**Prediction of fatigue damage region with the use of the notch critical plane approach for crack initiation and propagation**

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**Abstract**: The advanced volumetric approaches are reviewed briefly to state the stress/strain-based or energy-based parameters of fatigue damage region are critical. A new approach is proposed to distinguish crack initiation and propagation stage. The angles of crack initiation are predicted by notch critical plane (NCP) approach (Luo *et al.* *Fatigue Fract Eng Mater Struct*, 2019, 42(4): 854-870). NCP is the plane which passes through the fatigue critical point (FCP) and undergoes the maximum shear stress amplitude. The experiment of thin-walled round tube components is conducted to check these approaches. The direction of crack initiation is basically the same as that of NCP. Most of the predicted lives fall within an error factor of 3.

**Keywords**: notched specimen, crack initiation, complex fatigue, direction of crack initiation, length of crack initiation

**NOMENCLATURE**

|  |  |
| --- | --- |
| *a*1, *a*2 | Material constants which depend on the fracture strength |
| *A*, *B* | Material constants which rely on the material and stress ratio |
| D | Notched components with 2mm diameter circular hole |
| *E* | Young's Modulus |
| *F* | Geometry dependent constant |
| *f*-1 | The fully reverse tensional fatigue limit |
|  | Equivalent stress function. |
|  | Range of the fully reverse axial fatigue limit |
| *K*T | theoretical stress concentration coefficient of the notch |
| *K*f | The fatigue notch coefficient |
|  | Range of threshold value for fatigue crack propagation |
| *l*0 | Critical distance |
| *R* | Stress ratio |
| *r*eff | The radius of fatigue damage area of modified SFI |
| *t*-1 | The fully reverse torsional fatigue limit |
| V | Volume of fatigue damage area |
| Y | Notched specimens with waist round hole |
| *θ*2 | Angle of fatigue crack initiation |
| *θ*m | The mean value of crack initiation angle |
| *θ*p | The predicted angle of fatigue crack initiation |
| μ | Poisson's ratio |
|  | Notch radius |
| *σ*a | Normal stress amplitude of fatigue loading |
| *σ*b | Fracture strength |
|  | Stress field intensity |
|  | Maximum normal stress |
| *σ*r, *σθ*, *σrθ*, | Three stress components in the polar coordinate |
| *σ*y1 | Yield strength |
|  | Shear stress amplitude |
| *φ* | Phase angle |
|  | The weight function |
|  | Region of fatigue damage |

1. Introduction

**1.1 An introduction of advanced volumetric approaches**

Notched components are very common in mechanical engineering. Researchers pay close attention to fatigue life of notches because both stress concentration and stress multi-axiality exist at notch root [1-6]. In general, fatigue cracks initiate at notch root. That is meaningful to research the fatigue properties of notches under complex cyclic loading in engineering [7]. From the 1940s to 1950s, Neuber [8] and Peterson [9] have began to study the fatigue life estimation of notches. They considered the linear-elastic stress distribution within a certain region in the vicinity of the notch root and defined the fatigue notch reduction factor, *K*f, to assess fatigue lifetime of notches. Neuber's empirical formula and Peterson's empirical formula are as follows:



where, *K*T is the theoretical stress concentration coefficient, *ρ* is the radius of notches, whereas *a*1and *a*2 are the material constants which depend on the fracture strength, *σ*b.

The predicted fatigue life of notches could be conservative through regarding the maximum stress or strain in the vicinity of notch as the damage parameter. Neuber’s and Peterson's empirical equations are useful tools to design notched specimens under cyclic loading without the finite element analysis (FEA). A large number of researchers have been inspired by Neuber’s and Peterson’s idea of using stresses averaged in the vicinity of the notch root, and they have developed some notch fatigue analysis approaches which are called the “Advanced Volumetric Approaches”.

Lazzarin and Berto *et al.* [10-14] defined the average of the strain energy density (SED) over a well-defined control volume as the parameter to assess lifetime. Both the accuracy and reliability of the SED method was evaluated against experimental results [15]. Yao [16-17] conducted the stress field intensity (SFI) method to correct the maximum stress in the vicinity of notch to consider the influence of stress gradient, as shown in Eq. (2):



where  is the fatigue damage parameter of SFI,  is the area of fatigue damage, *V* is the volume of fatigue damage area,  is the equivalent stress,  is a vector from the fatigue critical point (FCP) to these point in the vicinity of notch,  is the weight parameter that means the different contribution of these point in the vicinity of notch to fatigue damage. The stress field intensity method is illustrated in Figure 1. The assumption of SFI approach is that the initiation of fatigue crack only depends on the contribution of fatigue damage of several or a dozen grains at notch root.



