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Decentralized event-triggered control of large-scale systems with saturated actuators

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Abstract—We consider a large-scale LTI system with multiple local communication networks connecting sensors, controllers, and actuators. The local networks operate asynchronously and independently of one another. The main novelty is that the decentralized controllers are subject to saturation. Our objective is to achieve a regional exponential stability providing a decentralized bound on the domain of attraction for each plant. We introduce a sampled-data event-triggering mechanism from sensors to controllers to reduce the amount of transmitted signals. Using the time-delay approach to networked control systems and appropriate Lyapunov-Krasovskii functionals, we derive linear matrix inequalities that allow to find the decentralized bounds on the domains of attraction for each plant. Numerical example of coupled cart-pendulums illustrates the efficiency of the method.

I. INTRODUCTION

Networked Control Systems (NCSs) are systems with spatially distributed sensors, actuators, and controllers that exchange data over a communication channel [1]. It is important to provide a stability and performance certificate that takes into account the network imperfections (such as variable sampling intervals, variable communication delays, etc.). Three main approaches have been used to study NCSs: a discrete-time [2], hybrid [3], and time-delay system approaches [4], [5], [6].

It is common place in industry that the total plant to be controlled consists of a large number of interacting subsystems [7]. Usually the control of the plant is designed in a decentralized manner with local control stations allocated to individual subsystems. In networked control of large-scale systems it is more efficient to use local controllers and local networks instead of the global ones. This leads to large-scale NCSs with independent and asynchronous local networks.

Decentralized networked control of large-scale interconnected systems with local independent networks was studied in the framework of hybrid systems [8], [9], [10], where variable sampling or/and small communication delays (meaning that they are smaller than transmission intervals) were taken into account. In [10] decentralized dynamic event-triggering mechanism was introduced to reduce the workload of the communication networks. To manage with large communication delays (that may be larger than the sampling intervals) in the presence of scheduling protocols from sensors to actuators, the *time-delay* approach to continuous-time decentralized NCS was suggested in [11] in the continuous-time and in [12] in the discrete-time setup.

For practical application of control laws, actuator saturations should be taken into account. This often leads to local results, where it is important to find a bound on the domain of attraction starting from which solutions of the closed-loop system asymptotically converge to zero. For large-scale systems with decentralized control laws, a decentralized bound on the domain of attraction for each plant should be an essential part of the design procedure. Such decentralized bounds have not been considered in the existing literature yet.

In the present paper, for a large-scale LTI system with multiple local communication networks (connecting sensors, controllers, and actuators) that operate asynchronously and independently of one another, we consider decentralized controllers that are subject to actuator saturation. Our objective is to achieve a regional exponential stability providing a decentralized bound on the domain of attraction for each plant. We introduce a sampled-data event-triggering mechanism from sensors to controllers to reduce the amount of transmitted signals. Using the time-delay approach to networked control systems and appropriate Lyapunov-Krasovskii functionals, we derive linear matrix inequalities that allow to find decentralized bounds on the domains of attraction for each plant. Numerical example of coupled cart-pendulums illustrates the efficiency of the method.

For simplicity (in order to avoid the bounds on the initial time interval [13] in the case of actuator saturation), we do not consider communication delays. Without actuator saturation, event-triggered decentralized control presented in this paper can be easily extended to the case of large communication delays by using standard Lyapunov-Krasovskii functionals for systems with time-varying delays [4].

Notations: $\mathbb{N}_0 = \{0\} \cup \mathbb{N}, P > 0$ means that $P \in \mathbb{R}^{n \times n}$ is symmetric and positive definite, the symmetric elements are denoted by *. For $K_j \in \mathbb{R}^{m_j \times l_j}, K_j^{(i)}$ denotes the *i*th row. For $0 < \bar{u} \in \mathbb{R}$, we define $\operatorname{sat}(u, \bar{u}) = \operatorname{sign}(u) \min(|u|, \bar{u})$. Given $\bar{u}_j = (\bar{u}_{1j}, \dots, \bar{u}_{mj})^T$, we denote $\operatorname{sat}(u_j, \bar{u}_j) = (\operatorname{sat}(u_{1j}, \bar{u}_{1j}), \dots, \operatorname{sat}(u_{mj}, \bar{u}_{mj}))^T$.

