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**Article:**

Schuster, Christian Stefano (2020) Analytical framework for the assessment and modelling of multi-junction solar cells in the outdoors. *Renewable Energy*. pp. 1367-1379. ISSN: 0960-1481

<https://doi.org/10.1016/j.renene.2020.01.002>

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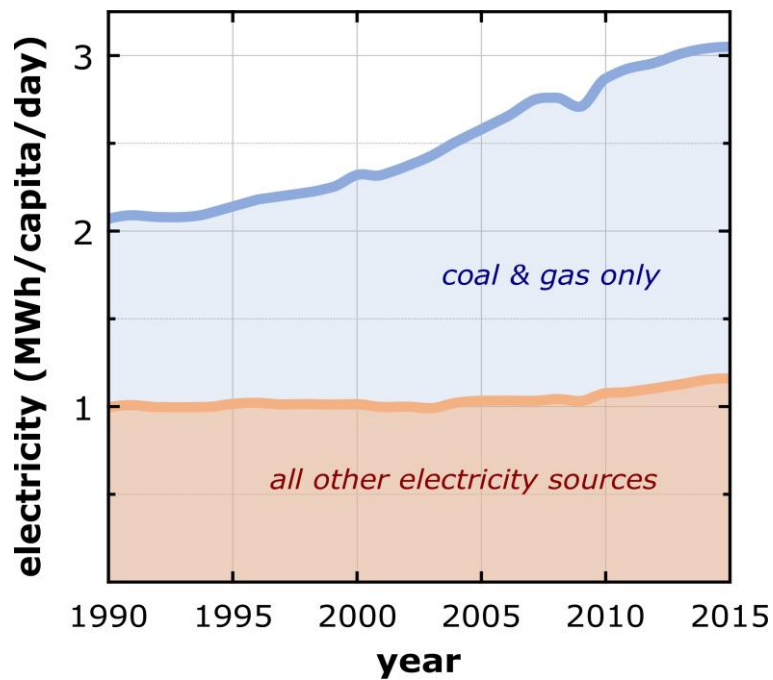


- 24 **Keywords:** Photovoltaics; circuit model; photovoltaic modeling; silicon solar cell; multi-junction
- 25 solar cells; efficiency limits

26 1. INTRODUCTION

27 Solar energy is by far the largest energy resource on Earth. Its enormous potential has kick-started  
28 great ambitions to replace conventional energy resources that were found as causes for climate and  
29 environment damaging effects [1]. However, even though solar photovoltaic (PV) has become the  
30 fastest growing renewable energy technology in the world [2], its share to the production of  
31 electricity has only been 1% in 2015. While coal and gas remain key to electricity production, solar PV  
32 still needs to prove higher conversion efficiencies at lower costs to compete with conventional  
33 technologies, see Fig. 1.

34



35

36 **Figure 1.** Daily electricity consumption per capita and production source. According to the International Energy  
37 Agency, world population grew by 40% from 1990 to 2015 whereas the demand for electricity increased by  
38 50% in the same quarter century [3]. Key to electricity production are coal and gas compared to other  
39 electricity sources such as oil, nuclear, hydro, wind and solar PV. While coal and gas generated 52% of the  
40 electricity mix in 1990, their contribution increased to 62% in 2015. Recently, solar PV became the fastest  
41 growing renewable energy technology in the world [2], albeit its share to the production of electricity was only  
42 1% in 2015 [3].

43 Today, major cost drivers of PV are linked to system components such as installation labour, racking,  
44 cabling and inverters [4]. Since most of these costs scale with the required space, a major increase in  
45 the conversion efficiency directly translates into a lower levelized cost of electricity, because the  
46 same amount of power can be produced by less area.

47 However, the PV industry is practically limited to a conversion efficiency of 26%, when it focuses on  
48 silicon as the only absorber material [5]. Higher efficiencies far beyond 30% were within reach, if  
49 different absorber materials are stacked on top of each other. By doing so, each layer converts a  
50 different part of the solar spectrum into electricity, thereby reducing optical as well as thermalization  
51 losses while increasing the overall power conversion efficiency [6].

52 Even though the multi-layer approach has been known since 1955 [7, 8], it has so far only been the  
53 selection of choice for space applications, where area is premium. Yet, the exciting achievements  
54 related to high-bandgap perovskites [9] as well as the discovery of innovative ways (e.g. mechanical  
55 stacking) to combine III/V materials with silicon [10, 11] have now launched a new development  
56 phase of multi-junction solar cell devices [12, 13, 14]. These rapid advancements could potentially  
57 have major impacts on generic terrestrial applications that require a reasonable balance between  
58 manufacturing costs and efficiency.

59 However, designing a novel multi-junction cell is a challenging task, because competing optical and  
60 electrical demands must be traded-off. For example, each material layer needs to be thick enough  
61 that photons in the corresponding wavelength range are absorbed but thin enough to guarantee the  
62 efficient collection of charge carriers. At the same time, each layer must deliver the maximum  
63 electrical power at the same electrical current (or voltage), which requires sophisticated and  
64 computational expensive numerical optimisation routines. For thin-films, the absorption will likely be  
65 split into multiple layers as well, which requires a demanding optical modelling approach. Finally,  
66 multi-junction cells also need a more careful evaluation to seasonal parameter changes [15], like in  
67 the temperature, daytime length, solar zenith and variations in the solar spectrum.

68 Here, these multiple issues are addressed with a general analytical framework. For arbitrary cell  
69 parameters, closed-form and *explicit* expressions are derived. It is also outlined how the absorption  
70 characteristics of a material layer stack can be calculated with geometric optics.

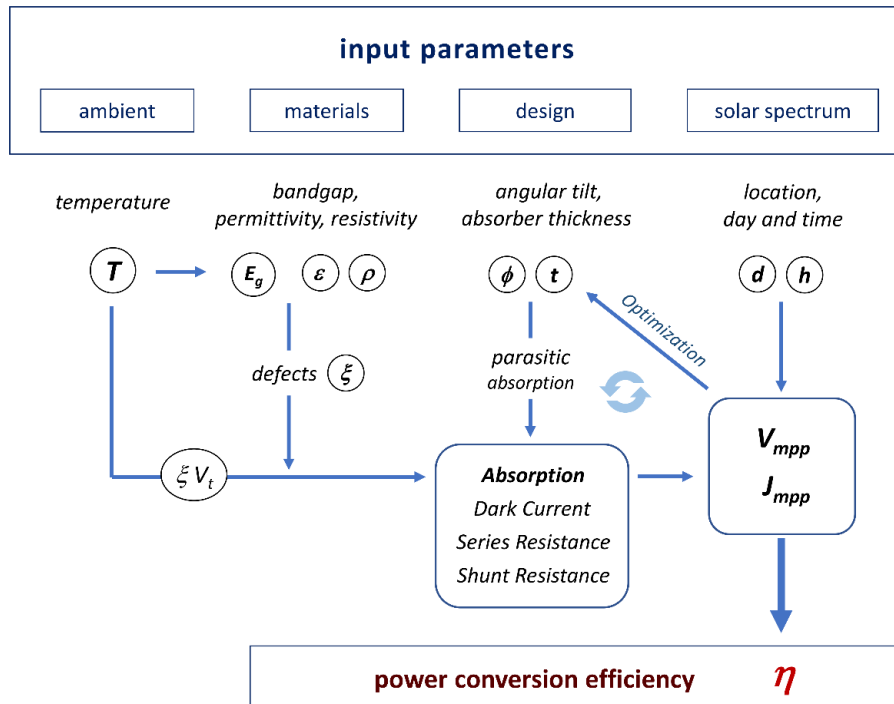
71 Neglecting coherent light effects is indeed without loss of generality for two reasons. Firstly,  
72 averaging Fabry-Perot interference fringes should not affect the integrated short circuit current.  
73 Secondly, the energy yield is a function of the angular-dependent incident global (hemispherical)  
74 solar spectrum throughout the day and year, so any coherent effects should again be averaged out.  
75 While seen as an acceptable simplification, non-coherence is found of greater importance in solar  
76 cell optimisations, according to Herman et al. [16]. Nevertheless, a coherent study of the materials'  
77 absorption can still be adopted.

78 The analytical framework is outlined in section 2 and then applied to an industrial solar panel in  
79 section 3.1 and to a perovskite-silicon tandem system in section 3.2. Since the analytical formalism  
80 enables a data-driven strategy, the examples in section 3 consider the impact of Earth's climate on  
81 the insolation. For the cities Trondheim (Norway), Paris (France), Cairo (Egypt) and Nairobi (Kenya),  
82 the **S**imple **M**odel of the **A**tmospheric **R**adiative **T**ransfer of **S**unshine (*SMARTS*) by C. Gueymard [17,  
83 18] is used to *minutely* track the total global (hemispherical) solar spectrum on a tilted plane due to  
84 changing atmospheric and meteorological conditions. All required data series were retrieved from  
85 free-accessible satellite-product services for a period of 14 years, as described in [19]. Finally, the  
86 theoretical maximum performance of single-, double- and triple-junction solar cells are established in  
87 section 3.3 for the same high-temporal, spectral dynamics.