Figure 1. The illustration of SFI approach

The strain field intensity method was developed by Shang *et al.* [18] to assess fatigue crack initiation life, and this method takes the effects of the local stress-strain gradient on fatigue damage at the notch into account. Qylafku *et al.* [19-20] took the influence of the metallic plastic deformation and the stress gradient in the vicinity of notch on fatigue lifetime into account. They modified the SFI method by taking the distance between the point with the minimum stress gradient and notch tip as the radius of damage area, which is illustrated in Figure 2 where the minimum stress gradient is point A. The abscissa *r*eff of point A is the radius of fatigue damage area. The modified SFI method is shown in the following Eq. (3) and can be formalized mathematically as follows:



Figure 2. Stress distribution and stress gradient in the vicinity of notch [19-20]



where *σ*y is the stress in the direction of external fatigue loading, *r*eff is the radius of fatigue damage region.

The most advanced version of the Theory of Critical Distance (TCD) has been presented independently by Tanaka [21] and Taylor [22-24]. The theoretical background of TCD is linear elastic fracture mechanics (LEFM). The mean value of the maximum stress within a characteristic point distance, a line distance, a plane or volume area is regarded as the fatigue damage parameter to predict fatigue life. TCD is illustrated in Figure 3. On the basis of the characteristics of the integration domain being used, TCD can be classified into four types. These equations formalizing the TCD are:



EL Haddad equation：

where，critical distance *l*0 is the key parameter which could be determined via EL Haddad’s empirical equation[25]. This equation is written via Eq.(5).  is the range of the fully reversed tensional fatigue limit and  is the threshold value range of the stress intensity factor.



Figure 3. The illustration of point method, line method and area method

The initial purpose of TCD proposed is to assess the fatigue limit of metals [23] , then Susmel and Taylor [26-27] corrected TCD to evaluate medium-cycle fatigue (MCF) lifetime by taking the assumption that there is a power relation between *l*0 and fatigue lifetime *N*f:



where *A* and *B* are material parameters which are determined by the material and the stress ratio *R*. The analysis shows a negative correlation between critical distance *l*0 and fatigue lifetime, which violates the hypothesis [23] that *l*0 is a material constant.

Susmel and Taylor [28-30] further came up with the elasto-plastic TCD to assess the low-cycle fatigue life of notches by combining the Manson-Coffin’s method [31] and S-W-T parameter [32]. Moreover, Susmel [33] conducted the experiments on the fatigue endurance of V-notched components under proportional loadings with different stress ratios *R*. Furthermore, Susmel [33] discussed the applicability of the multiaxial fatigue parameter in terms of the TCD, including Crossland’s parameter [34], Papadopoulos’s parameter [35-36], Matake’s parameter [37], McDiarmid’s parameter [38-39], and Susmel’s parameter [40]. In the end, the fatigue parameters based on the critical plane were taken as the only ones which could be coherently re-interpreted in terms of the TCD. The accuracy of the multiaxial version of the TCD (M-TCD) was evaluated by applying a lot of test data [7,41].

**1.2 Fatigue damage region of the advanced volumetric approaches**

The volumetric approaches have been the major method to evaluate the lifetime of notches under complex cyclic loading. The average values of the stress, strain or the strain energy density over a well-defined area are regarded as the fatigue damage parameters. Therefore, the key parameter of the above volumetric approaches is the radius of fatigue damage area which is summarized in the Table 1 according to the different definitions that are available in the literatures.

Table 1. Fatigue damage region of the advanced volumetric approaches

| Approaches | Authors | Fatigue damage region | Influence factors |
| --- | --- | --- | --- |
| Neuber's empirical formula | Neuber | Material constant *a*1 | *a*1 depends on fracture strength *σ*b |
| Peterson's empirical formula | Peterson | Material constant *a*2 | *a*2 depends on fracture strength *σ*b |
| SED approach | Lazzarin and Berto | Radius of critical volume (area) *R*0 | *R*0 depends on the threshold value of fatigue crack propagation, fatigue limit, Poisson’s ratio and shape of notch |
| SFI approach | Yao WX | Radius of fatigue damage field | depends on size of the grains |
| Modified SFI approach | Qylafku | Radius of fatigue damage field *r* | *r* depends on the metallic plastic deformation |
| TCD and M-TCD | Taylor and Susmel | Intrinsic crack length *l*0 | *l*0 depends on the threshold value of fatigue crack propagation, fatigue limit and stress ratio |

Not only the radius  but the critical distance *l*0 are vectors according the mechanical definition of SFI and TCD. The FCP is the initial point of these vectors. There are three steps to construct the fatigue damage parameter which can be used to predict the fatigue life of notch under multiaxial fatigue loading. 1) Locate the starting point of fatigue crack initiation at the notch edge, namely the FCP; 2) Determine the direction and length of the fatigue crack initiation path, namely the direction of the NCP and integral path of the volumetric approaches; 3) Select the proper multiaxial fatigue parameters based on critical plane theory.

