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EXISTENCE OF REGULAR NUT GRAPHS FOR DEGREE AT MOST 11

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Dedicated to the memory of Slobodan Simić.

Abstract

A nut graph is a singular graph with one-dimensional kernel and corresponding eigenvector with no zero elements. The problem of determining the orders n for which d-regular nut graphs exist was recently posed by Gauci, Pisanski and Sciriha. These orders are known for $d \leq 4$. Here we solve the problem for all remaining cases $d \leq 11$ and determine the complete lists of all d-regular nut graphs of order n for small values of d and n. The existence or non-existence of small regular nut graphs is determined by a computer search. The main tool is a construction that produces, for any d-regular nut graph of order n, another d-regular nut graph of order $n+2d$. If we are given a sufficient number of d-regular nut graphs of consecutive orders, called seed graphs, this construction may be applied in such a way that the existence of all d-regular nut graphs of higher orders is established. For even d the orders n are indeed consecutive, while for odd d the orders n are consecutive even numbers. Furthermore, necessary conditions for combinations of order and degree for vertex-transitive nut graphs are derived.

Keywords: nut graph, core graph, regular graph, nullity.

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1. Introduction

In this paper we consider graphs that are *simple*, that is, without loops or multiple edges. Given a labelled graph G of order n, its $0-1$ *adjacency matrix* $\mathbf{A} = \mathbf{A}(G)$ is the $n \times n$ matrix with entries a_{ij} (for $i, j \in \{1, \ldots, n\}$) such that $a_{ij} = 1$ if and only if there is an edge between the vertices i and j in G. If $\mathbf{A}(G)$ has an eigenvalue zero, the graph G is *singular* with nullity $\eta = \eta(G)$ equal to the multiplicity of the eigenvalue zero of $\mathbf{A}(G)$. In the sequel, we use terminology for a graph and its adjacency matrix interchangeably. A graph that has an all-non-zero eigenvector corresponding to the zero eigenvalue is a *core graph*. If, in addition, the dimension of the nullspace is one, the corresponding graph is a *nut graph* [10–14]. Some authors consider the isolated vertex to be a trivial case of a nut graph, and would consider nuts proper, or *non-trivial* nuts, to have $n > 1$. In the present paper, the only nut graphs that are considered are non-trivial, in this sense.

Two chemical motivations can be claimed for the study of nut graphs of maximum degree ≤ 3 (the *chemical* nut graphs). First, the eigenvalue zero is special in a chemical context, as it corresponds in Hückel Molecular Orbital Theory to the non-bonding energy level of a conjugated carbon π system, from which electrons are easily removed and to which they are easily added. If this happens to be the highest occupied molecular orbital, then partial occupation will correspond to a radical (molecule with unpaired spins) in which spin density and hence radical reactivity is distributed over all carbon atoms of the framework. Secondly, it has been shown [5] that nut graphs correspond exactly to the classes of distinct and strong omni–conductors of nullity one, according to the simplest model of molecular conduction. (A *distinct* omni–conductor is a molecule for which connection in a circuit via any distinct pair of atoms gives transmission; for a *strong* omni–conductor, the transmission occurs whether the connection atoms are distinct or not.)

Using a computer search, Fowler, Pickup, Todorova, Borg and Sciriha [5] determined all nut graphs up to 10 vertices, and all chemical nut graphs up to 16 vertices. Later Coolsaet, Fowler and Goedgebeur [4] designed a specialised algorithm to generate nut graphs and used it to determine all nut graphs up to 13 vertices and all chemical nut graphs up to 22 vertices. We refer the reader to [4] and [6] for a more detailed overview.

In [14] Sciriha and Gutman proved that the smallest non-trivial nut graphs have order 7 and that nut graphs exist for all orders at least 7. A natural question arises.

Question 1. *What is the smallest order for which a regular nut graph exists*?

A chemical motivation for the study of regular nut graphs is that 3-regular, 3-connected graphs are candidates as carbon cages, so that nut graphs of this type would themselves be candidates for omni-conductors and radical molecules of the kinds mentioned above.