88 In brief, this paper proposes an analytical framework for the analysis of multi-junction solar cells.  
89 Whereas non-analytical approaches may heavily rely on restrictive parameter assumptions, the here  
90 presented solution allows technical studies to collapse from computational expensive endeavours to  
91 a management strategy of large datasets. The analytical framework thus empowers data-driven  
92 investigations of how weather dynamics, non-ideal device properties and the thicknesses of  
93 individual sub-cells may impact on the overall annual energy yield of solar cells.

94 **2. METHODOLOGY**

95 The power conversion efficiency of a (multi-junction) solar cell depends on many parameters, such as  
 96 the used materials, surrounding ambient conditions and the incident solar spectrum, as indicated by  
 97 Fig. 2.



98

99 **Figure 2.** The conversion of solar energy into electricity is a complicated interplay of many parameters.  
 100 Sophisticated optimisation routines need to carefully evaluate the optimal design parameters for a given  
 101 absorber material, considering its interaction with the incident sunlight and surrounding materials. While the  
 102 light induced current  $J_{mpp}$  and voltage  $V_{mpp}$  are directly linked to the incident solar spectrum, and thus are  
 103 greatly affected by the position of the sun, the operating temperature influences the optical and electrical  
 104 properties of a solar cell. For a multi-junction device, the key to high efficiency is a coordinated fine-tuning  
 105 approach of the individual layers' optoelectronic properties to the incident solar spectrum.

106 For benchmarking purposes, most of the varying quantities were replaced with specific standards,  
 107 set by the **American Society for Testing and Materials (ASTM)**, like the angle of incidence, the solar  
 108 spectrum and the cell's temperature. While these simplifications led to useful design guidelines, they  
 109 impede the accurate prediction of outdoor performances. For example, the recommended global  
 110 solar spectrum distribution AM1.5G has already been found as unsuitable for performance  
 111 prediction of terrestrial PV cells [20, 21, 22, 15]. Additionally, if a cell's electrical current is only  
 112 *implicitly* given by its characteristic current-voltage relation [23], optimisation routines in a cell's  
 113 design process can quickly turn complex [24]. Therefore, many research studies have either adapted  
 114 a simplified cell model or used restrictive assumptions for the analysis [25] [26, 27, 28, 29, 30, 31, 32,  
 115 33, 34, 35].

116 In the following, *explicit* closed-form expressions of the current and voltage are derived at the  
 117 maximum power point that are free of restrictive parameter assumptions (section 2.1). Secondly, a  
 118 simple scattering matrix formalism is introduced to facilitate optimisation routines for the absorption  
 119 characteristics of a multi-layer stack (section 2.2). Finally, it is outlined how environmental factors can  
 120 be incorporated in the analysis of the harvesting efficiency (section 2.3).

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## 121 2.1. THE CONVERSION EFFICIENCY OF A SOLAR CELL

122 Solar cells are taken as electrically equivalent to a current source  $J_{ph}$  in parallel with a diode. A series  
 123  $r_s$  and parallel resistance  $r_p$  are further added to the circuit-model to simulate series and possible  
 124 shunt paths for the electrical current, respectively. If non-radiative recombination centres are  
 125 modelled as defects in the diode's space charge-region by the factor  $\xi$ , with  $\xi = 1$  as the defect-free  
 126 case, the current-voltage characteristic of an illuminated solar cell is given by [23]:

$$127 \quad J(U, J) = J_{ph} - J_o \cdot \left( \exp \left[ \frac{U + r_s \cdot J}{\xi U_t} \right] - 1 \right) - \frac{U + r_s \cdot J}{r_p}, \quad (1)$$

128 with  $U_t$  as the thermal voltage across the pn-junction of the diode. Since Shockley's diode equation  
 129 does not take photon recycling into account, the reverse-saturation dark current  $J_o$  must be  
 130 calculated according to the detailed balance theory outlined by Marti et al. [36].

131 Using the normalized quantities of Tab. 1, the current-voltage characteristic can be written in a  
 132 dimensionless format

$$133 \quad j(u) = q_{ph} - \frac{r}{r_p} \cdot u - \mathcal{W}(u), \quad (2)$$

134 via the LambertW-function  $\mathcal{LW}$ , defined as the inverse of the function  $\omega(u) = u \cdot e^u$ ,

$$135 \quad \mathcal{W}(u) = \mathcal{LW}\left(q_o \cdot \exp\left[q_{ph} + \frac{r}{r_s} \cdot u\right]\right). \quad (3)$$

general current	general voltage	specific resistances	dark factor	light factor
$j = \frac{r_s J}{\xi U_t}$	$u = \frac{U}{\xi U_t}$	$\frac{r}{r_p} = \frac{r_s}{r_s + r_p}, \frac{r}{r_s} = \frac{r_p}{r_s + r_p}$	$q_o = \frac{r \cdot J_o}{\xi U_t}$	$q_{ph} = \frac{r \cdot (J_{ph} + J_o)}{\xi U_t}$

136 **Table 1.** The dimensionless quantities used for the analytical assessment of solar cells.

137 The LambertW-function  $\mathcal{LW}$  is also known as the Omega-function, product logarithm or 'golden ratio  
 138 of exponentials' [37, 38, 39, 40], and it allows to write the previous *implicitly* defined current density  
 139 of Eq. 1 as an *explicit* function of the general voltage  $u$  (Eq. 2).

140 Solving for the voltage  $u_{mpp}$  at the maximum power point, taking  $\max(j(u) \cdot u)$  leads to:

$$141 \quad r_{ch} \cdot j(u_{mpp}) = r_s \cdot u_{mpp} \quad (4)$$

142 with the specific characteristic-resistance  $r_{ch}$  defined by

$$143 \quad \frac{r_s}{r_{ch}} = \frac{\mathcal{W}(u_{mpp}) + \frac{r}{r_p}}{\mathcal{W}(u_{mpp}) + 1} \leq 1. \quad (5)$$

144 The following two solutions can be found for Eq. 4:

$$146 \quad \mu_{max} = \kappa \cdot u_{oc} \left(\frac{r_p}{2}\right) \quad \text{for } \mathcal{W} \ll 1, \quad (6a)$$

$$145 \quad \mu_{min} = q_{ph}^* - \mathcal{LW}\left(q_o^* \cdot e^{q_{ph}^*}\right) \quad \text{for } \mathcal{W} \gg 1. \quad (6b)$$

147 The function  $u_{oc}$  stands for the  $r_s$ -independent open-circuit voltage, defined by Eq. 2 for  $j = 0$ :

$$148 \quad u_{oc}(r_p) = q_{ph}^\diamond - LW\left(q_o^\diamond \cdot e^{q_{ph}^\diamond}\right), \quad \text{with} \quad \lim_{r_p \rightarrow \infty} u_{oc} = \ln\left(\frac{q_{ph}}{q_o}\right). \quad (7)$$

149 The diamond  $\diamond$  or asterisk  $*$  sign reflect the fact that the resistance  $r$  was either replaced with  $r_p$  or

150  $r_p / \left(1 + \frac{r_p}{2r_s}\right)$ , respectively, in  $q_{ph}$  as well as  $q_o$ . The proportional constant  $\kappa$  describes the greatest

151 fraction of open-circuit voltage that can possibly be drawn by a load at the maximum power point:

$$152 \quad \kappa = \lim_{r_p \rightarrow \infty} \frac{LW e^{1+u_{oc}} - 1}{\ln e^{1+u_{oc}} - 1} = \frac{LW\left(\frac{q_{ph}}{q_o} \cdot e^1\right) - 1}{\ln\left(\frac{q_{ph}}{q_o} \cdot e^1\right) - 1} \xrightarrow{q_{ph} \gg q_o} \frac{LW\left(\frac{q_{ph}}{q_o}\right)}{\ln\left(\frac{q_{ph}}{q_o}\right)}. \quad (8)$$

153 One of the simplest and most popular maximum power point tracking methods is indeed based on a

154 fractional open circuit voltage technique, where  $\kappa$  is empirically found through extensive

155 characterizations of the PV cell and under varying meteorological conditions [41].

156 The solution of Eq. 4 is a logistic function and includes the two cases from Eq. 6:

$$157 \quad \mathbf{u}_{mpp} = \mathbf{\mu}_{max} - \mathbf{m} \cdot \mathbf{\mu}_{min} \quad \text{with} \quad m(r_p) = \frac{\mu_{max} - \frac{u_{oc}}{2}}{u_{oc}} = \kappa \cdot \frac{u_{oc}(r_p/2)}{u_{oc}(r_p)} - \frac{1}{2}. \quad (9a)$$

158 This equation is free of any parameter assumptions and directly links the five model parameters

159  $(\xi, r_s, r_p, J_{sc}, U_{oc})$  with the cell's maximum power operating point. Therefore, it is the heart of this

160 research paper and presents the key equation of the here proposed analytical framework. Its

161 accuracy has been extensively verified on a large and diverse set of reported data, see

162 supplementary material; a gnuplot-code is also provided for the reader's own measurement sets.