The axes of the principal stresses rotate during non-proportional cyclic loading, which lead to the difference of points bearing the maximum stress on the notch edge. The notch area in the presence of sharp notches enters the plastic deformation because of the severe stress concentration and residual stress. The stress mainly remains constant at notch root, thus the notch tip can be regarded as the FCP. Luo *et al.* [1,42] proposed the local stress response approach to determine the FCP under complex cyclic loading for blunt notches. The accuracy of this method was verified experimentally. In particular, it was demonstrated that the proposed approach can locate accurately the FCP in notches under complex cyclic loading. This is the first step to assess the fatigue lifetime of notches under multiaxial fatigue loading. For the third step, there are a large number of multiaxial fatigue damage parameters based on the critical plane theory which are summarized in literature [43-46]. So, the most important and difficult step is the second where the direction and length of crack initiation need to be researched. The length of fatigue crack initiation is the radius of damage area and the direction of damage area is both the orientation of the NCP and the integral path of the volumetric approaches.

It is worth noticing that the number of cycles to crack initiation is evaluated by combining the fatigue damage parameters and the *S*-*N* curve. The crack propagation life instead is evaluated by using Paris’ law. At the crack initiation stage, the local microstructure and the orientation of crystal are the main factors which affect the direction and length of crack initiation. LEFM and continuum mechanics cannot be used to model the crack initiation stage since the hypothesis of continuous material is no longer valid. Those methods based on mesoscopic mechanics and micromechanics can be used to investigate and model the changes in microstructure when cracks initiate. Nevertheless, in situations of practical interest it is very difficult to apply these very sophisticated methods.

On the basis of authors’ pervious work in Ref. [1] and Ref. [42] where local stress response approach was used to locate the fatigue critical point and NCP approach was proposed to evaluate the fatigue life of notches. This paper further check the accuracy and validity of NCP approach which is applied to predict the direction of crack initiation at notch root. In this paper, a brief review of the advanced volumetric approaches is presented. A new approach based on the crack length is proposed to distinguish crack initiation and crack propagation. In addition, an objective approach to measure the angle of fatigue crack initiation is presented. The NCP approach which is used to determine the direction of fatigue crack initiation is verified using constant amplitude multiaxial fatigue tests. The fatigue tests were conducted on the thin-wall round tube notches made of 2297 aluminum-lithium alloy. The angle of crack initiation was measured using an optical microscope after the test. Moreover, the angle of crack initiation and fatigue lives of notched specimens are predicted by the NCP approach. The results show that the direction of crack initiation is basically the same as that of the NCP and most of the predictive lifetimes fall within 3-time scatter band.

1. The difference between crack initiation and crack propagation
   1. Length and direction of fatigue crack initiation

The philosophical question here is: “how can the length of fatigue crack initiation at the notch root be defined?” [2]. It is very hard to judge whether cracks exist or not. Macroscopic cracks of a few millimeters can be observed visually. Microscopic cracks can be measured by using specific experimental techniques. As the magnification of the microscope increases, finer microstructure can be discovered, such as: persistent slip bands (PSB), distorted crystals, dislocations, cavities [47]. Some authors [48-49] have suggested to use Young's modulus to characterize fatigue crack initiation based on damage mechanics. In spite of the methods being used, there is no clear boundary between un-cracked material and cracked material.



Figure 4. The illustration of Kitagawa-Takahashi diagram

Kitagawa-Takahashi [50] diagram describes the relation between the range of the fatigue limit and the crack length, which is illustrated in Figure 4. EL Haddad *et al.* [25,51,52] introduced an effective crack length, *l*0, into the solutions of stress intensity factors and the J integral method of analysis to predict the behavior of short cracks by modifying elastic and elasto-plastic fracture mechanics solutions. The modified elastic stress intensity factor is calculated according to the following equation:



where △S is the range of applied nominal stress, *l*0 is a constant for a given material, and *F* is a geometry dependent constant.