The two main results of [6] can be stated as follows.

Theorem 2. *Cubic nut graphs on n vertices exist if and only if n is an even integer, $n \geq 12$ and $n \notin \{14, 16\}$.*

Theorem 3. *Quartic nut graphs on n vertices exist if and only if $n \in \{8, 10, 12\}$ or $n \geq 14$.*

The results of a computer search presented in Figure 2 have indeed shown that the answer to the above question is the following.

Remark 4 (Answer to Question 1)**.** The smallest order for which a regular nut graph exists is 8.

Since no nut d-regular graph exists for $d = 1$ or $d = 2$, the above results raise a number of related questions. (See also Section 4 "Conclusion").

Question 5. *Is it true, that for each $d > 2$ there exists a d-regular nut graph*?

In [6], the set $N(d)$ was defined as the set consisting of all integers n for which a d-regular nut graph of order n exists. Therefore we have $N(1) = N(2) = \emptyset$, $N(3) = \{12\} \cup \{2k : k \geq 9\}$, $N(4) = \{8, 10, 12\} \cup \{k : k \geq 14\}$.

Definition. Let us call a pair (n, d) *admissible* if there exists a d-regular (simple) graph of order n and let us denote by $A(d)$ the collection of integers n, such that there exists an (n, d)-admissible graph. Let us call a pair (n, d) *nut-realisable* if and only if there exists a d-regular nut graph of order n.

For d even, (n, d) is admissible if and only if $d < n$. For d odd, (n, d) is admissible if and only if $d < n$ and n is even. Clearly $N(d) \subset A(d)$. A valence d for which the set $A(d) \setminus N(d)$ is finite will be called *normal*.

Question 6. *Is it true, that for each $d > 2$ there are only a finite number of admissible orders that do not belong to $N(d)$? This is equivalent to saying, is the complement $A(d) \setminus N(d)$ finite? In other words, is it true that d is normal if and only if $d > 2$?*

In this note, we determine $N(d)$, and give a positive answer to the above question, for every $d \leq 11$. That is, we prove the next result.

Theorem 7. *The following holds.*

- $N(5) = \{2k : k \geq 5\}$,
- $N(6) = \{k : k \geq 12\}$,
- $N(7) = \{2k : k \geq 6\}$,
- $N(8) = \{12\} \cup \{k : k \geq 14\}$,
- $N(9) = \{2k : k \geq 8\}$,
- $N(10) = \{k : k \geq 15\}$,
- $N(11) = \{2k : k \geq 8\}$.

We use a construction that was proved and applied repeatedly in [6]. It extends any singular graph G at a vertex v of valency d, to give a graph $F(G, v)$ with the same nullity as G but with $2d$ more vertices (each of degree d), whilst all other vertices retain the degree they had in G. The process used in this extension, named in [6] the *Fowler Construction*, is depicted in Figure 1.

From the construction, a lemma follows immediately.

Lemma 8. *Let G be a nut graph on n vertices and let v be a vertex of G having valency d. Then there exists a nut graph $F(G, v)$ on $n + 2d$ vertices in which the $2d$ new vertices all have valency d.*

Starting from a nut graph of order n, the above lemma can be used repeatedly to produce an infinite sequence of nut graphs on $n, n+2d, n+4d, n+6d, \ldots$ vertices in which all of the newly introduced vertices have valency d. In particular, this argument can be applied to any d-regular nut graph on n vertices to obtain an infinite sequence of d-regular nut graphs.