Lemma 1 (Wirtinger inequality [14]): Let $a, b, \alpha \in \mathbb{R}$,

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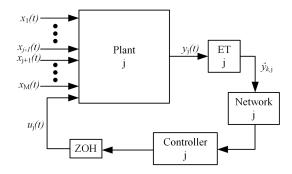


Fig. 1. Schematic representation of the jth subsystem

 $0 \leq W \in \mathbb{R}^{n \times n}$, and $f: [a, b] \to \mathbb{R}^n$ be an absolutely continuous function with a square integrable first derivative such that f(a) = 0 or f(b) = 0. Then

$$\int_{a}^{b} e^{2\alpha t} f^{T}(t) W f(t) dt \\ \leq e^{2|\alpha|(b-a)} \frac{4(b-a)^{2}}{\pi^{2}} \int_{a}^{b} e^{2\alpha t} \dot{f}^{T}(t) W \dot{f}(t) dt.$$

II. SYSTEM DESCRIPTION

We consider M interconnected systems described by (Fig. 1)

$$\dot{x}_{j}(t) = A_{j}x_{j}(t) + B_{j}u_{j}(t) + \sum_{\substack{i=1\\i\neq j}}^{M} F_{ij}x_{i}(t),$$

$$y_{j}(t) = C_{j}x_{j}(t), \qquad j = 1, \dots, M,$$
(1)

where $x_j \in \mathbb{R}^{n_j}$ are the states, $u_j \in \mathbb{R}^{m_j}$ are the control inputs, and $y_j \in \mathbb{R}^{l_j}$ are the measurements. For each $j = 1, \ldots, M$, we assume that

$$\exists K_j \in \mathbb{R}^{m_j \times l_j}: \quad A_j + B_j K_j C_j \text{ is Hurwitz.}$$
 (2)

That is, $u_j = K_j y_j$ stabilizes the system $\dot{x}_j = A_j x_j + B_j u_j$. We assume that only sampled in time measurement $y_j(t_{k,j})$ are transmitted to the controller, where $0 = t_{0,j} < t_{1,j} < \cdots$ are the sampling instants of the *j*th subsystem subject to

$$\lim_{k} t_{k,j} = \infty, \qquad t_{k+1,j} - t_{k,j} \le h_j$$

for $k \in \mathbb{N}_0$, $j = 1, \ldots, M$. To reduce the amount of transmitted measurement, we incorporate an event-triggering mechanism [15], [16]. The idea is to transmit only those measurements $y_j(t_{k,j})$ whose relative change is larger than some threshold, namely,

$$\hat{y}_{k,j} = \begin{cases} y_j(t_{k,j}), & (4) \text{ is true,} \\ \hat{y}_{k-1,j}, & (4) \text{ is false,} \end{cases}$$
(3)

where $\hat{y}_{k,j}$ are the transmitted measurements and the eventtriggering rule is given by

$$[y_{j}(t_{k,j}) - \hat{y}_{k-1,j}]^{T} \Omega_{j} [y_{j}(t_{k,j}) - \hat{y}_{k-1,j}] \\ \geq \sigma_{j}^{2} y_{j}^{T}(t_{k,j}) \Omega_{j} y_{j}(t_{k,j})$$
(4)

with $0 < \Omega_j \in \mathbb{R}^{l_j \times l_j}$ and $0 \le \sigma_j \in \mathbb{R}$.

In the next section, we derive the stability conditions for the system (1) under the saturated event-triggered control

$$u_j(t) = \operatorname{sat}(K_j \hat{y}_{k,j}, \bar{u}_j), \quad t \in [t_{k,j}, t_{k+1,j}),$$
 (5)

where $\bar{u}_i \in \mathbb{R}^{m_j}$ are the saturation levels.

