCGACC does not have an obvious jump in the recorded results of the average received packets. This is due to the sending frequency has been optimized based on contention delay and vehicles priorities to reach the optimal maximum of transmission rates. This leads to reducing the required time of delivering messages at the destination.

5.3.3. Average Number of Packets Loss

Figure 12 demonstrates the average number of lost messages in the highway scenario due to wireless channel congestion problem. It is clear that the NCGACC approach has less average number of lost messages as compared with the GTACC, NORAC, and FABRIC approaches. This is due to using an adaptive sending rate and choosing the optimal rates by considering contention delay in its utility function that improves the communication and decreases the competition among

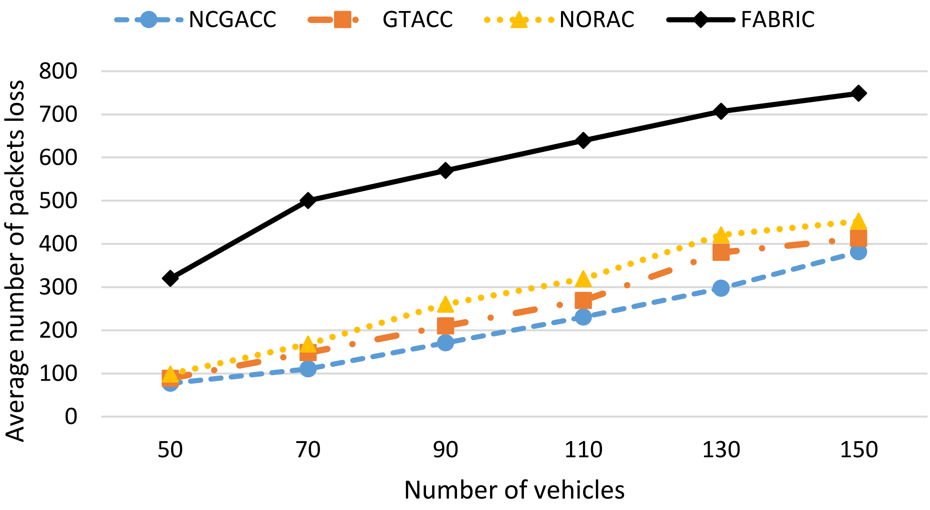

Figure 12: Total average lost packets in the highway scenario

the nodes to maintain the channel in VAENTs. This minimizes messages loss, despite the number of vehicles being communicated. Nevertheless, the FABRIC has several lost messages due to optimizing only non-safety messages. This causes an extra overhead communication once the safety messages are generated which increases the congestion in the wireless channel.

5.3.4. Average number of collision

Figure 13 shows the variations of the average number of collision with the utilized number of vehicles in this scenario. It depicts when the number of vehicles increases the collision among packets increases for all tested strategies. However, the NCGACC does not show a significant increase in the collision. This is due to the fact that it considers the contention delay parameter as a term in its optimization. Additionally, it can be seen that when 150 vehicles are considered in the simulation the collision avoidance for the proposed NCGACC approach is 126.27. It is worth noting that this is smaller than the value for the comparison approaches.

5.3.5. Average Channel Busy Time

Figure 14 shows the channel busy time variations with the utilized number of vehicles in this scenario. It is clear from Figure 14 that the channel busy time results are directly related to the number of sent messages. The channel busy time decreases by decreasing number of sent packets

