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Symbol-Independent Weight Magnitude Design for Antenna Array Based Directional Modulation☆

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Abstract

Directional modulation (DM) as a physical layer security technique has been studied from many different aspects. Recently, a constant magnitude constraint for all antennas of an array for a given modulation symbol was proposed. However, the proposed design does not work for multiple beams. In practice, multi-beam DM may often be required. As a compromise, instead of setting the same magnitude for all antennas for a given symbol, in this paper, a symbol independent magnitude constraint for each antenna is proposed for the first time. With the same magnitude for different symbols for each antenna, only phase changes between different symbols are needed, which can form multiple DM beams simultaneously, while still reducing the implementation complexity of the whole system. Design examples are provided to show the effectiveness of the proposed design.

Keywords: Directional modulation, linear antenna array, magnitude constraint, multiple beams.

1. Introduction

The Fifth Generation (5G) based technology has been studied widely [1, 2], and one of its critical technique is beamforming. Directional mod-

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ulation (DM) [3] as a beamforming based technique, which aims to transmit the information to the desired direction or directions with known constellation mappings, but with scrambled ones in other directions, has been studied extensively based on various antenna array configurations [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. To overcome the inherent limitation of DM, where eavesdroppers and the desired users share the same received signal when they lie in the same direction of the antenna array, positional modulation (PM) designs were proposed as a further extension of DM, with the aid of either a reflecting surface [14] or multiple antenna arrays [15]. Moreover, different methods for increasing the capacity of the system were also considered, such as those based on polarisation sensitive antenna arrays [16], and multiple frequencies [17], with the latter one leading to an orthogonal frequency division multiplexing (OFDM) type structure. The introduction of artificial noise (AN) has further advanced the directional modulation technology. Two methods for constructing AN were proposed: one is the orthogonal vector method [18, 19], and the other is the AN projection matrix method [20, 21].

Recently, an equal magnitude constraint for all antennas of an antenna array was proposed for a given DM symbol [22]. However, the proposed design does not work for multiple beams. In practice, multi-beam DM may be required in many applications. As a compromise, instead of setting the same magnitude for all antennas, in this paper, a symbol independent magnitude constraint for different symbols for each antenna is proposed for the first time. With the same magnitude for each antenna, only phase changes between different symbols are needed, which can form multiple DM beams simultaneously, while still reducing the implementation complexity of the whole system. The resultant non-convex constraint can be modified into a convex form, allowing the problem to be solved conveniently by existing convex optimisation toolboxes. Moreover, for the two separate but closely related minimisations in the design, we combine them into one single cost function by introducing a new variable λ as a trade-off factor.

The remaining part of this paper is structured as follows. A brief review of DM design based on a linear antenna array is given in Sec. 2. The proposed fixed magnitude constraint for each antenna working well for multiple beams is introduced with a solution to transform the non-convex problem into a convex one in Sec. 3. In Sec. 4, design examples are provided, following by conclusions in Sec. 5.

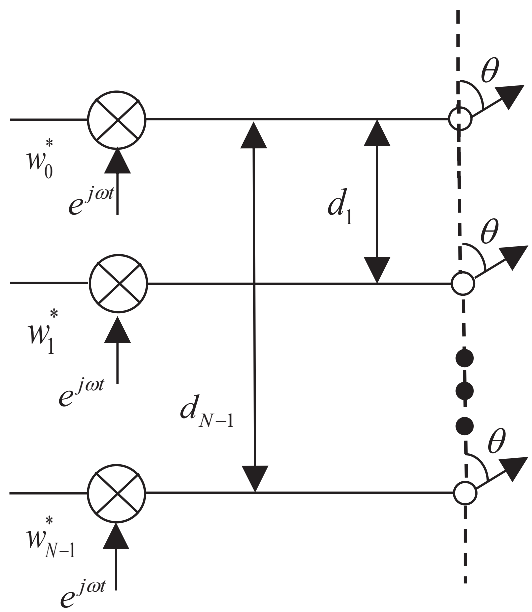

Figure 1: A narrowband transmit beamforming structure.

