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RESEARCH ARTICLE

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State-limiting PID controller for a class of nonlinear systems with constant uncertainties

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Summary

Proportional Integral Derivative (PID) controllers still represent the core control method for achieving output regulation of either linear or nonlinear systems in the majority of industrial applications. However, conventional PID control cannot guarantee specific state constraint requirements for the plant, when the system introduces uncertainties. In this paper, a novel nonlinear PID control that achieves output regulation and guarantees a desired state limitation below a given value for a wide class of nonlinear systems with constant uncertainties is proposed. Using nonlinear ultimate boundedness theory, it is shown that the proposed state-limiting PID (sl-PID) control maintains a given bound for the desired system states at all times, ie, even during transients, whereas an analytic method for selecting the controller gains is also presented to ensure closed-loop system stability and convergence at the desired equilibrium. Two nonlinear engineering examples that include an electric motor and a dc/dc converter are investigated using the conventional PID and the proposed sl-PID to validate the superiority of the proposed controller in achieving the desired output regulation with a given bounded state requirement.

KEYWORDS

nonlinear PID, stability, state constraints

1 | INTRODUCTION

Since its first design in the early 1900s, Proportional Integral Derivative (PID) control has been dominating the engineering industry when it is desired to achieve both asymptotic regulation and disturbance rejection. The control problem of linear plants using a PID controller is now well understood; however, its application to nonlinear systems to guarantee the desired regulation and closed-loop system stability still remains a challenge, particularly because of the integral dynamics.[1-4] The existing methods leverage on the properties of the system, ie, minimum phase, to guarantee either local or global stability; see the works of Mahmoud and Khalil,[5] Huang and Khalil,[6] and Khalil.[7] Nazrulla and Khalil[8] have proposed a robust nonlinear integral controller, based on high-gain observers, capable of stabilizing nonminimum phase dynamics with a desired output regulation, whereas in the work of Ma and Khalil,[9] the output feedback controller tracks references generated from an external source without using an internal model. The authors bring attention to the closed-loop performance deterioration when including an internal model in the controller structure. However, this problem can be circumvented by using a high-gain feedback controller and observer.[10,11]

While the aforementioned approaches offer global or semiglobal stability guarantees, they do not tackle the problem of constraint satisfaction (input or state constraints), which arises from safety requirements and actuator limitations in a real engineering system. Although the output regulation problem with input constraint satisfaction has been well studied and addressed via antiwindup methods[12-16] or bounded integral control,[17,18] a state constraint requirement often creates the need for more advanced control methods that change the traditional PID control architecture. Safety of operation and stability guarantees are essential in modern processes such as power networks, motor control applications, and chemical processes. In the former, Tilli and Conficoni[19] and Conficoni[20] list some of the potential pitfalls of not considering the power converter limits into the control strategy, ie, loss of stability, performance degradation, and operation outside the desired ranges because of system malfunctions. The challenge of limiting a system state increases when the plant introduces uncertainties in the model. For example, consider the one-dimensional system $\dot{x} = -wx^3 + u$, where w is a constant unknown uncertain parameter defined in the range $w \in [w_{\min}, w_{\max}]$. The objective is to regulate state x toward a desired constant nonzero reference r while satisfying state constraint $|x| \leq x^{\max}$, $\forall t \geq 0$. In general, the conventional PI controller of the form $u = k_P(r - x) + \sigma$, $\dot{\sigma} = k_I(r - x)$ can achieve the desired regulation but does not offer any guarantees when it comes to the desired state limitation. It can easily be shown that the solution may exhibit overshoots violating constraints. Therefore, an effective management of constraints may increase the operation ranges in both the transient and the steady-state regimes.

The problem of maintaining a given bound for the system states when the system is subject to disturbances or uncertainties, or when the state vector is (partially) unknown has inspired researchers since the late 1960s. In particular, when uncertain parameters or disturbances belong to some known compact set, several state-bounding methods have been developed to design simple time-varying sets (eg, ellipsoids and n-orthotopes) to guarantee that the state vector is constrained within these sets.[21-23] Most of these methods are designed for linear systems, whereas their extension to nonlinear systems is not trivial and still represents an active problem.[24,25] In the majority of the cases, the methodologies proposed rely on linearization around the state trajectory, which fails to produce reliable estimates when uncertainties are explicitly considered in the model.[26] For nonlinear systems with discrete dynamics, a design methodology for determining the controller parameters is presented in the work of Andonov et al,[25] where a desired system performance, including state limitation, is guaranteed. In a similar framework, techniques such as model predictive control (MPC),[27] for an excellent survey on its different properties, include the constraints into their formulation and aim to exploit the behavior of the system around the constraints. These methods, however, present limitations in their implementation because they require the solution of an optimization problem online. Jerez et al[28] propose an MPC controller for linear systems that operates at megahertz; however, this approach is aimed at linear system and quadratic performance objectives, which is at odds with nonlinear formulations of the problem. Overall, for nonlinear systems with continuous-time dynamics, the output regulation problem with guaranteed state limitation using the well-understood and widely applied PID control and without modifying the control concept is still an open problem.

To address this issue, in this paper, a novel nonlinear state-limiting PID (sl-PID) controller is proposed for a wide class of nonlinear systems with constant uncertainties to achieve output regulation with a desired state constraint satisfaction. The proposed approach can be applied to multi-input multi-output systems, and using nonlinear ultimate boundedness theory, it is analytically proven that a limitation on the desired system states can be guaranteed at all times, even during transients. Furthermore, a detailed methodology for selecting the proportional, integral, and derivative gains is provided to guarantee closed-loop system stability and convergence to the desired set point. Hence, the proposed sl-PID can replace the conventional PID control in applications where a state limitation is required, whereas the framework for selecting the controller gains presented in this paper can lead to a simple and effective design and implementation.

Overall, the novelties and contributions of this paper are summarized as follows:

1. The design of a novel nonlinear sl-PID controller for nonlinear systems with constant uncertainties capable of guaranteeing a desired state limitation on a number of the plant states.
2. Detailed stability analysis and design framework for the sl-PID controller states, and comparison with the conventional PID control scheme.

To better explain and validate the sl-PID control design procedure, we apply the proposed approach to two engineering examples: a motor control regulation problem and a dc/dc power converter. We compare the performance of the proposed algorithm to that of a conventional PID controller.

This paper is structured as follows. In Section 2, some key theoretic concepts for nonlinear system analysis are presented. In Section 3, the problem under investigation is stated and an analysis of the conventional PID controller is provided. The main result of this paper is presented in Section 4, where the sl-PID controller is designed and analyzed. A detailed stability analysis and the design framework for selecting the controller gains are provided as well. The proposed controller is applied to two different engineering applications in Section 5 and compared to the conventional approach to validate its effectiveness in a practical implementation. Finally, concluding remarks are provided in Section 6.

Notation. A finite set of natural numbers is denoted by $\mathbb{I}_n = \{1, \dots, n\}$; the set of integers is denoted by $\mathbb{Z}$.

2 | PRELIMINARIES

Consider the nonlinear system

$$\dot{x} = f(t, x, u), \tag{1}$$

where $f : [0, \infty) \times \mathcal{D}_x \times \mathcal{D}_u \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u, with $\mathcal{D}_x \subset \mathbb{R}^n$ and $\mathcal{D}_u \subset \mathbb{R}^m$ being open domains containing their respective origin, and $u(t)$ is a piecewise continuous bounded function.

Definition 1 (See the work of Khalil[1]). System (1) is said to be locally input-to-state stable if there exist $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}$, and positive constants $k_1, k_2 > 0$ such that for any initial state $x(t_0)$ with $\|x(t_0)\| < k_1$ and any input $u(t)$ with $\sup_{t \geq t_0} \|u(t)\| < k_2$, the solution $x(t)$ exists and satisfies

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma\left(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\|\right), \tag{2}$$

for all $t \geq t_0 \geq 0$. It is said to be input-to-state stable if $\mathcal{D}_x = \mathbb{R}^n$, $\mathcal{D}_u = \mathbb{R}^m$, and inequality (2) is satisfied for any initial $x(t_0)$ and for any bounded input $u(t)$.

Lemma 1 (See the work of Khalil[1]). *Suppose that, in some neighborhood of $(x, u) = (0, 0)$, function $f(t, x, u)$ is continuously differentiable and the Jacobian matrices $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial u}$ are bounded uniformly in t. If the unforced system*

$$\dot{x} = f(t, x, 0) \tag{3}$$

has a uniformly asymptotically stable equilibrium point at the origin $x = 0$, then system (1) is locally input-to-state stable.

Consider the interconnected system

$$\dot{x}_1 = f_1(t, x_1, x_2) \tag{4a}$$

$$\dot{x}_2 = f_2(t, x_2), \tag{4b}$$

where f_1 and f_2 satisfy similar conditions to f in (1).