163 If shunts can be neglected, hence when  $1/r_p \cong 0$ , only a small correction term is needed,

$$164 \quad u_{mpp} = \kappa \cdot u_{oc}(r_p) - \left(\kappa - \frac{1}{2}\right) \cdot \mu_{min} \quad \text{if} \quad 1/r_p \cong 0, \quad (9b)$$

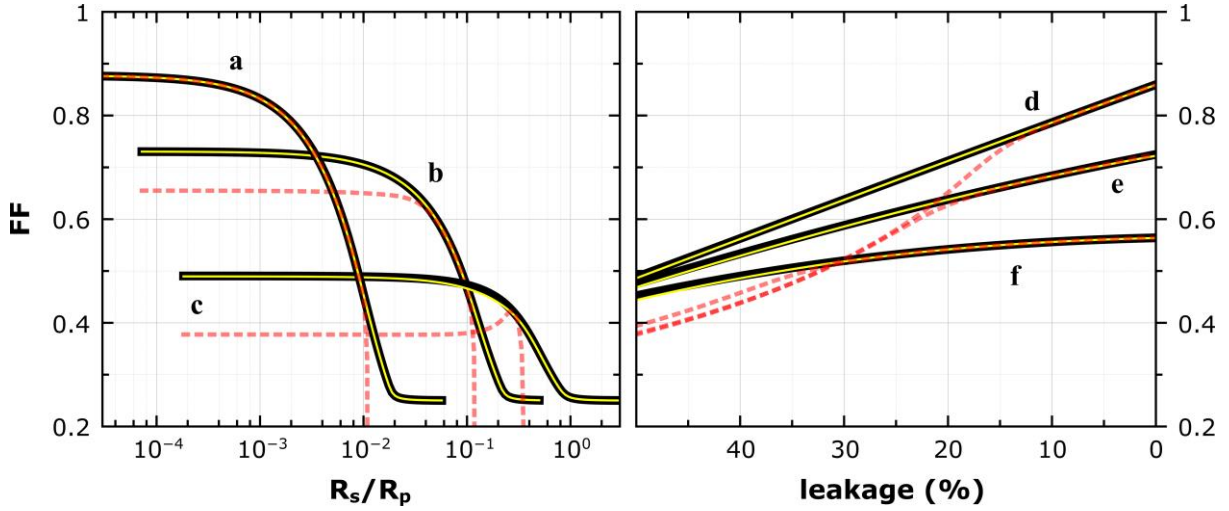
165 while  $u_{mpp} = \mu_{max}$  in the absence of series resistances, i.e. when  $r_s \cong 0$ .

166 Although the two special cases in Eq. 6 were derived with assumptions on the  $\mathcal{W}$ -function, the

167 general solution  $u_{mpp}$  is in good agreement with the numerical results. The fill-factor  $FF$ ,

$$168 \quad FF = \frac{j_{mpp}}{q_{ph} - LW\left(q_o \cdot e^{q_{ph}}\right)} \cdot \frac{u_{mpp}}{q_{ph}^\diamond - LW\left(q_o^\diamond \cdot e^{q_{ph}^\diamond}\right)}, \quad (10)$$

169 is even indistinguishable from the numerically computed values, as shown in Fig. 3. For practical  
 170 purposes, the evaluation of  $j(u_{mpp})$  via Eq. 4 is not recommended, because Eq. 5 is very sensitive to  
 171 small parameter changes. Therefore,  $j_{mpp}$  was calculated by inserting  $u_{mpp}$  of Eq. 9 into Eq. 2.



172

173 **Figure 3.** Excellent agreement is found between the analytical (yellow, thin solid line) and numerical (black,  
 174 thick solid line) computed solutions of Eq. 4, assuming a black body spectrum at 5800 K for the Sun. The  
 175 solutions proposed by Green [31] are included as red dashed lines for comparison. The left figure shows the fill  
 176 factor of a GaAs solar cell (1.4 eV bandgap) at a temperature of 300 K and as a function of its series resistance  
 177  $r_s$  for three different shunt values  $r_p$ , corresponding to a=2%, b=20% and c=50% leakage current. The right  
 178 figure shows the fill factor of the same GaAs cell as a function of its shunt resistance, expressed as a fraction of  
 179 leakage current by  $U_{oc}/(r_p J_{ph})$  for three different series resistances, d=10  $\Omega\text{cm}^2$ , e=50  $\Omega\text{cm}^2$  and f=100  $\Omega\text{cm}^2$ .  
 180 The FF was calculated for  $\xi=1$  and by inserting the analytical solution  $u_{mpp}$  from Eq. 9a into Eq. 2 and Eq. 10.

181 The maximum power output  $P$  of the solar cell is finally given by

$$182 \quad P = J_{mpp} \cdot U_{mpp} = \frac{j(u_{mpp}) \cdot u_{mpp}}{r_s} \cdot (\xi U_t)^2, \quad (11)$$

183 hence the cell's power output can be analytically calculated via Eq. 2 and 9, for arbitrary cell  
 184 parameters. Any asymptotic approximations of the LambertW-function are thereby not needed,  
 185 because it is a trivial matter to incorporate the LW-function into a non-specific software, for  
 186 example, as a User Defined Function in Microsoft Excel [40], see supplementary material.

187 In the theoretical limit, i.e. letting  $r_s \rightarrow 0$  and  $r_p \rightarrow \infty$  in Eq. 11, the power output  $P$  becomes

$$188 \quad P_{max} = \xi U_t \cdot J_{ph} \cdot \frac{\left[ \text{LW} \left( \frac{J_{ph}}{J_o} \cdot e^1 \right) - 1 \right]^2}{\text{LW} \left( \frac{J_{ph}}{J_o} \cdot e^1 \right)}, \quad (12)$$

189 which is known as the detailed balance limit derived by Shockley and Queisser [42, 43].  $P_{max}$  does  
190 not increase for greater  $\xi$  values, since the reverse-saturation dark current  $J_o$  strongly depends on  $\xi$ ,

$$191 \quad J_o = e \cdot \int_0^\infty EQE(\lambda) \cdot \frac{2\pi c}{\lambda^4 \cdot \exp\left(\frac{hc/\lambda}{\xi U_t}\right)} \cdot d\lambda. \quad (13)$$

192 The photocurrent  $J_{ph}$  is defined by the solar spectrum density  $GTI(\lambda)$ ,

$$193 \quad J_{ph} = e \cdot \int_0^\infty EQE(\lambda) \cdot \frac{GTI(\lambda)}{hc/\lambda} \cdot d\lambda, \quad (14)$$

194 as a function of the wavelength  $\lambda$ , elementary charge  $e$ , Planck constant  $h$  and the speed of light in  
195 vacuum  $c$ . While the external quantum efficiency  $EQE(\lambda)$  strongly depends on the solar elevation  
196 angle, surrounding media, layer thickness, resistivity and permittivity of the material, so does the  
197 incident solar spectrum  $GTI(\lambda)$  on the geographical location – the **Global Total** (hemispherical)  
198 Irradiance (GTI) is constantly changing during the day and seasons.

199 In the remaining part of the paper and when not otherwise stated, the dark current  $J_o$  is evaluated  
200 with  $EQE = 1$  in Eq. 13 for photon energies greater than the material bandgap (zero otherwise),  
201 while for  $J_{ph}$  the layer's absorption characteristics is used, i.e.  $EQE(\lambda) = A(\lambda)$  in Eq. 14.

202 As a final remark, Taretto et al. [29] noticed that the quantities  $q_{ph}$  and  $q_o$  are linked to the  
203 measured short-circuit current  $j_{sc}$  and open-circuit voltage  $u_{oc}$  of a solar cell. The characteristic  
204 current-voltage curve of a solar cell thus depends on only three non-directly measurable parameters,  
205 i.e.  $\xi$ ,  $r_s$  and  $r_p$ . These parameters can be extracted from experimental data by standard curve-fitting  
206 procedures. For such purposes, rearranging Eq. 2 with the auxiliary quantities of Tab. 2 yields a more  
207 practical expression:

$$208 \quad J(U) = \left( J_{sc} - \frac{U}{r_{tot}} \right) + \frac{1}{\alpha} \cdot \left( \alpha Q - \text{LW}(\alpha Q \cdot e^{\alpha Q + \beta U}) \right) \quad \text{with} \quad Q(\alpha, \beta, r_{tot}) = \frac{K}{\exp(k) - 1}. \quad (15)$$

209 The fitting parameters are  $\alpha$ ,  $\beta$  and  $r_{tot}$ , since they define the values of  $\xi$ ,  $r_s$  and  $r_p$ . The two factors  $k$   
 210 and  $K$  are closely related to the short-circuit current  $J_{sc}$  and open-circuit voltage  $U_{oc}$  of the solar cell,  
 211 whereas  $Q$  effectively plays the role of the dark-current  $J_o$ . Equation 15 is the explicit counterpart of  
 212 Eq. 1. If the key goal is to find the minimum of the root mean square error, Microsoft Excel's Solver  
 213 Add-in might already be able to extract the three unknown parameters.

reduced $J_{sc}$	reduced $U_{oc}$	current drop	voltage drop	total resistance
$K = J_{sc} - \frac{U_{oc}}{r_{tot}}$	$k = u_{oc} - j_{sc}$ $= \beta U_{oc} - \alpha K$	$\alpha = \frac{r_s}{\xi U_t}$	$\beta = \frac{r_p/r_{tot}}{\xi U_t}$	$r_{tot} = r_s + r_p$

214 **Table 2.** Auxiliary quantities used for the fitting of experimental current-voltage curves. The unknown  
 215 parameters are the current drop  $\alpha$  (due to a non-negligible series resistance), the voltage drop  $\beta$  (due to  
 216 potential shunt paths) and the total resistance  $r_{tot}$ .