2. Review of DM design based on linear antenna arrays

A narrowband linear antenna array with N omni-directional antennas for transmit beamforming is shown in Fig. 1, and the spacing between the first antenna to its subsequent antennas is represented by d_n ($n = 1, \ldots, N-1$). Each antenna has its corresponding weight coefficient w_n for $n = 0, 1, \ldots, N-1$. The transmission angle of the structure is represented by θ, and its range is $[0°, 180°]$. The steering vector of the array is a function of angular frequency ω and transmission angle θ, given by

$$\mathbf{s}(\omega, \theta) = [1, e^{j\omega d_1 \cos\theta/c}, \ldots, e^{j\omega d_{N-1} \cos\theta/c}]^T, \tag{1}$$

where $\{\cdot\}^T$ is the transpose operation, and c is the speed of propagation.

For effective directional modulation, the phases and magnitudes of weight coefficients for each symbol should be set properly. For M-ary signaling, such as quadrature amplitude modulation (QAM), there are M sets of desired array responses $p_m(\theta)$ ($m = 0, 1, \ldots, M-1$). The weight vector for the m-th symbol is represented by

$$\mathbf{w}_m = [w_{m,0}, w_{m,1}, \ldots, w_{m,N-1}]^T, \; m = 0, 1, \ldots, M-1. \tag{2}$$

Consider R transmission directions in the design, including r directions in the mainlobe and $R-r$ directions in the sidelobe, i.e.,

$$\begin{aligned}\theta_{ML} &= [\theta_0, \theta_1, \dots, \theta_{r-1}],\\ \theta_{SL} &= [\theta_r, \theta_{r+1}, \dots, \theta_{R-1}].\end{aligned} \tag{3}$$

Then, the desired responses in the mainlobe and sidelobe regions for the m-th symbol can be represented by

$$\begin{aligned}\mathbf{p}_{m,ML} &= [p_m(\omega, \theta_0), p_m(\omega, \theta_1), \dots, p_m(\omega, \theta_{r-1})],\\ \mathbf{p}_{m,SL} &= [p_m(\omega, \theta_r), p_m(\omega, \theta_{r+1}), \dots, p_m(\omega, \theta_{R-1})].\end{aligned} \tag{4}$$

Similarly, the steering matrix in mainlobe and sidelobe ranges can be expressed as

$$\begin{aligned}\mathbf{S}_{ML} &= [\mathbf{s}(\omega, \theta_0), \mathbf{s}(\omega, \theta_1), \dots, \mathbf{s}(\omega, \theta_{r-1})],\\ \mathbf{S}_{SL} &= [\mathbf{s}(\omega, \theta_r), \mathbf{s}(\omega, \theta_{r+1}), \dots, \mathbf{s}(\omega, \theta_{R-1})].\end{aligned} \tag{5}$$

Note that all symbols for a fixed θ share the same steering vector.

Then, for the m-th symbol, its corresponding weight coefficients for DM design can be obtained by solving the following problem

$$\begin{aligned}\min \quad & ||\mathbf{p}_{m,SL} - \mathbf{w}_m^H \mathbf{S}_{SL}||_2\\ \text{subject to} \quad & \mathbf{w}_m^H \mathbf{S}_{ML} = \mathbf{p}_{m,ML},\end{aligned} \tag{6}$$

where $\{\cdot\}^H$ represents the Hermitian transpose, and $||\cdot||_2$ denotes the l_2 norm. The above formulation aims to minimise the difference between desired and designed beam responses in sidelobe regions, while making sure the responses at the main lobe are the same as the required DM patterns.

3. Proposed magnitude constraint for DM design

For the same magnitude weight coefficients for all antennas, for the m-th symbol, as proposed in [22], the constraint is given by

$$|w_{m,0}| = |w_{m,1}| = \dots = |w_{m,N-1}|, \tag{7}$$

for $m = 0, 1, \dots, M-1$, and the corresponding DM design can be formulated as

$$\begin{aligned}\min \quad & ||\mathbf{P}_{SL} - \mathbf{W}^H \mathbf{S}_{SL}||_2\\ \text{subject to} \quad & \mathbf{W}^H \mathbf{S}_{ML} = \mathbf{P}_{ML}\\ & |w_{m,0}| = |w_{m,1}| = \dots = |w_{m,N-1}|,\\ & m = 0, 1, \dots, M-1,\end{aligned} \tag{8}$$

where

$$\mathbf{P}_{SL} = [\mathbf{p}_{0,SL}; \mathbf{p}_{1,SL}; \ldots, \mathbf{p}_{M-1,SL}], \tag{9}$$

$$\mathbf{P}_{ML} = [\mathbf{p}_{0,ML}; \mathbf{p}_{1,ML}; \ldots, \mathbf{p}_{M-1,ML}], \tag{10}$$

$$\mathbf{W} = [\mathbf{w}_0, \mathbf{w}_1, \ldots, \mathbf{w}_{M-1}]. \tag{11}$$