EL Haddad’s equation is empirical although the Kitagawa-Takahashi diagram can be fitted accurately by this equation. The material constant *l*0 is the virtual crack length which can characterize initial defects of the un-cracked material before cyclic loading. The cracks that are shorter than *l*0 are called fatigue short cracks and have different features [53-56]. In particular, the following considerations apply. (1) Short cracks have a much higher propagation rate than long cracks under the same far-field nominal stress. (2) The propagation rate of short cracks decreases with the increase of crack length within a certain range, and then accelerates when the short crack length exceeds a critical value. (3) Initiation and propagation of short cracks are deeply influenced by the material local microstructure. The sizes of grain, grain boundary and inclusions are closely related to the crack closure effect and zigzag effect in the process of short crack propagation, so the experimental data of short cracks are relatively dispersed. 4) The length of the short crack does not meet LEFM's requirements and traditional LEFM is no longer applicable for short crack.

Miller *et al.* [54-57] discussed the propagation process of fatigue cracks in detail, which is divided into three main categories, as shown in Figure 5. 1) Microstructurally short cracks (MSC); *l*<*d*1. In this category, continuum mechanic is no longer applicable. 2) Physically small cracks (PSC); *d*1<*l*<*d*2. 3) Highly stressed cracks with *d*2<*l*. In addition, Hong *et al.* [58] applied fractal dimension to characterize the crack length. The measurements of crack length reveal that the path of short cracks, compared to that of long cracks, possesses a more stable and relatively larger value of fractal dimension, which means that the paths of short cracks are more zigzag.



Figure 5.The three main categories of fatigue crack

MM: Microstructural mechanics; EPFM: Elastic plastic fracture mechanics

Previous researches [47,59,60] show that crack initiation is mainly driven by the shear stress while crack propagation is mainly driven by the normal stress. Many fatigue crack initiation models, such as slip-band extrusion and intrusion model [47,60,61,62], dislocation dipole accumulation model [63-65] and dislocation reaction model [66], are proposed based on micro-mechanism of fatigue damage. Wood’s model, i.e., one of the slip-band extrusion and intrusion approaches, has been widely used to explain fatigue crack initiation [67]-68] where fatigue cracking is regarded as a simple geometrical consequence of fine dip movements in the fatigue slip-bands.

The tests showed [69] that the direction of the fatigue crack initiation path is the orientation of the maximum shear stress amplitude for polycrystalline metallic materials under cyclic loading. It should be noted that the crack length to determine the initiation direction of fatigue cracks is usually of the order of several hundred microns. At a microscopic level, in a few microns, the path of crack initiation follows a zigzag trajectory due to the presence local crystals having different orientations.

* 1. The approach to distinguish crack initiation and propagation

For an ideal process of fatigue crack initiation and propagation, both the length of initiation crack *l*1 and the angle of initiation crack *θ*2 can be used to distinguish between the crack stage initiation and crack propagation stage. Point P is the FCP in Figure 6 which can be located by local stress response method[1,42]. Unfortunately, the crack path follows a zigzag trajectory in stage 1 because the Schmid factors are totally different for different crystals at the notch root [57-58].



Figure 6. The ideal process of fatigue crack initiation and propagation

Susmel and Meneghetti *et al.* [70-71] used the first crack transition to determine crack initiation. They found that cracks initiation and initial propagation are mixed mode governed [70-71]. The fatigue tests of U-notches under uniaxial fatigue loading were carried out to measure the length of initiation crack by Meneghetti*et al.* [71]. The results showed that the length of initiation crack ranges from 0.02mm to 0.26mm and the mean value of initiation length is 0.075mm, which is very close to the critical distance *l*0=0.08 [71]. However, Ref. [71] does not distinguish between positive and negative angles of crack initiation. In addition, the angle of the first crack transition depends on the magnification of the microscope.

The length of crack is objective and meaningful for distinguishing crack initiation and propagation in engineering. Shang *et al.* [72] defined the maximum length of the non-propagating crack in the vicinity of notch as the crack initiation size, namely *d*1 in Figure 5. However, a large number of parameters are necessary in the numerical calculation process, which increases the complexity of calculations since *d*1 is not strictly the maximum length of the non-propagating crack. A crack length shorter than *l*0 can be regarded as the initiation crack which cannot reduce the fatigue limit for most metallic materials. However it is very difficult to accurately measure the length of crack which strongly depends on the magnification [53,58].

A new approach proposed to distinguish between crack initiation and crack propagation is showed in Figure 7. The point O is the center of notch and point P is the FCP. Apply point P as the center and *l*0 as the radius of a circle to draw an arc intersecting with the crack at point Q. The cracks in circle P are considered initiation cracks and the angle *θ*2 is considered the angle of crack initiation. It should be noticed that the actual crack length between point P and point Q is larger than *l*0, so the fatigue life predicted by this approach is conservative. The approach in Figure 7 simplifies the crack initiation path as a straight line. The crack initiation life in the circle P is estimated by combining fatigue damage parameters and the *S*-*N* curve while the crack propagation life out of the circle P is estimated by Paris’ fatigue crack propagation law. The angle *θ*2 of crack initiation is called the orientation of the NCP [1]. It is very convenient to select a proper multiaxial parameter based on the NCP to assess the fatigue lifetime of notch [2,46].