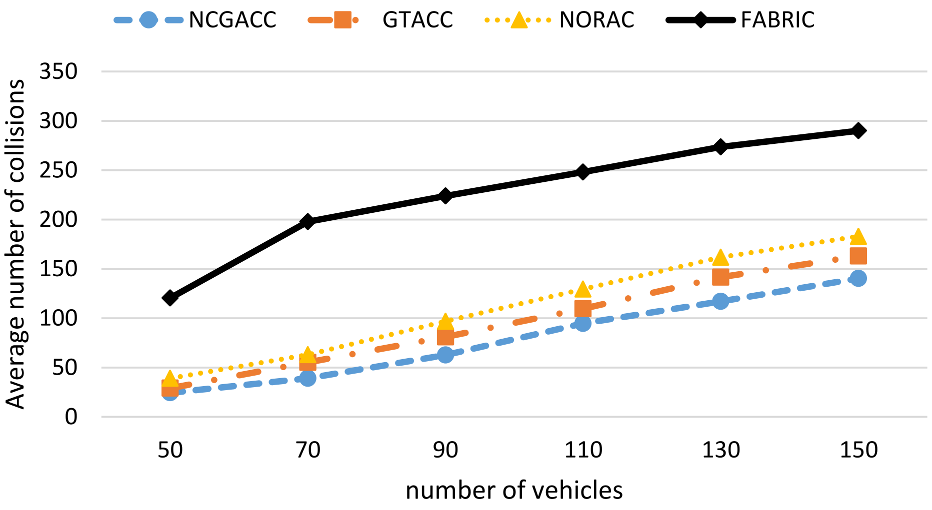

Figure 13: Collision number in the highway scenario

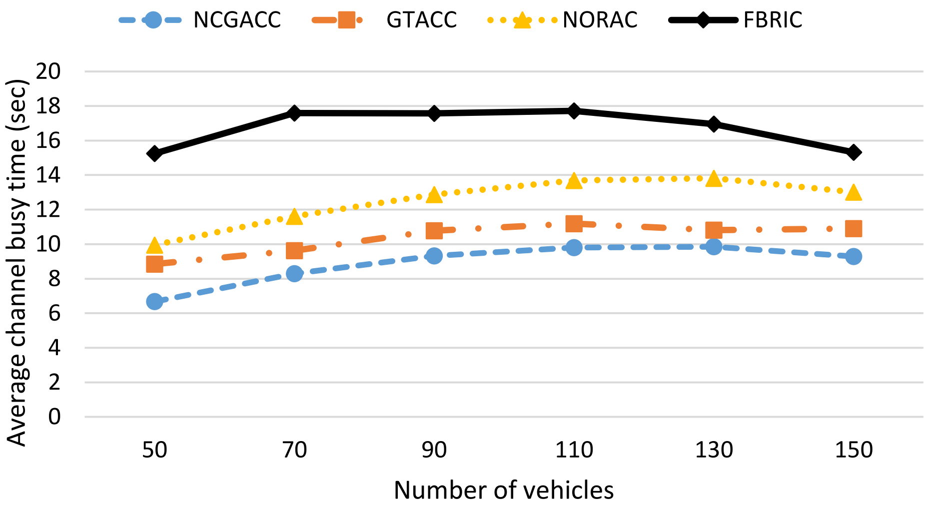

Figure 14: Channel busy time in the highway scenario

in the network. It is clear that the performance of the proposed approach in terms of channel busy time is better than that of the comparison approaches. This improvement in performance has been achieved by considering three parameters in the optimization process as compared to one and two parameters in FABRIC, GTACC and NORAC, respectively.

