Article:

Calastri, C, Hess, S orcid.org/0000-0002-3650-2518, Pinjari, AR et al. (1 more author) (2020) Accommodating correlation across days in multiple discrete-continuous models for time use. Transportmetrica B: Transport Dynamics, 8 (1). pp. 108-128. ISSN 2168-0566

https://doi.org/10.1080/21680566.2020.1721379

Accommodating correlation across days in multiple-discrete continuous models for time use

Chiara Calastri* Stephane Hess* Abdul Rawoof Pinjari †
Andrew Daly*

January 14, 2020

Abstract

The MDCEV modelling framework has established itself as the preferred method for modelling time allocation, with data very often collected through travel or activity diaries. However, standard implementations fail to recognise the fact that many of these datasets contain information on multiple days for the same individual, with possible correlations and substitution between days. This paper discusses how the theoretical accommodation of these effects is not straightforward, especially with budget constraints at the day and multi-day level. We rely on additive utility functions where we accommodate correlation between activities at the within-day and between-day level using a mixed MDCEV model, with multivariate random distributions. We illustrate our approach using a well-known time use datasets, confirming our theoretical points and highlighting the benefits of allowing for correlation across days in terms of model fit and behavioural insights.

Keywords: MDCEV; activity modelling; multi-day; time use

1 Introduction

Understanding and modelling the way in which individuals allocate time across different activities is a key topic in travel behaviour research. While much of the early research in this field focused on the analysis of how people allocate their time between different activities over a single randomly chosen day (Pas and Sundar, 1995), the literature gradually recognised that the way an individual allocates time may be poorly understood by this single day snapshot (Goodwin, 1981; Pas and Sundar, 1995). Several studies then started to make use of multi-day surveys of varying length, relying on various data collection methods, to investigate different research questions (Bhat et al., 2004; Jara-Díaz et al., 2008; Kang and Scott, 2010). Minnen et al. (2015) argues that multi-day

*Institute for Transport Studies and Choice Modelling Centre, University of Leeds (UK)

†Department of Civil Engineering and Center for Infrastructure, Sustainable Transportation and Urban Planning (CiSTUP), Indian Institute of Science (India)

data allow to observe temporal regularities and habitual behaviour while Kang and Scott (2010) use both descriptive analysis and structural equation modelling (SEM) to show that time-use patterns vary substantially across days, with marked differences between weekdays and weekends. SEM was also applied by Roorda and Ruiz (2008), who estimate a number of different models to test different hypotheses. The authors conclude not only that substitution and complementarity effects are at play between the activities and travel conducted within a single day, but also that that there are important relationships between activity behaviour across different days, with more marked similarities across weekdays. Interestingly, this paper also found that activity patterns during weekdays observed in different years are more similar than in consecutive weekday-weekend day pairs.

While the existing work established the building blocks to understand multi-day behaviour, the literature in this area is not abundant: as explained by Schlich and Axhausen (2003), collecting multi-day travel diaries is a very burdensome survey task for respondents and a trade-off between data quality and richness is often unavoidable.

Jara-Díaz and Rosales-Salas (2015), in their analysis of the duration of time diary data, recommend the collection of a week of data to model time allocation, or at least two or three days with appropriate weighting. Other studies focusing on other choices, such as mode choice, argued that one week is an appropriate time period, allowing to correctly analyse the day-to-day variability (Cherchi et al., 2017). Several studies have acknowledged the need to separate weekdays and weekend days in the analysis of time use (Bhat and Misra, 1999; Susilo and Kitamura, 2005) and hypothesised that individuals may select their travel pattern at the beginning of each week, and then possibly make small adjustments as later on (Hirsh et al., 1986).

A limited number of research efforts have attempted to capture the correlations between these different day types. As an example, Yamamoto and Kitamura (1999) do so by introducing error components within a Tobit model, although limiting their analysis to two discretionary activities only and distinguishing days simply between working and non-working.

The *Mobidrive* data used in this paper has been exploited by a range of studies looking at activity involvement over time. Cirillo and Axhausen (2010) propose the use of a mixed logit model where error components create correlation among choice options (activities) through the unobserved part of utility. Thanks to the long duration of the study, they also explore the effect of past behaviour on current choices.

From the modelling perspective, the behavioural context of an individual who, over the course of a day, allocates time to a set of different activities presents some challenges, as the amounts of time can differ across activities, and individuals can also decide to not engage in a particular activity on a given day. The choice process is thus one of selecting amongst different activities that are not mutually exclusive (as they would be in a discrete choice context) and determining a continuous time allocation for each.

Starting in the late 1950s, a number of different discrete-continuous econometric models have been put forward (e.g. De Jong, 1990; Dubin and McFadden, 1984; Heckman, 1977; Tobin, 1958; Train, 1986). Nowadays, the state-of-the-art model for accommodating both a discrete and a continuous element of choice is the Multiple Discrete-Continuous Extreme Value (MDCEV) model (cf. Bhat, 2008). This model is a generalisation of a multinomial logit model (MNL) for multiple discrete continuous choice contexts and has a simple closed-form probability which makes the model easy to use even with a large number of alternatives. The model is based on the Kuhn Tucker (KT) first-order conditions for constrained random utility maximisation, previously employed by Hanemann (1978) and Wales and Woodland (1983), which are used to derive the optimal consumption for the given random utility specification subject to a linear budget constraint. This model is preferred to its antecedents as it provides a comprehensive framework for modelling at the same time the discrete and continuous choice, relying on a CES-type utility function rather than on a statistical stitching of a discrete and continuous model. The framework provides a closed-form and easy to estimate likelihood function that is flexible in terms of the inclusion of explanatory variables both in the discrete and continuous part of the model.

The MDCEV model has become a popular tool for modelling time allocation (Kapur and Bhat, 2007; Wang and Li, 2011), given its flexibility and ease of application. A key characteristic of an MDCEV model is the budget component, which imposes a constraint on the cumulative consumption across alternatives. The use of MDCEV with a time budget as opposed to a money budget has traditionally be seen as an *easier* context (and has hence been the basis of many of the applications/developments of MDCEV) given that there are 24 hours in a day and that many time use surveys similarly operate at the day level. Whether or not data is available for a single day or multiple days, the modelling of time allocation at the day level can be seen as overly restrictive. At first sight, modelling each individual day on its own by assuming separate 24 hour budgets seems behaviourally reasonable, as it implies that, when choosing how to use their (e.g. weekly) time, decision makers think about days as units in which to allocate different activities. At the same time, this approach ignores the fact that time use behaviour may not be independent across days.

A number of previous applications of MDCEV have made use of such multi-day data without explicitly accommodating links across different days of the week, at best estimating day of the week-specific model coefficients to capture differences across days. Chikaraishi et al. (2010) show how different days affect the likelihood to perform different activities, and that part of the overall variability in the model can be attributed to temporal patterns. While not focusing on differences across days of the week, two papers looked into weekly patterns, applying week-level budgets and reaching different conclusions. Spissu et al. (2009) apply a panel mixed MDCEV model to analyse differences in activity patterns across different weeks, finding substantial week-to-week intra-individual variation and arguing that multi-week data might be required to allow a fuller understanding of time use behaviour. Habib et al. (2008) estimated different models for different weeks of the *Mobidrive* data, showing that while some variations were present, a random week of data can satisfactorily capture time use behaviour.

None of these studies captured the potential "links" across different days of the week. These can represent substitution or complementarity between activities, or they can simply materialise in correlations due to unobserved heterogeneity across individuals. For example, in certain households the parent who drops-off the children on one day may not do it on the next day (substitution), or a person who performs out-of home social activities on one day may also travel (complementarity). Also, the person who performs certain household obligations on one day might be more likely to also perform them on other days (positive correlation caused by common unobserved heterogeneity).

It is tempting to see that the solution to the above problem lies in modelling time allocation not at the day level but over a longer time horizon, with an example given for the 48 hour level in Calastri et al. (2017). This however creates a number of different problems. Firstly, while it becomes easy to create links over different days, such a specification would not allow us to understand differences in the utilities of activities across days and has substantial implications on the understanding of satiation. This can be understood by noting that the estimation of day-specific coefficients would also require day-specific budgets. By not imposing the latter, the analyst would neglect an essential constraint which is present in the data on which the model is based. Secondly, the question arises what unit of measurement should be used instead of the day level. While the specification may ultimately be driven by the length of the data collection period, an analyst will still need to decide between for example weekly or bi-weekly specifications. With either assumption, the old issue returns of no relationship across the different subsets defined by the budget.

The issue becomes even clearer when thinking about forecasting. In the first case, i.e. with day specific budgets, a change in the utility of an activity on one day would have no effect on the time spent on any activities on other days, as there is no link between days. This is quite unlikely in reality, as, for example, if an individual is unable to work on a certain day, he/she will try to make up for it by working a little longer on the previous/following day. In the second case, i.e. using an overall weekly budget, substitution between days becomes possible, but the model would not be constrained to allocating only 24 hours across all activities on a given day. Any forecasts would thus not allow us to understand what happens on individual days, but only at the aggregate level. These considerations highlight the fact that it is important to research solutions that allow the introduction of correlations across different days in MDCEV models of time use, while avoiding violations of the 24 hour budget constraint so as to be able to obtain consistent forecasts.

In this paper, we start by hypothesising that some activities are more similar than others, i.e. that there is correlation between days as well as within days (between different activities). We discuss past attempts to develop a model framework that allows an analyst to incorporate substitutions and complementarities in the MDCEV model across different days. This theoretical formulation is elegant but very difficult to apply in practice. To deal with these issues in a way that is computationally tractable, we instead explore the use of a mixed MDCEV model which directly accommodates correlations between activities within and across different days. The correlations that we estimate could be due to common heterogeneity, substitution (if the correlation is negative) or complementarity (if the correlation is positive). It is usually not possible to disentangle the different sources of correlation, but the analyst can make an informed guess on the primary source of correlation.The approach suggested in this paper goes some way towards accommodating the issue we identify without imposing excessive additional demands in terms of model complexity. We highlight the importance of the behavioural issue and thus hope to motivate further theoretical work.