Figure 7. The new approach to distinguish crack initiation and crack propagation

* 1. A brief review of the local stress response and notch critical plane (NCP) approach

The local stress response approach is proposed to locate the fatigue critical point of notches under complex cyclic loading [1,42]. Two-dimensional thin-walled structures, like aircraft skin, can be treated as the structure under plane stress state. The points on the notch edge are in uniaxial stress state even nominal loading is complex for the notched structure under plane stress state. Under plane strain state or for three-dimensional notched structures, not all the points on the notch edge are in uniaxial stress state, but cracks usually initiate in defects on the surface of materials. It is reasonable to regard the points both on the surface and on the notch edge as the uniaxial stress state for three-dimensional notched structures. It is very convenient to locate the fatigue critical point when the points on the notch edge are regarded as the uniaxial stress state, but local stress response approach also can be used to locate the fatigue critical point even the points on the notch edge are in multiaxial stress state.

There are two ways to locate the fatigue critical point for local stress response approach. One is directly to calculate the fatigue damage of all the points on the notch edge. The point with the maximum fatigue damage is the fatigue critical point. The other is to regard the uniaxial stress state as the special multiaxial stress state. Then multiaxial fatigue damage parameters can be used to calculate the fatigue damage of all the points on the notch edge. The fatigue damage is defined according to the Palmgren-Miner linear damage accumulation theory for the two ways [42].

The parameters based on the critical-plane method have been widely applied to evaluate the fatigue life of smooth specimens under complex cyclic loading [44-46]. The planes bearing the maximum range of shear stress, the maximum range of shear strain and the maximum fatigue damage are called the critical plane of smooth specimens (CPS). The critical plane method can assess both the fatigue lifetime of smooth components and the angle of crack initiated. Both the stress states and the angles of critical planes of these points are consistent. However, for notched specimens, both the stress states and the loading non-proportionalities of these points at the notch root are different [1]. Both the stress gradient and the gradient of the stress/strain non-proportionality exist at notch root [1,3,73]. In other words, the orientations of the critical planes at the notch root are different. That is the reason why the early stage of the crack initiation is mixed Mode governed [70-71].

The plane at the notch root that passes through the FCP and undergoes the maximum shear stress amplitude is regarded as the critical plane for notch components (CPN). Luo *et al.* [1] combined Susmel’s parameter [40] and TCD to predict the lifetime of notches.



Figure 8 . (*a*) The illustration of NCP (*b*) The maximum normal stress and the maximum shear stress amplitude on the CPN

** and *τ*a are the maximum normal stress and the amplitude of shear stress respectively in Figure 8. The mean value of the maximum shear stress amplitude  and the mean value of the maximum normal stress  on the NCP are expressed in Eq. (8) respectively.



The mean value of the maximum shear stress amplitude and the mean value of the maximum normal stress  are applied to Susmel’s parameter [40] and Zhang-Yao’s parameter [74] to assess the lifetime of notches. Modified Susmel’s parameter and modified Zhang-Yao’s parameter [74] are reported in Eq. (9) and Eq. (10), respectively:





where *t*-1 and *f*-1 are the fully reverse torsional and tensional fatigue limit, respectively. The plane bearing the maximum shear stress amplitude is regarded as the critical plane for the modified parameters.

It's worth mentioning that other fatigue damage parameters based on critical plane also can be used to evaluate the notch fatigue life by defining the NCP as their critical plane. Then the critical stress along the NCP can be averaged through the PM or LM of TCD, like Eq.(8).

In addition, the mean stress has significant effects on fatigue life. Positive mean normal stress can cause more fatigue damage and reduce the fatigue life, because the superimposed normal stress causes the crack surface to open. The effect of mean shear stress can be ignored as long as the maximum shear stress is lower than the material torsional yield stress. For Susmel’s and ZY’s fatigue damage parameters, the maximum normal stress on the critical plane,  which can be expressed:



where *σn,a* and *σn,m* are the amplitude and mean value of the normal stress on the critical plane respectively. The effect of the mean normal stress can be fully considered in M-Susmel’s and M-ZY’s parameters.

1. Experiments
   1. Material and specimens

2297 aluminum-lithium alloy is applied in aircraft structure widely due to high specific strength and specific stiffness. The chemical compositions and mechanical properties of this material are reported in Table 2 and Table 3, respectively. In addition, the critical distance is calculated according to the method proposed in Ref. [75]. The dimensions of the smooth specimen and two kinds of notched specimens are reported in Figure 9. The notches with 2-mm-diameter circular hole, and waist-round hole are referred to by using letters “D,” and “Y,” respectively.