III. REGIONAL STABILIZATION UNDER SATURATION

The control signals (5) can be presented as

$$u_j(t) = K_j y_j(t) - K_j v_j(t) - K_j e_{k,j} - \psi_{k,j}, t \in [t_{k,j}, t_{k+1,j}),$$

where $k \in \mathbb{N}_0$, j = 1, ..., M, and the errors $v_j(t)$, $e_{k,j}$, $\psi_{k,j}$ are given by

$$v_{j}(t) = y_{j}(t) - y_{j}(t_{k,j}), \quad t \in [t_{k,j}, t_{k+1,j}), e_{k,j} = y_{j}(t_{k,j}) - \hat{y}_{k,j}, \psi_{k,j} = K_{j}\hat{y}_{k,j} - \operatorname{sat}(K_{j}\hat{y}_{k,j}, \bar{u}_{j}).$$
(6)

Then the closed-loop system (1), (5) takes the form

$$\dot{x}_{j}(t) = (A_{j} + B_{j}K_{j}C_{j})x_{j}(t) - B_{j}K_{j}v_{j}(t) - B_{j}K_{j}e_{k,j} - B_{j}\psi_{k,j} + \sum_{\substack{i=1\\i\neq j}}^{M}F_{ij}x_{i}(t).$$
(7)

The "nominal" systems $\dot{x}_j = (A_j + B_j K_j C_j) x_j$ are stable by (2). The remaining terms are treated as disturbances. To compensate the errors due to sampling $v_j(t)$, we consider the functional

$$V(t) = \sum_{j=1}^{M} V_j(t), \quad V_j(t) = V_j^P(t) + V_j^W(t), \quad (8)$$

where

$$\begin{split} V_{j}^{P}(t) &= x_{j}^{T}(t)P_{j}x_{j}(t), \\ V_{j}^{W}(t) &= h_{j}^{2}e^{2\alpha h_{j}}\int_{t_{k,j}}^{t}e^{-2\alpha(t-s)}\dot{y}_{j}^{T}(s)W_{j}\dot{y}_{j}(s)ds, \\ &-\frac{\pi^{2}}{4}\int_{t_{k,j}}^{t}e^{-2\alpha(t-s)}v_{j}^{T}(s)W_{j}v_{j}(s)ds, \quad t \in [t_{k,j}, t_{k+1,j}) \end{split}$$

with positive definite $P_j \in \mathbb{R}^{n_j \times n_j}$ and $W_j \in \mathbb{R}^{l_j \times l_j}$. The piecewise continuous in time terms V_j^W are taken following [17], [14]. Note that the Wirtinger inequality (Lemma 1) guarantees $V_j^W \ge 0$. Moreover, the functionals $V_j(t)$ do not grow at the jumps since $V_j^W(t_{k,j}) = 0$, whereas $V_j^P(t)$ is continuous in time. Therefore

$$V_j(t) - V_j(t^-) \le 0, \quad t \ge 0.$$
 (9)

The event-triggering errors $e_{k,j}$ will be compensated using

$$0 \le \sigma_j^2 (y_j(t) - v_j(t))^T \Omega_j (y_j(t) - v_j(t)) - e_{k,j}^T \Omega_j e_{k,j},$$
(10)

which follows from (3), (4).

The errors due to saturation $\psi_{k,j}$ will be compensated using the following lemma.

Lemma 2 (Generalized sector condition [18]): Define

$$\mathscr{S}_{j} = \left\{ \hat{y}_{k,j} \in \mathbb{R}^{l_{j}} \left| |(K_{j}^{(i)} - G_{j}^{(i)})\hat{y}_{k,j}| \le \bar{u}_{ij}, i = 1, \dots, m_{j} \right\}$$
(11)

with some $G_j \in \mathbb{R}^{m_j \times l_j}$. If $\hat{y}_{k,j} \in \mathscr{S}_j$ then

$$\psi_{k,j}^{T} S_{j} \left[G_{j} \hat{y}_{k,j} - \psi_{k,j} \right] \ge 0$$
(12)

for any positive definite diagonal matrix $S_j \in \mathbb{R}^{m_j \times m_j}$.

To compensate the terms $F_{ij}x_i$ representing the interconnections, we will use the following lemma, which extends the results of [19]:

Lemma 3: For $0 < \epsilon < \alpha$ let V_j defined in (8) satisfy

$$\dot{V}_{j}(t) + 2\alpha V_{j}(t) - \sum_{i \neq j} \frac{2\epsilon}{M-1} V_{i}(t) \le 0,$$

 $t \ne t_{k,j}, \quad j = 1, \dots, M, \quad k = 0, 1, \dots$ (13)

Then V defined by (8) satisfies

$$V(t) \le e^{-2\delta t} V(0), \quad t \ge 0, \quad \delta = \alpha - \epsilon.$$
 (14)