The remainder of this paper is organised as follows. Section 2 discusses the limitations of the *"standard"* implementation of MDCEV models for multi-day data, highlights the complexity of working with non-additive utility functions and puts forward a mixed MDCEV solution within an additive framework. The modelling approach is illustrated using a well-known dataset in Section 3 and the implications of our results are discussed in Section 4. Finally, Section 5 summarises our findings and presents directions for future work.

2 Methodological considerations

In this section, we first discuss the limitations of the standard framework, before briefly looking at the use of a non-additive utility specification. We next put forward a mixed MDCEV model as a possible solution to the problem of working with multi-day data. Finally, we contrast these two solutions.

2.1 Base specification

The random utility specification of the MDCEV model, as introduced by Bhat (2008) and here described in terms of time use, is given by:

$$U(x) = \sum_{k} \frac{\gamma_k}{\alpha_k}[exp(\beta' z_k + \epsilon_k)] \left(\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \tag{1}$$

so that $U(x)$ is quasi-concave, increasing and continuously differentiable with respect to the vector of time amounts x_k, and $\psi_k = [exp(\beta' z_k + \epsilon_k)]$. ψ_k is the baseline utility of activity k, i.e. the marginal utility of the good at zero consumption. It is a function of z_k, i.e. the observed characteristics of the decision maker and of good k, which also includes a constant representing the generic preference for activity k. In this random specification, a multiplicative i.i.d. log-extreme value error term is introduced in the baseline utility.

The analyst can solve the optimal time allocation (with respect to the time spent in activities $1...K$):

$$Max\ U(e_1 \dots e_K) \quad s.t. \sum_{k=1}^{K} e_k^* = E \tag{2}$$

where e_k^* are the optimal amounts of time invested in activities $1...K$, that exhaust the budget E, and where $e_k = x_k p_k$. This problem is solved by forming the Lagrangian and applying the KT conditions, as detailed in Bhat (2008). The resulting probability of the expenditure pattern where M activities are chosen results in the closed-form expression below:

$$
\begin{aligned}
&P({e_1}^*, {e_2}^*, \ldots, {e_M}^*, 0, \ldots, 0)\\
=&\frac{1}{\sigma^{M-1}}\left(\prod_{i=1}^{M} c_i\right)\left(\sum_{i=1}^{M}\frac{1}{c_i}\right)\left(\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma}\right)^M}\right)(M-1)!
\end{aligned}
\tag{3}
$$

where σ is a scale parameter (not estimated in our case, as there is no price variation across products), $c_i = \frac{1-\alpha_i}{{e_i}^* + \gamma_i p_i}$ and $V_k = \beta' z_k + (\alpha_k - 1)\ln\left(\frac{{e_k}^*}{\gamma_k p_k} + 1\right) - \ln p_k$ $(k = 1, 2, 3, \ldots, K)$. As explained in Bhat (2008), equation 3 can also be expressed in terms of consumption quantities. In our case, the two forms are interchangeable because there is no price variation, and in the remainder of our discussion we will refer to consumption quantities (x_i).

To model multi-day time-use choices, it is desirable to formulate a unified multiple discrete-continuous choice model that simultaneously recognises two different aspects. Firstly, day-level (24 hour) constraints individuals face need to be enforced, with as many constraints as the number of days modelled. Secondly, it is important at the same time to account for the potential interactions between time allocation to activities across different days (such as the substitution, complementarity and heterogeneity discussed above). This forms the basis of the remainder of this paper.

2.2 Multi-day utility maximisation with day-level time constraints and non-additive utility functions

A first specification to model multi-day time use while accommodating day-level time constraints and interactions in time-use across different days is to use non-additive utility formulations that allow for explicit interactions between utility functions of different days while still imposing day-level constraints. One way to formulate non-additive utility functions is to begin with the additive utility form shown in Equation 1 and add multiplicative terms that create interactions between pairs of utilities corresponding to activity participation on different days, as in Bhat et al. (2015).

The parameters estimated on such multiplicative utility terms capture substitution and complementarity between the two choice alternatives being interacted. Specifically,

$$
U = \sum_{l=1}^{L}\sum_{k=1}^{K} u_{kl} + \sum_{k=1}^{K}\sum_{q,m=2,q\neq m}^{L} \theta_{kqm}[u_{kq} \cdot u_{km}] \tag{4}
$$

where K refers to activities, L to days and θ_{kqm} are the parameters estimated on the multiplicative utility terms $[u_{kq} \cdot u_{km}]$, interacting utility derived from activity k on two different days q and m. If the estimated θ_{kqm} terms are negative they indicate substitution between activity k on day q and on day m, while they capture complementarity if positive. While this model constitutes an interesting approach for accommodating these different relations, its complexity and limitations in terms of complementarity and substitution patterns (further discussed in Section 2.4) have thus far meant that there is no single empirical application in the literature.

2.3 Multi-day utility maximisation with day-level time constraints and correlated, additive utility functions

We will now look at our proposed use of a mixed MDCEV model to capture intra and inter-day correlations.

Let us consider the situation in which we have data from N separate individuals, where L_n days are observed for person n. On each day, individual n allocates time to K different activities ($k = 1, 2, \ldots, K$), where $K = 1$ is an activity that is performed by every person on every day (i.e. an outside good). In this specification, it is assumed that an individual makes his/her time use choices across different days to maximise the total utility derived from time allocation on all days under consideration; subject to as many day-level constraints as the number of days, where the time allocation to all activities on each day sums up to 24 hours. Specifically, $U = \sum_{l=1}^{L} \sum_{k=1}^{K} u_{kl}$ is maximised subject to L day-level time budget constraints $\sum_{k=1}^{K} x_{kl} = 24, \forall l = 1, 2, \ldots, L$.

In this specification, U is the total multi-day utility derived by the person, $u_{1l} = \psi_{1l} x_{1l}^{\alpha}$ is the utility from time allocation x_{1l} to the outside good on day l ($l = 1, 2, \ldots, L$), $u_{kl} = \frac{\psi_{kl}\gamma_{kl}}{\alpha}\left(\frac{x_{kl}}{\gamma_{kl}} + 1\right)^{\alpha} \forall k = 2, \ldots, K$ is the utility from time allocation x_{kl} to activity k on day l, ψ_{kl} is the corresponding baseline utility and γ_{kl} is the translation parameter, which allows for corner solutions as well as having a role in relation to satiation (with higher γ_{kl} implying lower satiation for activity k and day l). Finally, α is the generic satiation parameter. Note that the subscript for person n is suppressed for ease in notation.

In our theoretical discussions, we ignore the possibility of including socio-demographic effects (though we do so in the empirical work), meaning that the baseline utility for activity k on day l for person n is simply given by $\psi_{n,k,l} = e^{\delta_{k,d_{l_n}} + \varepsilon_{n,k,l}}$ where $\varepsilon_{n,k,l}$ is an extreme value error term for person n, activity k and day l, and where d_{l_n} is the day type for day l for person n . In addition to making the δ parameters activity and day-type specific, we do the same for the γ parameters. With D different day types, this would thus lead to the estimation of KD different δ parameters and KD different γ parameters. In practice, we do not set $D = 7$ but focus on the situation where weekdays (WD) are treated differently from Saturdays (SAT) and Sundays (SUN), i.e. $D = 3$.

A model of the form above would allow for different utilities for the same product across days and also different shapes for the indifference curves, but would fail to capture correlation across activities and across days. Furthermore, any heterogeneity across individual people would have to be captured via socio-demographic interactions in ψ and γ. We now instead define $\theta_n = \langle \delta_n \rangle$ to be a vector combining the individual δ parameters for person n, making these parameters individual-specific. As an example, we would have $\delta_n = \langle \delta_{n,1}, \ldots, \delta_{n,D} \rangle$, i.e. comprising itself different vectors where for example $\delta_{n,d} = \langle \delta_{n,d,1}, \ldots, \delta_{n,d,K} \rangle$, i.e. containing the constants used in the baseline utilities for activities on a day of type d by person n. The vector θ_n thus has KD elements, and we assume that it is distributed randomly across individuals, according to $\theta_n \sim f(\theta \mid \Omega)$.

Let $\theta_{n,d}$ be the subset of θ_n for days of type d. With $P_{n,l}(\theta_{n,d})$ giving the MDCEV probability (cf. Equation 3) for the consumption observed for individual n on day l (out of L_n), conditional on $\theta_{n,d}$, the unconditional probability for the observed sequence of day level consumptions for individual n is then given by:

$$P_n(\Omega) = \int_{\theta_n} \prod_{l=1}^{L_n} P_{n,l}\left(\theta_{n,d_{l_n}}\right) f(\theta \mid \Omega)\, \mathrm{d}\theta_n, \tag{5}$$

where d_{l_n} is the day type for observation l for respondent n, where the above notation ensures that the right subset of θ_n is used in $P_{n,l}\left(\theta_{n,d_{l_n}}\right)$. This model thus captures random heterogeneity across individual people in the utilities for different activities, where it is straightforward to extend this to include heterogeneity also in the γ parameters, of course leading to further demands on estimation and the data.

By carrying out the integration over the distribution of θ_n at the person level rather than at the day level, we already capture correlation across days for the same person and for the same activity if those days are of the same type - net of the extreme value term, the same baseline utility for work will be used for a given person across all five weekdays. However, the key flexibility arises if we allow for correlation between the different δ parameters.