Table 2. Chemical composition of 2297 Al-li alloy(wt.%)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Element | Cu | Li | Mn | H | Zr | Fe | Mg | Ti | Si |
| Percentage | 2.82 | 1.39 | 0.30 | 0.20 | 0.10 | 0.05 | 0.03 | 0.02 | 0.018 |

Table 3. Mechanical properties of 2297 Al-li alloy

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Young’s modulus *E*/GPa | Poisson ratio *μ* | Yield strength *σ*y1/MPa | Fracture strength *σ*b/MPa | The fully reverse torsional fatigue limit *t*-1/MPa | The fully reverse axial fatigue limit *f*-1/MPa | Critical distance *l*0/mm |
| 84.2 | 0.28 | 440 | 480 | 74.6 | 126 | 0.28 |



*a*) Smooth specimens



*b*) D=2mm, circular hole



*c*) waist-round hole

Figure 9. Dimensions of components made of 2297 Al-li alloy (“D” and “Y” represent the circular hole and waist-round hole respectively)

* 1. Experiments and results

All the experiments were carried out at room temperature using the MTS809 biaxial fatigue testing machine. The testing system was equipped with an electro-hydraulic servo control, computer control and data acquisition system. It has a capacity of ±100kN in tensional load and ±1100 in torsion. Force-controlled fatigue experiments were conducted with a sine wave whose frequency was 3Hz. The symmetric torsional fatigue tests were conducted for smooth components, which is listed in Table 4. The stress ratio *R* of multiaxial fatigue loading was 0.1 for notches. A camera monitors the crack near the notches online and continuously takes pictures at a frequency of 3Hz. The test stops when the crack length detected is greater than 1mm. The fatigue lifetime of notches was defined as the maximum loading cycle when the length of crack initiation at notch root reaches 0.28mm. The fatigue life of the notched specimens is listed in Table 5.

After these tests, the angles of fatigue crack were recorded with VHX-1000 3-DVM using the method proposed in Figure 7. Notches were displayed in Figure 10. The results of the measured crack initiation angles are summarized in Table 6 where the measured angles show greater dispersion than fatigue life even under the same fatigue loading. The reason is that the path of crack initiation was heavily influenced by the local microstructure around the notch root. Taking the mean value of the angles can weaken the dispersion to some extent.



Figure 10. The photos of the notches (Magnification: 25 times)

Table 4. Fatigue life of smooth components (stress ratio *R*=-1)

| Specimen No. | Maximum shear stress *τ*max*/*MPa | Fatigue life *N*f |
| --- | --- | --- |
| N-1 | 152.1 | 7068 |
| N-2 | 7540 |
| N-3 | 126.7 | 31811 |
| N-4 | 22679 |
| N-5 | 101.4 | 106425 |
| N-6 | 82780 |
| N-7 | 79.7 | 133181 |
| N-8 | 98531 |

Table 5. Fatigue life of notched components (stress ratio *R*=0.1)

| The type of notch | Specimen No. | Maximum normal stress *σ*max*/*MPa | Maximum shear stress *τ*max*/*MPa | Phase angle  *φ/*° | Fatigue life *N*f |
| --- | --- | --- | --- | --- | --- |
| D | D1 | 90 | 90 | 0 | 31980 |
| D2 | 26280 |
| D3 | 90 | 90 | 45 | 98760 |
| D4 | 76830 |
| D7 | 90 | 90 | 60 | 49650 |
| D8 | 104190 |
| D5 | 90 | 90 | 90 | 224172 |
| D6 | 100500 |
| D9 | 130 | 65 | 0 | 57095 |
| D10 | 89550 |
| D11 | 130 | 65 | 45 | 73530 |
| D12 | 67560 |
| D13 | 130 | 65 | 90 | 119460 |
| D14 | 112026 |
| D15 | 55 | 110 | 0 | 55620 |
| D16 | 80940 |
| D17 | 55 | 110 | 45 | 60090 |
| D18 | 61740 |
| D19 | 55 | 110 | 90 | 80550 |
| D20 | 89850 |
| Y6 | 150 | 75 | 0 | 54720 |
| Y7 | 54180 |
| Y8 | 150 | 75 | 45 | 40860 |
| Y9 | 99180 |
| Y12 | 150 | 75 | 90 | 171030 |
| Y14 | 134600 |
| Y11 | 95 | 95 | 0 | 89370 |
| Y13 | 77700 |
| Y15 | 95 | 95 | 45 | 65790 |
| Y16 | 28710 |
| Y19 | 95 | 95 | 60 | 360556 |
| Y20 | 91410 |
| Y21 | 171332 |
| Y17 | 95 | 95 | 90 | 117960 |
| Y18 | 101974 |