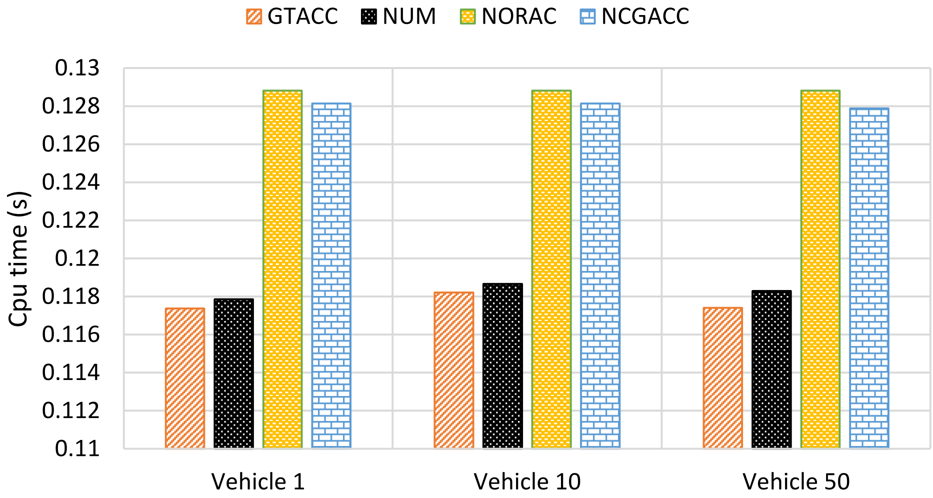

Figure 15: CPU time in the highway scenario

5.3.6. CPU Time

Figure 15 illustrates the average CPU time that has been estimated for a random number of vehicles for all tested approaches. It is obvious from Figure 15 that the GTACC and FABRIC require less CPU time as compared to NORAC and NCGACC. However, there is not a significant difference in CPU time among all tested approaches. NCGACC has a slight increase in computation time as compared to GTACC and FABRIC. This is due to the iterative nature of the Newton-Raphson method and the extra term (contention delay) in the utility function that is optimized. However, the increase has not been significant enough to affect the performance of the algorithm and real time application is still possible.

5.4. An Urban Scenario

An urban traffic scenario has been generated in SUMO to validate and examine the developed approach as shown in Figure 16. The parameters for this scenario are the same as the highway scenario (with the exception of the dimension of the problem area and speeds of the vehicles) and are summarized in Table 1.

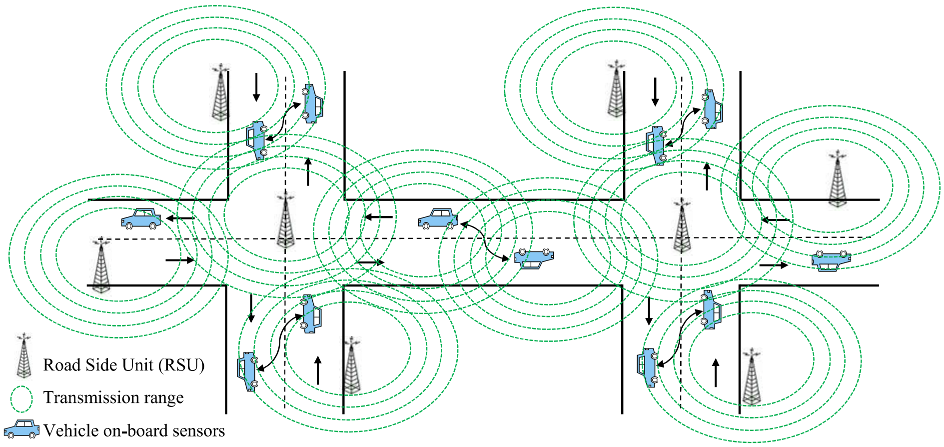

Figure 16: An urban scenario considered in SUMO

5.4.1. Average Number of Sent Packets

Figure 17 presents the obtained average sent packets by FABRIC, GTACC, NORAC, and NCGACC, respectively. It is clear that the NCGACC approach has significantly adapted the total sent messages as compared to the FABRIC, GTACC, and NORAC. This helps to decrease the channel overhead and alleviate the wireless channel congestion. The NCGACC adjusts the transmission rate of messages by choosing the optimal value by considering three parameters sending rate of vehicles, contention delay and vehicles priorities once the channel congestion is detected. On the other hand, the FABRIC, GTACC, and NORAC do not consider the contention delay in their optimization in order to alleviate the channel congestion of the VANETs. This generates many messages being transmitted at the same time in the network, which causes a data collision problem and increases the channel congestion.

5.4.2. Average Number of Received Packets

Figure 18 describes the received packets variation at the destination nodes against the number of vehicles. The increasing of the number of connected nodes on the road segment increases number of transmitted messages across the wireless communication channel. However, this increases contention among vehicles to access the wireless channel which in turn affects significantly the channel busy time. The results reveal that the NCGACC approach has less received messages as compared to the FABRIC, GTACC, and NORAC, respectively. However, this has not have a significant impact on the information accuracy.