Different possibilities arise. Let us assume without loss of generality that the multivariate distribution used for θ_n is characterised by a mean for each element (i.e. every δ term) along with a covariance matrix[1]. As a first step, we may want to focus on correlations in the baseline utilities for activities conducted on days of the same type. For day type d, we would now estimate K means for $\delta_{n,d}$, along with $\frac{K \cdot (K+1)}{2}$ covariance elements. This would for example allow us to understand in which way the baseline utilities for day type d are correlated with each other, e.g. whether a respondent who is more likely to take part in activity k_1 on day type d is also more likely to take part in activity k_2 on day type d. We may also want to allow for correlations across different day types in the baseline utilities for a given activity. This will imply the estimation of $K \frac{D \cdot (D-1)}{2}$ additional off-diagonal elements in the covariance matrix and would for example tell us whether respondents who are more likely to conduct leisure activities on a weekday are

[1] In the closed form MDCEV model, we would be estimating the KD mean values, while all elements of the covariance matrix would be fixed to zero. In a model allowing for simple independent heterogeneity for each element, we would in addition estimate KD variances.

less likely to do so on a Saturday. Allowing for correlations in baseline utilities within and across day types requires the estimation of up to $\frac{KD\cdot(KD+1)}{2}$ elements of the covariance matrix, in addition to the $2KD$ parameter means[2].

Conceptually, the above formulation belongs to the class of MDCEV models with multiple budget constraints. An individual has multiple 24 hour budget constraints, such that on day l, we have that $\sum_{k=1}^{K} x_i = 24$ and an overall $L \cdot 24$ budget constraint, such that $\sum_{l=1}^{L}\sum_{k=1}^{K} x_{kl} = L \cdot 24$ (if the individual level budgets are all met, then the multi-day one will be too). However, there is a subtle difference between our formulation and existing formulations with multiple budget constraints, such as those in Castro et al. (2012) and Pinjari and Sivaraman (2013). In most previous formulations, multiple budget constraints arise due to the use of multiple resources, such as time and money, for consuming the same choice alternative. Each activity would draw from both budgets.

In our formulation, however, activity participation on a day can draw only from the time available (24 hours) on that day. In other words, one cannot use time available on Sunday to work on a weekday. Since the 24 hour time budgets are not *fungible* across different days, as long as the utility functions are additively separable, it can be shown that the maximum multi-day utility derived by a person subject to multiple day-level constraints is the same as the sum (across all days) of maximum single-day utilities derived by the person subject to a single day's 24 hour time constraint. That is, $[Max(U), \text{subject to} \sum_{k=1}^{K} x_{kl} = 24\,\forall l] = \sum_{l=1}^{L}[Max(u_l), \text{subject to} \sum_{k=1}^{K} x_{kl} = 24]$.

Therefore, conditional on the mixing distributions used in the specification, the multi-day time-use MDC choice probability may be derived as a simple product of single-day MDC choice probabilities, with as many single-day probabilities in the product as the number of days being modelled. In short, conditional on the mixing distributions, the multi-day time-use probability may be derived as a product of independent single-day MDCEV probabilities. The unconditional probability is simply an integral of this product over the mixing distribution.

2.4 Discussion

We now provide some brief contrasts between the two approaches discussed in Sections 2.2 and 2.3. In comparison with the correlated, additive utility functions, the non-additive approach from Section 2.2 provides a more structural way to allow for interactions. A major disadvantage, however, is that such non-additive utility models are very difficult to estimate and apply in practice, substantially more so than our mixed MDCEV model even with a full covariance matrix, as the desirable optimisation properties of additive utility functions do not hold anymore. In our attempts to estimate the Bhat et al. (2015) formulation, only a limited number of interaction parameters (i.e. θ_{kqm}) could be estimated, limiting the scope of our investigation. Moreover, the pattern of complementarity

[2]As already mentioned, it is possible to also allow for heterogeneity in γ, and correlation within γ and δ and between γ and δ. We chose not to do so due to data limitations, but the reader may note that this would imply the estimation of a full covariance matrix of $\frac{2KD\cdot(2KD+1)}{2}$ elements on top of $2KD$ parameter means.

and substitution that the model estimation can give rise to is somehow constrained, so that complementarity must outweigh substitution.

An advantage of the additive utility specification is that it is easy to allow for correlations among baseline utility parameters, thereby differentiating substitution and/or complementarity effects in the discrete choice from those in continuous choice dimensions. The non-additive utility approach, on the other hand, does not offer an easy way to disentangle interactions among the translation parameters from that of baseline utility parameters. This is because the interactions in the latter approach are between the utility terms, not between the parameters.

Another advantage of the additive formulation in Section 2.3 is that it is a relatively simple implementation of the mixed-MDCEV formulation. A drawback, however, as discussed in the next section, is that the move from estimation to forecasting is not trivial. For example, if a person cannot work on a weekday due to some exogenous reasons, he/she will likely make up for the lost worktime by working more on a weekend day. Such substitution effects are not easy to accommodate when the model is applied for prediction, particularly when predictions are carried out separately for each day according to the estimated model. This is because the additively separable utility formulation does not incorporate explicit interactions between the utilities of time allocation across different days, except through correlations. Another disadvantage is that correlations between two utility functions may be either due to substitutive/complementarity relationships or simply due to common unobserved heterogeneity. The estimated correlation parameters typically capture a combined effect making it difficult to disentangle the source of correlation.

Given the difficulty of estimating and applying MDC models with non-additive utility functions, particularly those with multiple budget constraints, in this paper we employ the simpler model formulation discussed in Section 2.3 and explore the estimation challenges as well as the insights that can be gained from this model when the correlations are computed. Of course, formulating, estimating, and applying non-additive, multi-day utility models while considering day-level constraints is an important avenue for future research.

3 Empirical application

3.1 Data

This paper makes use of a well-known survey in the transport literature, the German travel survey *Mobidrive*.

The *Mobidrive* project conducted a six-week travel diary in the two German cities of Karlsruhe and Halle, with data collection taking place in the autumn of 1999. The availability of trip purpose allowed us to transform the travel diary into a time use diary. We only exploited two weeks of data, in particular we selected the second and third week recorded by respondents, to avoid bias due to any learning effects that may have occurred at the very beginning of the survey. Further information about the data collection protocol and the sample can be found in Axhausen et al. (2002).

We use a subset of the overall *Mobidrive* sample: we only included respondents who do not fail to report any activity for more than 4 days over the two weeks used for the analysis, ending up with a sample of 223 respondents. Corrections were applied in a few cases where the overall number of hours within a day would exceed 24h, for example if a respondent recorded the start of his/her activity/trip before midnight and the end during the early hours of the next day.

The study relied on a paper-and-pencil diary, where participants were free to specify the purpose of their trips (activities). For modelling purposes, these activities were subsequently grouped into a number of macro categories, depending on what participants reported and what was considered to be most relevant for the specific geographical and cultural context. Table 1 reports the sample averages for the discrete choice (percentage of people performing a given activity) and continuous choice (time invested in the different activities when this is performed, in hours). We present the statistics separately for weekdays, Saturday and Sunday.

	Share of individuals participating			*Average time*		
Activity	WD	SAT	SUN	WD	SAT	SUN
Basic Needs	100.00%	100.00%	100.00%	16.65	19.06	19.85
Work	41.75%	8.74%	3.59%	6.86	5.61	6.78
School	24.66%	2.02%	1.57%	5.30	3.01	5.85
Drop-off/Pick up	7.80%	7.85%	7.40%	0.71	0.42	0.54
Daily shopping	31.12%	36.10%	6.05%	0.60	0.69	0.29
Non daily shopping	14.53%	18.39%	2.47%	1.06	1.32	0.95
Social	24.08%	39.91%	42.15%	2.67	4.50	3.64
Leisure	19.46%	24.66%	29.15%	2.68	3.03	3.89
Personal business	31.21%	16.82%	11.88%	1.15	1.42	0.98
Travel	97.26%	87.67%	79.82%	1.29	1.24	1.18

Table 1: Average levels of continuous and discrete choice in the samples

The first activity, *Basic Needs*, includes sleeping, eating meals at home and spending time at home for everyday essential tasks. Everybody in the sample performs this activity every day, and this allows us to treat it as an "outside good" in our models.

Most of the categories in Table 1 are self-explanatory. *Work* refers to all work and work related activities. *School* refers to schooling and education activities. *Private business* includes personal errands, such as going to the bank, dentist, hairdresser with the addition of other personal activities.

3.2 Model specification and estimation

In our application, we work with two weeks of data per individual. Each individual thus contributes 14 observations to the likelihood of the model, where we enforce a 24h budget per observation.

In order to understand the effect of allowing for random heterogeneity across respondents and across days and assessing whether this brings an improvement in the ability of the model to explain the data, we estimate three sets of models, as shown in Table 2. As discussed in Section 2.3, the three sets differ in the level of correlation between model parameters allowed through mixing distributions in the model. We also test for the impact on findings if we allow for additional socio-demographic effects in the model. In Table 2, we indicate the models without socio-demographics with the subscript 1 and those incorporating such effects with the subscript 2. In the models with deterministic heterogeneity, we allowed for shifts (identical across days) in the baseline utility constants (δ) of each activity.

	No deterministic heterogeneity	**Deterministic heterogeneity**
Fixed parameters	A_1	A_2
Univariate random parameters	B_1	B_2
Multivariate random parameters	C_1	C_2

Table 2: Overview of estimated models

We start by estimating simple MDCEV models for time use (A_1 and A_2). A number of different specifications are possible with the MDCEV model, where our empirical work uses a generic α parameter across all activities including the outside good, which we (after testing) set to 0, along with activity-specific γ parameters for the $K-1$ activities that are not treated as outside goods. Separate parameters are estimated for the three types of days to allow for day-level differences. We do not introduce any random heterogeneity at this stage.

We then allow for random heterogeneity in the δ parameters (models B_1 and B_2), i.e. estimating μ_δ and σ_δ, which is the diagonal of Ω_δ with zero values on the off-diagonal. This allows us to account for heterogeneity across people in the activity choice for each day, where we use univariate Normal distributions (i.e. with a diagonal covariance matrix) - the use of Normals was found to be superior in empirical tests to other distributions.

Finally, we introduce additional flexibility by explicitly allowing for correlation between the δ parameters of different activities as well as of the same activity on different days (models C_1 and C_2), i.e. estimating some of the off-diagonal terms in the covariance matrix Ω_δ.