The above non-convex optimisation problem can be transformed into the following convex one [22]

$$\begin{aligned} \min \quad & ||\mathbf{P}_{SL} - \mathbf{W}^H \mathbf{S}_{SL}||_2 \\ \text{subject to} \quad & \mathbf{W}^H \mathbf{S}_{ML} = \mathbf{P}_{ML} \\ & ||\mathbf{w}_m||_\infty \leq \frac{|p_m(\omega, \theta_0)|}{N}, m = 0, 1, \ldots, M-1. \end{aligned} \tag{12}$$

where $||\cdot||_\infty$ represents the l_∞ norm (the maximum magnitude of the entries in the vector).

However, as proved in [22], the above solution can only work for a single beam DM transmission. For multi-beam DM, to keep part of benefit associated with the same magnitude constraint, instead of constraining the magnitude of all antenna coefficients with the same value, the following constraint is proposed

$$|w_{0,n}| = |w_{1,n}| = \ldots = |w_{M-1,n}|, \tag{13}$$

for $n = 0, 1, \ldots, N-1$. Thus, with a fixed coefficient magnitude for each antenna, only phase changes between different symbols are needed for the feed circuits of each antenna, reducing the implementation complexity of the whole system. Then, the corresponding DM design can be written as

$$\begin{aligned} \min \quad & ||\mathbf{P}_{SL} - \mathbf{W}^H \mathbf{S}_{SL}||_2 \\ \text{subject to} \quad & \mathbf{W}^H \mathbf{S}_{ML} = \mathbf{P}_{ML} \\ & |w_{0,n}| = |w_{1,n}| = \ldots = |w_{M-1,n}|, \\ & n = 0, 1, \ldots, N-1. \end{aligned} \tag{14}$$

However, the formulation (14) is non-convex, due to the equality constraint (13), which is difficult to solve. To find a solution, we consider the traditional MinMax problem, i.e. minimizing the maximum value among the M coefficients for each antenna as follows,

$$\min ||\tilde{\mathbf{w}}_n||_\infty, \tag{15}$$

where

$$\tilde{\mathbf{w}}_n = [w_{0,n}, w_{1,n}, \ldots, w_{M-1,n}], n = 0, 1, \ldots, N-1. \quad (16)$$

In this way, by minimizing the maximum value, all the M values will tend to have an equal magnitude value.

Since there are in total N antennas, we can minimize the sum of the maximum values, i.e.

$$\min \sum_{n=0}^{N-1} ||\tilde{\mathbf{w}}_n||_\infty. \quad (17)$$

Combining the above cost function with the original cost function in (14), we have the following new formulation of the problem:

$$\begin{aligned} \min \quad & \lambda||\mathbf{P}_{SL} - \mathbf{W}^H\mathbf{S}_{SL}||_2 + (1-\lambda)\sum_{n=0}^{N-1} ||\tilde{\mathbf{w}}_n||_\infty \\ \text{subject to} \quad & \mathbf{W}^H\mathbf{S}_{ML} = \mathbf{P}_{ML}, \end{aligned} \quad (18)$$

where $\lambda \in (0,1)$ is a trade-off factor between the two parts of the new cost function. A larger value close to one for λ will put more emphasis on minimizing the difference between the designed and desired beam patterns; on the other hand, a smaller value close to zero will tend to give equal-magnitude coefficients for different symbols at the cost of a larger error in the designed beam pattern. The above problems (18) can be solved by the CVX toolbox in MATLAB [23, 24].

Alternatively, we can treat the cost function in (14) as a constraint and minimize the cost function (17) as follows,

$$\begin{aligned} \min \quad & \sum_{n=0}^{N-1} ||\tilde{\mathbf{w}}_n||_\infty \\ \text{subject to} \quad & ||\mathbf{P}_{SL} - \mathbf{W}^H\mathbf{S}_{SL}||_2 \leq \gamma \\ & \mathbf{W}^H\mathbf{S}_{ML} = \mathbf{P}_{ML}, \end{aligned} \quad (19)$$

where γ is the allowed error between the design and desired beam patterns. However, a drawback with the above formulation is that it is difficult to find an appropriate value for γ allowing a final result with equal-magnitude weights. As a result, in our design examples, the formulation in (18) is used.