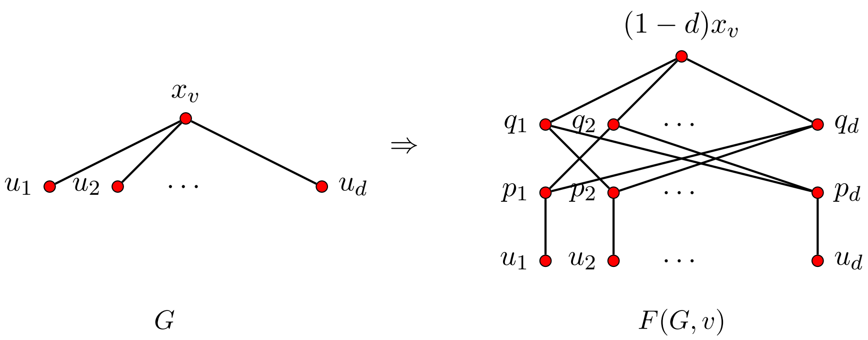

Figure 1. The 'Fowler Construction' [6]. Notation: x_v, u_i are entries in the kernel eigenvector of G, with entries u_i summing to zero; in $F(G,v)$, the two layers of extra vertices span a complete bipartite graph minus a perfect matching, with eigenvector entries $q_i = u_i$, $p_i = x_v$, and the entry on vertex v replaced by $(1-d)x_v$.

For even valency d, one has to establish the existence of d-regular nut graphs for $2d$ consecutive orders, starting with order n. By Lemma 8 above, this would then imply the existence of d-regular nut graphs of any order at least n. Smaller cases have to be checked by computer search or some other method. In the case of $d = 4$, such a run of eight consecutive orders was obtained starting at $n = 14$ [6].

By the well-known Handshaking Lemma, for odd valency d the order of a graph must be even. It therefore suffices to establish existence of d-regular nut graphs for d consecutive even orders, starting with order n. For $d = 3$ the run of three even orders started at $n = 18$ [6].

2. Existence of Regular Nut Graphs

In [4], a generation algorithm for nut graphs is presented. We could in principle use this generator and restrict the generation to d-regular graphs. However, it turned out to be more efficient to use Meringer's generator for regular graphs, *genreg* [9], to generate d-regular graphs and then test which are nut graphs (using the filter program from [3]).

Using this approach, we determined all 5-regular nut graphs up to 18 vertices. The counts can be found in Table 1. Using a similar approach (but using the generator from [2] for the cubic case), we also determined all 4-regular nut graphs up to 15 vertices and all 3-regular nut graphs up to 22 vertices, the counts for which can be found in Table 1. These graphs can also be downloaded from the *House of Graphs* [1] at https://hog.grinvin.org/Nuts.

We now proceed to the statement and proof of our main result.

3-regular nut graphs		4-regular nut graphs		5-regular nut graphs	
Order	Number of graphs	Order	Number of graphs	Order	Number of graphs
12	9	8	1	10	9
18	5 541	10	12	12	4
20	5	12	269	14	25
22	71	14	15 633	16	13 530
		15	1	18	665 456 900

6-regular nut graphs		7-regular nut graphs		8-regular nut graphs	
Order	Number of graphs	Order	Number of graphs	Order	Number of graphs
12	1 964	12	3	12	24
13	79	14	5 168 453	13	0
14	1 872			14	424 088

9-regular nut graphs		10-regular nut graphs		11-regular nut graphs	
Order	Number of graphs	Order	Number of graphs	Order	Number of graphs
14	0	14	0	14	0
16	> 0	15	173 650	16	3 316

Table 1. The numbers of d-regular nut graphs for $d \in \{3, \ldots, 11\}$. No such graphs exist for smaller orders.

Theorem 7. *The following holds.*

- $N(5) = \{2k : k \geq 5\}$,
- $N(6) = \{k : k \geq 12\}$,
- $N(7) = \{2k : k \geq 6\}$,
- $N(8) = \{12\} \cup \{k : k \geq 14\}$,
- $N(9) = \{2k : k \geq 8\}$,
- $N(10) = \{k : k \geq 15\}$,
- $N(11) = \{2k : k \geq 8\}$.