Lemma 2 (See the work of Khalil[1]). *If $\dot{x}_1 = f_1(t, x_1, x_2)$, with x_2 as input, is locally input-to-state stable and the origin of $\dot{x}_2 = f_2(t, x_2)$ is uniformly asymptotically stable, the origin of the interconnected system (4) is uniformly asymptotically stable.*

Definition 2 (ω-limit set[29]). The ω-limit set of a subset $\mathcal{B} \subset \mathbb{R}^n$, denoted by $\omega(\mathcal{B})$, for an autonomous system $\dot{x} = f(x)$ contains all points $x \in \mathbb{R}^n$ for which there exists a sequence of pairs $\{(x_k, t_k)\}_{k \in \mathbb{N}}$ with $x_k \in \mathcal{B}$ and $t_k \to \infty$ as $k \to \infty$ such that

$$\lim_{k \to \infty} \phi(t_k, x_k) = x,$$

where $\phi : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is the system flow.

Lemma 3. *If a solution $x(t) = \phi(t, x_0)$ of the autonomous system $\dot{x} = f(x)$ is bounded and belongs to $\mathcal{D}_x$ for $t \geq 0$, then for any compact set $\mathcal{B} \subset \mathcal{D}_x$, its positive ω-limit set $\omega(\mathcal{B})$ is a nonempty, compact, connected, invariant set. Moreover, $x(t) \to \omega(\mathcal{B})$ as $t \to \infty$.*

3 | PROBLEM DEFINITION

Consider the nonlinear plant of the form

$$\dot{z} = q(x, z, u, w), \tag{5a}$$

$$\dot{x}_i = f_i(x_i, w) + g_i(w)u_i, \tag{5b}$$

$$y_i = h_i(x_i), \tag{5c}$$

such that $i \in \mathbb{I}_n$ with $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, $z = (z_1, \dots, z_n) \in \mathbb{R}^m$, where $(x, z) \in \mathbb{R}^{n+m}$, $u = (u_1, \dots, u_n) \in \mathbb{R}^n$ and $y = (y_1, \dots, y_n) \in \mathbb{R}^n$ are the state, input, and output vectors, respectively; similarly, $w \in \mathbb{R}^l$ is a vector containing unknown constant parameters. The functions $f_i(\cdot,\cdot)$, $g_i(\cdot)$, $q(\cdot,\cdot,\cdot,\cdot)$, and $h_i(\cdot)$ are C^1 in (x, z, u) and continuous in w for $(x, z) \in \mathcal{D} \subset \mathbb{R}^{n+m}$, $u \in \mathbb{R}^n$, and $w \in \mathcal{D}_w \subset \mathbb{R}^l$, where $\mathcal{D}$ and $\mathcal{D}_w$ are open connected sets. Note that although the dynamics for the states x_i are given in the control-affine form (5b), the dynamics for the state vector z can have the generic nonlinear form (5a).

The main goal is to design a controller that regulates the output $y \in \mathbb{R}^n$ to a constant reference $r \in \mathcal{D}_r \subset \mathbb{R}^n$, where $r = (r_1, \dots, r_n)$ and $\mathcal{D}_r$ is an open connected set, while guaranteeing constraint satisfaction on the states x_i, $\forall i = 1, 2, \dots, n$, such that for some $x_i^{\max} > 0$, the flow satisfies

$$\left| x_i(t, x_{0,i}) \right| \le x_i^{\max}, \quad \forall t \ge 0, \tag{6}$$

when $|x_{0,i}| \le x_i^{\max}$. We invoke a set of assumption to make our formulation consistent. The first of these is related to the output behavior of the system.

Assumption 1 (State measurement)**.** The output function $h_i : \mathbb{R}^n \to \mathbb{R}^n$ for each $i \in \mathbb{I}_n$ is a local diffeomorphism satisfying $\frac{\partial h_i}{\partial x_i}(x_i) > 0$ for any neighborhood $\mathcal{N}_i(x_i) \subset \mathcal{D}_{x_i}$ of x_i, where $\mathcal{D}_{x_i} = \mathrm{Proj}_i \mathcal{D}_x$. In addition, state x_i is available for control design, ie, $h^{-1}(\cdot)$ can be found in explicit form in $\mathcal{D}_{x_i}$.

The next assumptions are concerned with the nature of $w \in \mathbb{R}^l$, which represent model uncertainties.

Assumption 2. The function $g : \mathbb{R}^l \to \mathbb{R}^n$ is positive, ie, $g_i(w) > 0$ for $i \in \mathbb{I}_n$.

Remark 1. Assumption 1 states that state x_i can be either measured or analytically calculated from the output y_i to be used in the control design. Assumption 2 is concerned with the controllability of system (5b) and states that the sign of $g_i(w)$ does not change independently of the values of uncertain parameter w.

Assumption 3 (Bounded parameter uncertainty)**.** Vector $w \in \mathbb{R}^l$ lies in a known compact and convex set, ie, $w \in \mathbb{W}$.

The system dynamics (5b)-(5c) satisfy the following.

Assumption 4. For each $i = 1, \dots, n$, there exists a continuously differentiable function $V_i(x_i, w) : \mathcal{D}_i \times \mathcal{D}_w \to R$, a constant c_i, two positive constants b_{1i}, b_{2i}, and two class $\mathcal{K}$ functions α_{1i} and α_{2i} such that

$$\alpha_{1i}(|x_i|) \le V_i(x_i, w) \le \alpha_{2i}(|x_i|) \tag{7a}$$

$$\frac{\partial V}{\partial x_i} f_i(x_i, w) \le c_i x_i^2 \tag{7b}$$

$$b_{1i} x_i \le \frac{\partial V}{\partial x_i} g_i(w) \le b_{2i} x_i. \tag{7c}$$

Although the plant dynamics and above assumptions can seem restrictive, it should be mentioned that *(i)* several engineering applications are described by (5b)-(5c), eg, power electronic converters[12] and *(ii)* a wide class of nonlinear systems can be brought in the form of (5b)-(5c) using partial feedback linearization.

Remark 2. Considering linear dynamics $f_i(x_i, w) = A_i(w)x_i$, then function $V_i = \frac{1}{2} p_i x_i^2$, with $p_i > 0$, guarantees all conditions (7a)-(7c) because according to Assumptions 2 and 3, $g_i(w)$ is positive with a given upper and lower bound. The unforced system ($u_i = 0$) can be unstable because c_i can take positive values, eg, $A_i(w) > 0$.

We are interested in the effect that exogenous inputs and set points have on the equilibria of the open-loop dynamics. We exploit the structure of these dynamics, mainly the fact that the system can be viewed as an interconnection between $\dot{x} = f(x, w) + g(w)u$ and $\dot{z} = q(z, x, w, u)$. We first characterize the steady state behavior of the upper system.

Proposition 1. *Suppose Assumptions 1 to 3 hold. There exists a continuous function* $\xi_i : \mathbb{R} \times \mathbb{R}^l \to \mathbb{R} \times \mathbb{R}$ *for each* $i \in \mathbb{I}_n$ *such that* $(x_i^{ss}, u_i^{ss}) = \xi_i(r_i, w)$ *defines the steady state for* $\dot{x}_i = f_i(x_i, w) + g_i(w)u_i$ *and* $y_i = h_i(x_i)$.

Proof. The proof is obtained by construction. Following Assumption 1, the output functions $h_i(\cdot)$ admit a differentiable inverse $h_i^{-1}(\cdot)$ such that at steady state $x_i = h_i^{-1}(r_i)$. On the other hand, Assumption 2 guarantees the existence of $g_i(w)^{-1} \leq \infty$, which is used to define the mapping $u_i = -g_i(w)^{-1} f_i(h_i^{-1}(r_i), w)$. The resulting steady state map $(x_i^{ss}, u_i^{ss}) = \xi_i(r_i, w) = (h_i^{-1}(r_i), -g_i(w)^{-1} f_i(h_i^{-1}(r_i), w))$ is continuous by construction. □

When the value $w \in \mathbb{W}$ is fixed, then $\xi_i(\cdot, \cdot)$ assigns to each set point a steady-state pair in an injective way; as a direct consequence of this, it is possible to find an interval such that $r_i \in [r_i^{min}, r_i^{max}]$ for which $|x_i^{ss}| \leq x_i^{max}$. For the proposed setting, both (x, u) can be considered as inputs to system (5a). A direct consequence of Proposition 1 is that for any compact set $\mathcal{R} \times \mathbb{W}$, the steady-state pairs also lie in a compact set; from Assumption 3, the uncertain parameters lie in a compact set $\mathbb{W}$, and $\mathcal{R}$ can be chosen such that $|x_i| \leq x_i^{max}$ for all $i \in \mathbb{I}_n$. Both the uncertain parameters and the set points can be realized via signal generators,[30] which allow us to characterize the steady-state locus, from constant set points and parameters to periodic ones, in terms of ω-limit sets.[29] Setting a compact set $\mathcal{Z} \subset \mathbb{R}^m$ containing the trajectories of (5a), Lemma 3 defines the steady-state behavior of the open-loop dynamics as $\omega(\mathcal{Z} \times \mathcal{R} \times \mathbb{W})$.