In practice, with finite data, it is not generally feasible to reliably estimate the full covariance matrix between all the individual random components, remembering that in our case this would imply the estimation of 378 terms in the covariance matrix. We instead focus on a number of selected activities, to understand their correlation across different day-types. Moreover, the correlations between δ parameters of different activities are estimated for a limited number of parameters, chosen after extensive testing because of their particularly interesting or meaningful correlations.

The estimation of a model of this type becomes rapidly infeasible in classical estimation and we thus use Hierarchical Bayes (HB) estimation techniques for our mixed MDCEV models (B_1, B_2, C_1 and C_2). As in classical estimation of mixed logit models, we make sample-level assumptions regarding the distribution of the δ parameters. We use noninformative (diffuse) priors for the parameters of the distribution, meaning that we do not rely on a priori expectations about the parameter values. We take repeated draws from the posterior distribution via "Gibbs sampling", and the analyst needs to establish how many iterations of this sampling to use. In our case, we use $600,000$ burn-in iterations to guarantee stable chains. We estimate our models in R (R Core Team, 2016) using the *RSGHB* (Dumont et al., 2015) and *Apollo* (Hess and Palma, 2019) packages.

3.3 Estimation results

Table 3 displays an overview of the estimated models, using the model names from Table 2.

Model name	**LL**	**N. param**	**Chi square test (p) with respect to the model above**	**Chi square test (p) comparing models with and w/o socios**
A_1	-26713.55	54		
B_1	-25122.96	81	0	
C_1	-24998.62	120	$2.38 * 10^{-32}$	
A_2	-25086.69	77		0
B_2	-24592.45	104	$6.4 * 10^{-191}$	$1.39 * 10^{-209}$
C_2	-24455.72	143	$5.66 * 10^{-37}$	$7.4 * 10^{-125}$

Table 3: Overview of estimated models and fit comparison

It is clear to see how the increased flexibility significantly improves model fit. As expected, we obtain a better log-likelihood (LL) for the models with socio-demographics, where the improvement in fit is highly significant, as shown by a likelihood ratio tests (right-most column of the table). In the following sections, we will focus in turn on the results of models B and C[3].

[3]The results of the models A_1 and A_2 are not reported in the paper for space reasons and given the limited insights that they provide with respect to more flexible models, but they are available upon request.

3.3.1 Mixed models without correlation across activities

The results of the models B_1 and B_2 are reported in Tables 4 and 5. While classical estimation produces estimates and standard errors, Bayesian estimation produces chains of draws from the posterior distributions for each model parameter. After discarding burn-in iterations, we can then calculate a mean and standard deviation across iterations for each of these chains, where these have similar properties to maximum likelihood estimates and standard errors (Train, 2001). For a given parameter β, we would thus report the mean of the posterior distribution and the standard deviation.

In Table 4, we look at the δ parameters and the socio-demographic shifts in those, labelled as Δ, where these are generic across days. In Table 5, we look at the γ parameters. For each parameter, we report the mean and standard deviation (sd) of the chains of posterior draws. In this model, the δ parameters follow univariate Normal distributions, and we thus obtain two parameters for each δ, namely a mean sensitivity (μ) and a standard deviation (σ), where these should not be confused with the means and standard deviations of the posteriors, which obviously exist for both. For the socio-demographic shifts in δ, i.e. Δ, and the γ terms, only a fixed parameter is estimated, i.e. no random heterogeneity.

Looking at the parameters for the model without socio-economic attributes (shorted as *socios* in our tables to save space) B_1, we observe that all the baseline utility parameters except for δ_{WD} for travel are negative, mainly reflecting the discrete choice and indicating that the outside good (used as a base) is always "preferred" with respect to the inside goods, as everybody in the sample always chooses it. The value of the δ coefficients can also be affected by the continuous choice, so that it is possible to obtain positive δ coefficients for popular inside goods. This also motivates the use of a Normal distribution for the δ parameters. The results clearly show substantially different sensitivities during weekday and weekend days. In addition, they reveal substantial random heterogeneity (the σ parameters), highlighting that different people have different sensitivities in terms of activity participation.

Turning to deterministic heterogeneity, the fixed shifts reported in Table 4 suggest that younger people (less than 25 year old) are less likely to work and to perform both daily and non daily shopping and more likely to go to school and be involved in social activities. People in the older age group (over 65) are less likely to work than those between 25 and 65 year old, they are also less likely to engage in educational activities and pick-up/drop-off other people, but they do perform more daily shopping and personal business. Men are less likely to perform both daily and non daily shopping and are more likely to work. As expected, being a student is associated with higher likelihood of attending education and a lower chance of working, carrying out personal business, social activities and travelling, while those with "other occupation" (as opposed to employed, so this category includes retirees, homemakers etc) are less likely to work, perform social activities and travel. Finally, parents are more likely to pick-up/drop-off others and carry out daily shopping. We believe these results, while illustrative and incorporating only a limited number of the variables that are likely to affect the complex picture of time use,

		Model w/o socios (B_1)				Model with socios (B_2)			
		μ (or fixed)		σ		μ (or fixed)		σ	
Activity	**Parameter**	mean post	sd post	mean post	sd post	mean post	sd post	mean post	sd post
Work	δ_{WD}	-4.57	0.27	3.23	0.25	-1.96	0.19	1.57	0.13
	δ_{SAT}	-6.22	0.39	1.62	0.33	-6.11	0.63	2.96	0.54
	δ_{SUN}	-8.06	1.06	2.06	0.73	-7.26	0.92	2.33	0.69
	Δ_{male}	-	-	-	-	0.74	0.57	-	-
	$\Delta_{age<25}$	-	-	-	-	-1.66	0.41	-	-
	$\Delta_{age>65}$	-	-	-	-	-1.57	0.36	-	-
	$\Delta_{student}$	-	-	-	-	-1.42	0.18	-	-
	$\Delta_{otherocc.}$	-	-	-	-	-1.45	0.14	-	-
School	δ_{WD}	-7.49	0.54	4.89	0.48	-7.49	0.35	1.98	0.19
	δ_{SAT}	-10.70	2.23	3.23	1.27	-17.07	2.51	5.46	1.37
	δ_{SUN}	-13.77	3.65	4.57	1.83	-16.79	2.81	4.43	1.44
	$\Delta_{age<25}$	-	-	-	-	1.52	0.22	-	-
	$\Delta_{age>65}$	-	-	-	-	-1.05	0.53	-	-
	$\Delta_{student}$	-	-	-	-	1.78	0.31	-	-
Drop-off/ Pick-up	δ_{WD}	-6.24	0.22	1.57	0.19	-6.33	0.23	1.49	0.19
	δ_{SAT}	-5.86	0.37	1.06	0.36	-5.87	0.29	0.88	0.26
	δ_{SUN}	-6.44	0.56	1.59	0.51	-6.67	0.57	1.61	0.47
	$\Delta_{age>65}$	-	-	-	-	-1.90	0.60	-	-
	Δ_{parent}	-	-	-	-	0.83	0.26	-	-
Daily shopping	δ_{WD}	-3.96	0.10	1.06	0.09	-3.68	0.14	0.88	0.08
	δ_{SAT}	-3.72	0.13	1.01	0.18	-3.46	0.18	0.98	0.21
	δ_{SUN}	-6.26	0.34	1.16	0.34	-6.43	0.53	1.51	0.50
	Δ_{male}	-	-	-	-	-1.18	0.36	-	-
	$\Delta_{age<25}$	-	-	-	-	-1.03	0.19	-	-
	$\Delta_{age>65}$	-	-	-	-	1.36	0.33	-	-
	Δ_{parent}	-	-	-	-	1.06	0.46	-	-
Non daily shopping	δ_{WD}	-4.77	0.09	0.72	0.10	-4.40	0.10	0.67	0.09
	δ_{SAT}	-4.61	0.17	0.81	0.21	-4.27	0.19	0.86	0.23
	δ_{SUN}	-7.28	0.58	1.18	0.51	-7.74	0.88	1.82	0.64
	Δ_{male}	-	-	-	-	-1.20	0.31	-	-
	$\Delta_{age<25}$	-	-	-	-	-1.05	0.25	-	-
Social	δ_{WD}	-4.29	0.09	1.01	0.09	-4.25	0.11	0.92	0.09
	δ_{SAT}	-3.53	0.11	0.62	0.14	-3.49	0.13	0.55	0.12
	δ_{SUN}	-3.44	0.12	0.83	0.15	-3.41	0.13	0.78	0.17
	$\Delta_{age<25}$	-	-	-	-	1.06	0.21	-	-
	$\Delta_{student}$	-	-	-	-	-1.04	0.36	-	-
	$\Delta_{otherocc.}$	-	-	-	-	-1.13	0.30	-	-
Leisure	δ_{WD}	-4.80	0.13	1.39	0.12	-4.80	0.13	1.37	0.11
	δ_{SAT}	-4.32	0.16	0.90	0.20	-4.28	0.16	0.86	0.23
	δ_{SUN}	-4.02	0.13	0.66	0.14	-4.00	0.14	0.66	0.16
Personal business	δ_{WD}	-3.82	0.08	0.82	0.07	-3.71	0.08	0.74	0.07
	δ_{SAT}	-4.95	0.25	1.15	0.28	-4.84	0.28	1.11	0.32
	δ_{SUN}	-5.65	0.36	1.44	0.37	-5.65	0.36	1.54	0.36
	$\Delta_{age>65}$	-	-	-	-	1.09	0.32	-	-
	$\Delta_{student}$	-	-	-	-	-1.20	0.22	-	-
Travel	δ_{WD}	0.92	0.14	0.37	0.04	1.02	0.13	0.32	0.04
	δ_{SAT}	-0.88	0.10	0.29	0.05	-0.66	0.16	0.30	0.04
	δ_{SUN}	-1.41	0.12	0.34	0.05	-1.22	0.14	0.34	0.06
	$\Delta_{student}$	-	-	-	-	-0.88	0.22	-	-
	$\Delta_{otherocc.}$	-	-	-	-	-0.91	0.18	-	-