Proof. It follows from Table 1 that the smallest 5-regular nut graphs have 10 vertices. In order to cover all orders using the construction, we need to present a 5-regular nut graph of every even order $10 \leq n \leq 18$. In the Appendix we give an example of a 5-regular nut graph of every such order n. In fact, the examples from the Appendix are chosen to exhibit the largest automorphism group size amongst the 5-regular nut graphs of that order.

The proof for the other cases of $N(d)$ for $6 \leq d \leq 11$ is completely analogous. That is, using Meringer's generator *genreg* [9] together with a filter which tests whether a given graph is a nut, we determined the smallest order for which a d-regular nut graph exists and excluded small orders for which d-regular nut graphs do not exist (e.g. to prove the nonexistence of an 8-regular nut graph on 13 vertices). See Table 1 for the counts of the smallest d-regular nuts for $6 \leq d \leq 11$. We have used *genreg* to establish the existence of a d-regular nut graph of order n for $2d$ consecutive orders (if d is even), or the existence of a d-regular nut graph for d consecutive even orders (if d is odd) which are then used as seeds for the construction.

The adjacency list of a d-regular seed graph of every required order can be found in the Appendix. These graphs can also be inspected in the database of interesting graphs from the *House of Graphs* [1] by searching with keywords "d-regular nut graph" (with $d \in \{5, \dots, 11\}$). ■

3. Vertex-Transitive Nut Graphs

A graph G such that all vertices are equivalent under the automorphism group $\mathrm{Aut}(G)$ is *vertex-transitive.* Requiring regular nut graphs to be vertex-transitive clearly imposes further restrictions, and it seems natural to ask the following question.

Question 9. *For what pairs (n, d) does a vertex-transitive nut graph of order n and degree d exist?*

Some straightforward observations give the following necessary conditions for the existence of a vertex-transitive nut graph, as summarised in the following theorem.

Theorem 10. *Let G be a vertex-transitive nut graph on n vertices, of degree d. Then n and d satisfy the following conditions. Either $d = 0 \pmod 4$, and $n = 0 \pmod 2$ and $n \geq d + 4$; or $d = 2 \pmod 4$, and $n = 0 \pmod 4$ and $n \geq d + 6$.*

Proof. We are going to prove the result in five steps.

Let $\mathbf{x}$ be a kernel eigenvector of a vertex-transitive nut graph G. As the zero eigenvalue is simple, $\mathbf{x}$ transforms as a non-degenerate irreducible representation of $\mathrm{Aut}(G)$. Hence, the eigenvector can be chosen to have all entries real. Under any given automorphism, each eigenvector entry x_i transforms to x_j or $-x_j$ for some j, and therefore all squared entries x_i^2 are equal. We can choose a normalisation where each entry in $\mathbf{x}$ is either $+1$ or -1.

A kernel eigenvector obeys the local adjacency condition

$$\sum_{i \sim j} x_j = 0 \tag{1}$$

for every vertex i, which implies that the neighbourhood of every vertex of a vertex-transitive nut graph contains an equal number of vertices bearing entries $+1$ and -1 in $\mathbf{x}$. Hence, a vertex-transitive nut graph is of even degree.

Next, note that a kernel eigenvector $\mathbf{x}$ of a d-regular graph obeys

$$\sum_{i=1}^{n} x_i = 0, \tag{2}$$

as $\mathbf{x}$ is orthogonal to the all-ones Perron eigenvector that corresponds to the maximum eigenvalue $\lambda_{\max} = d$ of a regular graph. Hence, for a vertex-transitive nut graph G, the sum over all of the $+1$ and -1 entries x_i is zero, from which we deduce that n is even.

Consider the subgraphs of G induced by the vertices with entries $x_i = +1$ and $x_i = -1$, respectively. Let H be the subgraph induced by the vertices with $x_i = +1$. Vertex i in G is of even degree, and in the neighbourhood of i in G there are $d/2$ vertices carrying entries of each sign. The graph H is therefore regular, of degree $d/2$ and of order $n/2$. Hence, if $d = 2 \pmod 4$, $d/2$ is odd and the order of H must be even. Therefore, $n/2$ is even and we have $n = 0 \pmod 4$ for $d = 2 \pmod 4$.