3.1 | Conventional PID control

Based on the previous conditions, the desired regulation scenario can be achieved using a conventional PID controller of the form

$$u_i = k_{Pi}\left(r_i - h_i(x_i)\right) + \sigma_i - k_{Di}\frac{\partial h_i}{\partial x_i}\dot{x}_i \tag{8a}$$

$$\dot{\sigma}_i = k_{Ii}\left(r_i - h_i(x_i)\right), \tag{8b}$$

where $k_{Pi}, k_{Ii} > 0$, $k_{Di} \geq 0$, and at the desired equilibrium point, there is $\sigma_i^e = u_{ie}$. The closed-loop system takes the form

$$\dot{z} = q(x, z, u(x), w). \tag{9a}$$

$$\left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right)\dot{x}_i = f_i(x_i, w) + g_i(w)k_{Pi}\left(r_i - h_i(x_i)\right) + g_i(w)\sigma_i, \tag{9b}$$

$$\dot{\sigma}_i = k_{Ii}\left(r_i - h_i(x_i)\right), \tag{9c}$$

where $i \in \mathbb{I}_n$. For system (9b)-(9c), stability of the equilibrium point (x_e, σ_e), where $\sigma = (\sigma_1 \ldots, \sigma_n)$, can be analyzed by investigating the Jacobian matrices around the equilibrium point

$$A_i = \begin{bmatrix} \dfrac{\left.\frac{\partial f_i}{\partial x_i}\right|_{x_i^e} - k_{Pi}g_i(w)\left.\frac{\partial h_i}{\partial x_i}\right|_{x_i^e}}{1 + g_i(w)k_{Di}\left.\frac{\partial h_i}{\partial x_i}\right|_{x_i^e}} & \dfrac{g_i(w)}{1 + g_i(w)k_{Di}\left.\frac{\partial h_i}{\partial x_i}\right|_{x_i^e}} \\ -k_{Ii}\left.\dfrac{\partial h_i}{\partial x_i}\right|_{x_i^e} & 0 \end{bmatrix},$$

which can be proven to be Hurwitz for a suitable selection of the proportional gain k_{Pi}, ie,

$$k_{Pi} > \max\left\{0, \frac{1}{\min\limits_{w \in \mathbb{W}}\{g_i(w)\}\left.\frac{\partial h_i}{\partial x_i}\right|_{x_i^e}}\left.\frac{\partial f_i}{\partial x_i}\right|_{x_i^e}\right\}, \tag{10}$$

and for any $k_{Ii} > 0$ and $k_{Di} \geq 0$. Now, by setting $v = v_e + \tilde{v}$, where $v = (x, \sigma)$, $v_e = (x_e, \sigma_e)$, $\tilde{v} = (\tilde{x}, \tilde{\sigma})$, and $z = z_e + \tilde{z}$, then (9b)-(9a) can be rewritten in the generic form

$$\dot{\tilde{z}} = \tilde{q}(\tilde{v}, \tilde{z}, w), \tag{11}$$

where the desired equilibrium has been shifted to the origin.

Assumption 5. The dynamics (11) are locally input-to-state stable when $\tilde{v}$ is considered as the input.

Based on the above assumption, when asymptotic stability at the equilibrium point v_e of (9b)-(9c) is guaranteed by the controller gains k_{pi}, then according to Lemma 2, the equilibrium point (x_e, σ_e, z_e) of the closed-loop system (9b)-(9a) is asymptotically stable.

Although the desired equilibrium point can be proven to be asymptotically stable using a conventional PID controller, it is not guaranteed that $|x_i| \leq x_i^{\max}$, $\forall t \geq 0$. This state constraint is crucial in several practical examples, such as power electronic converters and electromechanical systems,[12,19] where the current, voltage, or speed is required to remain bounded below a given value at all times to avoid damaging devices. To overcome this problem, a nonlinear sl-PID controller is proposed in the sequel.

4 | MAIN RESULT

In this section, we state the main contribution of this paper: the sl-PID. The motivation behind this approach is to guarantee constraint satisfaction for a PID scheme without the need of saturation units or antiwindup schemes. The formulation follows a similar pattern to that of the classic PID controller in terms of design and analysis. The resulting nonlinear controller attains both desired properties of constraint satisfaction and closed-loop stability.

4.1 | Proposed sl-PID control design

Consider the open-loop dynamics (5) subject to Assumptions 1 to 4; the control objective, as in the case of conventional PID, is to steer a number of states, $x \in \mathbb{R}^n$, to predefined constant set points $r \in \mathbb{R}^n$, which correspond to equilibrium triplets (x^e, z^e, u^e) such that $|x_i^e| \leq x_i^{\max}$, and $z^e \in \omega(\mathcal{Z} \times \mathcal{R} \times \mathbb{W})$. Assumption 1 guarantees the availability of the state for feedback purposes; hence, the proposed novel nonlinear sl-PID controller is

$$u_i(x_i) = -k_{Pi}x_i + M_i \sin\sigma_i - k_{Di}\frac{\partial h_i}{\partial x_i}\dot{x}_i \tag{12a}$$

$$M_i\dot{\sigma}_i = k_{Ii}\left(r_i - h_i(x_i)\right)\cos\sigma_i, \tag{12b}$$

where $M_i, k_{Pi}, k_{Ii} > 0$, $k_{Di} \geq 0$. By substituting the proposed controller (12a)-(12b) into the plant dynamics (5b)-(5c), the closed-loop system becomes

$$\dot{z} = \tilde{q}(x, \sigma, z, w), \tag{13a}$$

$$\left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right)\dot{x}_i = f_i(x_i, w) - g_i(w)k_{Pi}x_i + g_i(w)M_i\sin\sigma_i, \tag{13b}$$

$$M_i\dot{\sigma}_i = k_{Ii}\left(r_i - h_i(x_i)\right)\cos\sigma_i, \tag{13c}$$

where $i \in \mathbb{I}_n$. This resulting closed-loop dynamics, indeed, satisfies the desired state-limiting property under a suitable choice of controller gains $k_{Pi} > 0$, $k_{Ii} > 0$, and $k_{Di} > 0$. The following proposition gives guidelines for choosing K_{Pi} and M_i to achieve the desired objective.

Proposition 2. *Suppose Assumptions 1 to 4 hold. The trajectories of the closed-loop system (13) satisfy $|x_i(t)| \leq x_i^{\max}$ for all $t \geq 0$, if initially $|x_i(0)| \leq x_i^{\max}$ and the sl-PID control parameters M_i and k_{Pi} satisfy the inequality*

$$M_i \leq \alpha_{2i}^{-1}\left(\alpha_{1i}\left(x_i^{\max}\right)\right)\frac{\bar{k}_{Pi}b_{1i} - c_i}{b_{2i}}, \tag{14}$$

where $\frac{c_i}{b_{1i}} < \bar{k}_{Pi} \leq k_{Pi} - \epsilon_i$ for any arbitrarily small $\epsilon_i > 0$.

Proof. Consider a continuously differentiable function $V_i(x_i, w)$ for system (13b) satisfying conditions (7a)-(7c) of Assumption 4. Then,

$$\begin{aligned}\dot{V}_i &= \left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right)^{-1}\left[\frac{\partial V}{\partial x_i} f_i(x_i, w) - \frac{\partial V}{\partial x_i} g(w)k_{Pi}x_i + \frac{\partial V}{\partial x_i} g(w)M_i \sin\sigma_i\right] \\ &\leq \left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right)^{-1}\left[-(k_{Pi}b_{1i} - c_i)x_i^2 + b_{2i}x_iM_i \sin\sigma_i\right] \\ &\leq \left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right)^{-1}\left[-(k_{Pi}b_{1i} - c_i)|x_i|^2 + b_{2i}M_i\,|x_i|\right]. \end{aligned} \tag{15}$$

Because $\frac{c_i}{b_{1i}} < \bar{k}_{Pi} \leq k_{Pi} - \varepsilon_i$, then (15) can be rewritten as

$$\begin{aligned}\dot{V}_i &\leq \left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right)^{-1}\left[-(\bar{k}_{Pi}b_{1i} - c_i + \varepsilon_i)|x_i|^2 + b_{2i}M_i|x_i|\right] \\ &\leq -\varepsilon_i\left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right)^{-1}|x_i|^2 < 0,\ \forall |x_i| \geq \frac{b_{2i}M_i}{\bar{k}_{Pi}b_{1i} - c_i},\end{aligned}$$

which, according to Theorem 4.18 of Khalil,[1] proves that the solution $x_i(t)$ is uniformly ultimately bounded. In particular, considering a positive constant $\delta > 0$ such that $\mathcal{B}_\delta \subset \mathcal{D}_x$ satisfying $\frac{b_{2i}M_i}{\bar{k}_{Pi}b_{1i}-c_i} < \alpha_{2i}^{-1}(\alpha_{1i}(\delta))$, there exists $\beta \in \mathcal{KL}$ such that for every initial state $x_i(0)$ with $|x_i(0)| \leq \alpha_{2i}^{-1}(\alpha_{1i}(\delta))$, there is $T \geq 0$ such that

$$|x_i(t)| \leq \begin{cases} \beta_i(|x_i(0)|, t), & 0 \leq t < T \\ \alpha_{1i}^{-1}\left(\alpha_{2i}\left(\frac{b_{2i}M_i}{\bar{k}_{Pi}b_{1i}-c_i}\right)\right), & t \geq T. \end{cases} \tag{16}$$

