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# Opportunities to learn foundational number sense in three Swedish year one textbooks: implications for the importation of overseas-authored materials

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**ABSTRACT**

In this paper we present statistical analyses of three textbooks used by Swedish teachers to support year one children's learning of mathematics. One, Eldorado, is authored by Swedish teachers, another, Favorit, is a Swedish adaptation of a popular Finnish series and the third, Singma, is a Swedish adaptation of a Singapore series. Data were coded against the eight categories of foundational number sense, which are the number-related competences literature has shown to be essential for the later mathematical success of year one learners. Two analyses were undertaken; the first was a frequency analysis of the tasks coded for a particular category, the second was a time-series analysis highlighting the temporal location of such opportunities. The frequency analyses identified statistically significant differences with respect to children's opportunities to acquire foundational number sense. Additionally, the time series showed substantial differences in the ways in which such tasks were located in the structure of the textbooks. Such differences, we argue, offer substantial didactical challenges to teachers trying to adapt their practices to the expectations of such imports.

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**KEYWORDS**
Sweden; mathematics textbooks; comparative research; year one children; foundational number sense

## 1. Introduction

Analyzing and comparing school textbooks is not a new activity, at least from the perspective of US educational research. For example, Patty and Painter (1931), implicitly asserting variability in the quality and relevance of US school textbooks, offered teachers critical guidelines for selecting textbooks. Some years later, also from the specific perspective of school mathematics, Schutter and Spreckelmeyer (1959) compared US and European arithmetic textbooks, while Williams and Shuff (1963) compared textbooks from modern and traditional US series and Pinker (1981) a range of US textbooks with respect to their presentations of sets, a core topic of the new mathematics current at that time. In short, comparing textbooks, both with within and across educational systems, is a long-standing

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research tradition, a tradition well represented in the recent pages of IJMEST (Bütüner, 2018; Glasnovic Gracin, 2018; Kajander & Lovric, 2017; Sangwin, 2019). More recently, however, many studies have been motivated by a desire to understand how educational systems more successful than those of the writers, typically the US, present mathematical ideas (Ding, 2016; Li, Chen, & An, 2009; Yang, Reys, & Wu, 2010). This latter tradition has also been well represented in the recent pages of IJMEST (Avcu, 2019; Barcelos Amaral & Hollebrands, 2017; Kar, Güler, Şen, & Özdemir, 2018; Son & Hu, 2016).

The analysis and comparison of the textbooks used in Swedish mathematics classrooms, the site of the research presented in this paper, is no exception and draws on similar warrants. Firstly, following several decades of increasingly light-touch regulation (Andersson & Nilsson, 2000), processes of decentralization and marketization (Daun & Siminou, 2005; Wiborg, 2013) led to textbook production being fully deregulated by 1991 (Ahl, 2016) and a widespread perception of diminishing academic mathematical standards (Brandell, Hemmi, & Thunberg, 2008). Secondly, perceptions of Swedish students' failure on the first and second international mathematics studies (Brown, 1996; Robitaille, 1990) and, more recently, TIMSS and PISA have reinforced this perception. One consequence of this perceived systemic failure, fuelled by an invited OECD critique of the country's education system (OECD, 2015), has been the importation and adaptation of foreign textbooks from both Finland and Singapore, systems perceived to be better-functioning, into Swedish mathematics classrooms.

However, peceptions that PISA, for example, offers a reliable measure of a system's performance may be tempered by cultural factors typically ignored in the OECD's discourse. For example, due to laws governing the residential status of many low-paid workers, Singapore's achievements may be inflated by the systematic exclusion of the children of around a third of the country's workforce (Petersson, Sayers, Rosenqvist, & Andrews, under review). In similar vein, Finnish PISA success, a success not matched on TIMSS, may be more a consequence of unique characteristics relating to what it is to be Finnish than the quality teaching (Andrews, 2014; Andrews, Ryve, Hemmi, & Sayers, 2014; Carlgren, Klette, Mýrdal, Schnack, & Simola, 2006). In other words, without some form of systematic evaluation, the importation of textbooks from such countries may, at best, be unwarranted and, at worst, damaging.

The warrant for the introduction of such textbooks in Sweden has been made problematic by evidence that Swedish students' achievements on both PISA and TIMSS may not be as they seem. First, because they do not take such tests seriously, Swedish students typically make less effort than their international colleagues (Eklöf, 2007; Eklöf, Pavešič, & Grønmo, 2014; Skolverket, 2015). Moreover, while both TIMSS and PISA indicate that Swedish students lack both competence and familiarity with linear equations, independently conducted studies show year nine students with high levels of competence (Petersson, 2018; Skolverket, 2011), a competence they continue to demonstrate when they transfer to upper secondary school (Szabo,[1] private communication). In short, the results of equations-related counter-examples and Swedish students' lack of commitment to international tests challenge not only the legitimacy of such tests but also the warrants for the importations of Finnish and Singaporean textbooks. In this paper, therefore, we compare the number-related content of three year-one mathematics textbooks used in Swedish classrooms. The first is a respected Swedish-authored series, while the other two are adaptations of Finnish and Singaporean series respectively.

The analysis of textbooks is widespread in Swedish mathematics education research, with, typically, two foci. On the one hand, at all levels of the system, colleagues have examined how textbooks are exploited in the teaching and learning of mathematics (see, for example, Hemmi, Krzywacki, & Liljekvist, 2019; Johansson, 2007; Lithner, 2003; Sidenvall, Lithner, & Jäder, 2015). On the other hand, colleagues have focused on textbooks' suitability for supporting students' mathematical learning and concluded, again at all levels of the system, that Swedish textbooks are cognitively undemanding. For example, university textbooks typically promote low levels of mathematical reasoning (Lithner, 2004) and upper secondary textbooks, particularly in relation to their Finnish counterparts (Bergwall & Hemmi, 2017), offer few genuine opportunities for students to engage in mathematical problem solving (Brehmer, Ryve, & Van Steenbrugge, 2016) or proof (Nordström & Löfwall, 2005). With respect to compulsory school, studies have found limited opportunities for students to engage with proportional reasoning (Lundberg, 2011), particularly in relation to what research has identified as cognitively and didactically important (Ahl, 2016). In short, the limited available evidence suggests that Swedish textbooks may offer few opportunities for students to make connections and engage in mathematical reasoning. However, little is known of how textbooks written for young children present mathematical ideas and even less is known about the impact on one system's didactical traditions when a textbook is imported from one cultural context to another. Such matters are particularly significant in the context of Sweden, where 89% of Swedish compulsory school students experience instruction structured by textbooks (Mullis, Martin, Foy, & Arora, 2012).