Finally, note a limitation on the smallest vertex-transitive nut graph of given degree d. A d-regular graph has at least $(d+1)$ vertices. Given that both degree and order of a vertex-transitive nut graph are even, the order of a vertex-transitive nut graph obeys $n \geq d+2$. However, a vertex-transitive nut graph with $n = d+2$ is not possible. For even n and d, there is a unique vertex-transitive graph, H', of order $d+2$ [8], which is not a nut graph. Indeed, as every vertex of H' is adjacent to all but one of the others, we can choose a labelling such that vertices i and $d+1-i$ of H' are duplicates for all $0 \leq i \leq n-1$. In particular, 0 and $d+1$ have the same neighbourhood, $\{1, \ldots, d\}$. The subgraph induced by this neighbourhood is K_d minus a perfect matching, as edges between j and $d+1-j$ are missing for $1 \leq j \leq d$. This is the well known cocktail-party graph on d vertices, $CP(d/2)$. It has $d/2$ distinct pairs of non-adjacent vertices. A kernel eigenvector for H' can be constructed as follows: assign values $+1$ to vertex 0, -1 to vertex $d+1$, and zero to all others. Hence, H' is not a nut graph, and the stated limits on n follow. ■

Calculations using the catalogue of vertex-transitive graphs created by Holt and Royle [7] indicate that all pairs (n, d) compatible with the above conditions and with $n \leq 42$ correspond to at least one vertex-transitive nut graph.

See, for example, the blue entries in Figure 2. This figure gives an overview of the existence of d-regular and vertex-transitive nut graphs of order $n \leq 30$.

Our question is therefore refined to the following.

Question 11. *For what pairs* (n, d) *obeying Theorem* 10 *does a vertex-transitive nut graph of order* n *and degree* d *exist*? (*In other words: what pairs are realisable, in the sense that a pair* (n, d) *is* realisable *if there exists at least one vertex-transitive nut graph with these parameters*?)

We do know that there is at least one infinite family of realisable pairs (n, d) of vertex-transitive graphs, as all antiprisms with largest ring size not divisible by 3 are nut graphs; hence all pairs $(n, 4)$ with $n \neq 0 \pmod 6$ are vertex-transitive-nut-realisable.

d \ n	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
3		X		X		S		X		X		C		S		S		C		C		C		C
4	X	S	X	S	X	S	X	S	S	C	S	C	S	C	S	C	C	C	C	C	C	C	C	C
5		X		S		S		S		S		S		C		C		C		C		C		C
6		X	X	X	X	S	S	S	S	S	S	S	S	S	S	S	S	C	C	C	C	C	C	C
7				X		S		S		S		S		S		S		S		C		C		C
8				X	X	S	X	S	S	S	S	S	S	S	S	S	S	S	S	S	S	C	S	C
9						X		X		S		S		S		S		S		S		S		S
10						X	X	X	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
11								X		S		S		S		S		S		S		S		S
12								X	X	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
13										X		S		S		S		S		S		S		S
14										X	X	X	S	S	S	S	S	S	S	S	S	S	S	S
15												X		S		S		S		S		S		S
16												X	X	S	S	S	S	S	S	S	S	S	?	S
17														X		S		S		S		S		?
18														X	X	S	S	S	S	S	S	S	?	?
19																X		S		S		S		S
20																X	X	S	?	S	?	S	?	S
21																		X		?		?		S
22																		X	X	S	?	S	?	?
23																				X		?		?
24																				X	X	S	?	S
25																						X		?
26																						X	X	S
27																								X
28																								X

Figure 2. Snapshot of small regular nut graphs: calculations and proven results for order n and degree d for $n \leq 30$. Symbols: blank spaces and red crosses indicate non-existence; question marks indicate open cases; the symbol S indicates a seed graph; the symbol C indicates that at least one example of a regular nut graph can be obtained by the Fowler construction; blue (respectively, green) symbols indicate the existence (respectively, non-existence) of a vertex-transitive nut graph.