Note that if the initial state $x_i(0)$ satisfies

$$|x_i(0)| \leq \alpha_{1i}^{-1}\left(\alpha_{2i}\left(\frac{b_{2i}M_i}{\bar{k}_{Pi}b_{1i} - c_i}\right)\right),$$

then the solution $x(t)$ will remain in this range for all future time, ie,

$$|x_i(t)| \leq \alpha_{1i}^{-1}\left(\alpha_{2i}\left(\frac{b_{2i}M_i}{\bar{k}_{Pi}b_{1i} - c_i}\right)\right),\ \forall t \geq 0.$$

Hence, if M_i and k_{Pi} satisfy (14), then

$$|x_i(t)| \leq x_i^{\max},\ \forall t \geq 0,$$

which verifies the state-limiting property of the proposed controller. □

Consider the typical zero initial condition of the integral state σ_i, ie, $\sigma_i(0) = 0$; then, the controller state remains, following the equilibria of (13c), within interval $\sigma_i(t) \in [-\frac{\pi}{2}, \frac{\pi}{2}], \forall t \geq 0$. Whenever $\sigma(t) \to \pm\frac{\pi}{2}$, then $\dot{\sigma}_i \to 0$, which means that σ_i will converge to the upper or lower limit ($\pm\frac{\pi}{2}$) independently of the term $r_i - h_i(x_i)$.

This also implies an inherent antiwindup property of the proposed sl-PID because the integration slows down near the two limits of σ_i without the need for adding any antiwindup mechanisms, which often result in changes of the original controller dynamics, as in the conventional antiwindup PID control design. Similarly, if initially $\sigma_i(0)$ is selected in the range $[-\frac{\pi}{2} + k\pi, +\frac{\pi}{2} + k\pi]$, for all $k \in \mathbb{Z}$, then it will remain within this range thereafter. This allows us to characterize the region of attraction (see Figure 1 and Section 4.2) for the composite dynamics (x, σ) as $\mathcal{X}_{i,k} = \{(x, \sigma) \in \mathbb{R}^2 : |x_i| \leq x_i^{\max},\ \sigma_i \in [-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi]\}$ for any $k \in \mathbb{Z}$ determined by the initial conditions of the integrator. An additional consequence of using the proposed control law is that the sets $\mathcal{X}_{i,k}$ are positively invariant for the closed-loop dynamics, formalized in the following proposition.

Proposition 3. *The set* $\mathcal{X}_{i,k} = [-x_i^{\max}, x_i^{\max}] \times [-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi]$ *is positively invariant for the closed-loop dynamics (13b) and (13c) when* $\sigma_i(0) \in [-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi]$ *for any* $k \in \mathbb{Z}$.

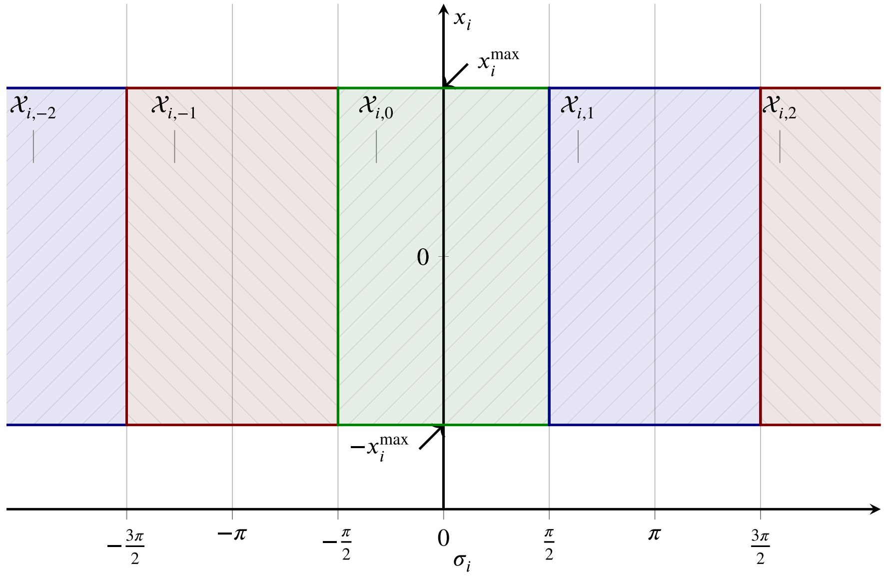

FIGURE 1 Region of attraction for the closed-loop dynamics (13b) and (13c). Each region corresponds to an initial condition for the integrator, ie, $\sigma_i(0) = 0$ implies that the corresponding region of attraction, $\mathcal{X}_{i,0}$, is positively invariant [Colour figure can be viewed at wileyonlinelibrary.com]

It has been proven in Proposition 2 that the proposed sl-PID (12a)-(12b) guarantees a symmetric constraint $|x_i(t)| \leq x_i^{\max}$ for the closed-loop system (13). In the case where a nonsymmetric constraint is required, eg, $x_i^{\min} \leq x_i \leq x_i^{\max}$, the following Remark is provided.

Remark 3. Consider system (5a)-(5c) with the desired state constraint $x_i^{\min} \leq x_i \leq x_i^{\max}$, $\forall i = 1, 2, \dots, n$, where Assumptions 1 to 3 hold and Assumption 4 is true with respect to $\bar{x}_i = x_i - x_i^m$, with $x_i^m = \frac{x_i^{\min}+x_i^{\max}}{2}$, ie, there exists $V_i(\bar{x}_i, w)$ that satisfies (7a)-(7c) with respect to $\bar{x}_i$. Then, the problem can be transformed into a problem with symmetric state constraints of the form $|\bar{x}_i(t)| \leq \Delta x_i^m$, where $\Delta x_i^m = \frac{x_i^{\min}-x_i^{\max}}{2}$. Hence, the sl-PID can take the form

$$u_i(\bar{x}_i) = -k_{Pi}\bar{x}_i + M_i \sin\sigma_i - k_{Di}\frac{\partial h_i}{\partial \bar{x}_i}\dot{\bar{x}}_i \tag{17}$$

with σ_i dynamics (12b) and guarantee the desired state limitation, following the same analysis provided in Proposition 2.

4.2 | Stability analysis

Although the desired x_i state limitation is guaranteed, the stability of the desired equilibrium point (x_i^e, σ_i^e) with $x_e \in \mathcal{D}_x$, still needs to be investigated. Following Proposition 1, the map $(x_e, u_e) = \xi(r, w)$ for a constant $w \in \mathbb{W}$ defines injectively a steady-state pair, which allows us to characterize the equilibrium points.

Proposition 4. *Suppose that Assumptions 1 to 3 hold, and* $\frac{\partial f_i}{\partial x_i}(x_i, w) - k_{Pi}g_i(w) \leq 0$ *for all* $i \in \mathbb{I}_n$ *and* $|x_i| \leq x_i^{\max}$. *The equilibrium points* (x_i^e, σ_i^e) *for the closed-loop system (13b) and (13c) for all* $i \in \mathbb{I}_n$ *satisfy* $(x_i^e, \sigma_i^e) \in (-x_i^{\max}, x_i^{\max}) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ *when* $r_i \in (-h_i^{-1}(x_i^{\max}), h_i^{-1}(x_i^{\max}))$ *and* $(x_i^e, \sigma_i^e) = \left(\pm x_i^{\max}, \pm\frac{\pi}{2} + k\pi\right)$ *when* $|r_i| \geq h_i^{-1}(x_i^{\max})$.