---

## 217 2.2. ABSORPTION OF INCOHERENT SUNLIGHT

218 According to Herman et al. [16], tuning the absorption capability to incoherent sunlight is of greater  
 219 importance for PV applications. Therefore, the scattering matrix formalism introduced by Centurioni  
 220 [44, 45] is here refined for the absorption of incoherent light by a multi-layer system, such as a stack  
 221 of  $n$  individual solar cells.

222 Let  $X_i$  be the interface matrix, describing the light reflection and transmission at the  $i$ -th interface,  
 223 and  $L_i$  the layer matrix, describing the transmittance of the absorber layer  $i$ :

$$224 \quad X_i = \begin{pmatrix} 1 & -R_i^- \\ R_i^+ & 1 - R_i^- - R_i^+ \end{pmatrix} \quad L_i = \begin{pmatrix} 1/T_i^+ & 0 \\ 0 & T_i^- \end{pmatrix} \quad \mathcal{M}_i = X_i \cdot L_i \quad (16)$$

225  $R$  and  $T$  stand for the reflectance and transmittance of the incident and transmitted power,  
 226 respectively. While both are wavelength, polarization and angular dependent quantities,  $L$  also  
 227 depends on the total layer thickness. The lower index refers to the layer number with  $n$  as the last  
 228 layer. The upper index indicates either downwards (-) or upwards (+) travelling light. For perfect anti-  
 229 reflective properties ( $R = 0$ ) or for transparent media ( $T = 1$ ), the identity matrix is obtained.

230 The scattering matrix of the system  $\mathcal{S}$  is now given by the product of the scattering matrices  $\mathcal{M}_i$

$$231 \quad \mathcal{S} = \prod_{i=1}^n \mathcal{M}_i, \quad s = \mathcal{S}_{11} + R_{n+1}^+ \cdot \mathcal{S}_{12}. \quad (17)$$

232 The total absorption of light in layer  $i$  can now be determined via the absorption matrix  $\mathcal{A}_i$ :

$$233 \quad \mathcal{A}_i = \prod_{j=1}^i (1 - R_j^+) \cdot (L_i - \mathbb{1}) \cdot \mathcal{Q}_i \quad \text{with } \mathcal{Q}_i = \frac{1}{s} \prod_{j=i+1}^n \mathcal{M}_j \text{ and } \mathcal{M}_{n+1} = \mathbb{1}. \quad (18)$$

234 Computing the energy flux  $\Phi_i$  in layer  $i$

$$235 \quad \Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^- \end{pmatrix} = \varphi_i \cdot \left[ \mathcal{A}_i \cdot \begin{pmatrix} 1 \\ R_{n+1}^+ \end{pmatrix} \right] \quad \text{with } \varphi_i = \frac{\text{Re}(\gamma_i)}{\prod_{j=1}^i \text{Re}\left(\frac{\gamma_j}{\gamma_{j-1}}\right)} \quad (19)$$

236 finally allows to calculate the total absorption in layer  $i$

$$237 \quad A_i = \phi_i^+ - \phi_i^-. \quad (20)$$

238 The  $\gamma$ -factor in Eq. 19 is defined for layer  $i$  by

$$239 \quad \gamma_i = \frac{N_i^{(*)}}{N_0} \cdot \frac{\cos \vartheta_i}{\cos \vartheta_0} \quad (21)$$

240 and describes the undergoing effects of the energy flux in the direction normal to the  $i$ -th interface  
 241 for s-polarized light ( $N_i$ ) or p-polarized light ( $N_i^*$ ). The first factor reflects the respective change in  
 242 velocity, expressed by the complex indices of refraction  $N$ ; the second term accounts for variations in  
 243 the area cross-section, expressed by the incident and refracted angle  $\vartheta_0$  and  $\vartheta_i$ , respectively.

---

### 244 2.3. AMBIENTAL AND ATMOSPHERIC EFFECTS

245 Since solar cells are encapsulated in PV modules, their operating temperature  $T$  is in general higher  
 246 than the ambient temperature  $T_{amb}$ . This is especially the case in the afternoon, when more heat is  
 247 radiated out by Earth's surface, once the local insolation has passed its peak value. However, higher  
 248 temperatures can lead to significant increases in  $J_0$  (see Eq. 13) and in turn to a reduction in the  
 249 power conversion efficiency.

250 A simple and widely used way to estimate the operating cell temperature is given by [46]

$$251 \quad T = T_{amb} + (NOCT - 20) \cdot \frac{P_{sun}}{800}, \quad (22)$$

252 where  $P_{sun}$  stands for the total incident solar irradiance in  $W/m^2$  and *NOCT* for the **Nominal**  
253 **Operating Cell Temperature**, which is typically around 48 °C for silicon. The *NOCT* is defined as the  
254 mean solar cell junction temperature within an open-rack mounted module in a standard reference  
255 environment: tilt angle at normal incidence to the direct solar beam at local solar noon; total  
256 irradiance of 800  $W/m^2$ ; ambient temperature of 20 °C; wind speed of 1 m/s and nil electrical load. It  
257 is an important parameter in module characterisation, since it is a reference of how the module will  
258 work when operating in real conditions.

259 Although records for  $P_{sun}$  are widely available, the irradiance is a spectrally integrated quantity and,  
260 as such, cannot resolve the impact of spectral variations on a solar energy technology. For multi-  
261 junction solar cells, the actual solar spectrum is therefore needed. But while the sunshine received  
262 by a terrestrial solar panel continuously changes due to Earth's rotation and revolution, the solar  
263 spectrum also depends on the chemical composition and meteorological condition of the  
264 atmosphere – both being subject to fluctuations on a minutely time scale. In order to account for  
265 these dynamics, *minutely* time series of historical, global (hemispherical) solar spectra between 2004  
266 and 2018 were reconstructed from multiple satellite-retrieved datasets via the open-source program  
267 SMARTS [17, 18]; the method is in detail described in [19]. This sequence of spectra was then used as  
268 the solar resource data  $GTI(\lambda)$  in the integral of Eq. 14.

### 269 3. RESULTS & DISCUSSION

270 The analytical formalism of Section 2 allows to calculate the maximum power of a solar cell for  
271 arbitrary model parameters. As such, the incident solar spectrum could be treated as a variable  
272 quantity now. The following three examples are based on this idea and highlight how one can study  
273 the impact of a highly-variable solar spectrum on the potential energy yield.

274 By minutely tracking any variations in the Sun's position, sunshine duration, meteorological condition  
275 and atmospheric chemistry, a series of historical, global (hemispherical) solar spectra at one-minute

276 intervals were retrieved from 2004 to 2018 for the four different climatic zones on Earth, as in [19],  
277 represented by Trondheim (Norway), Paris (France), Cairo (Egypt) and Nairobi (Kenya). All here  
278 considered examples use these sequences of spectra as the solar resource data  $GI(\lambda)$  in the integral  
279 of Eq. 14. The management and analysis of the large datasets is performed by the software Maple  
280 2017 from Maplesoft on the **York Advanced Research Computing Cluster (YARCC)**.

281 First, since Eq. 9 and Eq. 11 analytically link the maximum output power to the incident solar  
282 spectrum for arbitrary electrical parameters, a typical industrial cell is considered in Section 3.1. The  
283 model parameters are listed in Tab. 3 and were extracted from SunPower datasheets; its optical  
284 response has been reported as the  $EQE$  of a SunPower's Maxeon™ II solar cell. Furthermore, to  
285 clarify whether changes in the solar spectrum or ambient temperature have the greatest effect on  
286 the annual energy yield, their influences are separately assessed.

287 Second, choosing the optimal thickness combination of different absorber materials is often a  
288 challenging task. However, since Eq. 12 and Eq. 20 now directly link the maximum output power of a  
289 multi-junction solar cell to the individual layers' thicknesses, the analytically approach simplifies the  
290 analysis with material functions and is here applied to a perovskite-silicon tandem device in Section  
291 3.2.

292 Finally, the theoretical (detailed balance) limit for terrestrial solar cells is derived in Section 3.3 by  
293 using a sequence of multi-year solar spectra instead of a standard solar spectrum [47].

---

### 294 3.1. PV PERFORMANCE OF CURRENT'S INDUSTRIAL STATE-OF-THE-ART

295 The annual energy yield of a typical industrial solar cell is derived for the following cases:

- 296 1. The cell is kept at a constant temperature.
- 297 2. The cell and the surrounding ambient have the same temperature.
- 298 3. The cell experiences the elevated temperature of a module according to Eq. 22.
- 299 4. As case 2, but the absorption is 100% for photons beyond the silicon bandgap and 0% otherwise.
- 300 5. As case 3, but the absorption is 100% for photons beyond the silicon bandgap and 0% otherwise.