Moreover, if $V_j(0) \leq \beta$ for j = 1, ..., M with $\beta > 0$, then

$$V_j(t) < \beta \left(1 + \frac{\epsilon M}{\alpha(M-1)} \right), \quad t \ge 0.$$
 (15)

Proof 1: We have

$$\dot{V} + 2\delta V \stackrel{(8)}{=} \sum_{j=1}^{M} \left[\dot{V}_j + 2\alpha V_j - \frac{2\epsilon}{M-1} \sum_{i \neq j} V_i \right] \stackrel{(13)}{\leq} 0.$$

The latter inequality together with (9) yields (14) implying

 $V(t) \le V(0).$

If $V_j(0) \leq \beta$ then

$$\sum_{i \neq j} V_i(t) \stackrel{(8)}{\leq} V(t) \leq V(0) \stackrel{(8)}{\leq} M\beta.$$
(16)

By the comparison principle,

$$V_{j}(t) \stackrel{(13)}{\leq} e^{-2\alpha t} V_{j}(0) + \frac{2\epsilon}{M-1} \int_{0}^{t} e^{-2\alpha(t-s)} \left[\sum_{i \neq j} V_{i}(s) \right] ds$$

$$\stackrel{(16)}{\leq} \beta + \frac{\epsilon M}{\alpha(M-1)} \beta.$$

Now we are in a position to formulate our main result.

Theorem 1: Consider the system (1) under the eventtriggered saturated control (5) with sampling periods h_j , event-triggering thresholds σ_j , and saturation levels \bar{u}_j , where $j = 1, \ldots, M$. For given tuning parameters $0 < \epsilon < \alpha$ and matrices $G_j \in \mathbb{R}^{m_j \times l_j}$ let there exist positive definite matrices $P_j \in \mathbb{R}^{n_j \times n_j}, W_j \in \mathbb{R}^{l_j \times l_j}, \Omega_j \in \mathbb{R}^{l_j \times l_j}$, positive definite diagonal matrix $S_j \in \mathbb{R}^{m_j \times m_j}$, and scalars $\rho_j > 0$ such that for $j = 1, \ldots, M$,

$$P_j \le \rho_j I \tag{17}$$

$$\begin{bmatrix} P_j & C_j^T \left(K_j^{(i)} - G_j^{(i)} \right)^T \bar{u}_{ij}^{-1} \\ * & 1 \end{bmatrix} \ge 0, \quad i = 1, \dots, m_j,$$
(18)

$$\begin{bmatrix} \mathcal{P}_j & \mathcal{F}_j \\ * & \mathcal{E}_j \end{bmatrix} \le 0, \tag{19}$$

where

$$\mathcal{F}_{j} = \operatorname{row} \left\{ \begin{bmatrix} P_{j}F_{ij} \\ 0 \\ 0 \\ 0 \\ h_{j}e^{\alpha h_{j}}W_{j}C_{j}F_{ij} \end{bmatrix} \right\}_{\substack{i=1,\dots,M\\ i\neq j}}$$
(20)

$$\mathcal{E}_j = \frac{-2\epsilon}{M-1} \operatorname{diag} \left\{ P_1, \dots, P_{j-1}, P_{j+1}, \dots, P_M \right\}, \quad (21)$$

and \mathcal{P}_j are the symmetric matrices composed from

$$\begin{split} \mathcal{P}_{j}^{11} &= P_{j}(A_{j} + B_{j}K_{j}C_{j}) + (A_{j} + B_{j}K_{j}C_{j})^{T}P_{j} + 2\alpha P_{j}, \\ \mathcal{P}_{j}^{12} &= \mathcal{P}_{j}^{13} = -P_{j}B_{j}K_{j}, \\ \mathcal{P}_{j}^{14} &= -P_{j}B_{j} + (S_{j}G_{j}C_{j})^{T}, \\ \mathcal{P}_{j}^{15} &= \sigma_{j}C_{j}^{T}\Omega_{j}, \\ \mathcal{P}_{j}^{16} &= h_{j}e^{\alpha h_{j}}(A_{j} + B_{j}K_{j}C_{j})^{T}C_{j}^{T}W_{j}, \\ \mathcal{P}_{j}^{22} &= -\frac{\pi^{2}}{4}W_{j}, \\ \mathcal{P}_{j}^{24} &= \mathcal{P}_{j}^{34} = -G_{j}^{T}S_{j}, \\ \mathcal{P}_{j}^{25} &= -\sigma_{j}\Omega_{j}, \\ \mathcal{P}_{j}^{26} &= \mathcal{P}_{j}^{36} = -h_{j}e^{\alpha h_{j}}(C_{j}B_{j}K_{j})^{T}W_{j}, \\ \mathcal{P}_{j}^{33} &= -\Omega_{j}, \\ \mathcal{P}_{j}^{44} &= -2S_{j}, \\ \mathcal{P}_{j}^{46} &= -h_{j}e^{\alpha h_{j}}(C_{j}B_{j})^{T}W_{j}, \\ \mathcal{P}_{j}^{55} &= -\Omega_{j}, \\ \mathcal{P}_{j}^{66} &= -W_{j}. \end{split}$$