4. Design Examples

In this section, we consider an $N = 18$ ULA with $d_n - d_{n-1} = \lambda/2$ for $n = 0, 1, \ldots, N-1$. Two main beams point to $\theta_{ML} = [60^\circ \cup 150^\circ]$

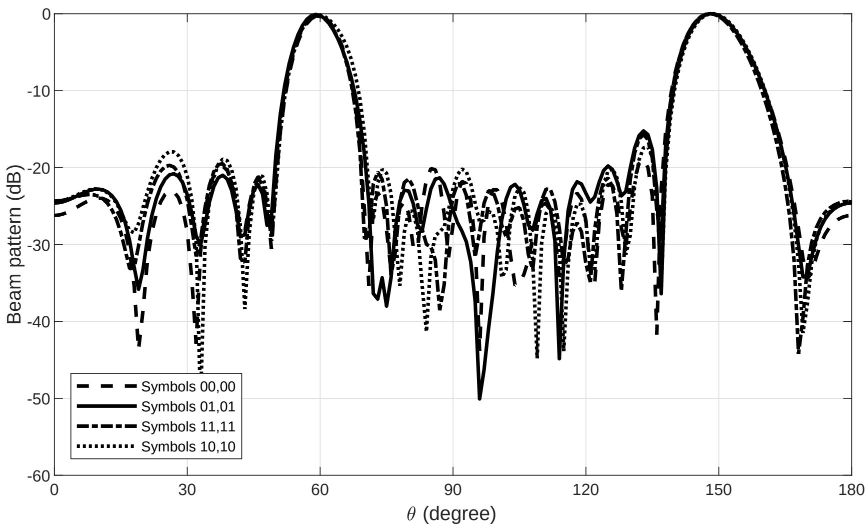

Figure 2: Resultant beam responses without magnitude constraint in (6), transmitting same symbols in two main beams.

and $\theta_{SL} \in [0°, 50°] \cup [70°, 140°] \cup [160°, 180°]$. Without loss of generality, we assume the desired response in the desired directions θ_{ML} follows the standard QPSK modulation scheme, i.e.,

$$\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2} \tag{20}$$

for symbols '00', '01', '11', '10', while the response in the sidelobe range θ_{SL} has a magnitude of 0.1 with random phase. The value of $\lambda = 0.22$ is chosen by trial and error.

Two design scenarios are considered: 1, same symbols are transmitted to two different main beam directions, i.e., '00,00', '01,01', '11,11' and '10,10'; 2, different symbols are transmitted to two different main beam directions, i.e., '00,01', '01,11', '11,10' and '10,00'.

To verify the performance of the proposed design, the beam and phase patterns for the design with and without magnitude constraint are provided.

For the design ($\theta_{ML} = 60° \cup 150°$) without constant magnitude constraint in (6), the beam and phase patterns for symbols '00,00', '01,01', '11,11' and '10,10' are shown in Figs. 2 and 3. It can be seen that all main beams are pointed to 60° and 150° with a low sidelobe level, and the phases in the desired directions follow the standard QPSK scheme, but random for the rest of the angles. The magnitude of all antennas for all different symbols