301 These differentiations allow to distinguish the impact of local temperatures from variations in the  
 302 solar spectrum, established by the site-specific meteorological and atmospheric dynamics.

303 The cell's parameters are listed in Tab. 3, whereas the front reflection  $R_1$  is assumed to follow the  
 304 correction factor chosen by Ramirez [48] for the cases 1-3,

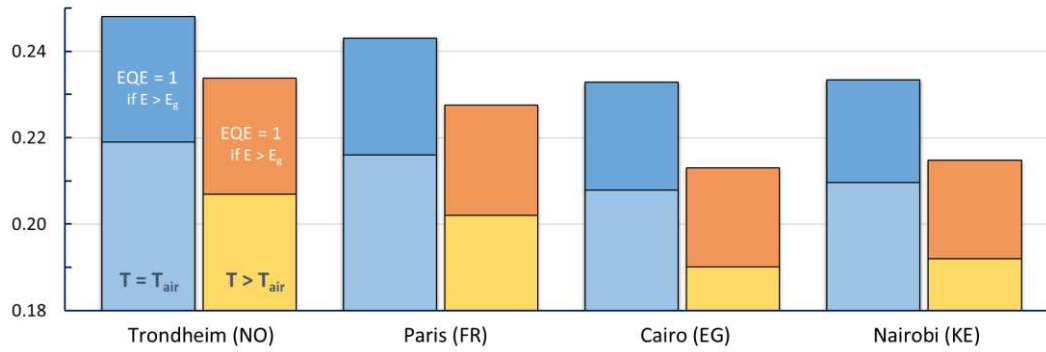
$$305 \quad R_1(\vartheta_0) = 0.1 \cdot \left( \frac{1}{\cos \vartheta_0} - 1 \right), \quad (23)$$

306 with the angle of incidence  $\vartheta_0$ , and  $R_1 = 0$  otherwise.

307 Figure 4 summarizes the outcomes and shows how the performance of a typical industrial solar cell  
 308 depends on the geographical location. While the cell at Cairo may not work as efficient as at  
 309 Trondheim, Cairo's insolation level is still twice as much compared to Trondheim and, therefore,  
 310 enables a far higher energy yield. The comparison shows, that if solar cells can absorb more sunlight  
 311 at the same temperature, the efficiency gains will exceed those achieved by passive cooling methods  
 312 alone. Industrial solar panels thus have the potential to increase efficiency levels by 4% in absolute  
 313 by combining radiative cooling methods [49] with absorption enhancement schemes [50, 51, 52, 53,  
 314 54, 55, 56]. Consequently, while large area silicon cells are approaching their practical Shockley-  
 315 Queisser-limit of 26% in the lab/fab [5], more R&D efforts are needed to boost their performances to  
 316 the same level in the outdoors.

$J_{sc}$	$V_{oc}$	$\eta$	$\xi$	$J_o$	$r_s$	$r_p$	<i>NOCT</i>
41.66 mA/cm <sup>2</sup>	0.68 V	22.1 %	1.02	193 fA/cm <sup>2</sup>	1.14 Ω.cm <sup>2</sup>	4.87 kΩ.cm <sup>2</sup>	48 °C

317 **Table 3.** Representative parameter set for industrial silicon solar cells. The dark current  $J_o$ , non-ideality factor  $\xi$   
 318 and specific series  $r_s$  and shunt  $r_p$  resistances were extracted from the IV data-curve of a SunPower® E20/333  
 319 solar panel at standard test conditions, whereas the short-circuit current  $J_{sc}$ , open-circuit voltage  $V_{oc}$  and power  
 320 conversion efficiency  $\eta$  were derived from the AM 1.5G solar spectrum [57], using the external quantum  
 321 efficiency (*EQE*) of a SunPower's Maxeon™ II solar cell. All quantities are normalized to the average cell area of  
 322 170 cm<sup>2</sup>, i.e. to the total SunPower panel area (1.63 m<sup>2</sup>) divided by the number of interconnected Maxeon cells  
 323 (96). In addition, a more typical value of 48 °C is assumed for the nominal operating cell temperature *NOCT*  
 324 instead of SunPower's certified 45 °C.



325

326

**Figure 4.** The performance of a typical industrial silicon solar cell at four climatic-distinctive cities. The overall conversion efficiency (ordinate) is defined by the ratio of the total energy yield to the total insolation received between February 2004 and February 2018. Using a dynamic solar spectrum with a 1 nm spectral and 1 min temporal resolution, the latitude-tilted solar cell of Tab. 3 is modelled to follow either the actual ambient temperature ( $T=T_{air}$ ) or elevated temperature ( $T>T_{air}$ ) according to Eq. 22. If the absorption were unity up to the absorption edge, the absolute efficiency gain would be greater at Trondheim (+2.7 %) than at Nairobi (+2.3 %).

332

In contrast, passive cooling techniques have a greater effect at Cairo than at Trondheim, translating into an absolute efficiency gain of +1.8 % and +1.2 %, respectively. When combining the two effects, cooling and unity quantum yield, the absolute gain becomes almost independent of the location ( $+4.1 \pm 0.1$  %).

335

Additional increases in the overall conversion efficiency are then only possible by reducing series resistances, shunt paths, non-radiative recombination centres and electrical noise. For comparison, the power conversion efficiency is

337

0.221 under standard test conditions.

---

### 338 3.2. THE PEROVSKITE-SILICON TANDEM CELL

339 One way of improving the overall efficiency of a silicon solar cell is the inclusion of a top absorber  
340 layer with a higher energy bandgap than silicon. If the high energy photons are all absorbed in the  
341 top layer, thermalization losses would be greatly reduced, because they are caused by the blue part  
342 of the solar spectrum in silicon. In principal, a silicon-based tandem cell then operates at a lower  
343 temperature.

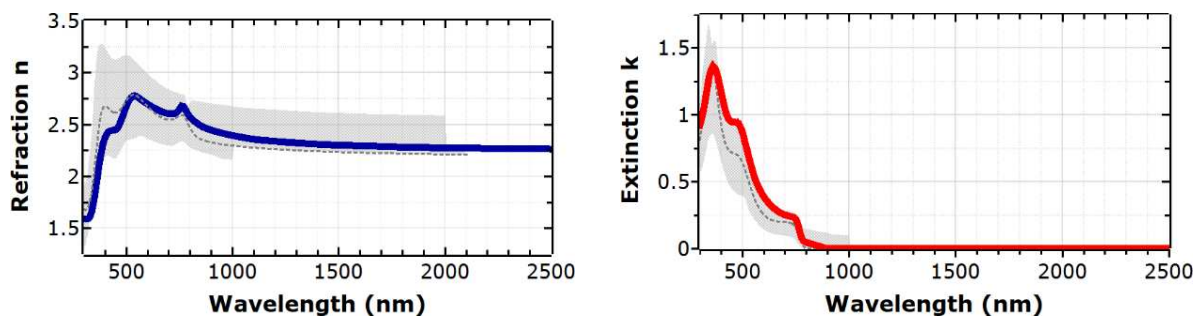
344 Yamaguchi et al. reviewed the progresses and challenges for integrating silicon with other materials  
345 [58]. The authors quote perovskite materials as a promising candidate for this endeavour – like many  
346 others [15, 59, 60]. In fact, despite their extraordinary short history as PV material [59], rapid  
347 developments already enabled 27% efficient silicon-perovskite tandem cells [60, 61], exceeding the  
348 26.7% efficiency of the current world-record single-junction silicon solar cell [62]. However, the  
349 efficiency testing of some high-performance perovskite-based cells is often completed under the  
350 inert atmosphere in a glove box.

351 Nevertheless, Hoerantner and Snaith [63] modelled the silicon-perovskite tandem performance  
352 under the most typical outdoor conditions. Since the authors used a constant silicon substrate  
353 thickness of 3.5 mm for the yield optimisation, instead of a usual wafer thickness between 100 and  
354 300  $\mu\text{m}$ , their approach neglects the optical interplay between the perovskite and silicon material.  
355 Therefore, the scattering-matrix formalism for incoherent sunlight (see Section 2.2) is here applied to  
356 establish the theoretical upper limit of a silicon-perovskite tandem cell under actual solar spectra.  
357 The reflection between air and the perovskite's front interface thereby follows Ramirez correction  
358 factor, according to Eq. 23, whereas the back interface of the substrate is assumed to be 100%  
359 reflective to imitate the presence of a high-reflective mirror. The reflectances of all other interfaces  
360 are given by Fresnel's energy equations, i.e. not by the ratio of the waves' electric (or magnetic) field  
361 amplitudes but their absolute squares.