4. CONCLUSION

We divide the set of nut graphs into two disjoint subsets. If a graph can be obtained from the Fowler construction, we call it a *C-graph*. If it cannot be obtained in this way, we call it a seed graph or *S-graph*. This carries over to nut-realisable pairs (n, d). Such a pair is a *C-pair* if there exists at least one (n, d)-graph that is a C-graph. On the other hand, if all (n, d)-graphs are seed graphs, we call the pair (n, d) an *S-pair* or *seed pair.*

Proposition 12. *If d is a normal valence, then the following is true. If d is odd, then there are exactly d seed pairs* (n, d) *and if d is even, then there are exactly* $2d$ *such seed pairs. In the former case each seed value of n has a different (even) remainder mod* $2d$ *while in the latter case each n gives a different remainder* (mod $2d$).

d	$n_0(d)$	$n_1(d)$
3	12	18
4	8	14
5	10	10
6	12	12
7	12	12
8	12	14
9	16	16
10	15	15
11	16	16
12	16	≥ 16
13	18	≥ 18
14	19	≥ 19
15	20	≥ 20
16	20	≥ 20
17	22	≥ 22
18	22	≥ 22
19	24	≥ 24
20	24	≥ 24
21	≥ 26	≥ 26
22	26	≥ 26
23	≥ 28	≥ 28
24	28	≥ 28
25	≥ 30	≥ 30
26	30	≥ 30

Table 2. Known values of $n_0(d)$ and $n_1(d)$. An entry of the form $n_0(d) \geq p$, indicates that a complete search of all feasible smaller cases of order less than p has not revealed an example of a d-regular nut graph.

Let d be a normal valence. We can define two numbers: $n_0(d)$ and $n_1(d)$. Here $n_0(d)$ represents the smallest value of n such that the pair (n, d) is nut-realisable. On the other hand let $n_1(d)$ represent the smallest value such that admissible pairs (n, d) with $n \geq n_1(d)$ are nut-realisable.

Our results from Figure 2 lead to the values of $n_0(d)$ and $n_1(d)$ in Table 2.

Variants of the original question suggest themselves, for example the following.

Question 13. *Determine the set $N(p, g)$ such that there exists a p-regular nut graph of order n and and girth (at least) g if and only if $n \in N(p, g)$.*

Question 14. *Determine the set $N(p)$ such that there exists a q-regular nut graph of order n for all degrees $3 \leq q \leq p$ if and only if $n \in N(p)$.*

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Appendix

Adjacency lists of regular nut graphs

The graphs in this section act as seed graphs in the proof of Theorem 7. For completeness we also list the seed graphs for $d = 3$ and $d = 4$. These graphs can also be inspected at the database of interesting graphs from the *House of Graphs* [1] by searching for the keywords "d-regular nut graph" (with $d \in \{3, \ldots, 11\}$).

Adjacency lists of 3-regular nut graphs

- Order 12: {0: 1 2 3; 1: 0 4 5; 2: 0 6 7; 3: 0 8 9; 4: 1 6 8; 5: 1 7 9; 6: 2 4 10; 7: 2 5 11; 8: 3 4 11; 9: 3 5 10; 10: 6 9 11; 11: 7 8 10}
- Order 20: {0: 1 2 3; 1: 0 4 5; 2: 0 6 7; 3: 0 8 9; 4: 1 6 10; 5: 1 8 11; 6: 2 4 12; 7: 2 11 13; 8: 3 5 14; 9: 3 12 15; 10: 4 13 16; 11: 5 7 17; 12: 6 9 16; 13: 7 10 18; 14: 8 15 19; 15: 9 14 18; 16: 10 12 19; 17: 11 18 19; 18: 13 15 17; 19: 14 16 17}
- Order 22: {0: 9 14 15; 1: 10 16 18; 2: 11 13 14; 3: 11 13 15; 4: 12 18 21; 5: 12 20 21; 6: 15 16 17; 7: 16 19 20; 8: 17 19 20; 9: 0 14 19; 10: 1 17 18; 11: 2 3 13; 12: 4 5 21; 13: 2 3 11; 14: 0 2 9; 15: 0 3 6; 16: 1 6 7; 17: 6 8 10; 18: 1 4 10; 19: 7 8 9; 20: 5 7 8; 21: 4 5 12}