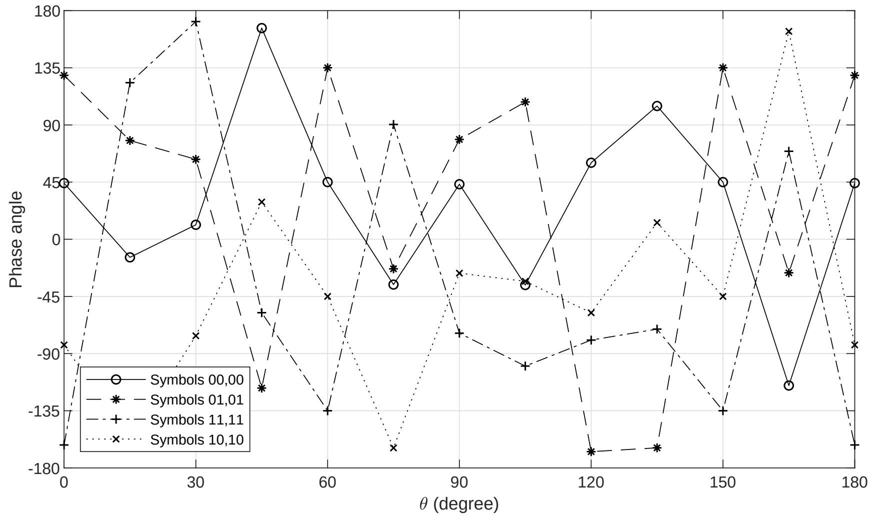

Figure 3: Resultant phase responses without magnitude constraint in (6), transmitting same symbols in two main beams.

Table 1: Magnitude of weight coefficient of each antenna for all symbols without magnitude constraint in (6), transmitting same symbols in two main beams.

	SYM '00,00'	SYM '01,01'	SYM '11,11'	SYM '10,10'
ANT #0	0.0737	0.0700	0.0721	0.0806
ANT #1	0.0631	0.0550	0.0665	0.0673
ANT #2	0.0334	0.0344	0.0273	0.0301
ANT #3	0.1065	0.1061	0.1215	0.1232
ANT #4	0.1052	0.1020	0.0961	0.1055
ANT #5	0.0365	0.0462	0.0405	0.0583
ANT #6	0.1556	0.1622	0.1451	0.1523
ANT #7	0.1246	0.1164	0.1266	0.1284
ANT #8	0.0178	0.0407	0.0324	0.0376
ANT #9	0.1420	0.1423	0.1403	0.1349
ANT #10	0.1130	0.1125	0.1219	0.1130
ANT #11	0.0090	0.0073	0.0041	0.0053
ANT #12	0.1023	0.0916	0.0895	0.0848
ANT #13	0.0820	0.0994	0.0915	0.0752
ANT #14	0.0268	0.0271	0.0279	0.0388
ANT #15	0.0545	0.0442	0.0538	0.0427
ANT #16	0.0447	0.0616	0.0455	0.0573
ANT #17	0.0425	0.0424	0.0519	0.0526

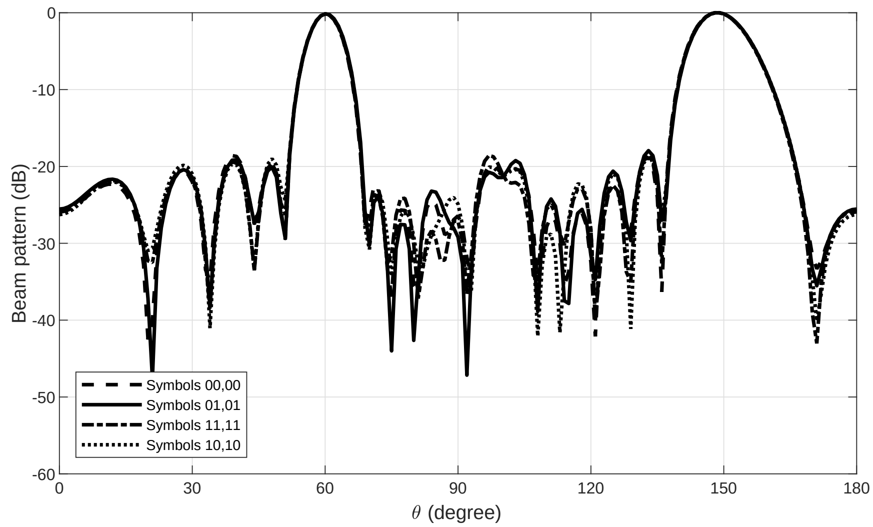

Figure 4: Resultant beam responses with the magnitude constraint in (18) for design 1.