---

### 3.2.1 THE MATERIAL FUNCTION OF A MAPI LAYER

A  $\text{CH}_3\text{NH}_3\text{PbI}_3$  (MAPI) layer with an energy bandgap of 1.55 eV is placed onto a crystalline silicon substrate. MAPI is currently one of the most widespread perovskite compositions found in the literature [64, 65, 66, 67, 68, 69, 70, 71, 72, 73] [74, 75, 76, 77], yet its material function shows significant variations, see Fig. 5. Here, the data by Jiang et al. [71] are chosen, because the authors characterised the film properties in detail over a wide wavelength range (from 300 nm to 2500 nm) by combining the measurement results of variable angle spectroscopic ellipsometry, spectrophotometry and atomic force microscopy.



**Figure 5.** The refractive index (left) and extinction coefficient (right) of planar  $\text{CH}_3\text{NH}_3\text{PbI}_3$  (MAPI) layers. The solid line refers to the data published by Jiang et al. [71], used in this study, whereas the thin dashed line refers to the data by Löper et al. [68]. The results are based on a film thickness of 200 nm and 300 nm, respectively. Although both studies analysed the MAPI layer over a wide range of wavelengths via multiple techniques, the coefficients do not overlap. Generally, large differences can be found in the literature [64, 65, 66, 67, 68, 69, 70, 71, 72] for the dielectric function of MAPI, as highlighted by the grey shaded area.

---

### 3.2.2 THE TEMPERATURE DEPENDENCE OF A MAPI-SI TANDEM CELL

While a MAPI-Si tandem cell could potentially operate at a lower temperature than a silicon solar cell, the tandem cell is still considered to experience the same elevated temperature of a typical silicon-only device, according to Eq. 22. This particular choice is motivated by both the lack of underlying data available [78], and the many ongoing challenges related to the outdoor deployment of perovskite modules [79]. For example, Dupre et al. [80] showed that a “thermal benefit” strongly

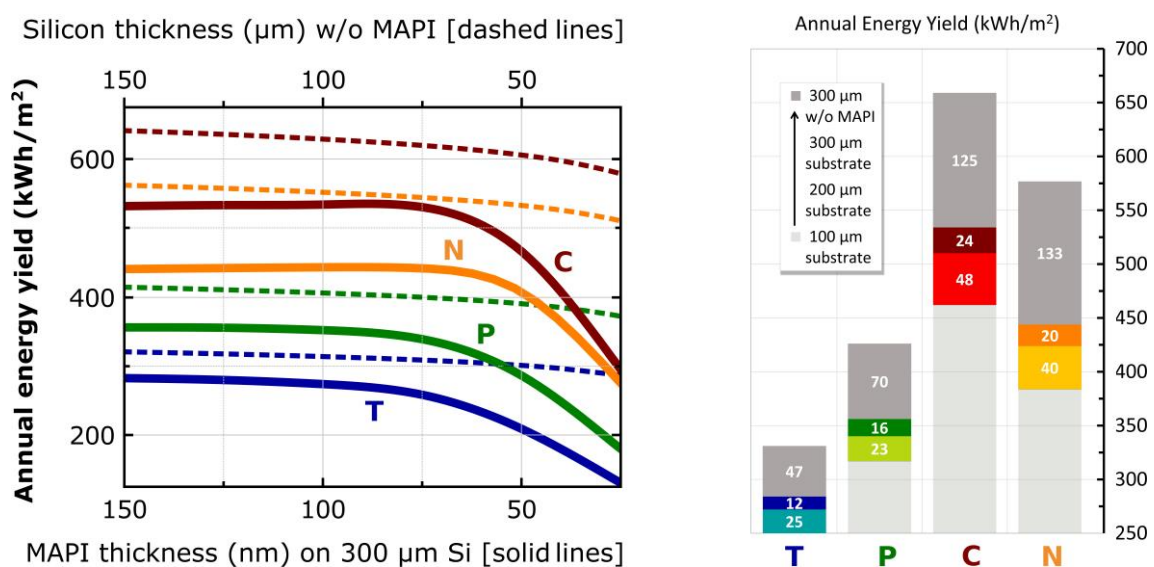
383 depends on the global heat transfer mechanisms between the module and its (outdoor) surrounding.  
384 Yet, the multiple interfaces of a perovskite solar cell stack can lead to mechanical failure during  
385 temperature cycling [81]. Cai et al. [82] also noticed a 30% efficiency drop by an increased contact  
386 resistance when going from 1 cm<sup>2</sup> cells to 25 cm<sup>2</sup> modules, whereas Stoichkov et al. [83] observed a  
387 rapid degradation of perovskite mini-modules under outdoor conditions, caused by breaches of the  
388 edge sealant. Finally, while most of the research has only been done on solution-processed  
389 perovskites, a spin-coating deposition technique is seen as incompatible with high-volume  
390 manufacturing methods. Therefore, perovskite layers ultimately employed in the field might show  
391 different physical properties as those currently associated with MAPI. Reference [79] gives a  
392 comprehensive discussion about the major impediment to highly efficient, stable and low-cost  
393 perovskites.

---

### 394 3.2.3 THE LAYER THICKNESS OF MAPI FOR SI-BASED TANDEM CELLS

395 Figure 6 compares the thickness dependence of the MAPI layer for different silicon substrates.  
396 Firstly, the annual energy yield scales with the substrate thickness, because the two cells are series-  
397 connected. The photo-generated current in the silicon layer limits the total electrical current of the  
398 tandem device. Secondly, the MAPI thickness scales with the location's latitude. As the solar  
399 irradiance decreases in the visible part of the spectrum, so does the photocurrent of the MAPI film.  
400 However, the near-infrared part of the spectrum is less affected by latitudinal changes, such that a  
401 thicker MAPI layer better mitigates current mismatches with the silicon bottom cell at higher  
402 latitudes. Thirdly, the application of a MAPI layer always reduces the overall performance, if series  
403 connected [76], hence the highest yield is obtained by a single-junction silicon solar cell. The  
404 bandgap combination of MAPI and silicon is the reason for this observed setback, as shown in  
405 section 3.3: In theory, if all photons with energy above the bandgap were to contribute to the  
406 individual cells' electrical currents (ideal scenario), shifting the bandgap of the top cell from 1.55 eV  
407 to 1.70 eV could dramatically increase the overall conversion efficiency of a silicon-based tandem cell

408 from ca. 30% to over 40%; in practice, however, the higher the bandgap of the top cell, the more  
 409 photons are transmitted to the silicon substrate, and thus the thicker the top layer must become to  
 410 mitigate the effects of an increased photocurrent of the bottom cell.  
 411 Since parasitic influences by the absorber and other important cell materials have so far been  
 412 ignored, making thick – i.e. more than 1  $\mu\text{m}$  thick layers [63] – and high-quality crystallized  
 413 perovskites might become crucial for the application of high-bandgap perovskites in silicon-based  
 414 tandem cells.



415 MAPI thickness (nm) on 300  $\mu\text{m}$  Si [solid lines]  
 416 **Figure 6.** Annual energy yields of latitude-tilted solar cells. Neither shunts nor series resistances and only  
 417 radiative recombinations were assumed for the analysis. The left figure compares the outdoor performance of  
 418 silicon solar cells [dashed lines] to perovskite-silicon tandem devices [solid lines] at Trondheim (T), Paris (P),  
 419 Cairo (C) and Nairobi (N). Apparently, the deposition of a MAPI layer on crystalline silicon reduces the energy  
 420 yield of a silicon solar cell. The coloured bars in the right figure quantify the increases in annual energy yield  
 421 when going from a 100  $\mu\text{m}$  to a 300  $\mu\text{m}$  thick silicon substrate for a 200 nm (T), 150 nm (P), 100 nm (C) and 75  
 422 nm (N) MAPI coating, and after its removal (dark grey).

423 As one of the lead movers in this technology, Snaith's group already made a transition from MAPI to  
424 mixed cation and mixed anion materials [84], such as  $\text{Cs}_x(\text{MA}_{0.17}\text{FA}_{0.83})_{1-x}\text{Pb}(\text{I}_{0.83}\text{Br}_{0.17})_3$ . The bandgap  
425 of this triple-cation perovskite could potentially be increased beyond 1.65 eV by raising the Cs  
426 content  $x$  [85]. Yang et al. [86] thoroughly discuss the strategies and challenges for achieving high-  
427 bandgap perovskite materials for multi-junction solar cells.