## 2. Textbook borrowing

Across the globe, with the goal of fashioning school systems to rival those of their economic competitors (Alexander, 2012), governments are adopting policies, philosophies and concepts from abroad (Phillips & Ochs, 2004). Fuelled in particular by international tests of achievement, policy borrowing, as it is known, has become commonplace. Unfortunately, those responsible for policy importation too often ignore the 'support structures that ensured the success of the policies, the different cultural contexts in which they were situated (and) the effect of policy borrowing on the coherence of existing ... education provision' (Cantley, 2019, p. 1202). In other words, when looking towards these *reference* societies, too many policy borrowers ignore those historical, cultural, societal, and political traditions that are unlikely to be replicable elsewhere (Andrews, 2016; Chung, 2010). In a related vein, the importation of textbooks from one cultural context to another can be construed as a variant of policy borrowing. In some instances, this importation may occur at the level of the system, while in others it may be a consequence of predatory publishers seeing an opportunity to exploit the media- and test-fuelled anxieties of teachers. In this paper we analyze three year one textbooks used in Sweden. One is a Swedish-authored series of long-standing, while two are imports, one from Finland and the other from Singapore, countries generally acknowledged as successful on international tests of mathematics achievement, particularly PISA. Analyses are framed against the eight categories of foundational number sense (FoNS) (Andrews & Sayers, 2015), each of which is a literature-derived competence necessary for the later learning of mathematics.

**Table 1.** Summary of the eight FoNS categories.

| | FoNS Category | Teachers encourage, in relation to the numbers 0–20, learners to |
|---|---|---|
| 1 | Number recognition | Identify, name and write particular number symbol |
| 2 | Systematic counting | Count systematically, forwards and backwards, from arbitrary starting points |
| 3 | Number and quantity | Understand the one-to-one correspondence between number and quantity |
| 4 | Quantity discrimination | Compare magnitudes and deploy language like 'bigger than' or 'smaller than' |
| 5 | Different representations | Recognise and make connections between different representations of number |
| 6 | Estimation | Estimate, whether it be the size of a set or an object |
| 7 | Simple arithmetic | Perform simple addition and subtraction operations |
| 8 | Number patterns | Recognise and extend number patterns, identify a missing number |

## 3. The foundational number sense (FoNS) framework

The eight categories of the foundational number sense (FoNS) framework were developed to bridge the gap between those number-related understandings innate to all humans and forms of number sense typically associated with functional numeracy. It is focused on the number competences that all year one children need to acquire if they are to succeed as learners of mathematics. Unlike earlier studies, in which large numbers of such competences were identified (Berch, 2005; Howell & Kemp, 2006; Purpura & Lonigan, 2013), the FoNS team's goal was to develop and implement a simple to operationalize framework for analyzing FoNS-related opportunities in different cultural contexts (Andrews & Sayers, 2015). In this respect, the FoNS framework has already been used successfully for comparing classroom practices cross-culturally (Sayers, Andrews, & Björklund Boistrup, 2016; Löwenhielm, Marschall, Sayers, & Andrews, 2017) and the textbooks used in English schools (Petersson et al., under review). The framework was the result of a constant comparison analysis of almost 400 peer-reviewed articles, an appropriate approach when new perspectives on a construct are derived from 'previously identified possibilities' (Brod, Tesler, & Christensen, 2009, p. 1268). The eight categories identified by this process, all of which are curriculum independent and implicated in the later learning of mathematics, are summarized in Table 1. However, rather than discuss this literature here and appear repetitive it is used to inform the discussion below. Importantly, the framework's international origins make it an appropriate tool for comparing textbooks in much the same way as that devised by Bergwall and Hemmi (2017) for analyzing proof-related opportunities in different textbooks. Their goal, in augmenting an earlier analytical framework developed for use with US high school textbooks (Thompson, Senk, & Johnson, 2012) was to create a tool appropriate for analysis any high school textbook. A Similar process was employed by Glasnovic Gracin (2018), who adapted the earlier work of Zhu and Fan (2006) to develop a five-dimensional framework for general mathematical task analysis.

## 4. Methods

In this paper we present analyses of the FoNS-related learning opportunities in three year-one mathematics textbooks currently being used in Sweden. One of these, Eldorado, can be described as a typical Swedish-authored textbook. The others, Favorit and Singma, are adaptations of Finnish and Singaporean textbook series respectively. The focus of the analyses was solely on tasks that expected action on the part of the student. All tasks in the textbooks and workbooks in each of the three textbook series were coded, each by at least

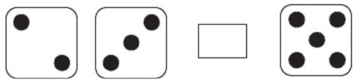

**Figure 1.** A task from a Swedish year one mathematics textbook.

two members of the project team, for the opportunities offered for the development of FoNS, and any disagreements resolved. For example, Figure 1 shows a Swedish textbook task. Here, students were invited to 'compare the number of dots' and then 'write either = or ≠' in the box. This particular task, which occurred before children had been introduced to addition, was expected to be completed by counting. Thus, it was coded for *systematic counting*. The expectation that students would address issues of equality or inequality led to the task also being coded for *quantity discrimination*. Moreover, because the task could be construed as encouraging students to subitise, it was coded for *awareness of the relationship between number and quantity*. In this manner, each textbook was construed as a sequence of tasks and each task coded for the presence (1) or absence (0) of each FoNS category. Thus, every task was coded as a series of 1s and 0s according to the presence or absence of the eight FoNS categories. Importantly, the process yielded data that facilitated a meaningful comparison of the three textbooks.