Table 4: Models B_1 and B_2: parameters for baseline utilities

Activity	Parameter	***Model w/o socios (B_1)*** mean post	sd post	***Model with socios (B_2)*** mean post	sd post
Work	γ_{WD}	0.94	0.08	0.94	0.08
	γ_{SAT}	1.57	0.36	1.01	0.29
	γ_{SUN}	1.98	0.91	0.88	0.40
School	γ_{WD}	0.88	0.10	0.87	0.09
	γ_{SAT}	0.74	0.46	0.82	0.48
	γ_{SUN}	0.35	0.31	0.29	0.22
Drop-off/ Pick-up	γ_{WD}	0.58	0.08	0.58	0.08
	γ_{SAT}	0.43	0.12	0.44	0.12
	γ_{SUN}	0.38	0.14	0.40	0.14
Daily shopping	γ_{WD}	0.74	0.05	0.73	0.05
	γ_{SAT}	0.60	0.09	0.58	0.10
	γ_{SUN}	0.32	0.10	0.29	0.09
Non daily shopping	γ_{WD}	0.83	0.07	0.82	0.07
	γ_{SAT}	0.80	0.17	0.77	0.14
	γ_{SUN}	0.87	0.40	0.81	0.33
Social	γ_{WD}	0.93	0.07	0.92	0.07
	γ_{SAT}	1.07	0.15	1.03	0.16
	γ_{SUN}	0.87	0.12	0.87	0.14
Leisure	γ_{WD}	0.91	0.08	0.91	0.08
	γ_{SAT}	0.95	0.17	0.89	0.13
	γ_{SUN}	0.85	0.15	0.81	0.13
Personal business	γ_{WD}	0.82	0.05	0.82	0.05
	γ_{SAT}	0.63	0.12	0.62	0.14
	γ_{SUN}	0.44	0.13	0.44	0.13
Travel	γ_{WD}	0.24	0.03	0.25	0.03
	γ_{SAT}	0.38	0.04	0.37	0.06
	γ_{SUN}	0.46	0.06	0.45	0.06

Table 5: Models B_1 and B_2: translation parameters

are in line with expectations.

The means for the δ parameters (μ) differ across models in those cases where there are socio-demographic effects as the μ parameter alone now relates to a specific socio-demographic subgroup. What is more interesting is to study the differences in the random heterogeneity, i.e. σ. Overall, we see that these values are lower in B_2 with respect to B_1, indicating that some of the heterogeneity is now captured by incorporating the socio-demographic shifts.

The γ parameters in Table 5 mainly describe the continuous choice, indicating that people spend most time in, i.e. they get less satiated by, working and engaging in social activities, especially on Saturday (this is reflected in the data on average time spent in different activities presented in Table 1), while for school on Saturday and Sunday the results seem to diverge from the data. For these two sets of values, very few observations exist in the data and this is reflected in high standard deviations for the posterior, equivalent to low significance in classical estimation. The γ parameters also highlight a difference in the satiation accrued from the same activities on different days. This has implications for the time spent in each activity, i.e. the amounts of time dedicated to each activity differs between weekdays and weekends.

A comparison between the model with and without socio-demographics highlights that the values of the parameters are in line, with a few exceptions: we observe higher satiation (i.e. lower time spent) from work on Saturdays and Sundays in B_2, as well as a 17% decrease in γ_{SUN} for school, indicating again a reduction in time allocation.

3.3.2 Mixed models with correlation across activities

We next turn to the results of models C_1 and C_2, i.e. the MDCEV models where we accommodate correlations between some of the δ parameters. These results are reported across three separate tables. Table 6 is the analogue of Table 4 for models B_1 and B_2, reporting the means and standard deviations of the δ parameters, as well as the socio-demographic shifts in these baseline utilities, given by the non-random Δ terms. Table 7 is the analogue of Table 5 for models B_1 and B_2, reporting the translation parameters. Finally, Table 8 reports the correlations between those δ parameters for which the off-diagonal elements of the covariance matrix were not set to zero.

The results of models C_1 and C_2 are behaviourally in line with those of models B_1 and B_2, in the sense that the sign of the coefficients does not change across the models. Nevertheless, some differences in the absolute values of the parameters can be observed. In terms of baseline δ parameters, the mean values of δ_{SAT} for school and for travel are heightened in both C_1 and C_2 when compared to their values in B_1 and B_2. There are also a number of changes in the socio-demographic effects, most of which reinforce our expectations. For example, in relation to the school activity, we see a 20% increase in the coefficient indicating the likelihood of someone younger than 25 to attend school and a 130% increase in the coefficient related to the job category “student”. We see instead a lower absolute value for the coefficients related to the likelihood of people aged over 65 (-42%) and of parents (-20%) to pick-up and drop-off others, with the first coefficients remaining negative and the second one positive. Other coefficients which see a reduction are the ones measuring the effect of being a student and of being older than 65 on the likelihood of conducting personal business, as well as the likelihood of travelling for students and people with "other" occupation.

In the C models, we allow for correlation between some of the δ parameters, and this not only improves model fit as shown in Table 3, but also changes the patterns of heterogeneity. By looking at the values of the posterior mean for the σ parameters (i.e. the heterogeneity) in both $C1$ and $C2$, we see that most of the ones that undergo a change of more than 15-20% actually show a decrease with respect to their values in B_1 and B_2. This is possibly an indication of overstated heterogeneity in the models where we do not allow for correlation between different activities, which can happen in the presence of unaccounted for positive correlation (Hess and Train, 2011). However, a comparison between the coefficients of $C1$ and $C2$ also shows that the introduction of the socio-demographics in the presence of correlation does not have as strong a dampening effect on the heterogeneity as in model $B2$. This makes sense as, in $B2$, the socio-demographics were the only way for the model to capture correlation across activities, while now, correlation is modelled explicitly. The translation parameters in $C1$ and $C2$ (see Table 7) are largely in line. We see a few reductions in γ_{SUN} for social (-19%) and leisure (-12%), while γ_{SAT} for work is higher in the model with socio-demographics, indicating higher time allocation to this activity.

We finally look at the correlations between individual δ parameters, as reported in Table 8. As mentioned earlier, the models with a diagonal covariance matrix ($B1$ and $B2$) implicitly captures correlation for the same activity across days of the same type given the random heterogeneity. In models $C1$ and $C2$, we additionally capture correlation between different activities and across day-types. After a careful exploration of a large number of possible correlations between different model coefficients, we retained a subset that were large in size and meaningful from a behavioural perspective, as well as showing stable posterior distributions. Most of these are correlations across day-types for the same activity, where in addition, we look at three inter-activity correlations.

The correlations in 8 can be interpreted on the basis of their sign. While there is some scope for confounding of roles, positive correlations could be interpreted as a complementarity effect. If for example the correlation between WD and SAT is positive, this could mean that people who (do not) perform the activity on weekdays also (do not) perform it on Saturday. Differently, a negative correlation may indicate a substitution effect: if the given activity is performed on a weekday, it will not be performed on a Saturday (and vice versa).

Looking at the correlations from model $C1$ first, we can see that the baseline utility constant for work shows high levels of correlation across different day-types, in particular between Saturday and Sunday, possibly suggesting that people who work on the weekend tend to do so on both days, and people who do not work on weekends will not work on either day. High levels of positive correlation are also found for other activities, for example for leisure and social (where this could give an indication of the fact that people who engage in leisure activities tend to do so on different days) and drop-off/pick-up or daily shopping (which could capture the fact that a specific person in the household is in charge of given activities), even if in the latter case the values are slightly lower. The correlations between different activities reported in the last three lines of the table can be interpreted similarly: as an example, we observe that there is a positive correlation between performing social activities and travel on a Sunday, which hints at the complementarity between the two activities.

As could have been expected, we see some significant changes in the correlations between model $C1$ and $C2$. Focussing on a few examples, we can see that the high levels of correlation shown in the baseline utility constant for work are reduced[4] Several socio-demographic variables have a strong impact on the utility of work, as shown in Table 7, so we expect them to capture some of the variability that would otherwise be attributed to correlation. We see a similar reduction in most of the other activities, while in other cases the correlations are stable between the model with and without socio-demographics, like in the case of leisure on weekdays and Sunday and social and travel on a Sunday. There are also some cases (e.g. see the correlation between $\delta_{\text{daily shopping}_{WD}}$ and $\delta_{\text{daily shopping}_{SUN}}$) where the introduction of socio-demographics actually increases rather than reduces the correlation in the unobserved heterogeneity. While reductions would be expected (if the deterministic heterogeneity explains that given people are more likely to behave consistently across days), the fact that we see the opposite here shows that the introduction of deterministic heterogeneity may allow the model to capture different patterns of correlation that are otherwise masked.

4 Implications of the difference across models

As shown in Table 3, the models incorporating correlations across parameters provide improvements in model fit, but the difference with the other models goes beyond model fit. We first look at how the shape of the utility profiles (i.e. the overall utility accrued by different amounts of time) of different activities changes across specifications. The number of alternatives in our models is large and we focus here on a small subset by looking at the utility of two activities, work and leisure, on weekdays. These results are summarised in Figure 1, where, to allow for comparison, we rescaled the results to be the same value for the utility of working for four hours. It is evident how the shape of the utility profiles changes across the different models, as a result of incorporating socio-demographics, random heterogeneity and finally correlation between the random terms. The *true* shape is of course not known but the differences in fit clearly point towards model $C2$ and these results show that there are quite substantial differences in implied behaviour. It is also interesting to note that the impact of introducing the socio-demographics on the shape of the utility profiles is far less substantial in the C models.

[4] The correlation between $\delta_{work_{WD}}$ and $\delta_{work_{SAT}}$ becomes negative in model $C2$, but given the high standard error of this parameter we believe it not to be different from zero.