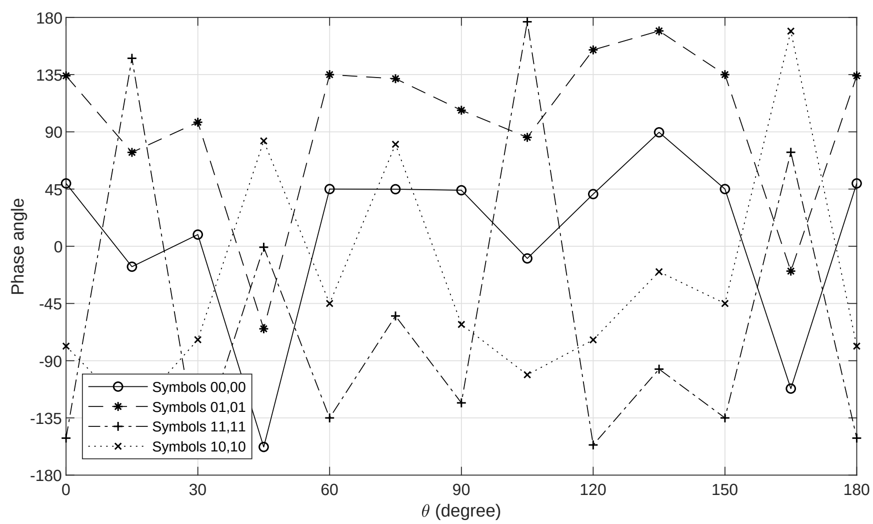

Figure 5: Resultant phase responses with the magnitude constraint in (18) for design 1.

Table 2: Magnitude of weight coefficient of each antennas for all symbols with the magnitude constraint in (18) for design 1.

	SYM '00,00'	SYM '01,01'	SYM '11,11'	SYM '10,10'
ANT #0	0.0735	0.0735	0.0735	0.0735
ANT #1	0.0436	0.0436	0.0436	0.0436
ANT #2	0.0147	0.0147	0.0147	0.0147
ANT #3	0.1012	0.1012	0.1012	0.1012
ANT #4	0.0790	0.0790	0.0790	0.0790
ANT #5	0.0189	0.0189	0.0189	0.0189
ANT #6	0.1395	0.1395	0.1395	0.1395
ANT #7	0.1078	0.1078	0.1078	0.1078
ANT #8	0.0080	0.0080	0.0080	0.0080
ANT #9	0.1402	0.1402	0.1402	0.1402
ANT #10	0.1159	0.1159	0.1159	0.1159
ANT #11	0.0000	0.0000	0.0000	0.0000
ANT #12	0.1095	0.1095	0.1095	0.1095
ANT #13	0.1013	0.1013	0.1013	0.1013
ANT #14	0.0000	0.0000	0.0000	0.0000
ANT #15	0.0754	0.0754	0.0754	0.0754
ANT #16	0.0663	0.0663	0.0663	0.0663
ANT #17	0.0099	0.0099	0.0099	0.0099

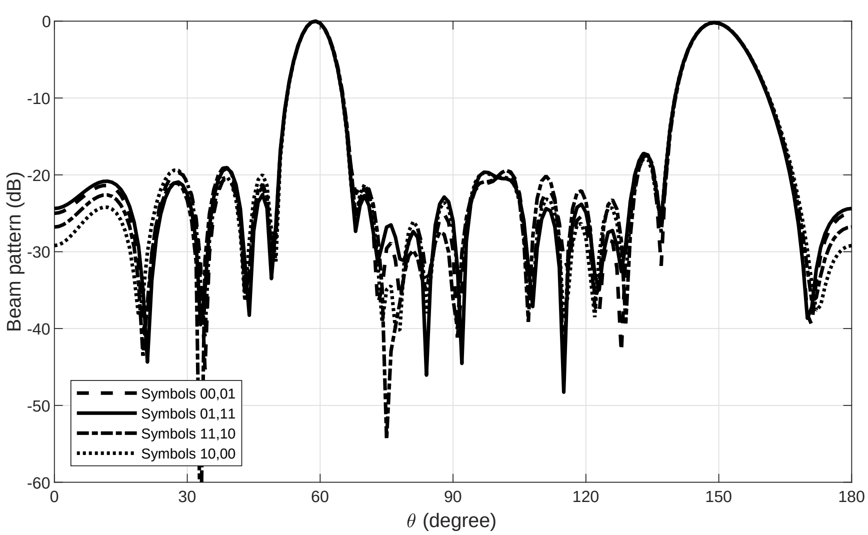

Figure 6: Resultant beam responses with the magnitude constraint in (18) for design 2.