With respect to textbook analyses, many researchers have incorporated some form of frequency analysis (Borba & Selva, 2013), with others additionally comparing the proportions of the tasks under scrutiny (Ding & Li, 2010) and even presenting them graphically as proportions per semester (Ding, 2016). Other frequency analysis studies have gone a step further by employing statistical tests to determine the significance of any differences (Löwenhielm et al., 2017). Others, concerned that frequencies alone offer too limited a narrative, have examined the first position of particular tasks in the sequence of all tasks (Fujita, 2001) or the percentage of all tasks before such an occurrence (Alajmi, 2012), while others have presented each task of interest as a dot in the timeline of all tasks (Huntley & Terrell, 2014). Still others have supplemented frequencies with qualitative descriptions of tasks (Barcelos Amaral & Hollebrands, 2017). All these approaches allow for straightforward and meaningful comparisons and, while Huntley and Terrell's (2014) approach offers an indication of where tasks occur in the overall structure of a textbook, none show how children's learning is structured on a day-to-day, week-to-week or month-to-month basis. In this paper, alongside conventional frequency analyses and their associated statistical significance, we demonstrate how moving averages as a hitherto unconsidered approach to the analysis of textbooks offer important insights with respect to the temporal location and emphases of different forms of mathematical tasks.

The use of moving averages is a well-established procedure for showing visually how particular characteristics of a data-set evolve over time (Fan & Yao, 2003) and is frequently used to analyze trends in, for example, temperature over time. Our view, as we show below, is that textbooks' tasks form a temporal sequence of events and are, therefore, amenable to moving averages as a means of identifying their structural emphases at different points in time. Procedurally, this means that single data points are replaced by the arithmetical mean of several data points, drawn from before the point of interest, the point of interest itself and after. This process smooths out short-term fluctuations so that longer-term patterns become visible and the influence of outliers is eliminated. Mathematically, this means replacing a single data point $(t_k, y_k)$ with $(t_k, \hat{y}_k)$, where $\hat{y}_k$ is the arithmetic mean of its neighbouring data points $y_j$ as in equation 1. Importantly, if the time period selected for the moving average is too short, then its associated graph becomes noisy and trends may be lost. Similarly, if the time period is too long then important details may be lost (Wakaura & Ogata, 2007). Thus, the choice of time interval is key to the successful use of the approach.

$$\hat{y}_k = \sum_{j=k-n}^{k+n} \frac{1}{2n+1} y_j. \tag{1}$$

In this latter respect, the desired time interval determines the size of the divisor, $2n + 1$, of equation 1 For example, in the context of mathematics textbooks, the width, $2n + 1$, of this window could be determined by the number of tasks that an average student is expected to complete each day, week or month. In this way, the width of the window could be determined by dividing the total number of tasks in a book by the number of weeks in the school year. This would give an approximation to a single week's workload across the year. This has the consequence that whenever the moving average shows 'above zero' any student who has completed all the available tasks would have met that coded property during that week. In this sense, the higher the graph the more frequently the code is likely to have been experienced. In the analyses presented here, we have chosen to use a window related to the estimated workload of a week, as this reduces the 'noise' yielded by a daily time period and eliminates the impact of isolated occurrences.

## 5. Results

In the following we present the two analyses described above for each of the eight FoNS codes. With respect to frequencies, the tables below show that Favorit comprises more tasks than Eldorado, which comprises more than Singma. Indeed, the total number of tasks in Favorit is nearly two and a half times that of Singma, with Eldorado lying between the two. Moreover, the same tables show the results of chi square tests run to determine the extent to which variation in codes' distribution can be attributed to chance. In this respect, all tests reject the null hypothesis at unequivocal levels of probability, confirming that differences in the distribution of codes can be attributable to textbooks' writers' different emphases. Furthermore, the tables show both absolute and proportional values, as the former may reflect author perspectives on workload and the latter a sense of the 'value' placed on particular forms of mathematical activity within the overall scheme of the particular book. Finally, with respect to the second analyses, shown in the figures below, a solid line represents the moving averages for the codes in Favorit, a dashed line those in Eldorado and a dotted line those in Singma.

### *5.1. FoNS category 1: number recognition*

In absolute terms, the three books offered different perspectives on *number recognition*. The figures of Table 2 show that Favorit comprises twice as many such tasks as Eldorado and nearly six times as many as Singma. These figures are somewhat reflected in the proportions found in each of the three books, which, despite major variation, were the highest proportions of any FoNS category. In other words, despite extreme variation in absolute terms,

**Table 2.** Frequency and percentage distribution of FoNS 1 across the three textbooks.

| FoNS1 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 1718 | 921 | 2639 | 65 |
| Favorit | 3877 | 1258 | 5135 | 76 |
| Singma | 614 | 1079 | 1693 | 36 |
| Total | 6209 | 3258 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 869 | 2 | $< 0.0005$ | |

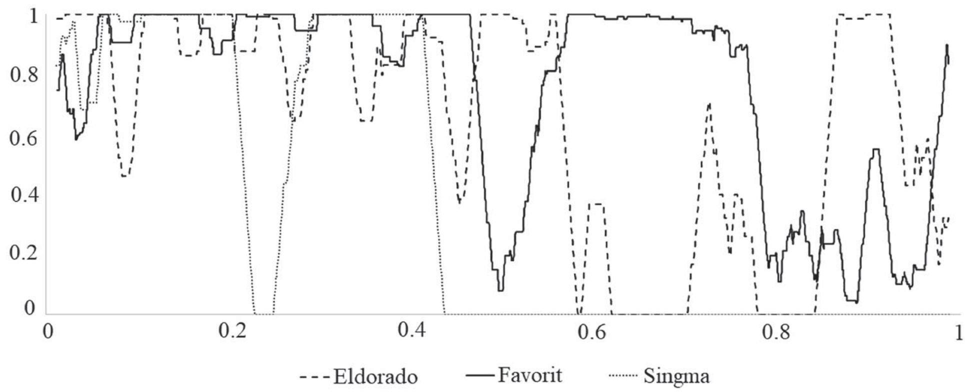


**Figure 2.** Moving average of FoNS 1-related tasks per month.

each of the three books yielded a higher proportion of tasks coded for *number recognition* than any other FoNS category, although it is also clear that the two Nordic books place a much greater emphasis on it than the Singaporean.