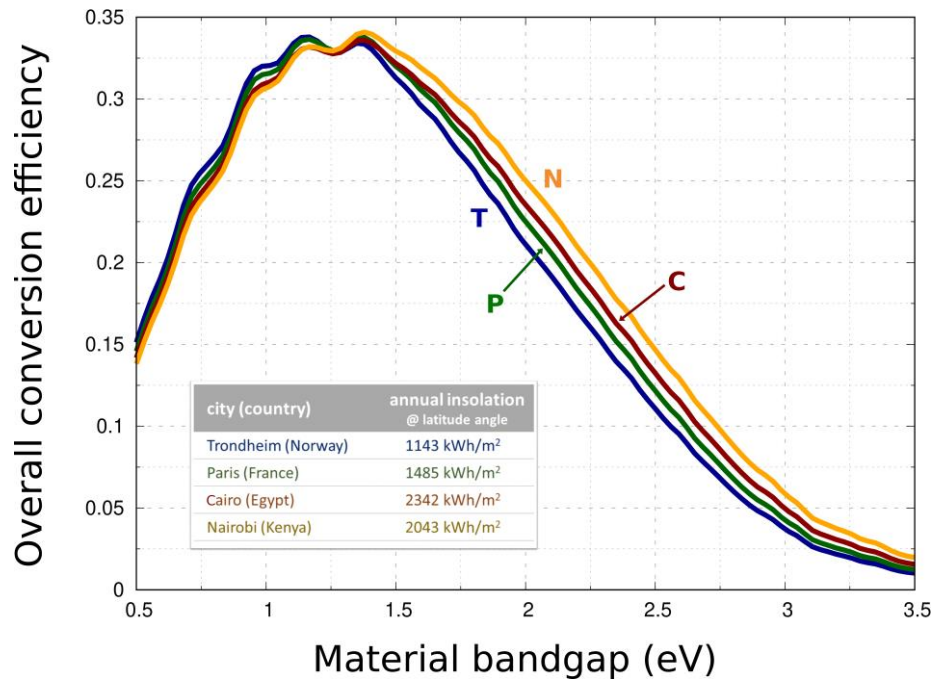
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### 428 3.3. LIMITING EFFICIENCIES OF TERRESTRIAL SOLAR CELLS UNDER DYNAMIC 429 SOLAR SPECTRA

430 For general terrestrial applications, the harvesting or overall efficiency of a solar cell is defined by the  
431 ratio of the useful electricity produced and the total insolation received over the same time window.  
432 Following the method set out in [19], a 14-year time series of minutely terrestrial global  
433 (hemispherical) solar spectra was reconstructed to establish the limiting efficiencies of latitude-tilted  
434 (ideal) solar cells at four distinct climatic zones. The commonly definition of ideal solar cells is  
435 adopted by the following qualities:

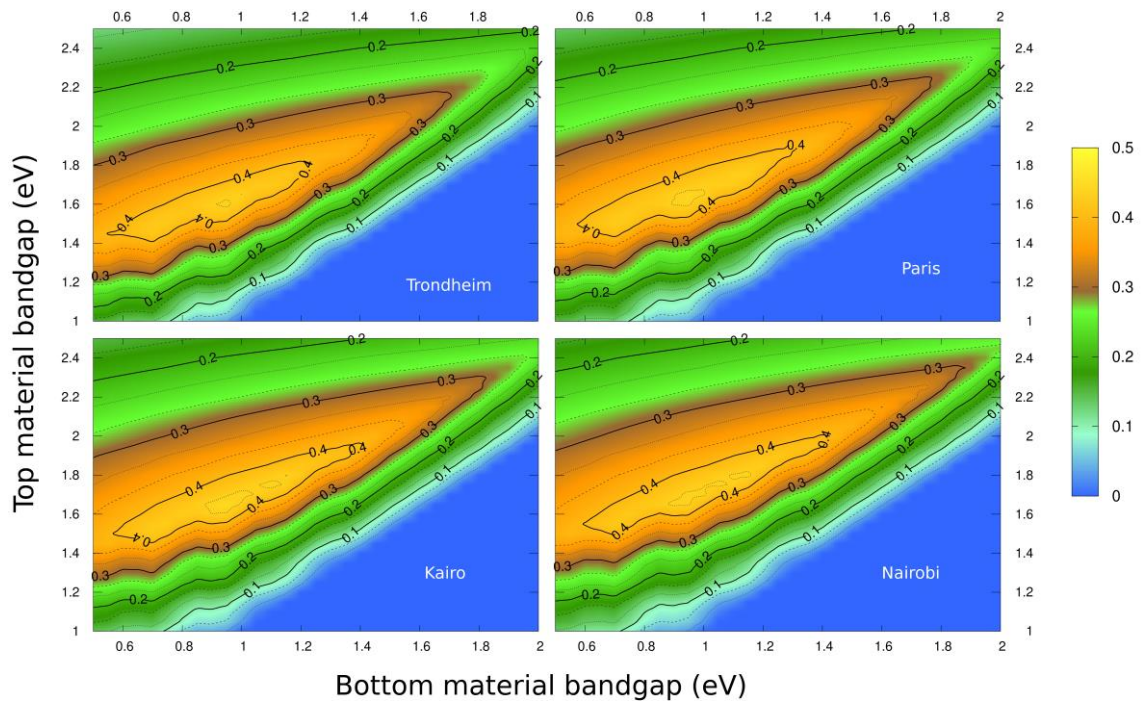
- 436 1. no reflection losses for all angles of incidence,
- 437 2. 100% absorption for photons above the energy bandgap (0% otherwise),
- 438 3. only direct bandgap transitions,
- 439 4. only radiative charge-carrier recombinations,
- 440 5. no electrical shunts nor series resistance effects,
- 441 6. a vanishing absorber thickness,
- 442 7. operating at ambient temperature.

443 The limiting efficiencies of a single, tandem and triple energy bandgap cell are shown in Fig. 7, 8 and  
444 9, respectively. While the optimum bandgap shows a weak site-dependency, using real material  
445 properties and thickness values could severely reduce the limiting efficiencies, as discussed in section  
446 3.2.3.



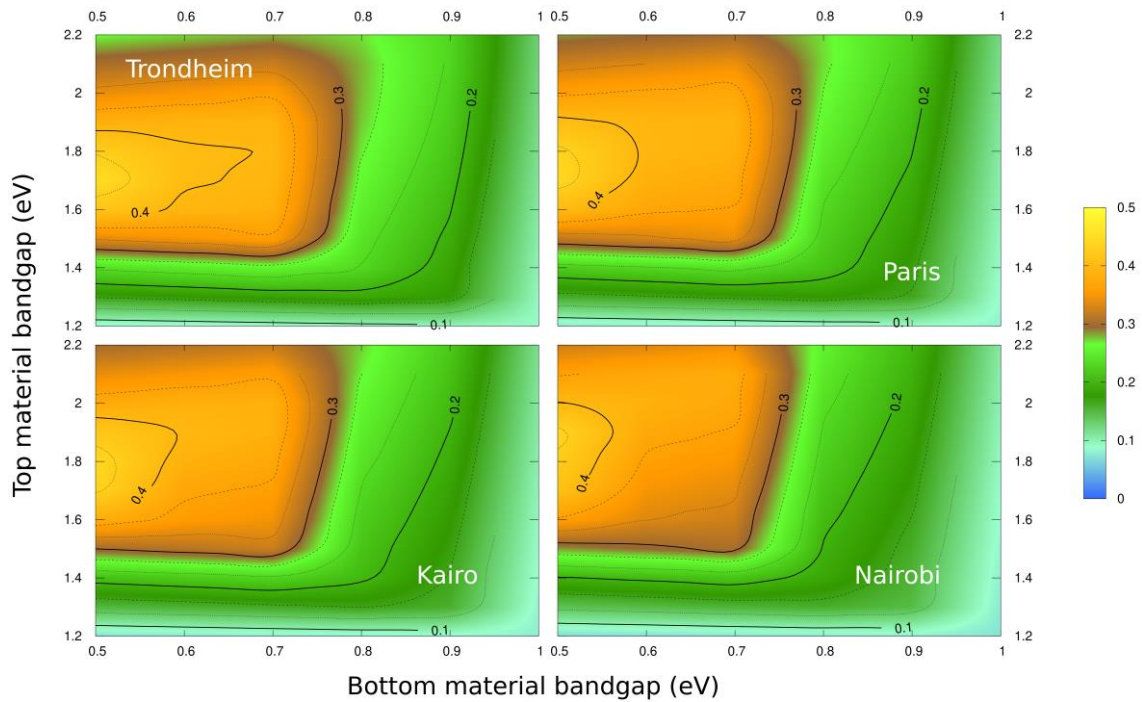
447

448 **Figure 7.** Limiting conversion efficiency of latitude-tilted, idealised single-junction cells as a function of the  
 449 material bandgap. For every minute, the produced electricity and the received insolation were derived from  
 450 satellite data series between 2004 and 2018 for the locations Trondheim (T), Paris (P), Cairo (C) and Nairobi (N),  
 451 following the method set out in [19]. After integration, the ratio of the total generated electricity and total  
 452 received solar insolation defines the overall conversion efficiency.



453

454 **Figure 8.** Limiting conversion efficiency of latitude-tilted, idealised double-junction solar cells as a function of  
 455 the top and bottom material bandgap. For every minute, the produced electricity and the received insolation  
 456 were derived from satellite data series between 2004 and 2018, following the method set out in [19]. After  
 457 integration, the ratio of the total generated electricity and total received solar insolation defines the overall  
 458 conversion efficiency.