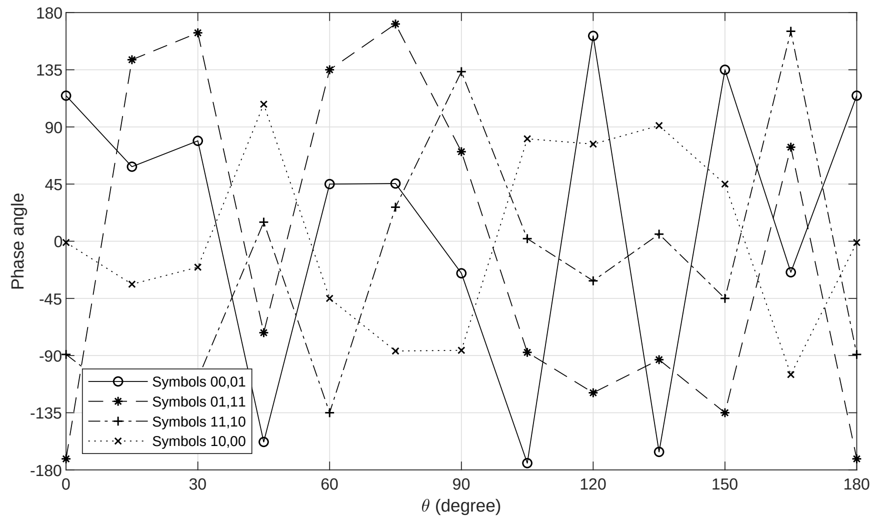

Figure 7: Resultant phase responses with the magnitude constraint in (18) for design 2.

Table 3: Magnitude of weight coefficient of each antennas for all symbols with the magnitude constraint in (18) for design 2.

	SYM'00,01'	SYM'01,11'	SYM'11,10'	SYM'10,00'
ANT #0	0.0395	0.0395	0.0395	0.0395
ANT #1	0.0780	0.0780	0.0780	0.0780
ANT #2	0.0129	0.0129	0.0129	0.0129
ANT #3	0.0480	0.0480	0.0480	0.0480
ANT #4	0.1183	0.1183	0.1183	0.1183
ANT #5	0.0433	0.0433	0.0433	0.0433
ANT #6	0.0575	0.0575	0.0575	0.0575
ANT #7	0.1450	0.1450	0.1450	0.1450
ANT #8	0.0845	0.0845	0.0845	0.0845
ANT #9	0.0336	0.0336	0.0336	0.0336
ANT #10	0.1426	0.1426	0.1426	0.1426
ANT #11	0.1067	0.1067	0.1067	0.1067
ANT #12	0.0053	0.0053	0.0053	0.0053
ANT #13	0.1180	0.1180	0.1180	0.1180
ANT #14	0.0854	0.0854	0.0854	0.0854
ANT #15	0.0000	0.0000	0.0000	0.0000
ANT #16	0.0692	0.0692	0.0692	0.0692
ANT #17	0.0609	0.0609	0.0609	0.0609

are shown in Table 1, where we can see that the magnitudes of weight coefficients of each antenna for all symbols are not the same. In other words, the requirement for the same magnitude of each antenna is not satisfied.

By comparison, for design 1, Figs. 4 and 5 show the corresponding beam and phase patterns, where symbols '00,00', '01,01', '11,11' and '10,10' are transmitted, satisfying the DM design. The magnitude of the antenna array for all symbols are shown in Table 2, showing the same magnitude for each antenna for all symbols.

For design 2 where different symbols are transmitted in two main beams with the fixed magnitude constraint in (18), Figs. 6 and 7 show the corresponding beam and phase patterns. It can be seen that '00,01', '01,11', '11,10' and '10,00' are transmitted, matching the DM design requirement. The magnitude of the antenna array for all symbols are shown in Table 3, also showing the same magnitude for each antenna for all symbols.

5. Conclusions

A new DM design with symbol-independent coefficient magnitude for each antenna has been proposed. For the two separate but closely related minimisations in the design, we combine them into one single cost function by introducing a variable λ as a trade-off factor. Due to the same coefficient magnitude for each antenna, only phase change of each antenna's coefficient between different symbols is needed, which can form multiple DM beams simultaneously, while still reducing the implementation complexity of the whole system. This is different from a previously proposed design, which can form the same coefficient magnitude for different antennas for a given symbol, but can only work for the single-beam scenario.

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