However, frequencies alone say little about the distribution of these opportunities across the school year. In this respect, the monthly moving average (Figure 2) shows clearly that *number recognition* opportunities occur in different ways within the textbooks. For example, the dotted line of Singma shows two major emphases in the first half of the school year before disappearing completely in the second half. By way of contrast there are similarities with respect to Eldorado and Favorit. Both begin the year with high levels of sustained activity before diverging for the second half. In this respect, the dashed line of Eldorado shows further repeated emphases throughout the second half, while the solid line of Favorit highlights an extended emphasis during the whole of the third quarter before dropping for the last quarter. Overall, not only does Favorit include many more opportunities for children to engage with *number recognition* that the other books, it also maintains high concentrations of such opportunities across the school year.

### 5.2. FoNS category 2: systematic counting

In absolute terms, the figures of Table 3 show that Favorit comprises approximately twice as many opportunities for children to *count systematically* than the other two books. However, despite having fewest such opportunities, Singma incorporates the highest proportion of tasks so coded, while Eldorado and Favorit have effectively identical and meaningful

**Table 3.** Frequency and percentage distribution of FoNS 2 across the three textbooks.

| FoNS2 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 225 | 2414 | 2639 | 9 |
| Favorit | 415 | 4720 | 5135 | 8 |
| Singma | 214 | 1479 | 1693 | 13 |
| Total | 854 | 8613 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 33 | 2 | $< 0.0005$ | |

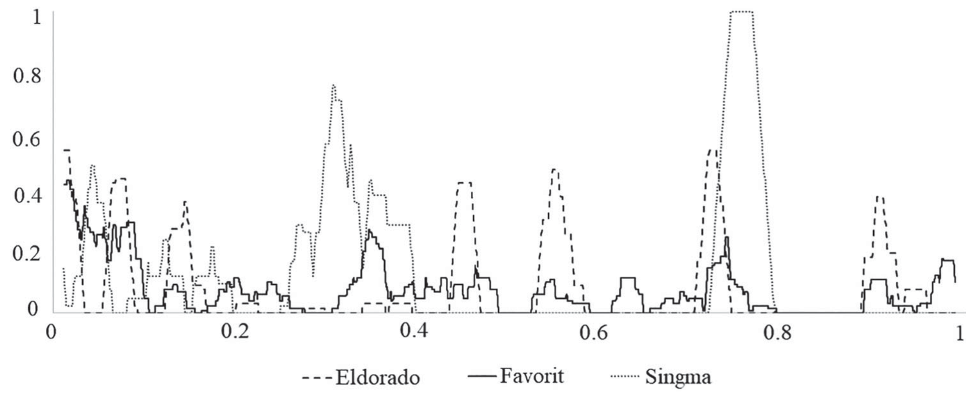


**Figure 3.** Moving average of FoNS 2-related tasks per month.

proportions. Moreover, as shown in Figure 3, Favorit offers a low-level continuous emphasis across the year, while Eldorado has several moderately high repetitions. Singma, after a low-key introduction to the year, returns to the topic twice with increasingly strong emphases. In sum, while both Eldorado and Favorit present frequent opportunities at what might be described as sustained levels, Singma's emphases become increasingly stronger over the year.

### 5.3. FoNS category 3: relationship between number and quantity

With respect to the *relationship between number and quantity*, the figures of Table 4 show that Singma offers fewer than a half the opportunities found in Eldorado and barely a quarter of those found in Favorit. However, proportionally, these differences are less apparent, being between 25% and 29% of all tasks. The graphs of Figure 4 offer alternative perspectives on the roles of such tasks in the didactical structures of the books. Firstly, all three books begin the year with several short opportunities of, effectively, diminishing emphasis. After the midpoint, however, differences emerge. On the one hand, Singma offers no further opportunities for students to work on the *relationship between number and quantity,* while, on the other hand, both Eldorado and Favorit continue to offer such tasks, although at different levels of intensity, Favorit being lower than Eldorado.

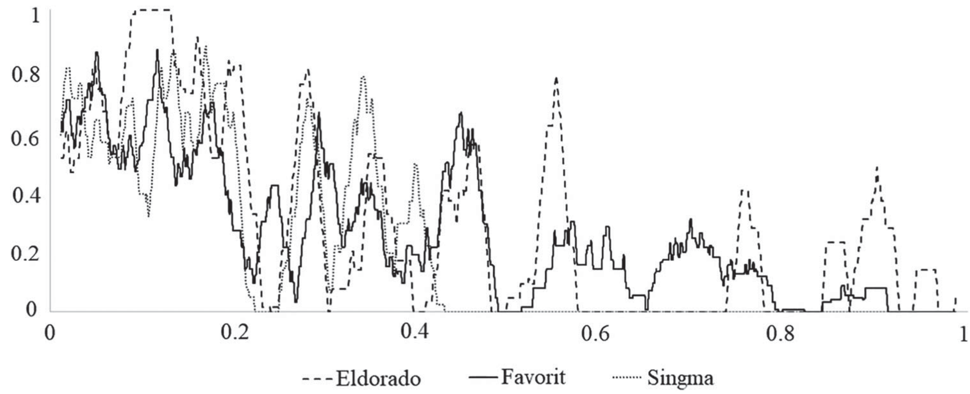


**Figure 4.** Moving average of FoNS 3-related tasks per month.

**Table 4.** Frequency and percentage distribution of FoNS 3 across the three textbooks.

| FoNS3 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 771 | 1868 | 2639 | 29 |
| Favorit | 1305 | 3830 | 5135 | 25 |
| Singma | 335 | 1358 | 1693 | 20 |
| Total | 2411 | 7056 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 48 | 2 | $< 0.0005$ | |