Adjacency lists of 4-regular nut graphs

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- Order 19: {0: 1 2 3 4 5 6; 1: 0 2 3 4 5 6; 2: 0 1 3 4 5 6; 3: 0 1 2 4 5 6; 4: 0 1 2 3 5 6; 5: 0 1 2 3 4 7; 6: 0 1 2 3 4 8; 7: 5 8 9 10 11 12; 8: 6 7 9 10 13 14; 9: 7 8 10 11 12 15; 10: 7 8 9 13 16 17; 11: 7 9 12 14 15 16; 12: 7 9 11 15 17 18; 13: 8 10 14 16 17 18; 14: 8 11 13

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Adjacency lists of 7-regular nut graphs

- Order 12: {0: 1 2 3 4 5 6 7; 1: 0 2 3 4 5 8 9; 2: 0 1 3 4 6 10 11; 3: 0 1 2 5 8 10 11; 4: 0 1 2 7 8 9 10; 5: 0 1 3 6 7 9 11; 6: 0 2 5 7 9 10 11; 7: 0 4 5 6 8 9 10; 8: 1 3 4 7 9 10 11; 9: 1 4 5 6 7 8 11; 10: 2 3 4 6 7 8 11; 11: 2 3 5 6 8 9 10}
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- Order 33: {0: 1 2 3 4 5 6 7 8 9 10; 1: 0 2 3 4 5 6 7 8 9 10; 2: 0 1 3 4 5 6 7 8 9 10; 3: 0 1 2 4 5 6 7 8 9 10; 4: 0 1 2 3 5 6 7 8 9 10; 5: 0 1 2 3 4 6 7 8 9 10; 6: 0 1 2 3 4 5 7 8 11 12; 7: 0 1 2 3 4 5 6 11 12 13; 8: 0 1 2 3 4 5 6 11 13 14; 9: 0 1 2 3 4 5 11 14 15 16; 10: 0 1 2

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- Order 24: {0: 1 2 3 4 5 6 7 8 9 10 11; 1: 0 2 3 4 5 6 7 8 9 10 11; 2: 0 1 3 4 5 6 7 8 9 10 11; 3: 0 1 2 4 5 6 7 8 9 10 11; 4: 0 1 2 3 5 6 7 8 9 10 11; 5: 0 1 2 3 4 6 7 8 9 10 12; 6: 0 1 2 3 4 5 7 8 9 13 14; 7: 0 1 2 3 4 5 6 10 15 16 17; 8: 0 1 2 3 4 5 6 10 15 16 18; 9: 0 1 2 3 4 5 6 10 15 19 20; 10: 0 1 2 3 4 5 7 8 9 13 21; 11: 0 1 2 3 4 13 14 17 18 22 23; 12: 5 13 14 15 16 17 19 20 21 22 23; 13: 6 10 11 12 17 18 19 20 21 22 23; 14: 6 11 12 15 16 17 18 19 20 21 22; 15: 7 8 9 12 14 16 17 19 20 22 23; 16: 7 8 12 14 15 17 18 19 21 22 23; 17: 7 11 12 13 14 15 16 18 20 21 23; 18: 8 11 13 14 16 17 19 20 21 22 23; 19: 9 12 13 14 15 16 18 20 21 22 23; 20: 9 12 13 14 15 17 18 19 21 22 23; 21: 10 12 13 14 16 17 18 19 20 22 23; 22: 11 12 13 14 15 16 18 19 20 21 23; 23: 11 12 13 15 16 17 18 19 20 21 22}
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