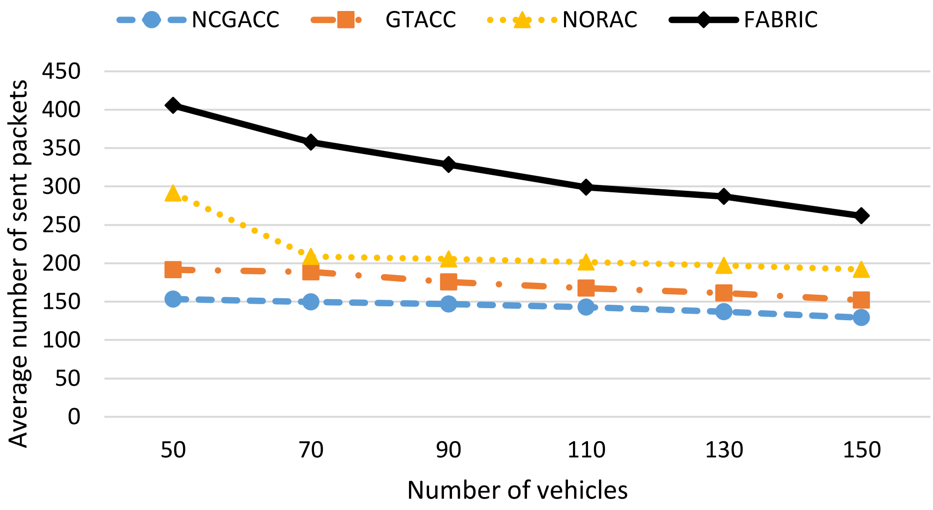

Figure 17: Total average sent packets in the urban scenario

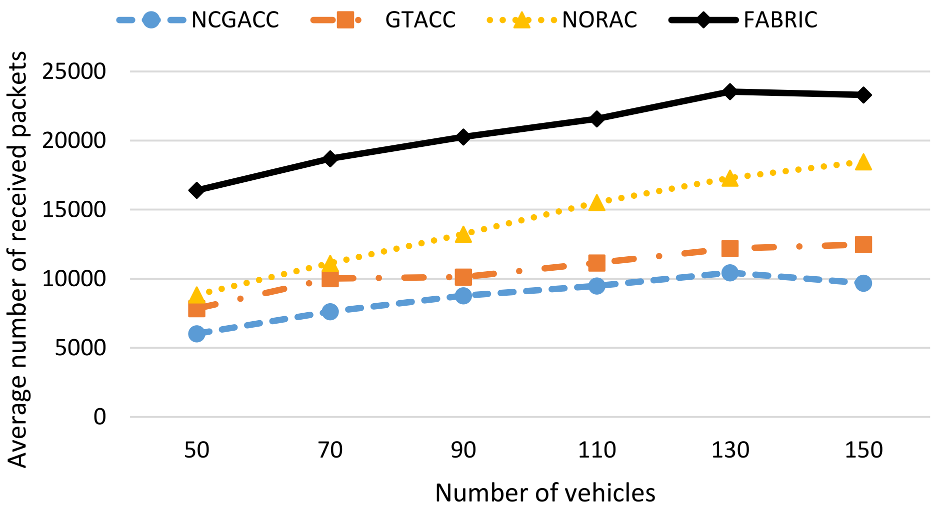

Figure 18: Total average received packets in the urban scenario

5.4.3. Average Number of Packets Loss

Figure 19 demonstrates the number of lost messages in the highway scenario due to wireless channel congestion problem. It is clear that the NCGACC approach has less number of lost messages as compared with the GTACC, NORAC, and FABRIC approaches. This is due to using an adaptive sending rate and choosing the optimal rates by considering contention delay in its utility function that improves the communication and decreases the competition among the nodes to maintain