459

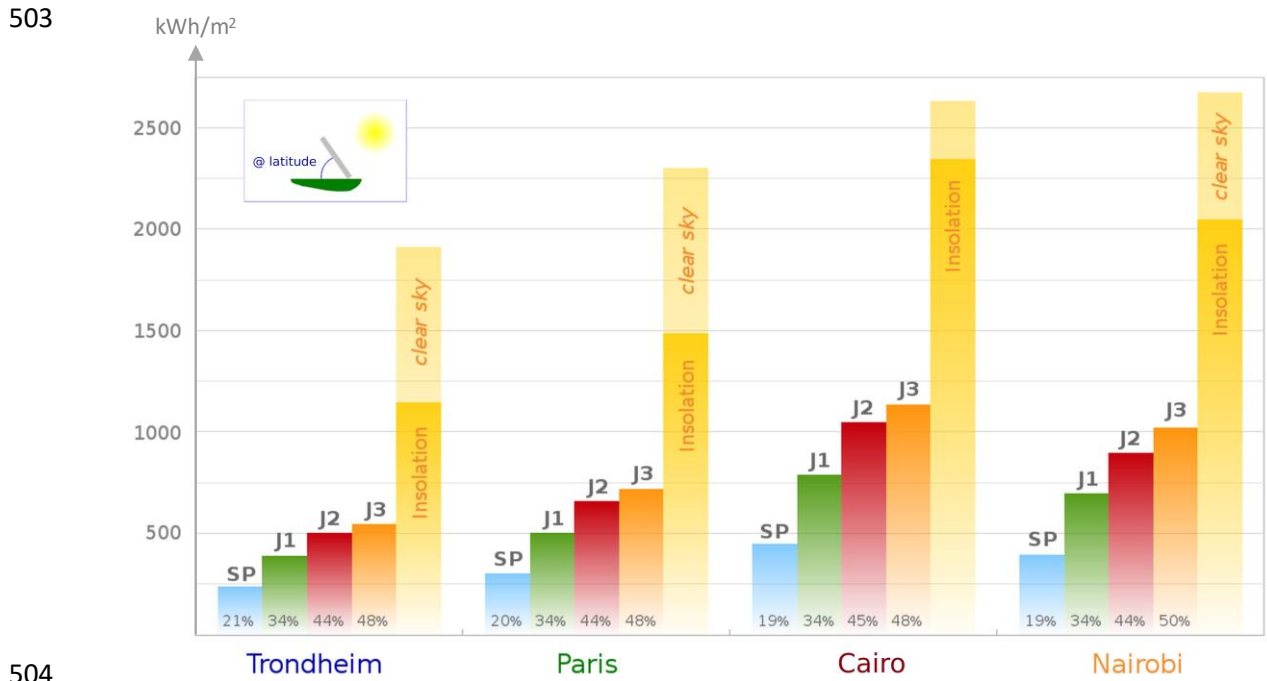
460 **Figure 9.** Limiting conversion efficiency of latitude-tilted, idealised triple-junction solar cells as a function of the  
 461 top and bottom material bandgap. The middle cell is assumed to be made of a bandgap of 1.1 eV,  
 462 representative for silicon. For every minute, the produced electricity and the received insolation were derived  
 463 from satellite data series between 2004 and 2018, following the method set out in [19]. After integration, the  
 464 ratio of the total generated electricity and total received solar insolation defines the overall conversion  
 465 efficiency.

#### 466 4. SUMMARY AND CONCLUSIONS

467 Driven by the need to replace conventional but environmentally damaging electricity resources, such  
 468 as coal and gas, solar PV emerged as the fastest growing renewable energy technology in the world.  
 469 Its enormous potential, however, may not be unlocked if solar cells are based on only one absorber  
 470 material. Therefore, multi-junction technologies are increasingly appealing as a pathway to go, due  
 471 to the rapid developments of high-bandgap perovskites and the progresses in combining III/V  
 472 materials with silicon. Multi-junction solar cells, however, require a more careful evaluation, because  
 473 they are more susceptible to spectral variations [15].

474 Here, the impact of weather/climate effects on the energy yield of solar cells is rigorously analysed  
475 by modelling the incident spectrum at a high spectral and temporal resolution, i.e. at 1 nm  
476 wavelength intervals and at 1 min time steps, over a long-time window (14 years) and for the main  
477 climatic zones. Therefore, a scattering-matrix treatment is formulated based on incoherent sunlight,  
478 as it is seen as the more relevant case for photovoltaics. Secondly, while the modelling of solar cells  
479 has previously relied on numerical solutions for the maximum electrical power, often at the cost of  
480 computational demanding operations or numerical stability issues, a framework is here proposed  
481 that is suitable for the analytical assessment of multi-junction solar cells. For arbitrary cell  
482 parameters, closed-form and *explicit* expressions are derived to facilitate both design optimisation  
483 routines and the accurate modelling of PV outdoor performances with observational datasets.  
484 The current industrial state-of-the-art, its limiting potential and the ongoing developments are thus  
485 reviewed from a different perspective. In contrast to previous studies that have only focused on the  
486 (spectrally integrated) irradiance, multiple satellite-product services are used to retrieve long-time  
487 series of historical, global (hemispherical) solar spectra. As major issues relevant to the deployment  
488 of solar cells can thereby be quantified, key differences between the performance in the lab and  
489 under the effects of a dynamic solar spectrum are apparent. For example, if passive cooling and  
490 advanced light management techniques were combined, a 26% harvesting efficiency is found as a  
491 more realistic limit for conventional silicon cells, but which is almost 10% in absolute lower than the  
492 theoretical radiative limit (34%) derived from idealised conditions.  
493 In case of perovskite-silicon tandem cells, the perovskite's bandgap will need to be carefully tuned to  
494 a desirable thick silicon substrate, though its layer thickness and material quality would also need to  
495 be traded off: Since the bandgap of  $\text{CH}_3\text{NH}_3\text{PbI}_3$  is too low (1.55 eV), depositing a MAPbI<sub>3</sub> film onto a  
496 300  $\mu\text{m}$  thick silicon substrate reduces the harvesting efficiency of a silicon-only device (29%) by 4%  
497 (Trondheim) to 7% (Nairobi) in absolute. In contrast, efficiencies higher than 40% are more likely  
498 obtained by a high-crystalline but thick ( $> 1 \mu\text{m}$ ) perovskite layer with bandgap of 1.70 eV.  
499 Alternatively, if the silicon were used as the middle cell of a triple-junction approach, the bandgap

500 sensitivity issue becomes relaxed and would make a thin MAPI layer (< 200 nm) more acceptable as  
 501 top cell. Fig. 10 summarizes the differences, challenges and opportunities for multi-junction solar  
 502 cells in different climatic zones.



504 **Figure 10.** Comparison of an industrial standard solar panel (SP) to the theoretical maximum annual energy  
 505 yield of idealised multi-junctions in four distinct climatic zones; J1, J2 and J3 indicate the number of junctions.  
 506 The insolation is taken as 100% for the overall efficiency, shown at the bottom of the bars. The modelled clear-  
 507 sky insolation is included as a light-coloured bar to the insolation, for comparison. All surfaces are latitude-  
 508 tilted, facing toward the Equator. While the harvesting efficiencies are almost site-independent, the energy  
 509 yield is sensitive to the geographical location and all-sky conditions. For example, a single-junction cell at Cairo  
 510 can produce more electricity than a multi-junction cell at Trondheim or Paris. While Cairo and Nairobi have  
 511 approximately the same clear-sky conditions, Nairobi still receives less sunshine than Cairo due to a more  
 512 frequent cloud formation and higher air pollution level.  
 513

514 Since the active area is assumed to be free of any debris in this study, the effect of snow cover  
515 periods and the impact of ice, shadings or dirt along with any induced material degradations are left  
516 out. The potential energy yield should also be judged by the AC-DC conversion losses over the  
517 lifetime of the PV system, but is neglected here. Finally, the temperature will likely not be uniformly  
518 distributed in multi-junction cells. So future work would need to consider the impact of thermal  
519 gradients as well as the temperature dependency of the dielectric function.

520 In summary, an analytical solution is derived for the maximum electrical power of a solar cell. Since  
521 the general solution (Eq. 9a) is free of restrictive parameter assumptions, technical studies now could  
522 collapse from computational expensive endeavours to simple data management strategies. Here,  
523 three data-driven examples are indeed based on this approach, as they break free from solving a  
524 complicated or transcendental equation numerically. Consequently, the application of the framework  
525 may become a crucial factor in the response analysis of solar modules, because it effectively enables  
526 to model the behaviour of all its interconnected sub-cells analytically. In conclusion, this paper not  
527 only indicates critical aspects for the deployment of multi-junction technologies in the outdoors, but  
528 it does also give new theoretical insights about the impact of the cell's parameters on the conversion  
529 efficiency and thus presents a powerful analytical tool for the design and assessment of more  
530 efficient solar cells.

## 531 5. ACKNOWLEDGEMENTS

532 This research did not receive any specific grant from funding agencies in the public, commercial, or  
533 not-for-profit sectors. Thanks are given to Andrea Canino, Alberto Jimenez, Stefan Kremer, Kezheng  
534 Li, Manuel Mendes, Claudio Padilha, Dario Rapisarda, Mark Scullion and Brian Smith for their  
535 thorough and helpful critiques. The author wants to thank the Institute of Physics for the carer and  
536 beneficiary fund, but also to Sara Castillo Avila for her patience, encouragements and continuous  
537 support over the past two years, in particular.

538 6. ADDITIONAL INFORMATION

539 **Competing interests:** The author declares no competing financial and non-financial interests in  
540 relation to the work described.

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