		Model w/o socios (B_1)				***Model with socios (B_2)***			
		μ (or fixed)		**σ**		**μ (or fixed)**		**σ**	
Activity	**Parameter**	mean post	sd post	mean post	sd post	mean post	sd post	mean post	sd post
Work	δ_{WD}	-4.46	0.25	3.21	0.25	-1.82	0.17	1.65	0.10
	δ_{SAT}	-7.39	0.19	2.54	0.34	-5.96	0.23	2.46	0.18
	δ_{SUN}	-8.44	0.21	1.63	0.23	-7.47	0.43	2.36	0.19
	Δ_{male}	-	-	-	-	0.12	0.25	-	-
	$\Delta_{age<25}$	-	-	-	-	-1.62	0.14	-	-
	$\Delta_{age>65}$	-	-	-	-	-2.47	0.24	-	-
	$\Delta_{student}$	-	-	-	-	-4.55	0.17	-	-
	$\Delta_{otherocc.}$	-	-	-	-	-3.77	0.13	-	-
School	δ_{WD}	-7.53	0.57	4.96	0.53	-7.22	0.24	1.86	0.17
	δ_{SAT}	-8.35	0.29	1.23	0.40	-12.04	0.27	2.36	0.34
	δ_{SUN}	-11.92	3.62	3.52	1.78	-14.55	1.93	3.16	1.32
	$\Delta_{age<25}$	-	-	-	-	3.61	0.37	-	-
	$\Delta_{age>65}$	-	-	-	-	-1.04	0.22	-	-
	$\Delta_{student}$	-	-	-	-	2.16	0.36	-	-
Drop-off/ Pick-up	δ_{WD}	-6.31	0.14	1.77	0.12	-6.26	0.18	1.51	0.23
	δ_{SAT}	-5.99	0.13	1.35	0.10	-5.92	0.22	0.85	0.13
	δ_{SUN}	-6.29	0.24	1.17	0.19	-6.96	0.18	1.69	0.42
	$\Delta_{age>65}$	-	-	-	-	-1.08	0.13	-	-
	Δ_{parent}	-	-	-	-	0.66	0.14	-	-
Daily shopping	δ_{WD}	-3.99	0.08	1.11	0.10	-3.68	0.17	0.90	0.08
	δ_{SAT}	-3.81	0.15	0.91	0.27	-3.47	0.16	1.22	0.13
	δ_{SUN}	-7.32	0.32	2.24	0.23	-6.73	0.17	1.15	0.15
	Δ_{male}	-	-	-	-	-0.55	0.14	-	-
	$\Delta_{age<25}$	-	-	-	-	-0.72	0.15	-	-
	$\Delta_{age>65}$	-	-	-	-	0.44	0.11	-	-
	Δ_{parent}	-	-	-	-	0.38	0.13	-	-
Non daily shopping	δ_{WD}	-4.78	0.09	0.74	0.10	-4.44	0.10	0.65	0.10
	δ_{SAT}	-4.63	0.17	1.13	0.15	-4.35	0.13	0.97	0.15
	δ_{SUN}	-7.06	0.61	0.96	0.48	-7.35	0.60	1.40	0.50
	Δ_{male}	-	-	-	-	-0.37	0.13	-	-
	$\Delta_{age<25}$	-	-	-	-	-0.66	0.10	-	-
Social	δ_{WD}	-4.30	0.09	1.02	0.09	-4.30	0.10	0.91	0.08
	δ_{SAT}	-3.57	0.12	0.67	0.09	-3.55	0.14	0.62	0.09
	δ_{SUN}	-3.53	0.12	0.67	0.14	-3.51	0.13	0.64	0.17
	$\Delta_{age<25}$	-	-	-	-	0.69	0.13	-	-
	$\Delta_{student}$	-	-	-	-	-0.34	0.12	-	-
	$\Delta_{otherocc.}$	-	-	-	-	-0.32	0.09		
Leisure	δ_{WD}	-4.77	0.10	1.43	0.12	-4.80	0.09	1.34	0.10
	δ_{SAT}	-4.31	0.15	1.13	0.10	-4.39	0.13	1.17	0.21
	δ_{SUN}	-4.06	0.09	0.96	0.11	-4.22	0.13	0.87	0.11
Personal business	δ_{WD}	-3.82	0.08	0.81	0.07	-3.73	0.08	0.74	0.07
	δ_{SAT}	-4.91	0.26	1.09	0.32	-4.84	0.22	1.05	0.26
	δ_{SUN}	-5.59	0.33	1.37	0.34	-5.70	0.37	1.56	0.38
	$\Delta_{age>65}$	-	-	-	-	0.51	0.16	-	-
	$\Delta_{student}$	-	-	-	-	-0.74	0.14	-	-
Travel	δ_{WD}	0.89	0.11	0.37	0.04	1.01	0.15	0.32	0.04
	δ_{SAT}	-0.73	0.12	0.30	0.04	-0.70	0.12	0.30	0.05
	δ_{SUN}	-1.44	0.11	0.49	0.11	-1.29	0.13	0.84	0.15
	$\Delta_{student}$	-	-	-	-	-0.25	0.07	-	-
	$\Delta_{otherocc.}$	-	-	-	-	-0.33	0.05	-	-

Table 6: Models C_1 and C_2: parameters for the baseline utilities

Activity	Parameter	***Model w/o socios (B_1)*** mean post	sd post	***Model with socios (B_2)*** mean post	sd post
Work	γ_{WD}	1.69	0.09	1.82	0.09
	γ_{SAT}	2.98	0.45	4.43	0.20
	γ_{SUN}	10.69	0.32	9.46	0.49
School	γ_{WD}	0.96	0.13	1.02	0.09
	γ_{SAT}	4.19	0.35	1.71	0.31
	γ_{SUN}	17.51	0.21	16.83	0.20
Drop-off/ Pick-up	γ_{WD}	0.24	0.03	0.24	0.03
	γ_{SAT}	0.22	0.06	0.23	0.06
	γ_{SUN}	0.17	0.05	0.16	0.05
Daily shopping	γ_{WD}	0.25	0.02	0.25	0.01
	γ_{SAT}	0.25	0.04	0.25	0.04
	γ_{SUN}	0.14	0.05	0.17	0.05
Non daily shopping	γ_{WD}	0.62	0.06	0.61	0.05
	γ_{SAT}	0.68	0.13	0.68	0.10
	γ_{SUN}	1.44	0.19	1.46	0.11
Social	γ_{WD}	1.63	0.08	1.69	0.11
	γ_{SAT}	2.65	0.19	2.93	0.19
	γ_{SUN}	1.76	0.22	1.41	0.15
Leisure	γ_{WD}	1.46	0.10	1.49	0.10
	γ_{SAT}	2.18	0.26	2.32	0.20
	γ_{SUN}	2.56	0.19	2.23	0.18
Personal business	γ_{WD}	0.45	0.03	0.45	0.03
	γ_{SAT}	0.64	0.17	0.60	0.08
	γ_{SUN}	0.29	0.09	0.27	0.08
Travel	γ_{WD}	0.03	0.00	0.03	0.00
	γ_{SAT}	0.09	0.01	0.10	0.01
	γ_{SUN}	0.15	0.02	0.15	0.02

Table 7: Models C_1 and C_2: translation parameters

Activity	Day	*Model w/o socios (C_1)* mean post	sd post	*Model with socios (C_2)* mean post	sd post
Work	WD-SAT	0.76	0.03	0.67	0.04
	WD-SUN	0.54	0.11	-0.15	0.09
	SAT-SUN	0.93	0.06	0.59	0.10
Drop-off/ Pick-up	WD-SAT	0.94	0.04	0.87	0.08
	WD-SUN	0.57	0.11	0.39	0.12
	SAT-SUN	0.49	0.14	0.66	0.13
Daily shopping	WD-SAT	0.79	0.11	0.51	0.10
	WD-SUN	0.33	0.06	0.69	0.06
	SAT-SUN	0.18	0.11	0.42	0.09
Social	WD-SAT	0.74	0.10	0.91	0.09
	WD-SUN	0.71	0.17	0.70	0.18
	SAT-SUN	0.88	0.10	0.78	0.15
Leisure	WD-SAT	0.84	0.08	0.92	0.07
	WD-SUN	0.95	0.05	0.95	0.06
	SAT-SUN	0.86	0.10	0.95	0.05
Social & Travel	SUN-SUN	0.93	0.19	0.92	0.12
Work & Non daily shopping	SAT-SAT	0.39	0.32	-0.24	0.83
School & Daily shopping	SAT-SAT	-0.97	0.03	0.12	0.07

Table 8: Key correlations between different δ parameters

A key interest of developing behavioural models is of course to predict future behaviour. To this extent, we have applied the Pinjari and Bhat (2010) routine to forecast time use across two weeks (aggregating the forecasts across the 14 individual days) using the six models presented in this paper, with the aim of assessing the implications of our modelling specification when the model is applied.

We first perform a base forecast, i.e. predicting the time allocation without changing the data. This is summarised in Tables 9-11. Table 9 shows the base forecast for the discrete part of the model, i.e. the share of people in the sample who perform each activity. Table 10 and 11 both report the base prediction in term of time allocation, but the former reports average time allocations to all activities across the sample while the latter only include the time allocation of respondents who actually perform the activity. Therefore, the numbers in Table 11 are the most "realistic" in terms of behaviour. We note very small differences between the models with and without socio-demographics within a given model group (e.g. A1 vs A2). This is however not surprising at the aggregate level, which is what we present here. Introducing covariate interactions for constants will have an impact on the market shares within each socio-demographic group, but the overall picture should remain unaffected.