Table 6. Angles of fatigue crack initiation of notches

| The type of notch | Specimen No. | Test value *θ*2/° | | Mean value *θ*m/° |
| --- | --- | --- | --- | --- |
| Left | Right |
| D | D1 | 28.3 | --- | 19.4 |
| D2 | 12.3 | 17.6 |
| D3 | 37 | 11.6 | 12.2 |
| D4 | 0 | 0 |
| D7 | 17.2 | 24 | 25.3 |
| D8 | 42.3 | 17.6 |
| D5 | 0 | 38.2 | 29.4 |
| D6 | 20.2 | 59.1 |
| D9 | 12.2 | 19.5 | 13.0 |
| D10 | 14 | 6.2 |
| D11 | 45 | 0 | 22.7 |
| D12 | 16.6 | 29.2 |
| D13 | --- | --- | 15.7 |
| D14 | 0 | 31.4 |
| D15 | 0 | 14.4 | 8.2 |
| D16 | -4.8 | 23 |
| D17 | 0 | 16.3 | 9.5 |
| D18 | --- | 12.2 |
| D19 | 4.2 | 46.8 | 18.8 |
| D20 | 0 | 24.2 |
| Y | Y6 | 43.3 | 0 | 13.4 |
| Y7 | 0 | 10.1 |
| Y8 | 0 | 0 | 16.0 |
| Y9 | 21.7 | 42.2 |
| Y12 | 44.5 | 34.5 | 41.6 |
| Y14 | 45.9 | --- |
| Y11 | 0 | 9.1 | 4.3 |
| Y13 | 0 | 8 |
| Y15 | 22.7 | 27.4 | 14.7 |
| Y16 | 8.8 | 0 |
| Y19 | --- | --- | 18.8 |
| Y20 | 33.7 | 32.7 |
| Y21 | 28.4 | 12.8 |
| Y17 | 0 | 0 | 1.1 |
| Y18 | 0 | 4.2 |

Note:“---” means that no cracks were observed.

1. Fatigue life and angle of crack initiation evaluation
   1. Stress field solution at notch root by FEA

The linear-elastic constitutive relation is applied in FEA of notches because only a small region around notch root enters plasticity. For the 2D circular hole notches under normal and shear stress in Figure 11, the analytical solution of the stress field at notch root is derived by Ernst Gustav Kirsch in 1898, as is shown in Eq.(12). In addition, the radius of notch in Figure 11 is *a*. FEA is adopted to obtain the stress filed at notch root of waist-round hole notch.





Figure 11. Two dimensional circular notches under normal and shear stress

It should be noticed that the non-proportional fatigue loadings can’t be directly added to the finite element model. These loadings should be dispersed into several loading cases uniformly [1]. Afterwards, the stress components at the notch root were obtained by software Patran 2012. In addition, a linear constitutive relation and 2D shell elements were applied. The minimum size of mesh was 0.02 mm to obtain the numerical stress field distribution at notch root. The meshes around the notch are reported in Figure 12.



Figure 12. The meshes of waist-round hole notch

* 1. Fatigue life evaluation by NCP

The fatigue lives of the two notches were predicted by combing Eq.(8) and Eq.(9), and by combing Eq.(8) and Eq.(10). The relations between test lifetime, *N*Exp, and predictive lifetime, *N*Cal, with adoption of M-Susmel’s parameter and M-ZY’s parameter are reported in Figure 13 and Figure 14, respectively.

It is concluded both M-PM and M-LM of M-Susmel’s parameter can accurately assess the fatigue lifetime of the two notches, which further shows that Susmel’s parameter can accurately characterize the multiaxial fatigue damage of metallic materials. Most of the predicted lives are on the safe side in Figure 13. 71% and 76% of the predictive lifetime using M-PM fall within 4-time and 3-time scatter band, respectively. What’s more, 94% and 100% of the predictive lifetime using the M-PM fall within 4-time and 3-time scatter band, respectively. Furthermore, the predictive results of the M-LM are more accurate than those of the M-PM due to the fact that the M-LM uses more points along the NCP to predict fatigue lives, which is consistent with the conclusion in the Ref. [76]. However, the M-PM is simpler to apply in engineering than the M-LM.