Proof. The steady-state locus for (13b) and (13c) for constant set points is given by the solutions of the system of nonlinear equations

$$0 = f_i(x_i, w) - g_i(w)k_{Pi}x_i + g_i(w)M_i \sin\sigma_i,$$
$$0 = (r_i - h_i(x_i))\cos\sigma_i,$$

which can be translated into the set

$$\sin\sigma_i^e = (M_i g_i(w))^{-1}\left(k_{Pi}g_i(w)x_i^e - f_i\left(x_i^e, w\right)\right) \cap \left(\sigma_i^e = \pm\frac{\pi}{2} + k\pi \cup x_i^e = h_i^{-1}(r_i)\right), \tag{18}$$

for $k \in \mathbb{Z}$. As a result, the steady-state locus can be characterized completely by two types of behavior: when $\sigma_i^e = \pm\frac{\pi}{2}+k\pi$, the state satisfies $f_i(x_i^e, w) - k_{Pi}g_i(w)x_i^e + M_i g_i(w) = 0$, and on the other hand, when $x_i^e = h_i^{-1}(r_i)$, the equilibrium for the integrator state can be obtained as

$$\sigma_i^e = \arcsin\left(\frac{k_{Pi}g_i(w)h_i^{-1}(r_i) - f_i\left(h_i^{-1}(r_i), w\right)}{M_i g_i(w)}\right).$$

This implies that the following inequality holds:

$$-M_i g_i(w) \leq k_{Pi}g_i(w)h_i^{-1}(r_i) - f_i\left(h_i^{-1}(r_i), w\right) \leq M_i g_i(w),$$

which in turn allows us to define two functions

$$\begin{aligned} F_{1,i}(r_i, w) &= f_i\left(h_i^{-1}(r_i), w\right) - k_{Pi}g_i(w)h_i^{-1}(r_i) + M_i g_i(w) \geq 0 \\ F_{2,i}(r_i, w) &= f_i\left(h_i^{-1}(r_i), w\right) - k_{Pi}g_i(w)h_i^{-1}(r_i) - M_i g_i(w) \leq 0 \end{aligned}$$

that, under the hypothesis $\frac{\partial f_i}{\partial x_i}(x_i, w) - k_{Pi}g_i(w) \leq 0$, are both nonincreasing functions of the reference points. For the first case, the value of r_i for which $F_{1,i}(r_i, w) = 0$ is paired with $\sigma_i^e = \frac{\pi}{2}$; in addition, because $F_{1,i}(\cdot,\cdot)$ is nonincreasing, this value corresponds to $r_i^{\max}$. A similar argument follows for $F_{2,i}(\cdot,\cdot)$ and $(-r_i^{\max}, -\frac{\pi}{2})$. In the case that $r_i \in (-h_i(x_i^{\max}), h_i(x_i^{\max}))$, then, the equilibrium point satisfies $x_i^e \in (-x_i^{\max}, x_i^{\max})$ and $\sigma_i^e \in (-\frac{\pi}{2}, \frac{\pi}{2})$. □

A direct consequence of the previous result is the existence of equilibrium points inside the region of attraction $\mathcal{X}_{i,k}$:

Corollary 1. *The set $\mathcal{X}_{i,k}$ for all $i \in \mathbb{I}_n$ and any $k \in \mathbb{Z}$ contains three equilibrium points: (x_i^e, σ_i^e), $(x_i^{\max}, \frac{\pi}{2} + k\pi)$, and $(-x_i^{\max}, -\frac{\pi}{2} + k\pi)$.*

For the desired equilibrium point $x_i^e \in (-x_i^{\max}, x_i^{\max})$, $\sigma_i^e \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\forall i \in \mathbb{I}_n$, the Jacobian matrix of system (13b)-(13c) is

$$A_i = \begin{bmatrix} \dfrac{\left.\frac{\partial f_i}{\partial x_i}\right|_{x_i^e} - k_{Pi}g_i(w)}{1 + g_i(w)k_{Di}\left.\frac{\partial h_i}{\partial x_i}\right|_{x_i^e}} & \dfrac{g_i(w)M_i\cos\sigma_i^e}{1 + g_i(w)k_{Di}\left.\frac{\partial h_i}{\partial x_i}\right|_{x_i^e}} \\ -\frac{k_{Ii}}{M_i}\cos\sigma_i^e\left.\frac{\partial h_i}{\partial x_i}\right|_{x_i^e} & 0 \end{bmatrix}.$$

Thus, the equilibrium point (x_e, σ_e) will be asymptotically stable when

$$k_{Pi} > \max\left\{0, \left.\frac{\partial f_i}{\partial x_i}\right|_{x_i^e} \frac{1}{\min\limits_{w \in \mathbb{W}}\{g_i(w)\}}\right\} \tag{19}$$

and for any $k_{Ii} > 0$ and $k_{Di} \geq 0$. Hence, by selecting k_{Pi} according to (19), then M_i can be chosen to satisfy (14). For the remaining dynamics, similarly to the analysis of the conventional PID controller, (13a) can be rewritten in the generic form

$$\dot{\tilde{z}} = \tilde{q}(\tilde{v}, \tilde{z}, w), \tag{20}$$

with the desired equilibrium being shifted at the origin $\tilde{z} = 0$. As in Section 3.1, if Assumption 5 holds for system (20), according to Lemma 2, the equilibrium point (x_e, σ_e, z_e) of the closed-loop system (13b)-(13a) is asymptotically stable.

The asymptotic stability of the desired equilibrium point (x_e, σ_e) has been proven in a neighborhood of the equilibrium point, as in the case of the conventional PID control when system (5a)-(5c) is nonlinear. However, in the proposed approach, a detailed methodology for selecting the sl-PID controller gains can be applied to prove that the asymptotic stability holds in the entire constrained range $x_i \in [-x_i^{\max}, x_i^{\max}]$, as explained in the following theorem.

Theorem 1. *Suppose that Assumptions 1 to 4 hold. The desired equilibrium point (x_e, σ_e) of the closed-loop system (13b)-(13c) with $x_i^e \in (-x_i^{\max}, x_i^{\max})$, $\sigma_i^e \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\forall i \in \mathbb{I}_n$, is asymptotically stable and every solution $(x_i(t), \sigma_i(t))$ with* *initial conditions* $x_i(0) \in (-x_i^{\max}, x_i^{\max})$, $\sigma_i(0) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\forall i = \mathbb{I}_n$ *converges to* (x_i^e, σ_i^e) *as* $t \to \infty$ *when* k_{Pi}, k_{Ii}, *and* $k_{Di} \geq 0$ *satisfy*

$$k_{Pi} > \max\left\{0, \max_{|x_i|\leq x_i^{\max}}\left\{\frac{\partial f_i}{\partial x_i}\right\}\frac{1}{\min\limits_{w\in\mathbb{W}} g_i(w)}\right\}, \tag{21}$$

$$k_{Ii} < \left[\min_{|x_i|\leq x_i^{\max}, w\in\mathbb{W}}\left\{\left(k_{Pi}g_i(w) - \frac{\partial f_i}{\partial x_i}\right)\alpha + g_i(w)k_{Di}\left|\frac{\partial^2 h_i}{\partial x_i^2}\left(f_i(x_i, w) - g_i(w)k_{Pi}x_i\right)\right|\right\}\right]\frac{M_i}{\max\limits_{|x_i|\leq x_i^{\max}}\left\{|r_i - h_i(x_i)|\,\alpha^2\right\}}, \tag{22}$$

where $\alpha = 1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}$.

Proof. Because (21) is satisfied for all $|x_i| < x_i^{\max}$, then (19) holds true, and as a result, the desired equilibrium point (x_e, σ_e) of the closed-loop system with $x_i^e \in (-x_i^{\max}, x_i^{\max})$, $\sigma_i^e \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is asymptotically stable. The extension of this result to the entire bounded range follows from an analysis of the closed-loop dynamics (13b)-(13c), which describe a second-order system for each $i \in \mathbb{I}_n$ of the form

$$\dot{x}_i = \tilde{f}_i(x_i, \sigma_i) \tag{23}$$

$$\dot{\sigma}_i = \tilde{g}_i(x_i, \sigma_i, r_i). \tag{24}$$

First, we ensure that no limit cycles exist within the constraint set $(x_i, \sigma_i) \in \mathcal{X}_{i,k}$; this assertion follows from the Bendixon theorem[31] such that the divergence of the vector field does not vanish nor change sign, ie, $\frac{\partial \tilde{f}_i}{\partial x_i} + \frac{\partial \tilde{g}_i}{\partial \sigma_i} \neq 0$ for all $(x_i, \sigma_i) \in \mathcal{X}_{i,k}$. Computing the derivatives yields the expression

$$\frac{\left(\frac{\partial f_i}{\partial x_i} - k_{Pi}g_i(w)\right)\left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right) - g_i(w)k_{Di}\frac{\partial^2 h_i}{\partial x_i^2}\left(f_i(x_i, w) - g_i(w)k_{Pi}x_i\right)}{\left(1 + g_i(w)k_{Di}\frac{\partial h_i}{\partial x_i}\right)^2} - \frac{k_{Ii}}{M_i}(r_i - h_i(x_i))\sin\sigma_i \neq 0.$$

Therefore, no limit cycles will exist in the bounded range if

$$\frac{\left(\frac{\partial f_i}{\partial x_i} - k_{Pi}g_i(w)\right)\alpha - g_i(w)k_{Di}\frac{\partial^2 h_i}{\partial x_i^2}\left(f_i(x_i, w) - g_i(w)k_{Pi}x_i\right)}{\alpha^2} + \frac{k_{Ii}}{M_i}\max_{|x_i|\leq x_i^{\max}}|r_i - h_i(x_i)| < 0, \forall t \geq 0. \tag{25}$$

From (21) and (22), the above expression does not change sign nor vanish. This condition implies from Lemma 3 that any bounded trajectory $(x_i(t), \sigma_i(t))$ within $\mathcal{X}_{i,k}$ will converge to the omega limit set $\omega(\mathcal{X}_{i,k})$ of (24), which cannot include a limit cycle.