In terms of the continuous time allocation (Tables 10 and 11), it is clear to see that models $B1$ and $B2$ substantially overpredict the amount of travel conducted, where in the data, this was on average 1.19 and 1.24 hours, respectively. There is also some overprediction, much less so, for the A and C models. This is a direct result of the much

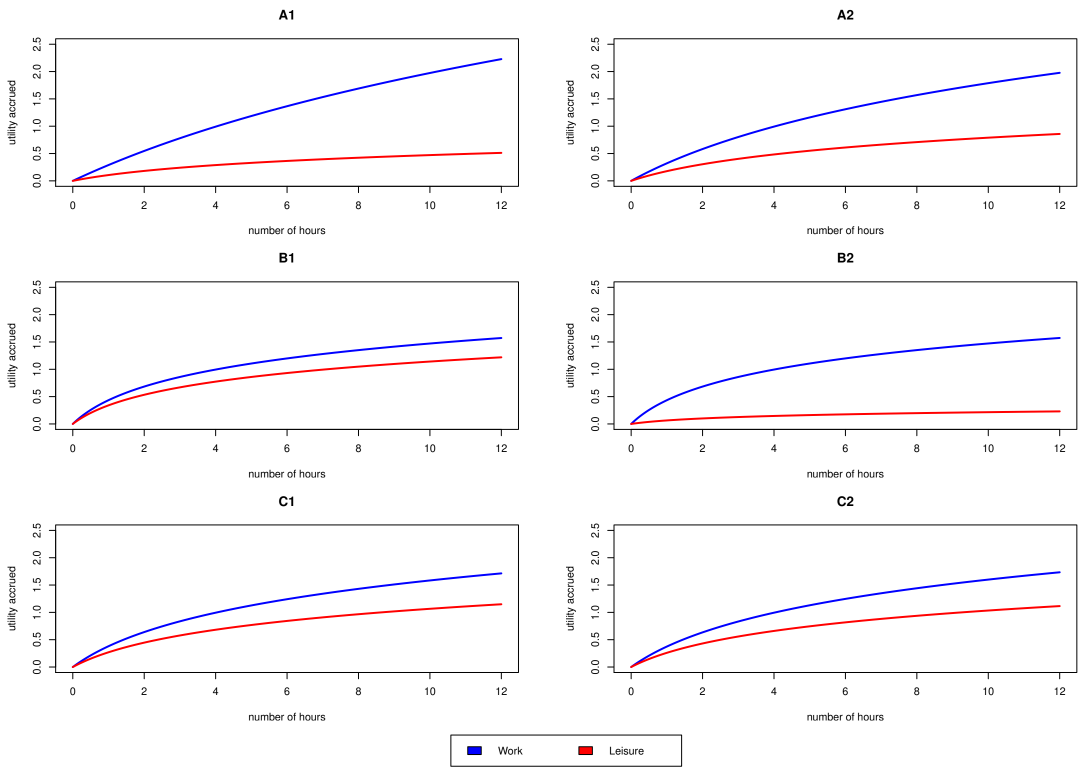

Figure 1: Utility profiles of the work and leisure activities

higher estimate for the translation parameter in the B models (cf. Table 5). This could suggest that incorporating independent random heterogeneity, i.e. without correlation, can lead to confounding impacts on the shape of the utility. A similar results is observed in the paper by Cherchi and Cirillo (2011), where the authors analysed the Mobidrive dataset to explore the model validation and forecasting issues associated with longitudinal diaries using logit and mixed logit models. Interestingly, they also reported no benefits in predictive performance of the mixed model accommodating random heterogeneity without capturing correlations between different alternatives. This reinforces our finding that such correlations should be accommodated in the model, so as to improve prediction while better accommodating behavioural realism with respect to simpler models. Furthermore, while the Cherchi and Cirillo (2011) work allows for correlation across choices at the level of individual marginal utility coefficients, we allow for correlations between different activities within and across days."

In the forecasting exercise, we looked at the simplistic case where a respondent does not work for a period of two weeks (therefore this activity is unavailable) and looked at how time is reallocated. The results of this process in terms of redistribution to other activities are represented in Figure 2, which highlights the differences in the forecasts

	Data	A1	A2	B1	B2	C1	C2
Outside good	100%	100%	100%	100%	100%	100%	100%
Work	18%	24%	26%	20%	32%	24%	26%
School	9%	14%	15%	11%	5%	13%	14%
Drop-off/pick-up	8%	6%	6%	5%	5%	6%	6%
Daily shopping	24%	22%	22%	17%	19%	21%	21%
Non-daily shopping	12%	10%	10%	7%	6%	10%	10%
Social	35%	23%	23%	19%	17%	23%	22%
Leisure	24%	17%	17%	14%	15%	17%	16%
Personal business	20%	20%	20%	15%	17%	20%	20%
Travel	88%	94%	94%	93%	92%	92%	93%

Table 9: Results of forecasts for base scenario (share of people performing each activity)

	Data	A1	A2	B1	B2	C1	C2
Outside good	17.45	16.24	16.16	12.99	13.85	15.76	15.88
Work	2.15	2.00	2.07	1.32	1.81	2.25	2.33
School	0.96	0.97	1.01	0.94	0.22	1.17	1.06
Drop-off/pick-up	0.05	0.06	0.06	0.08	0.09	0.07	0.06
Daily shopping	0.17	0.29	0.29	0.42	0.54	0.28	0.29
Non-daily shopping	0.15	0.20	0.20	0.17	0.14	0.20	0.20
Social	0.94	1.05	1.05	0.56	0.51	1.04	1.02
Leisure	0.64	0.74	0.74	0.39	0.41	0.76	0.72
Personal business	0.31	0.38	0.37	0.38	0.44	0.37	0.37
Travel	1.19	2.07	2.05	6.74	5.97	2.09	2.08

Table 10: Results of forecasts for base scenario (time allocation in hours per day)

	Data	A1	A2	B1	B2	C1	C2
Outside good	18.52	16.24	16.16	12.99	13.85	15.76	15.88
Work	6.42	8.19	7.93	6.75	5.64	9.34	8.89
School	4.72	7.18	6.69	8.28	4.04	9.11	7.63
Drop-off/pick-up	0.55	1.04	1.02	1.82	1.85	1.12	1.06
Daily shopping	0.52	1.33	1.33	2.48	2.82	1.37	1.36
Non-daily shopping	1.11	2.00	2.00	2.32	2.32	2.05	2.02
Social	3.60	4.55	4.53	3.02	2.97	4.62	4.56
Leisure	3.20	4.40	4.38	2.85	2.83	4.44	4.36
Personal business	1.18	1.86	1.85	2.45	2.61	1.88	1.87
Travel	1.24	2.21	2.19	7.25	6.49	2.27	2.23

Table 11: Results of forecasts for base scenario (time allocation in hours per day when the activity is performed)

generated by the different models, implying that incorporating random parameters and correlations will yield substantially different shares for the time allocation. A number of observations can be made. With the exception of school, we see that the models incorporating socio-demographics show a higher redistribution of time, which is a result of a higher predicted share for work in these models. We also see that the changes implied by the C models are more similar to those obtained from the A models than those from the B models, where the latter are lower. This is a direct result of the earlier observation relating to the overprediction of the travel activity in the B models, and hence the lower amount of time available for other activities. The differences between the A and C models are small, but this is likely in part a result of conducting forecasting at the day level, i.e. missing out on the impact of inter-day correlations, a point we return to in the conclusions.

5 Conclusions

The MDCEV modelling framework has established itself as a preferred method for modelling time allocation, with data very often coming from travel or activity diaries. However, while many of such datasets contain information on multiple days for the same individual, the standard modelling approach has treated each day in isolation. This paper has made the case that this practice misses out on important links between days, potentially leading to reduced insights from the modelling.

Our paper discusses several possible ways of accommodating links across days within an MDCEV framework. While the implementation of a non-additive utility function would be the theoretically correct way to accommodate the complementarities and substitutions across days, such an approach is very difficult to estimate and apply in practice, especially with budget constraints at the day and multi-day level. We instead rely on

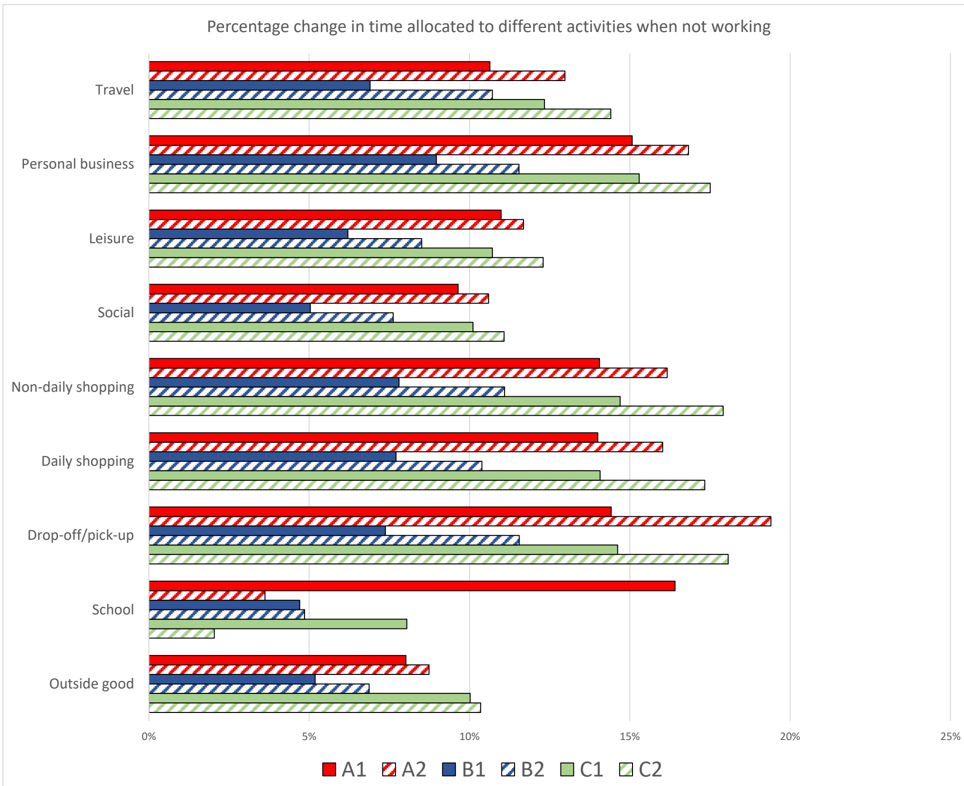

Figure 2: Redistribution of time when work is not available

additive utility functions where we accommodate correlation between activities at the within-day and between-day level through correlated random heterogeneity. The use of such a mixed MDCEV model allows us to capture correlations within and across days without the use of non-additive utility functions and by relying on a simple day-level budget.