The predictive lifetimes of M-PM are more accurate than those of the M-LM in Figure 14. We can find that most of the predictive lifetime of M-PM fall within 3-time scatter band while 59% of the predictive lifetime of M-LM fall within 3-time scatter band in Figure 14. One reason is that Zhang-Yao model only can be used to evaluate the high-cycle fatigue life which is near the fatigue limit of material. The other reason is that the critical distance *l*0 of TCD is different between PM and LM [77-78]. Santus *et al*. came to the conclusion that the lengths of PM were larger than the corresponding LM values [77]. The critical distance *l*0 is much smaller than the radius of circular hole and the radius of waist-round hole in the multiaxial fatigue test.



Figure 13. The relation between test life and predicted life of M-Susmel



Figure 14. The relation between test life and predicted life of M-ZY

* 1. Angle of crack initiation evaluation by NCP

The crack initiation angles of the two notches are evaluated by the NCP where the plane at the notch root that passes through the FCP and undergoes the maximum shear stress amplitude is regarded as the critical plane. The test mean values and the predicted values of the crack initiation angles are listed in Table 7 and Figure 15. According to Table 7 and Figure 15, we can find that the maximum predicted error of circular hole is -13.9° and the maximum predicted error of waist-round hole is -19.2°. The predicted errors distribute symmetrically on both sides of the zero-error line and most of the predicted errors are within 10°, which shows that the direction of crack initiation is basically the same as that of the NCP. Moreover, both the experimental results and the predicted results of the crack initiation angles support the opinion that crack initiation at notch root is mixed mode governed.

Table 7. The mean values and the predicted values of the crack initiation angles

| Specimen No. | Loading No. | Mean value *θ*m/° | Predicted value *θ*p/° | Predicted error *E*r /°  *E*r=*θ*m-*θ*p |
| --- | --- | --- | --- | --- |
| D1 | 1 | 19.4 | 18.1 | 1.3 |
| D2 |
| D3 | 2 | 12.2 | 15.0 | -2.8 |
| D4 |
| D7 | 3 | 25.3 | 23.4 | 1.9 |
| D8 |
| D5 | 4 | 29.4 | 25.9 | 3.5 |
| D6 |
| D9 | 5 | 13.0 | 12.5 | 0.5 |
| D10 |
| D11 | 6 | 22.7 | 20.8 | 1.9 |
| D12 |
| D13 | 7 | 15.7 | 26.8 | -11.1 |
| D14 |
| D15 | 8 | 8.2 | 15.2 | -7 |
| D16 |
| D17 | 9 | 9.5 | 14.3 | -4.8 |
| D18 |
| D19 | 10 | 18.8 | 32.7 | -13.9 |
| D20 |
| Y6 | 11 | 13.4 | 30.6 | -17.2 |
| Y7 |
| Y8 | 12 | 16.0 | 35.2 | -19.2 |
| Y9 |
| Y12 | 13 | 41.6 | 25.7 | 15.9 |
| Y14 |
| Y11 | 14 | 4.3 | 7.6 | -3.3 |
| Y13 |
| Y15 | 15 | 14.7 | 16.5 | -1.8 |
| Y16 |
| Y19 | 16 | 18.8 | 30.9 | -12.1 |
| Y20 |
| Y21 |
| Y17 | 17 | 1.1 | 5.3 | -4.2 |
| Y18 |



Figure 15. The relations between mean values and the predicted values

1. Conclusion

The advanced volumetric approaches like the TCD, SFI approach, and SED approach are briefly reviewed to show that the radius of fatigue damage region are important and they have different physical definition from each other. Essentially, the fatigue damage region is the area of crack initiation. A method based on the crack length is presented to distinguish the crack initiation stage and the crack propagation stage. The crack length which does not affect the fatigue limit of the material is regarded as the crack initiation stage. In addition, a rigorous approach to distinguish the angle of fatigue crack initiation is presented. The angle of crack initiation is predicted by the NCP which is the plane at the notch root that passes through the FCP and undergoes the maximum shear stress amplitude. The fatigue test is conducted to check the approaches. It is concluded that the direction of crack initiation is basically the same as that of NCP and most of the predicted lives based on NCP fall within 3-time scatter band.

1. The advanced volumetric approaches have been the major approaches to assess the fatigue life of notches under multiaxial cyclic loading. The predicted fatigue lives of these approaches are very sensitive to the radius of fatigue damage region.
2. A new method based on the crack length is proposed to distinguish crack initiation and crack propagation, which regards the crack length shorter than *l*0 as the crack initiation stage. An objective approach to measure the angle of fatigue crack initiation is presented.
3. Both the angle of crack initiation and the fatigue lives of notched specimens can be accurately predicted by NCP approach when the dispersion of the material is taken into account.
4. The experimental results and predicted results show that the NCP approach can explain the reason why crack initiation in notched specimens is mixed mode governed.
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