However, except from the desired equilibrium point (x_e, σ_e), which is asymptotically stable, according to Corollary 1, there exist two more equilibrium points in the bounded range, ie, $(x_i^e, \sigma_i^e) = \left(x_i^{\max}, \frac{\pi}{2}\right)$ and $(x_i^e, \sigma_i^e) = \left(-x_i^{\max}, -\frac{\pi}{2}\right)$. The proof that these two equilibrium points are unstable follows by contradiction. In particular, consider that for any (x_{i0}, σ_{i0}) with $x_{i0} \in (-x_i^{\max}, x_i^{\max})$ and $\sigma_{i0} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, there is $(x_i(t), \sigma_i(t)) \to \left(x_i^{\max}, \frac{\pi}{2}\right)$ as $t \to \infty$. However, for a point in $\mathcal{X}_{i,0}$ arbitrarily close to the equilibrium point $(x_i^e, \sigma_i^e) = \left(x_i^{\max}, \frac{\pi}{2}\right)$, because Assumption 1 holds in $\mathcal{X}_{i,0}$, then $h(x_i) < h(x_i^e)$ and $x_i < x_i^e$ hold. As a result, $\dot{\sigma}_i < 0$, which implies that σ_i will decrease and diverge from $\sigma_i^e = \frac{\pi}{2}$, ie, consequently, the solution $(x_i(t), \sigma_i(t))$ will diverge from $\left(x_i^{\max}, \frac{\pi}{2}\right)$ taking into account (13b) and Corollary 1, leading to a contradiction. Hence, the equilibrium point $\left(x_i^{\max}, \frac{\pi}{2}\right)$ is unstable. A similar procedure holds for $(x_i^e, \sigma_i^e) = \left(-x_i^{\max}, -\frac{\pi}{2}\right)$. As a result, the desired equilibrium point (x_i^e, σ_i^e) with $x_i^e \in (-x_i^{\max}, x_i^{\max})$ and $\sigma_i^e \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ represents the only positive limit point in the omega limit set $\omega(\mathcal{X}_{i,0})$; thus, every solution $(x_i(t), \sigma_i(t))$ starting in the range $(x_i(0), \sigma_i(0)) \in \mathcal{X}_{i,0}$, $\forall i \in \mathbb{I}_n$, converges to (x_i^e, σ_i^e) as $t \to \infty$ when k_{Pi}, k_{Ii}, and $k_{Di} \geq 0$ satisfy (21)-(22), thus completing the proof. □

Although the condition of the integral gain selection, ie, condition (22), might seem complicated to verify, it should be noted that it can be significantly simplified for the case of the sl-PI controller, as given in the following remark.

Remark 4. For the design of a sl-PI controller, the derivative gain is zero, ie, $k_{Di} = 0$, and hence, condition (22) is simplified to the expression:

$$k_{Ii} < \min_{|x_i| \le x_i^{\max}, w \in \mathbb{W}} \left\{ k_{Pi} g_i(w) - \frac{\partial f_i}{\partial x_i} \right\} \frac{M_i}{\max\limits_{|x_i| \le x_i^{\max}} \{|r_i - h_i(x_i)|\}}. \tag{26}$$

Remark 5. From the closed-loop dynamics, the set point r_i acts as a bifurcation parameter in the sense that its value changes the dynamic properties of the system. For any $|h_i^{-1}(r_i)| < x_i^{\max}$, following Corollary 1, there are three distinct equilibrium points: two unstable at the boundary of $\mathcal{X}_{i,k}$ and one stable in the interior of $\mathcal{X}_{i,k}$. As the set point approaches $h_i(x_i^{\max})$, the stable set point also approaches the boundary of $\mathcal{X}_{i,k}$; eventually, when $|r_i| \ge h_i(x_i^{\max})$, there are only two equilibrium points.

To summarize the design procedure of the proposed sl-PID controller, the following steps can be followed.

1. Check that the system under investigation is in form of (5a)-(5c) or whether it can be brought into this form using partial feedback linearization.
2. Check that Assumptions 1 to 5 hold.
3. Select k_{Pi} to satisfy (21) and $k_{Di} \ge 0$.
4. Select M_i according to (14).
5. Select k_{Ii} satisfying (22).
6. Design the sl-PID controller according to (12a)-(12b).

Although, in this paper, the state constraint $|x_i| \le x_i^{\max}$ is the main requirement for the closed-loop system equipped with the proposed sl-PID, it is worth mentioning the proposed design in the PI control format, ie, sl-PI controller, can additionally guarantee an input constraint of the form $|u_i| \le u_i^{\max}$, $u_i^{\max} > 0$ when required. This is explained in the following remark.

Remark 6. The proposed sl-PI control (12a)-(12b) with $k_{Di} = 0$ guarantees a desired input constraint $|u_i| \le u_i^{\max}$ in addition to the state constraint $|x_i| \le x_i^{\max}$ when M_i is selected as

$$M_i \le \min \left\{ u_i^{\max} - k_{Pi} x_i^{\max}, \alpha_{2i}^{-1} \left(\alpha_{1i} \left(x_i^{\max} \right) \right) \frac{\bar{k}_{Pi} b_{1i} - c_i}{b_{2i}} \right\} \tag{27}$$

when $k_{Pi} < u_i^{\max} / x_i^{\max}$.

The above remark and the additional conditions for M_i and k_{Pi} can be easily obtained by taking into account the control structure (12a), the desired input and state constraints, and $M_i > 0$. It should be noted that even though the sl-PI controller can be designed based on Remark 6 to guarantee both state and input constraints, the selection of the controller parameters may lead to a slow response in a real application because the upper limits for M_i and k_{Pi} have been obtained to guarantee the desired input constraint for the worst-case scenario, ie, when $|x_i| \le x_i^{\max}$ and $|\sin(\sigma_i)| \le 1$.

5 | ENGINEERING EXAMPLES

5.1 | Motor feeding a constant power load

5.1.1 | System and controller design

Consider first the case of a motor feeding a constant power load (CPL), ie, the load torque is represented as $T_L = \frac{P_L}{\omega}$, where $P_L > 0$ is the power of the load, which is constant, and ω is the speed of the motor. Example of these loads includes machine tools and center winders in the industry. Hence, the motor dynamics are given in the form

$$J \frac{d\omega}{dt} = -\beta\omega + T - \frac{P_L}{\omega}, \tag{28}$$

where J is the motor inertia, β is the motor damping, and T is the electrical torque applied to the motor, representing the control input of the system. It is assumed that the motor parameters and the load are not accurately known but can vary around their nominal values, ie, they satisfy $J \in [J_n - \Delta J, J_n + \Delta J] > 0$, $\beta \in [\beta_n - \Delta\beta, \beta_n + \Delta\beta] > 0$, and $P_L \in [P_{Ln} - \Delta P_L, P_{Ln} + \Delta P_L] > 0$ (Assumption 3). The main task is to regulate the speed of the motor to a desired reference ω^{ref}, while guaranteeing that during the transient response, the speed never violates an upper limit, ie, $|\omega| \leq \omega^{\max}$, $\forall t \geq 0$. Note that because a CPL is connected at the motor, the motor speed should take strictly positive values, ie, consider that $\omega \geq \omega_{\min} > 0$. The plant is in the form of (5b) with $y = \omega$. Hence, $g(w) = \frac{1}{J} > 0$ and $h(\omega) = \omega$, ie, $\frac{\partial h}{\partial \omega} = 1 > 0$, which confirm that both Assumptions 1 and 2 hold. For system (28), consider the continuous and differentiable function $V = \frac{1}{2}J\omega^2$, which satisfies (7a). In addition,

$$\frac{\partial V}{\partial \omega} f = -\beta\omega^2 - P_L \leq -(\beta_n - \Delta\beta)\omega^2$$
$$\frac{\partial V}{\partial \omega} g = \omega,$$

which verifies that (7b) and (7c) are also satisfied with $c = -\beta_n + \Delta\beta$ and $b_1 = b_2 = 1$, yielding that Assumption 4 holds as well. As a result, both the conventional PID and the sl-PID can be applied to regulate the motor speed. For the conventional PID, the proportional gain k_P can be selected according to (10), whereas the integral and derivative gains can take any positive values k_I and k_D. For the sl-PID design, the proportional gain should be selected according to (21) as

$$k_P > \frac{P_{Ln} + \Delta P_L}{\omega_{\min}^2} - \beta_n + \Delta\beta.$$

Consequently, parameter M when chosen according to (14) satisfies

$$M_i \leq \omega^{\max}(k_P + \beta_n - \Delta\beta - \varepsilon),$$

for an arbitrarily small $\varepsilon > 0$ such that $k_P + \beta_n - \Delta\beta - \varepsilon > 0$. The proportional and derivative gains can take any positive value k_P and nonnegative value k_D, respectively, whereas the integral gain can be selected according to condition (22), taking into account that $h(\omega) = \omega$, ie, $\frac{\partial h}{\partial \omega} = 1$, $\frac{\partial^2 h}{\partial \omega^2} = 0$:

$$k_I < \min_{\substack{\omega_{\min} \leq \omega \leq \omega^{\max} \\ w \in \mathbb{W}}} \left[\frac{1}{J}\left(k_P + \beta - \frac{P_{Ln}}{\omega^2}\right)\right] \frac{M}{\max\limits_{\substack{\omega_{\min} \leq \omega \leq \omega^{\max} \\ w \in \mathbb{W}}} \left\{ |\omega^{\text{ref}} - \omega| \left(1 + \frac{k_D}{J}\right) \right\}}.$$