We illustrate the benefit of this approach using a two week sample from the *Mobidrive* dataset. Our estimation work confirms the presence of deterministic and random heterogeneity, and crucially in the context of the present paper, correlations both at the within-day and between-day level. We find that allowing for these correlations leads to improved model fit, but also substantive changes in model estimates. Overall, the changes that we observe are meaningful and the results of the model allowing for correlations are behaviourally realistic and seem to indicate that not allowing for the correlation can lead to misguided results in terms of heterogeneity, both deterministic and random. We have also illustrated how these differences in parameter values lead to different model forecasts as well as utility profiles for the activities.

While the MDCEV model represent the state-of-the-art framework for modelling discrete-continuous choices, we acknowledge that its structure implies some limitation in the type of variables that can be included in the model. Namely, as the overall duration of each activity within the specified time budget is modelled, it is not possible to include any variable about the characteristics of specific occurrences of each activity, such as location, start and end time, people involved.

As always, there is substantial scope for further work. A refined model specification with additional covariates could give a more detailed representation of the behavioural process at stake. Additionally, it would be possible to allow for random heterogeneity in the translation parameters, but this would, as described earlier, require a larger dataset. We have also presented a simple forecasting example. This showed differences across models in the implied redistribution of time when an activity becomes unavailable. It should be noted that our forecasting example worked at the level of individual days and there is thus no redistribution across days. This shows that even if making predictions for individual days, the use of a model that captures a richer pattern of heterogeneity by incorporating inter-day effects will still lead to differences in prediction. Of course, a further benefit of this model assumption would arise in forecasts that can allow for inter-day substitution[5]. The Pinjari and Bhat (2010) algorithm for model forecasting however cannot accommodate the impact of the correlations between parameters in terms of substitution across days if conducting the forecasting at the day level. This could only be accommodated by relaxing the 24 hours budget constraint to allow time redistribution across different days, although this would result in time allocation higher than 24 hours for some days and require ex-post corrections. The full impact of the correlation thus requires the development of new forecasting approaches, which is an important topic for future work.

Acknowledgments

The authors are grateful to Kay Axhausen for making the Mobidrive data available for this paper and to David Palma (University of Leeds) for useful discussions. The Leeds authors acknowledge the financial support by the European Research Council through the consolidator grant 615596-DECISIONS.

References

Axhausen, K. W., Zimmermann, A., Schönfelder, S., Rindsfüser, G., Haupt, T., 2002. Observing the rhythms of daily life: A six-week travel diary. Transportation 29 (2), 95–124.

Bhat, C. R., 2008. The multiple discrete-continuous extreme value (mdcev) model: role

[5] By making work *unavailable* on all days, the need for inter-day substitution of course largely disappears.

of utility function parameters, identification considerations, and model extensions. Transportation Research Part B: Methodological 42 (3), 274–303.

Bhat, C. R., Castro, M., Pinjari, A. R., 2015. Allowing for complementarity and rich substitution patterns in multiple discrete–continuous models. Transportation Research Part B: Methodological 81, 59–77.

Bhat, C. R., Frusti, T., Zhao, H., Schönfelder, S., Axhausen, K. W., 2004. Intershopping duration: an analysis using multiweek data. Transportation Research Part B: Methodological 38 (1), 39 – 60.
URL http://www.sciencedirect.com/science/article/pii/S0191261502000930

Bhat, C. R., Misra, R., 1999. Discretionary activity time allocation of individuals between in-home and out-of-home and between weekdays and weekends. Transportation 26 (2), 193–229.

Calastri, C., Hess, S., Daly, A., Carrasco, J. A., 2017. Does the social context help with understanding and predicting the choice of activity type and duration? an application of the multiple discrete-continuous nested extreme value model to activity diary data. Transportation Research Part A: Policy and Practice 104, 1–20.

Castro, M., Bhat, C. R., Pendyala, R. M., Jara-Díaz, S. R., 2012. Accommodating multiple constraints in the multiple discrete–continuous extreme value (mdcev) choice model. Transportation Research Part B: Methodological 46 (6), 729–743.

Cherchi, E., Cirillo, C., 2011. Validation and forecasts in models estimated from multiday travel survey. Transportation Research Record 2175 (1), 57–64.

Cherchi, E., Cirillo, C., de Dios Ortúzar, J., 2017. Modelling correlation patterns in mode choice models estimated on multiday travel data. Transportation Research Part A: Policy and Practice 96, 146–153.

Chikaraishi, M., Zhang, J., Fujiwara, A., Axhausen, K., 2010. Exploring variation properties of time use behavior on the basis of a multilevel multiple discrete-continuous extreme value model. Transportation Research Record: Journal of the Transportation Research Board (2156), 101–110.

Cirillo, C., Axhausen, K. W., 2010. Dynamic model of activity-type choice and scheduling. Transportation 37 (1), 15–38.

De Jong, G. C., 1990. An indirect utility model of car ownership and private car use. European Economic Review 34 (5), 971–985.

Dubin, J. A., McFadden, D. L., 1984. An econometric analysis of residential electric appliance holdings and consumption. Econometrica: Journal of the Econometric Society, 345–362.

Dumont, J., Keller, J., Carpenter, C., 2015. Rsghb: Functions for hierarchical bayesian estimation: A flexible approach. R package version 1.1.2, URL http://CRAN.R-project.org/package=RSGHB.

Goodwin, P., 1981. The usefulness of travel budgets. Transportation Research Part A: General 15 (1), 97 – 106.
URL http://www.sciencedirect.com/science/article/pii/0191260783900195

Habib, K. M. N., Miller, E. J., Axhausen, K. W., 2008. Weekly rhythm in joint time expenditure for all at-home and out-of-home activities: application of kuhn-tucker demand system model using multiweek travel diary data. Transportation Research Record 2054 (1), 64–73.

Hanemann, W. M., 1978. A methodological and empirical study of the recreation benefits from water quality improvement. Department of Agricultural and Resource Economics, University of California.

Heckman, J. J., 1977. Sample selection bias as a specification error (with an application to the estimation of labor supply functions).

Hess, S., Palma, D., 2019. Apollo: a flexible, powerful and customisable freeware package for choice model estimation and application. www.apollochoicemodelling.com.

Hess, S., Train, K. E., 2011. Recovery of inter-and intra-personal heterogeneity using mixed logit models. Transportation Research Part B: Methodological 45 (7), 973–990.

Hirsh, M., Prashkea, J. N., Ben-Akiva, M., 1986. Dynamic model of weekly activity pattern. Transportation Science 20 (1), 24–36.

Jara-Díaz, S., Rosales-Salas, J., 2015. Understanding time use: Daily or weekly data? Transportation Research Part A: Policy and Practice 76, 38–57.

Jara-Díaz, S. R., Munizaga, M. A., Greeven, P., Guerra, R., Axhausen, K., 2008. Estimating the value of leisure from a time allocation model. Transportation Research Part B: Methodological 42 (10), 946–957.

Kang, H., Scott, D. M., 2010. Exploring day-to-day variability in time use for household members. Transportation Research Part A: Policy and Practice 44 (8), 609–619.

Kapur, A., Bhat, C., 2007. Modeling adults' weekend day-time use by activity purpose and accompaniment arrangement. Transportation Research Record: Journal of the Transportation Research Board (2021), 18–27.

Minnen, J., Glorieux, I., van Tienoven, T. P., 2015. Transportation habits: Evidence from time diary data. Transportation Research Part A: Policy and Practice 76, 25–37.

Pas, E. I., Sundar, S., May 1995. Intrapersonal variability in daily urban travel behavior: Some additional evidence. Transportation 22 (2), 135–150.
URL https://doi.org/10.1007/BF01099436

Pinjari, A. R., Bhat, C., 2010. An efficient forecasting procedure for kuhn-tucker consumer demand model systems. Technical paper. Department of Civil & Environmental Engineering, University of South Florida.

Pinjari, A. R., Sivaraman, V., 2013. Discrete-continuous choice model with multiple budget constraints, imperfect and perfect substitutes, and price variation: Application to household vacation travel behavior. In: Transportation Research Board 92nd Annual Meeting. No. 13-1885.

R Core Team, 2016. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria.
URL https://www.R-project.org/

Roorda, M. J., Ruiz, T., 2008. Long-and short-term dynamics in activity scheduling: a structural equations approach. Transportation Research Part A: Policy and Practice 42 (3), 545–562.

Schlich, R., Axhausen, K. W., 2003. Habitual travel behaviour: evidence from a six-week travel diary. Transportation 30 (1), 13–36.

Spissu, E., Pinjari, A. R., Bhat, C. R., Pendyala, R. M., Axhausen, K. W., 2009. An analysis of weekly out-of-home discretionary activity participation and time-use behavior. Transportation 36 (5), 483–510.

Susilo, Y., Kitamura, R., 2005. Analysis of day-to-day variability in an individual's action space: exploration of 6-week mobidrive travel diary data. Transportation Research Record: Journal of the Transportation Research Board (1902), 124–133.

Tobin, J., 1958. Estimation of relationships for limited dependent variables. Econometrica: journal of the Econometric Society, 24–36.

Train, K., 1986. Qualitative choice analysis: Theory, econometrics, and an application to automobile demand. Vol. 10. MIT press.

Train, K., 2001. A comparison of hierarchical bayes and maximum simulated likelihood for mixed logit. University of California, Berkeley, 1–13.

Wales, T. J., Woodland, A. D., 1983. Estimation of consumer demand systems with binding non-negativity constraints. Journal of Econometrics 21 (3), 263–285.

Wang, D., Li, J., 2011. A two-level multiple discrete-continuous model of time allocation to virtual and physical activities. Transportmetrica 7 (6), 395–416.

Yamamoto, T., Kitamura, R., 1999. An analysis of time allocation to in-home and out-of-home discretionary activities across working days and non-working days. Transportation 26 (2), 231–250.