Then, for the initial conditions satisfying

$$|x_j(0)|^2 \le \rho_j^{-1} \left(1 + \frac{\epsilon M}{\alpha(M-1)}\right)^{-1}, \quad j = 1, \dots, M,$$
 (22)

the system (1), (5) is exponentially stable with the decay rate $\delta = \alpha - \epsilon$.

Proof 2: We divide the proof into two parts. First, we show that (13) holds for V_i defined in (8) if

$$\hat{y}_{k,j} \in \mathscr{S}_j, \quad k \in \mathbb{N}_0, \quad j = 1, \dots, M.$$
 (23)

Then, we show that (23) holds if $x_j(0)$ satisfies (22).

I. Proof of (13) under (23)

For V_j^P and V_j^W defined below (8), we have

$$\begin{split} \dot{V}_{j}^{P} + 2\alpha V_{j}^{P} \stackrel{(7)}{=} 2x_{j}^{T} P_{j} \Big[(A_{j} + B_{j}K_{j}C_{j})x_{j} - B_{j}K_{j}v_{j} \\ -B_{j}K_{j}e_{k,j} - B_{j}\psi_{k,j} + \sum_{\substack{i=1\\i\neq j}}^{M} F_{ij}x_{i} \Big] + 2\alpha x_{j}^{T} P_{j}x_{j}, \\ \dot{V}_{j}^{W} + 2\alpha V_{j}^{W} = h_{j}^{2}e^{2\alpha h_{j}}\dot{y}_{j}^{T}W_{j}\dot{y}_{j} - \frac{\pi^{2}}{4}v_{j}^{T}W_{j}v_{j}. \end{split}$$
(24)

Under the condition (23), Lemma 2 implies (12), which we rewrite using (6) as

$$0 \le \psi_{k,j}^T S_j \left[G_j y_j - G_j v_j - G_j e_{k,j} - \psi_{k,j} \right].$$
(25)

Summing up the right-hand sides of (10), (24), and (25), we obtain

$$\dot{V}_{j}(t) + 2\alpha V_{j}(t) - \sum_{\substack{i=1\\i\neq j}}^{M} \frac{2\epsilon}{M-1} V_{i}^{P}(t) \leq \phi_{j}^{T} \begin{bmatrix} \overline{\mathcal{P}}_{j} & \overline{\mathcal{F}}_{j} \\ * & \mathcal{E}_{j} \end{bmatrix} \phi_{j} + h_{j}^{2} e^{2\alpha h_{j}} \dot{y}_{j}^{T} W_{j} \dot{y}_{j} + \sigma_{j}^{2} [y_{j} - v_{j}]^{T} \Omega_{j} [y_{j} - v_{j}]$$

$$(26)$$

where

$$\phi_j = \operatorname{col}\{x_j, v_j, e_{k,j}, \psi_{k,j}, x_1, \cdots, x_{j-1}, x_{j+1}, \cdots, x_M\},$$
(27)

 $\overline{\mathcal{P}}_j$ is obtained from \mathcal{P}_j by removing the last two blockcolumns and block-rows, $\overline{\mathcal{F}}_j$ is obtained from \mathcal{F}_j by removing the last two block-rows. Substituting the expression (7) into $\dot{y}_j = C_j \dot{x}_j$ and using the Schur complement, we find that (19) guarantees (13).