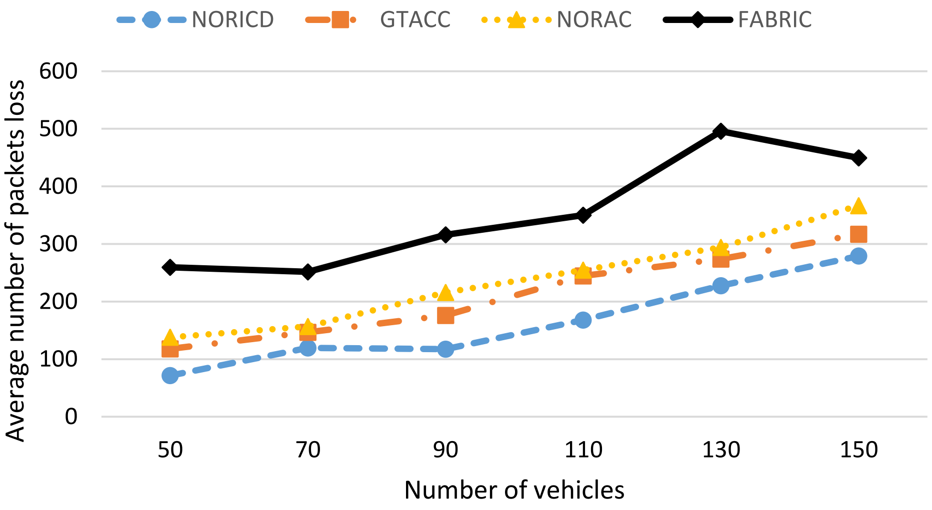

Figure 19: Total average of lost packets in the urban scenario

the channel in VAENTs. This minimizes messages loss, despite the number of vehicles being communicated.

5.4.4. Average Number of Collision

Figure 20 shows the variations of collision with the utilized number of vehicles in this scenario. It is similar to the highway scenario when the number of vehicles increases the collision number increases for the tested FABRIC, GTACC, and NORAC approaches. However, the NCGACC does not show a significant increase in the collision messages. This is due to the fact that it considers the contention delay parameter as a term in its optimization.

5.4.5. Average Channel Busy Time

Figure 21 shows the channel busy time variations with the utilized number of vehicles in this scenario. The channel busy time decreases by decreasing number of sent packets in the network. It is clear from the Figure 21 that the performance of the proposed approach in terms of channel busy time is better than that of the comparison approaches. This is due to the improvement of the average sent packets, average lost packets and number of collision via the improved approach for avoiding wireless channel congestion.