**Table 5.** Frequency and percentage distribution of FoNS 4 across the three textbooks.

| FoNS4 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 233 | 2406 | 2639 | 9 |
| Favorit | 291 | 4844 | 5135 | 6 |
| Singma | 110 | 1583 | 1693 | 6 |
| Total | 634 | 8833 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 28 | 2 | $< 0.0005$ | |

### 5.4. FoNS category 4: quantity discrimination

The figures of Table 5 show, across the three books, low but not unimportant emphases on *quantity discrimination*. In absolute terms, Favorit comprises more such tasks than either Eldorado or Singma, with two and a half as many as the latter. However, proportionally, Favorit and Singma are comparable at around six per cent, while Eldorado comprises half as much again. The graphs in Figure 5 show, on the one hand, Singma with two peaks in the first third of the year and nothing after. On the other hand, both Eldorado and Favorit offer several opportunities of relatively low emphasis throughout the year.

### 5.5. FoNS category 5: different representations of number

In absolute terms, Table 6 shows that Favorit offers more *different representations of number* opportunities than either Eldorado or Singma, four times as many in the latter. However,

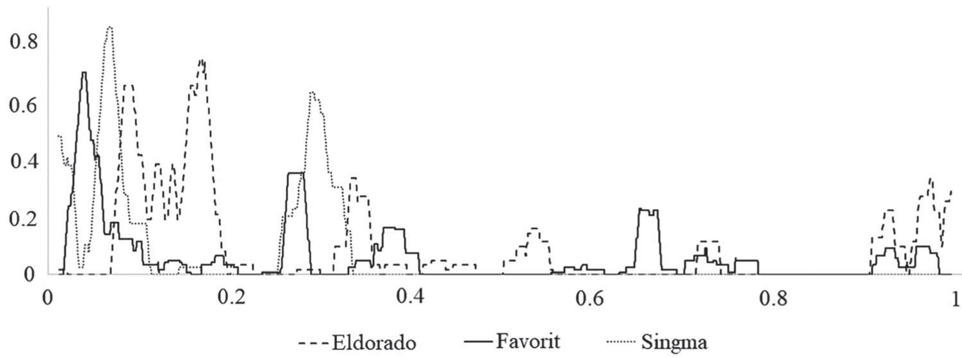


**Figure 5.** Moving average of FoNS 4-related tasks per month.

**Table 6.** Frequency and percentage distribution of FoNS 5 across the three textbooks.

| FoNS5 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 1035 | 1604 | 2639 | 39 |
| Favorit | 1402 | 3733 | 5135 | 27 |
| Singma | 346 | 1347 | 1693 | 20 |
| Total | 2783 | 6684 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 199 | 2 | $< 0.0005$ | |

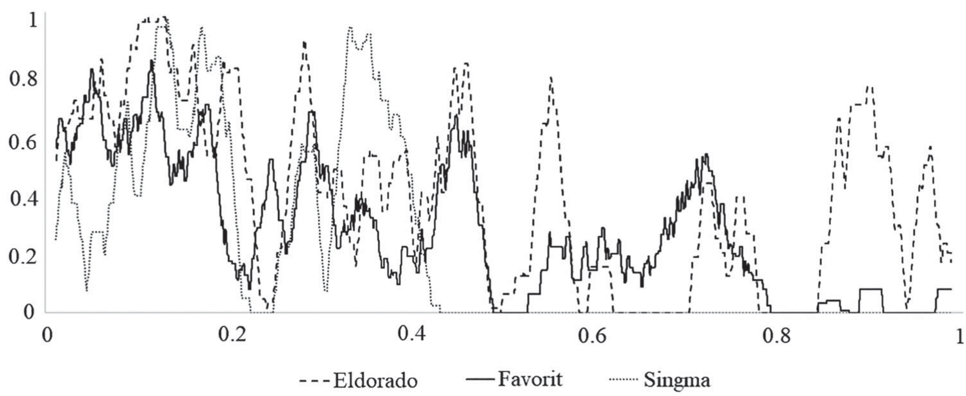


**Figure 6.** Moving average of FoNS 5-related tasks per month.

Eldorado has a much higher proportion of such tasks, two-fifths compared with Singma's one fifth, than the other two books. The graphs in Figure 6 yield additional interpretations. Firstly, Favorit offer a series of opportunities, slowly diminishing in intensity, across the first three-quarters of the year. Secondly, Eldorado offers several repetitions across the whole year, all of which have moderate to high intensity. Thirdly, Singma presents, effectively, two high intensity opportunities in the first half of the year and nothing in the second.

**Table 7.** Frequency and percentage distribution of FoNS 6 across the three textbooks.

| FoNS6 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 24 | 2615 | 2639 | 1 |
| Favorit | 0 | 5135 | 5135 | 0 |
| Singma | 0 | 1693 | 1693 | 0 |
| Total | 24 | 9443 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 62 | 2 | $< 0.0005$ | |

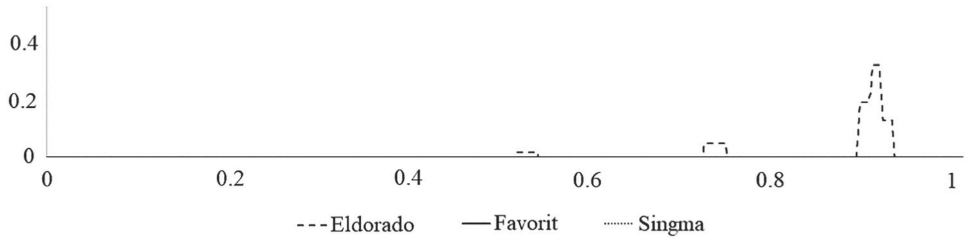


**Figure 7.** Moving average of FoNS 6-related tasks per month.

### 5.6. FoNS category 6: estimation

*Estimation*, as seen in Table 7, is absent from both Favorit and Singma, and only nominally present in Eldorado. Figure 7 shows that such tasks occur only in second half of the year at increasing but low levels of intensity. In short, *estimation* is effectively absent from all three books.