II. Proof of (23) under (22)

By the Schur complement, (18) implies

$$x_j^T C_j^T (K_j^{(i)} - G_j^{(i)})^T (K_j^{(i)} - G_j^{(i)}) C_j x_j \le x_j^T P_j x_j \bar{u}_j^2.$$
(28)

Since $x_j^T(t)P_jx_j(t) \leq V_j(t)$, if

$$V_j(t) \le 1, \quad t \ge 0, \quad j = 1, \dots, M$$
 (29)

then (28) implies $y_j(t) \in \mathscr{S}_j$ for $t \ge 0, j = 1, ..., M$ and, in particular, (23) is true. Thus, it suffices to prove (29). Let

$$\beta = \left(1 + \frac{\epsilon M}{\alpha(M-1)}\right)^{-1}$$

Relation (29) holds for t = 0 since

$$V_j(0) = x_j^T(0) P_j x_j(0) \stackrel{(17)}{\leq} \rho_j |x_j(0)|^2 \stackrel{(22)}{\leq} \beta < 1.$$

Let (29) be false for some t > 0. Since all $V_j(t)$ can have only negative jumps at $t_{k,j}$ and are continuous elsewhere, there must be t_* such that (29) holds on $[0, t_*]$ for all j = 1, ..., Mand $V_q(t_*) = 1$ for some $q \in \{1, ..., M\}$. Then, on $[0, t_*]$ we have:

$$(29) \Rightarrow (28) \Rightarrow (23) \Rightarrow (13).$$

By Lemma 3,

$$V_q(t_*) < \beta \left(1 + \frac{\epsilon M}{\alpha(M-1)}\right) = 1.$$

This contradicts the definition of t_* . Thus, (29) holds for $t \ge 0$ implying (13). Lemma 3 guarantees $V(t) \le e^{-2\delta t}V(0)$ that implies exponential stability of the system (1), (5).

IV. EXAMPLE: COUPLED CART-PENDULUMS

Consider two coupled inverted pendulums on carts [19], [20] whose dynamics are given by (1) with

$$A_{j} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2.9156 & 0 & -0.0005 & 0 \\ 0 & 0 & 0 & 1 \\ -1.6663 & 0 & 0.0002 & 0 \end{bmatrix}, \quad B_{j} = \begin{bmatrix} 0 \\ -0.0042 \\ 0 \\ 0.0167 \end{bmatrix},$$
$$C_{j} = I_{4}, \quad F_{12} = F_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0011 & 0 & 0.0005 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0003 & 0 & -0.0002 & 0 \end{bmatrix}$$

for j = 1, 2. The controllers are given by (5) with

$$K_1 = \begin{bmatrix} 11396 & 7196.2 & 573.96 & 1199 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 29241 & 18135 & 2875.3 & 3693.9 \end{bmatrix}$$

and the saturation levels $\bar{u}_1 = \bar{u}_2 = 10^5$. The conditions of Theorem 1 are feasible for $\alpha = 0.5$, $\epsilon = 0.05$, $G_j = 0.5 \cdot K_j$,

 $\sigma = 0$, and h = 0.09, where ρ is a decision variable. This implies that the sampled-data saturated controllers (5) (without the event-triggering mechanism) stabilize the system (1) for the initial conditions |x(0)| < 0.4467 (calculated using (22)). Note that $G_i = 0$ lead to a smaller domain |x(0)| < 0.2759.

Sampled-data controllers (5) without the event-triggering mechanism ($\sigma = 0$) require to transmit $\lfloor \frac{20}{0.09} \rfloor + 1 = 223$ signals during 20 seconds of simulations. The conditions of Theorem 1 remain feasible for $\sigma = 0.2$, h = 0.05. For these values, the event-triggered controllers (5) stabilize the system (1) with the same decay rate requiring to transmit 116 signals. This value was found performing numerical simulations for 20 randomly chosen initial conditions satisfying |x(0)| < 0.4467. Thus, the event-triggering mechanism reduces the amount of transmitted signals by almost 50%.

V. CONCLUSION

This paper introduced decentralized control in the presence of saturated actuators for large-scale systems with independent networks. The time-delay approach to sampleddata control and event-triggered control led to efficient LMIbased conditions for regional exponential stability. By using plant-dependent Lyapunov-Krasovskii functionals, decentralized bounds on the domain of attraction were derived. A numerical example showed that the generalized sector condition introduced in [18] allowed to enlarge the domain of attraction.

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