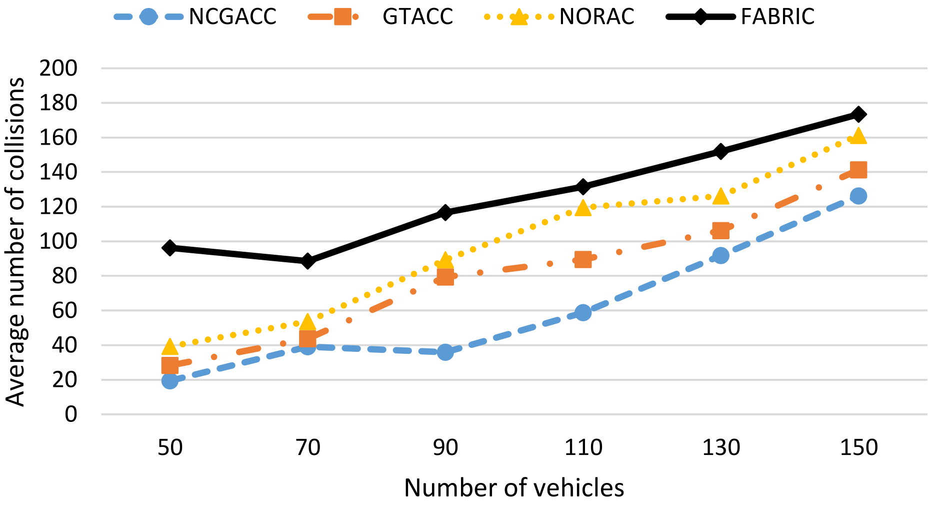

Figure 20: Collision number in the urban scenario

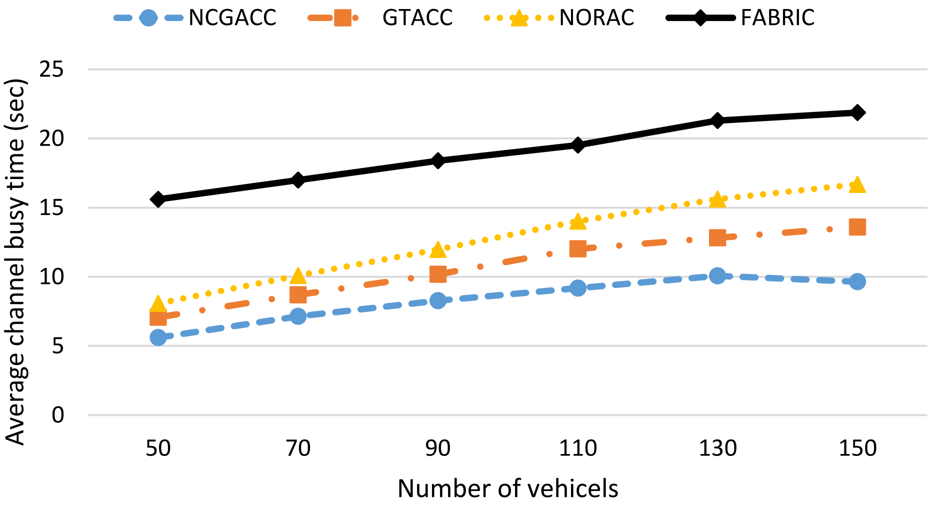

Figure 21: Channel busy time in the urban scenario

6. Conclusion

The number of transmitted messages has a proportional relationship with the number of vehicles being communicated on a road network. Therefore, sending many messages at a high rate in dense vehicular environments leads to a wireless channel congestion problem. This paper develops a new approach to alleviate the channel congestion problem in VANETs by using a non-cooperative game theory. Every vehicle behaves like a player in the game and demands a high information flow in a greedy behavior. Additionally, the utility function is based on three factors (sending rate, contention delay and priority of each vehicle). Then, the Newton-Raphson method has been used to give the optimal transmission rates. Simulation results reveal that the performance of NCGACC is better as compared to FABRIC, GTACC and NORAC strategies, respectively. As stated from the highway scenario, it is revealed that the developed approach enhances the Quality of Service elements such as packets loss, channel busy time and number of collision messages by an overall average of 35%, 30% and 37.17%, respectively as compared to FABRIC, GTACC and NORAC strategies. In future works, we will consider the mobility of vehicles and investigate its effect on the disconnection and data rate of vehicles. For future works, other networks such as 5G or Long Term Evolution-Vehicle (LTE-V) side-link are another Internet of vehicles protocols for supporting vehicular communication systems can be investigated. Scenarios such as London city map can be applied to test and evaluate the used approach.

Appendix A. Derivation of NCGACC Utility Function

- The first derivative of $\Theta_i(r_i, r_{-i})$ with respect to r_i is:

$$\begin{aligned} \frac{\partial\Theta_i}{\partial r_i} &= \frac{\alpha_i}{r_i+1} + \beta_i[\frac{(r_{in}\times 0)-(1\times N)}{(r_{in})^2}] \\ -\frac{\pi_i}{p_i}, & \\ &= \frac{\alpha_i}{r_i+1} + \frac{\beta_i\times N}{(r_{in})^2} - \frac{\pi_i}{p_i}. \end{aligned} \tag{A.1}$$

- The first derivative of $\Theta_i(r_i, r_{-i})$ with respect to r_j is:

$$\begin{aligned} \frac{\partial\Theta_i}{\partial r_j} &= \frac{\alpha_i\times 0}{r_i+1} + \beta_i[\frac{(r_{in}\times 0)-(1\times N)}{(r_{in})^2}] - \frac{\pi_i\times 0}{p_i}, \\ &= 0 + \frac{\beta_i\times N}{(r_{in})^2} - 0 \\ &= \frac{\beta_i N}{(r_{in})^2}. \end{aligned} \tag{A.2}$$