### 5.7. FoNS category 7: simple addition and subtraction

FoNS category 7, *simple addition and subtraction*, receives very different emphases in the three books. In absolute terms, the figures of Table 8 show that Favorit comprises more than seven times as many such opportunities as Singma and more than twice as many as Eldorado. Proportionally these differences are profound. In comparison with Singma, where a quarter of all tasks address simple arithmetic, nearly two-thirds of the tasks in Favorit can be so described, with those in Eldorado close behind. The moving averages shown in Figure 8 reflect these differences. The tasks found in Singma comprise two high intensity opportunities that are completed by the mid-point of the year. Eldorado has, effectively, a single extended major emphasis in the first half of the year, followed by a period of low emphasis before a final flourish in the last few weeks. Thirdly, Favorit, like Eldorado, begins the year with a single, high intensity and extended opportunity that lasts until the middle of the school year but continues, after a short dip, with a second, high intensity and extended opportunity during the third quarter. In sum, Favorit maintains an intense arithmetical emphasis throughout the school year, while Singma offers just two short and intense periods during the first half. Eldorado falls between the two, with an intense and continuous first half of the year and a less intense period of repeated opportunities in the second.

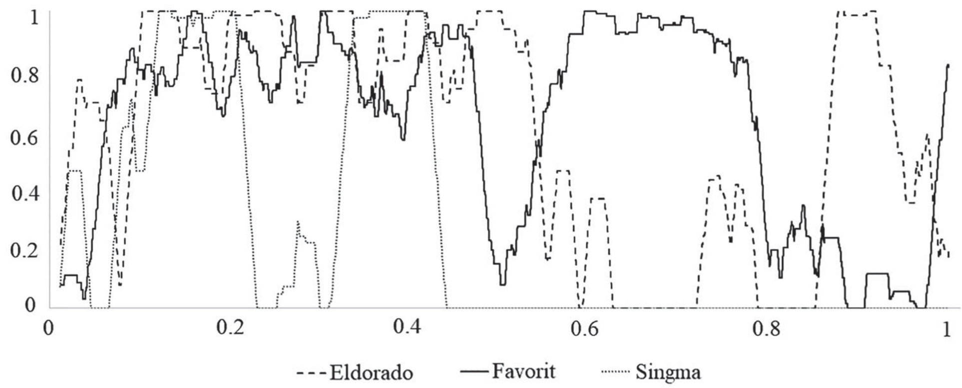


**Figure 8.** Moving average of FoNS 7-related tasks per month.

**Table 8.** Frequency and percentage distribution of FoNS 7 across the three textbooks.

| FoNS7 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 1536 | 1103 | 2639 | 58 |
| Favorit | 3217 | 1918 | 5135 | 63 |
| Singma | 415 | 1278 | 1693 | 25 |
| Total | 5168 | 4299 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 766 | 2 | $< 0.0005$ | |

**Table 9.** Frequency and percentage distribution of FoNS 8 across the three textbooks.

| FoNS8 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 170 | 2469 | 2639 | 6 |
| Favorit | 101 | 5034 | 5135 | 2 |
| Singma | 0 | 1693 | 1693 | 0 |
| Total | 271 | 9196 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 186 | 2 | $< 0.0005$ | |

### *5.8. FoNS category 8: number patterns*

Finally, the eighth FoNS category, *number patterns*, yields an interesting point of divergence. On the one hand, the figures of Table 9 show the category was absent from Singma, nominally present in Favorit and meaningfully present only in Eldorado. The graphs of Figure 9 show that the *number patterns*-related tasks found in Eldorado occur at regular intervals during the school year, with its peaks being at the end of each quarter. The few opportunities in Favorit could be found in the second half of the year on three low intensity occasions.

### *5.9. FoNS category 9: other number-related opportunities*

Finally, to better understand how opportunities for children to acquire FoNS-related competences are structured, it seems appropriate to consider what additional opportunities

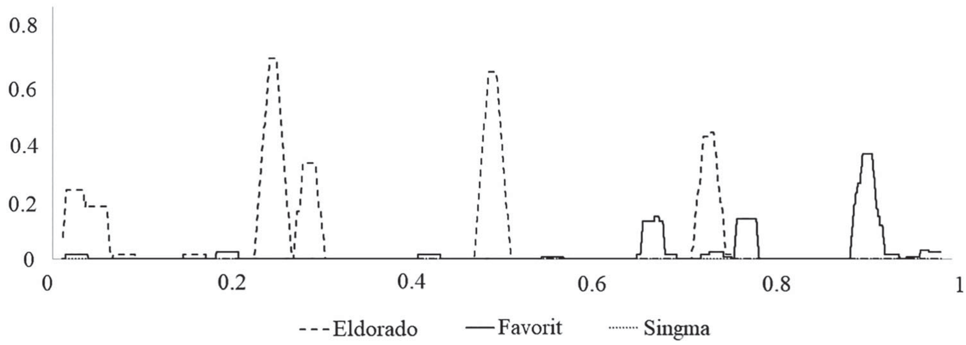


**Figure 9.** Moving average of FoNS 8-related tasks per month.

**Table 10.** Number and proportion of tasks coded for FoNS 9.

| FoNS9 | Present | Absent | Total | Present (%) |
|---|---|---|---|---|
| Eldorado | 804 | 1835 | 2639 | 30 |
| Favorit | 1214 | 3921 | 5135 | 24 |
| Singma | 968 | 725 | 1693 | 57 |
| Total | 2986 | 6481 | 9467 | |
| | $\chi^2$ | df | $p$ | |
| | 665 | 2 | $< 0.0005$ | |

exist in the three books. In this respect, the figures of Table 10 show the total number of tasks not coded for any FoNS category. These include, for example, number-related tasks outside the FoNS domains and other tasks relating to, say, measurement (time, weight, length and so on).