Based on the given ranges of the uncertain parameters and the proven limit of the state ω below $\omega^{\max}$, which together with $|\omega^{\text{ref}}| \leq \omega^{\max}$ yields $\max\{|\omega^{\text{ref}} - \omega|\} = 2\omega^{\max}$, then, the range of k_I becomes

$$k_I < \frac{\left(k_p + \beta_n - \Delta\beta - \frac{P_{Ln} + \Delta P_L}{\omega_{\min}^2}\right)(k_P + \beta_n - \Delta\beta - \varepsilon)}{2(J_n + \Delta J + k_D)}.$$

Note that because the nominal power of the load P_{Ln} is known and the motor speed ω is measured, an additional term $\frac{P_{Ln}}{\omega}$ can be introduced in the control input torque T together with the proposed sl-PID to reduce system nonlinearity. In this case, k_P should satisfy $k_P > \frac{\Delta_{P_L}}{\omega_{\min}^2} - \beta_n + \Delta\beta$ and parameters M and k_I can be defined accordingly. However, this special case is not applied in the simulation that follows in order to keep consistency with the generic case of the system and the control design procedure presented in the theory.

5.1.2 | Simulation results

To validate the proposed sl-PID control performance and compare it with the conventional PID control, the motor system with parameters given in Table 1 is simulated. The main goal is to regulate motor speed ω to the desired value ω^{ref} without violating the upper limit $\omega^{\max} = 200[\text{rad s}^{-1}]$, whereas at the steady state, the motor speed always stays above

TABLE 1 Motor parameters

Parameters	Values	Parameters	Values
J	0.22 kg·m^2	J_n	0.2 kg·m^2
ΔJ	0.1 kg·m^2	β	0.0012 ·m/s rad^{-1}
β_n	0.001 ·m/s rad^{-1}	$\Delta\beta$	0.0005 N·m/s rad^{-1}
P_L, P_{Ln}	1 kW	ΔP	0.2 kW

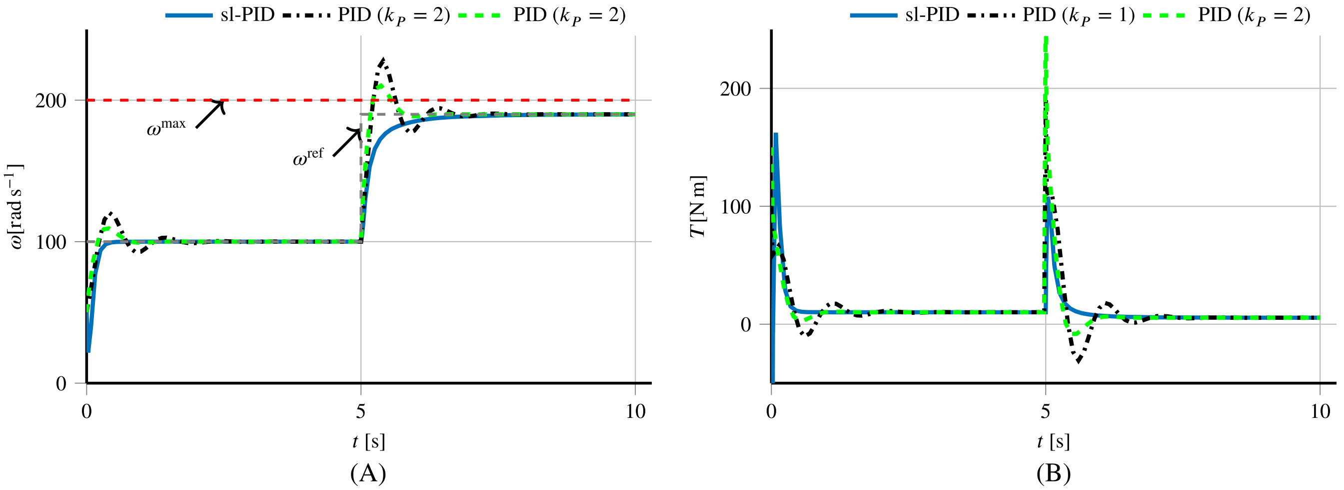

FIGURE 2 Simulation results of a motor feeding a constant power load using the conventional PID and the proposed sl-PID control. A, Motor speed; B, Input torque [Colour figure can be viewed at wileyonlinelibrary.com]

$\omega_{\min} = 30$[rad s^{-1}]. It is underlined that according to Table 1, the system parameters can vary up to 50% from the nominal values to investigate a scenario with extreme level of uncertainties. The conventional PID controller is tested with two different values of $k_P = 1$ and $k_P = 2$, whereas the integral and derivative gains take the values $k_I = 10$ and $k_D = 0.01$, respectively. For the proposed sl-PID, the controller gains can be computed according to the analysis presented in the previous subsection leading to $k_P = 8$, $k_I = 85$, and $k_D = 0.01$.

Initially, the desired speed is set to $\omega^{\text{ref}} = 100$[rad s^{-1}] and at the time instant $t = 5$[s], it changes to 190[rad s^{-1}]. As it can be observed from the response in Figure 2A, both the conventional PID and the proposed sl-PID can achieve the desired regulation as expected from the theoretical analysis. However, during the transient, the conventional PID control forces the motor speed to exceed the upper limit $\omega^{\max}$, whereas the proposed sl-PID maintains the desired state limitation at all times. Note, however, that for larger values of the proportional gain k_P, the conventional PID may possibly maintain the speed to values lower than $\omega^{\max}$. Nevertheless, the conventional PID gains are designed based on the linearized model, ie, the desired state limitation cannot be guaranteed for a generic nonlinear system with uncertainties, whereas the state limitation for the proposed sl-PID has been proven using ultimate boundedness theory for the generic nonlinear model. In addition, as it can be seen from the response of the input torque in Figure 2B, larger values of the proportional gain k_P will lead to unrealistically large values of the torque (eg, higher than 160[N m]), whereas the proposed sl-PID results in a much smoother input response.

5.2 | A dc/dc boost power converter

5.2.1 | System and controller design

Consider the dynamic equations of the dc/dc boost converter connected to a resistive load R, given as in Konstantopoulos and Zhong[32]:

$$L\frac{di}{dt} = -ri - (1-u)v + V_{in} \tag{29a}$$

$$C\frac{dv}{dt} = (1-u)i - \frac{v}{R}, \tag{29b}$$

where $V_{in} > 0$ is the dc input voltage, L is the converter inductance with a series resistance r, C is the converter capacitance, $\bar{x} = (i, v)$ is the state vector, and u is the control input describing the duty-ratio input of the converter. The dc/dc converter can achieve a higher dc voltage v at its output compared to the input voltage V_{in}. The main task is to regulate the converter power $P = V_{\text{in}} i$ to a constant reference P^{ref}, while maintaining a desired constraint $|i| \leq i^{\max}$, where $i^{\max} > 0$ represents the maximum allowed current of the converter to avoid damage of the device. It is assumed that L, C, r, and R are not accurately known, ie, $L \in [L_n - \Delta L, L_n + \Delta L] > 0$, $C \in [C_n - \Delta C, C_n + \Delta C] > 0$, $r \in [r_n - \Delta r, r_n + \Delta r] \geq 0$, and $R \in [R_n - \Delta R, R_n + \Delta R] > 0$, where L_n, C_n, r_n, and R_n are the corresponding nominal quantities, which are considered known together with the maximum deviations of the parameters (Assumption 3). By defining the control input u as

$$u = 1 - \frac{V_{in} - \bar{u}}{v} \tag{30}$$

when $v \geq V_{\text{in}} > 0$, which is a physical property of DC/DC boost converters, and replacing it in (29a)-(29b), the converter dynamics take the form