- Here, taking the second partial derivative of $\Theta_i(r_i, r_{-i})$ in (A.1) and (A.2) gives:

1. Taking the second partial derivative of $\frac{\partial \Theta_i}{\partial r_i}$ with respect to r_i we will have the following equation:

$$\begin{aligned} \frac{\partial^2 \Theta_i}{\partial r_i \partial r_j} &= \frac{((r_i+1)\times 0)-(\alpha_i \times 1)}{(r_i+1)^2}+ \\ &\frac{(r_{in}\times 0)-(\beta_i N \times 2r_i}{(r_{in})^4}-0, \\ &= -\frac{\alpha_i}{(r_i+1)^2}-\frac{2\times \beta_i \times N \times r_i}{(r_{in})^4} \\ &= -\frac{\alpha_i}{(r_i+1)^2}-\frac{2N\beta_i r_i}{(r_{in})^4}. \end{aligned} \tag{A.3}$$

2. Taking the second partial derivative of $\frac{\partial \Theta_i}{\partial r_j}$ with respect to r_j we will have the following equation:

$$\begin{aligned} \frac{\partial^2 \Theta_i}{\partial r_i \partial r_j} &= \frac{((r_i+1)\times 0)-(\alpha_i \times 0)}{(r_i+1)^2}+ \\ &\frac{(r_{in}\times 0)-(\beta_i N \times 2r_j}{(r_{in})^4}-0, \\ &= 0-\frac{2\times \beta_i \times N \times r_j}{(r_{in})^4} \\ &= -\frac{2N\beta_i r_j}{(r_{in})^4}. \end{aligned} \tag{A.4}$$

From equations (A.3) and (A.4) we will have equation (9) as follows:

$$f''_{i,j} = \begin{cases} -\frac{\alpha_i}{(r_i+1)^2}-\frac{2N\beta_i r_i}{(r_{in})^4} & \text{if } i=j; \forall i,j \in V \\ -\frac{2N\beta_i r_j}{(r_{in})^4} & \text{if } i \neq j; \forall i,j \in V \end{cases}. \tag{A.5}$$

Appendix B. Derivation of GTACC optimal solution

The problem in (14) has three unknowns (r_i, λ_i and ξ_i). In order to solve the problem, three cases are considered based on complementarity conditions:

Case 1: $r_i = 0$ and $\xi_i = 0$:

$$\alpha_i - \frac{\pi_i}{p_i} + \lambda_i = 0$$

$$\lambda_i = \frac{\pi_i}{p_i} - \alpha_i.$$

The solution $r_i = 0$ is feasible, if the condition ($\lambda_i \geq 0$) holds and it is as follows:

$$\frac{\pi_i}{p_i} \geq \alpha_i \qquad \text{gives condition 1}$$

<u>Case 2</u>: $r_i = r_i^{max}$ and $\lambda_i \geq 0$:

$$\frac{\alpha_i}{r_i^{max}+1} - \frac{\pi_i}{p_i} - \xi_i = 0$$

$$\xi_i = \frac{\alpha_i}{r_i^{max}+1} - \frac{\pi_i}{p_i}.$$

The solution $r_i = r_i^{max}$ is feasible, if the condition ($\xi_i > 0$) holds and it is as follows:

$$\frac{\pi_i}{p_i} \leq \frac{\alpha_i}{r_i^{max}+1} \qquad \text{gives condition 2}$$

<u>Case 3</u>: $\lambda_i \geq 0$, $\xi_i = 0$ and ($0 < r_i < r_i^{max}$)

$$\frac{\alpha_i}{r_i+1} - \frac{\pi_i}{p_i} = 0$$

$$r_i = \frac{\alpha_i p_i}{\pi_i} - 1.$$

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