The figures of Table 10 show that all three books comprise many tasks outside the FoNS framework. Interestingly, they also show one of only two occasions, the other being *systematic counting*, when Singma comprises a higher proportion of such tasks than either Eldorado or Favorit. Indeed, it comprises almost twice the proportion of tasks as Eldorado and two and a half times the proportions of Favorit. The graphs of Figure 10 highlight other major structural differences. On the one hand, they show that while neither Eldorado nor Favorit comprises more than an occasional FoNS 9-related activity in the first half of the year, both have repeated high levels of activity in the second half. On the other hand, Singma, dotted line, not only has a peak roughly a quarter of the way through the school year but also, maintains an almost uninterrupted peak throughout the second half.

## 6. Discussion

Internationally, textbooks are not only the principal resource for lesson planning and student tasks but also the means by which the curriculum is realized (Stacey & Vincent, 2009; Stein & Kim, 2009; Tarr, Chávez, Reys, & Reys, 2006). Such a situation is common in Sweden (Ahl, 2016; Sidenvall et al., 2015), although the loose specification of the Swedish curriculum (Pansell, & Andrews, 2017) gives teachers freedom to choose how a textbook is used (Hemmi et al., 2019; Johansson, 2007). Thus, textbook analyses in such contexts are in

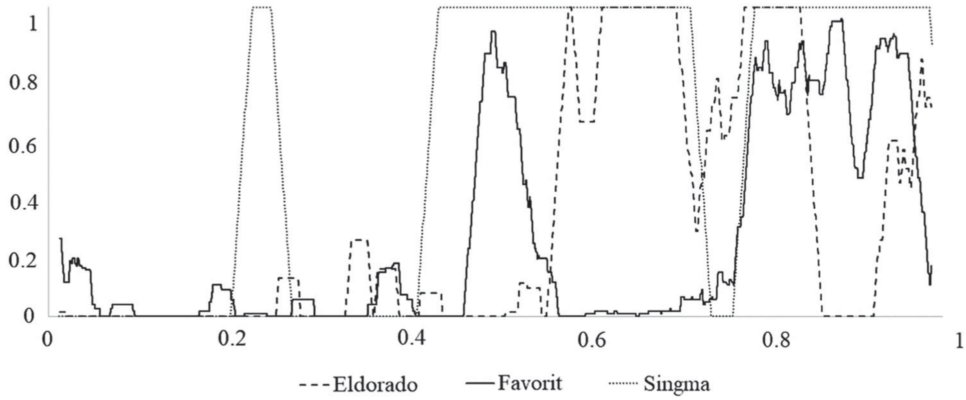


**Figure 10.** Moving average of FoNS 9-related tasks per month.

some sense probabilistic, as there is no certainty that any child will complete all a textbook's tasks (Mesa, 2004). That being said, understanding how textbooks structure children's opportunities to learn, particularly in light of textbook importation, seems important. In this paper, therefore, we have analyzed, in two different ways, the opportunities found in three mathematics textbooks for Swedish year one children to acquire foundational number sense (FoNS). Every one of the eight FoNS categories has been implicated in children's later learning of mathematics (Andrews & Sayers, 2015) and yet the three series analyzed here, one Swedish-authored, one Finnish-authored and one Singaporean-authored, offer very different opportunities.

The frequency analyses identified both similarities and differences in the three books. The most obvious difference, the total number of tasks, suggests that the textbooks' authors have differing views on workload. For example, the Finnish-authored Favorit comprises one and a half times as many tasks as the Swedish-authored Eldorado and two and a half times as many as the Singaporean-authored Singma. Thus, if s major role of a textbook is to provide opportunities for students to consolidate their learning, then Favorit seems to offer most, while Singma offers least. However, workload, as measured by the number of tasks, is not necessarily an indicator of the quality of a student's opportunity to learn. In the following, we focus principally on the proportions of tasks found in the books, as a measure of the emphases they receive.

The most obvious similarities concerned the low proportions of tasks addressing *estimation* and *number patterns*. The omission of estimation seems problematic, not least because it is a determinant of later arithmetical competence, particularly in respect of novel situations (Booth & Siegler, 2008; Gersten, Jordan, & Flojo, 2005; Holloway & Ansari, 2009; Libertus, Feigenson, & Halberda, 2013). Tasks concerning *number patterns*, which underpin later arithmetical competence (Van Luit & Schopman, 2000) and, importantly, facilitate the diagnosis of children at risk of later mathematical difficulties (Clarke & Shinn, 2004; Gersten et al., 2005; Lembke & Foegen, 2009), were meaningfully present only in the Swedish-authored Eldorado. A third category, *quantity discrimination*, was also relatively rare, particularly in Favorit and Singma. Both of these, in comparison with Swedish-authored Eldorado's nine per cent, or one task in eleven, offered only six per cent of such tasks. Acknowledging that quantity discrimination is a strong predictor of later mathematical achievement (Aunio & Niemivirta, 2010; De Smedt, Noël, Gilmore, & Ansari, 2013; Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Holloway & Ansari, 2009; Stock, Desoete, & Roeyers, 2010), the low proportions in both Favorit and Singma may be problematic. Finally, with respect to the more lowly occurring categories, Singma offered a higher proportion of tasks involving *systematic counting* than either Eldorado or Favorit. Despite this variation, however, the proportions in all three books indicate that *systematic counting*, which underpins arithmetical competence in general (Gersten et al., 2005; Passolunghi, Vercelloni, & Schadee, 2007; Stock et al., 2010) and mental arithmetical competence in particular (Lyons & Beilock, 2011), is given a significant emphasis within each book. Overall, with respect to the less visible categories, and with the exception of estimation, Eldorado seems the most consistent of the three books with respect to tasks focused on *number patterns*, *quantity discrimination* and *systematic counting*, with Singma seeming the least consistent.

The remaining four categories, *number recognition*, *the relationship between number and quantity*, *different representations of number* and *simple arithmetic*, discriminated between the three books. *Number recognition*, a necessary competence for multi-digit arithmetic (Desoete et al., 2012; Krajewski & Schneider, 2009) and predictor of later mathematical problems (Lembke & Foegen, 2009; Stock et al., 2010), was proportionally the most frequently addressed category in both Eldorado and Favorit. Indeed, in contrast with a much lower proportion of such tasks in Singma, a substantial majority of tasks in both books were construed as including such an emphasis. In similar vein, the proportional analyses showed, in contrast with the quarter of all tasks found in Singma, that around three fifths of all tasks in Eldorado and Favorit were focused on *simple arithmetical operations*. In other words, acknowledging that such competence is a stronger predictor of later mathematical success than measures of general intelligence (Geary, Bailey, & Hoard, 2009; Krajewski & Schneider, 2009), it seems that Singma offers a much-reduced opportunity in comparison with the Nordic-authored books.

Finally, in terms of the *relationship between number and quantity* and *different representations of number*, all three books offer proportions of tasks indicative of strong emphases. That being said, with respect to both codes, Singma offers the lowest proportion of such tasks and Eldorado the highest, with Favorit somewhere between the two. Thus, acknowledging that children who have difficulty relating number to quantity tend to experience later mathematical difficulties (Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009; Mazzocco, Feigenson, & Halberda, 2011) and that the better the connections between *different representations* the more likely a child is to become arithmetically competent (Mundy & Gilmore, 2009; Van Nes & Van Eerde, 2010), the evidence suggests that Eldorado may offer the most productive opportunities.

With respect to the second analyses, the moving averages proved significant in highlighting how the three textbooks structure the learning opportunities they offer. The most obvious difference was that FoNS-related tasks were typically located only in the first half of Singma, but distributed throughout the year, albeit in different ways, in both Favorit and Eldorado. The single exception concerned tasks relating to *systematic counting*, which occurred throughout the year in all three books. The structural role of the early-in-the-year emphases in Singma were further emphasized by the number and proportions of tasks falling outside the FoNS categorization. Admittedly, the absolute number of such tasks found in Singma did not exceed that of Favorit, but the proportion was at least double that found in either of the other two books. Moreover, the structural importance of FoNS9 in Singma was clearly shown in the moving average graphs, which showed a unique emphasis in the first third of the year and a barely interrupted second half. There were also important distributive differences between Eldorado and Favorit. The key distinction being that while Favorit emphasized *number recognition*, *different representations of number* and *simple addition and subtraction* constantly throughout the year, Eldorado addressed them in periodic bursts. Indeed, with the exception of *different representations of number*, which occurred similarly early in both books, Eldorado offered periodic emphases on all FoNS categories, including *estimation* and *number patterns*, throughout the year.

Overall, if Swedish-authored Eldorado is construed as an appropriate manifestation of the loosely specified Swedish national curriculum, and its popularity would suggest it is, then both Favorit and Singma are likely to challenge, albeit in very different ways, teachers' conceptions of not only what should be taught to year one children but also how it should be taught. For example, comparing Eldorado with Favorit, despite similar proportions of tasks focused on *systematic counting* and *understanding the relation between number and quantity*, one can discern important differences in how their authors conceptualize number. On the one hand, the significantly higher proportions of Favorit's tasks focused on *number recognition* and *simple arithmetic* confirm emphases on procedural competence, emphases that compare favourably with earlier studies of, for example, English textbooks (Newton & Newton, 2007; Park & Leung, 2006). On the other hand, Favorit comprises significantly lower proportions of tasks focused on *quantity discrimination*, *different representations of number* and *number patterns*, indicating a lower emphasis than Eldorado on those FoNS categories likely to deepen children's conceptual understanding of number. Moreover, the structure of Eldorado, with its regular revisitation of most FoNS-related forms of task offers a very different didactical approach from Favorit, with its continuous exposure to several FoNS categories.

In similar vein, Singma comprised lower proportions of tasks than Eldorado on every FoNS category except *systematic counting*, confirming the procedural emphases of earlier studies of Singaporean textbooks (Yang et al., 2010). However, the key difference is that while all FoNS categories could be found repeatedly throughout Eldorado's school year, they were all completed by the year's midpoint in Singma. This was emphasized not only by the proportion of Singma's tasks falling outside the FoNS framework but also the dominance of such tasks in the second half of the school year. That is, Singma appears to have a steeper gradient of difficulty than Eldorado (and Favorit), a hypothesis supported by the moving averages. Such distinctions, when set against the concerns raised earlier about the PISA-driven relevance of textbook imports, seem to confirm that the uncritical importation of curriculum materials from systems that international tests allege are successful is unwise. Indeed, acknowledging that the FoNS framework was developed independently of any culturally normative expectations of number learning, having been driven by a systematic review of the literature from various fields (Author 4 & Author 1, 2015), it is important to acknowledge that of the three books analyzed above, Swedish-authored Eldorado offers the most consistent and extensive FoNS-related learning opportunities. In other words, the importation of both Favorit and Singma may be attempts to fix something that is not broken.

In sum, assuming that Swedish teachers work within 'culturally determined patterns of belief and behaviour, frequently beneath articulation, that distinguish one set of teachers from their culturally different colleagues' (Andrews & Sayers, 2013, p. 133), and evidence suggests they do (Andrews & Larson, 2017a, 2017b), then the deployment of either Favorit and Singma is likely to prove problematic. Both books present different emphases on the core FoNS competences found in Eldorado, particularly with respect to the breadth of opportunities found uniquely in within it, and, importantly, culturally situation didactical perspectives that differ majorly from the Swedish, whether its shallow gradient of difficulty or expectation of regular exposure to core ideas throughout the year.

## Note

1. Szabo is responsible for the development and implementation of a diagnostic test for the several thousand Stockholm students who transfer to upper secondary school annually. His data, over a five-year period, show a consistent attainment at around 85% on linear equations with the unknown on one side of the equation.

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