$$\frac{di}{dt} = -\frac{r}{L} i + \frac{1}{L} \bar{u} \tag{31a}$$

$$\frac{dv}{dt} = \frac{V_{in} - \bar{u}}{Cv} i - \frac{v}{CR} \tag{31b}$$

$$y = V_{\text{in}} i, \tag{31c}$$

which is in the form of (5b)-(5c) considering the control input $\bar{u}$. Note that $g(w) = \frac{1}{L} > 0$ and $h(i) = V_{\text{in}} i$, ie, $\frac{\partial h}{\partial i} = V_{\text{in}} > 0$, which confirm that both Assumptions 1 and 2 hold. Because (31a) is linear and time invariant, then by considering the quadratic function $V = \frac{1}{2} L i^2$, which satisfies (7a),

$$\frac{\partial V}{\partial i} f = -r i^2$$
$$\frac{\partial V}{\partial i} g = i,$$

hence, (7b) and (7c) are also satisfied with $c = -r_n + \Delta r$ and $b_1 = b_2 = 1$, yielding that Assumption 4 holds as well. As a result, both the conventional PID controller and the proposed nonlinear sl-PID controller can be implemented to regulate $\bar{x}$ to the desired unique equilibrium $\bar{x}_e = (i_e, v_e)$, where $i_e = i^{\text{ref}} \in [-i^{\max}, i^{\max}]$ and $v_e = \sqrt{R i^{\text{ref}} (V_{in} - r i^{\text{ref}})}$.

Because of its physical properties, the boost converter output voltage is always higher than the input voltage, ie, $v \geq V_{in} > 0$. For any proportional gain $k_P > 0$, the equilibrium point $i_e = i^{\text{ref}}$ of (31a) will be asymptotically stable because conditions (10) and (19) will be satisfied for both the conventional and the proposed sl-PID controller. For the proposed controller, from (14), parameter M should satisfy

$$M \leq i_{\max} (k_P + r_n - \Delta r - \varepsilon), \tag{32}$$

for an arbitrarily small $\varepsilon > 0$ such that $k_P + r_n - \Delta r - \varepsilon > 0$. Finally, from (22), the integral gain k_I should satisfy

$$k_I < \min_{w \in \mathbb{W}} \left\{ \frac{k_P + r}{L} \right\} \frac{M}{V_{\text{in}} \max\limits_{|i| \leq i^{\max}} |i^{\text{ref}} - i|}. \tag{33}$$

Because the proposed controller guarantees the state constraint $|i| \leq i^{\max}$ according to the analysis in Subsection 4.1 and $|i^{\text{ref}}| < i^{\max}$, then $\max |i^{\text{ref}} - i| = 2 i^{\max}$. As a result, taking into account the choice of M from (32) and the range of the uncertain parameters $L \in [L_n - \Delta L, L_n + \Delta L] > 0$ and $r \in [r_n - \Delta r, r_n + \Delta r] \geq 0$, inequality (33) becomes

$$k_I < \frac{(k_P + r_n - \Delta r)(k_P + r_n - \Delta r - \varepsilon)}{2(L_n + \Delta L) V_{\text{in}}}. \tag{34}$$

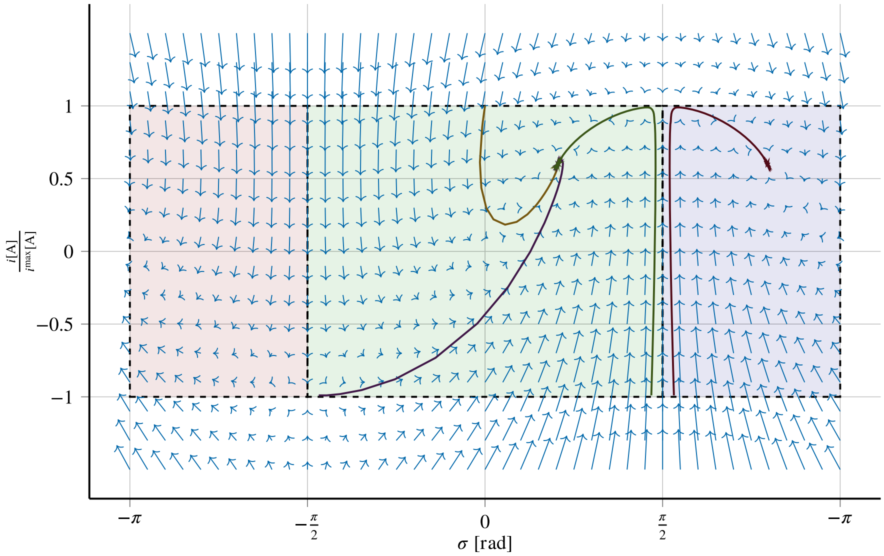

FIGURE 3 Vector field of the current dynamics in closed loop with the proposed state-limiting proportional integral derivative controller. The vector field shows the positive invariance of set $\mathcal{X}_{1,k}$; furthermore, the trajectories emanating from points in the vicinity of $(-i^{\max}, -\frac{\pi}{2})$ and $(i^{\max}, \frac{\pi}{2})$ diverge from these points and converge to the single stable equilibrium point [Colour figure can be viewed at wileyonlinelibrary.com]

By setting $i = i_e + \tilde{i}$, $v = v_e + \tilde{v}$, and $\sigma = \sigma_e + \tilde{\sigma}$, then (31b) can be expressed as

$$\begin{aligned}\frac{d\tilde{v}}{dt} &= \frac{V_{in} + k_P(i_e + \tilde{i}) - M\sin(\sigma_e + \tilde{\sigma})}{C(v_e + \tilde{v})}(i_e + \tilde{i}) - \frac{v_e + \tilde{v}}{CR} \\ &= \tilde{q}(\tilde{x}, \tilde{v}, w).\end{aligned} \tag{35}$$

Considering a set $D_{\tilde{v}}$ for $\tilde{v}$, where $\tilde{v} > -v_e$ in $D_{\tilde{v}}$, then both $\frac{\partial \tilde{q}}{\partial \tilde{v}}$ and $\frac{\partial \tilde{q}}{\partial \tilde{x}}$ are bounded in $D_{\tilde{v}}$ where $\tilde{x} = (\tilde{i}, \tilde{\sigma})$. In addition, the unforced system, ie, for $\tilde{x} = 0$, has the form

$$\frac{d\tilde{v}}{dt} = \frac{V_{in} i_e}{C(v_e + \tilde{v})} - \frac{v_e + \tilde{v}}{CR}. \tag{36}$$

The Jacobian of system (36) results in

$$A_v = -\frac{V_{in} i_e}{C v_e^2} - \frac{1}{CR} = -\frac{1}{CR}\left(\frac{V_{in}}{V_{in} - r i^{\mathrm{ref}}} + 1\right) < 0,$$

and therefore, the origin of (36) is asymptotically stable. Then, according to Lemma 1, system (35) is locally input-to-state stable. As a result, the desired equilibrium point (i_e, σ_e, v_e) of system (31a)- (31b) with both the conventional PID and the proposed sl-PID controller will be asymptotically stable. In Figure 3, we illustrate the behavior of the system dynamics under the proposed control law. We can see that the vector field and any trajectory starting inside $\mathcal{X}_{1,0}$ remain within. This illustrates the nature of the equilibria (see Corollary 1); for those on the boundary of $\mathcal{X}_{1,0}$, ie, $\left(-i^{\max}, -\frac{\pi}{2}\right)$ and $\left(i^{\max}, \frac{\pi}{2}\right)$, the closed-loop system is unstable and any trajectory starting arbitrarily closed to them diverges, whereas point (i^e, σ^e) is stable.

5.2.2 | Simulation results

To demonstrate the performance of the proposed nonlinear sl-PID controller in comparison with the conventional PID control, the dc/dc converter system of (29a)-(29b) was simulated using the parameters shown in Table 2. For the conventional PID controller, the integral gain is selected as $k_I = 200$, whereas two different values are tested for the proportional gain $k_P = 0.4$ and 0.6, with $\mathrm{k_D} = 0$. For the proposed nonlinear sl-PID controller gains, the design procedure mentioned in the previous subsection is followed, providing the selection $k_P = 25$, $k_I = 2083$ and $\mathrm{k_D} = 0$. The desired scenario is for the converter to regulate initially the power P to a desired value $P^{\mathrm{ref}} = 20$[W], whereas at time instant $t = 0.1$[s], the reference power changes to $P^{\mathrm{ref}} = 38$[W]. It is required that the converter current i remains limited below $i^{\max} = 4$[A] at all times. As it is illustrated in Figure 4A, both the proposed and the conventional PID controllers (with both proportional gains)

TABLE 2 Converter parameters

Parameters	Values	Parameters	Values
L	12 mH	L_n	10 mH
ΔL	5 mH	r	8 mH
r_n	10 mΩ	Δr	10 mΩ
C	120 μF	C_n	100 μF
ΔC	50 μF	R, R_n	10 MΩ
V_{in}	10 V	ΔR	5 MΩ

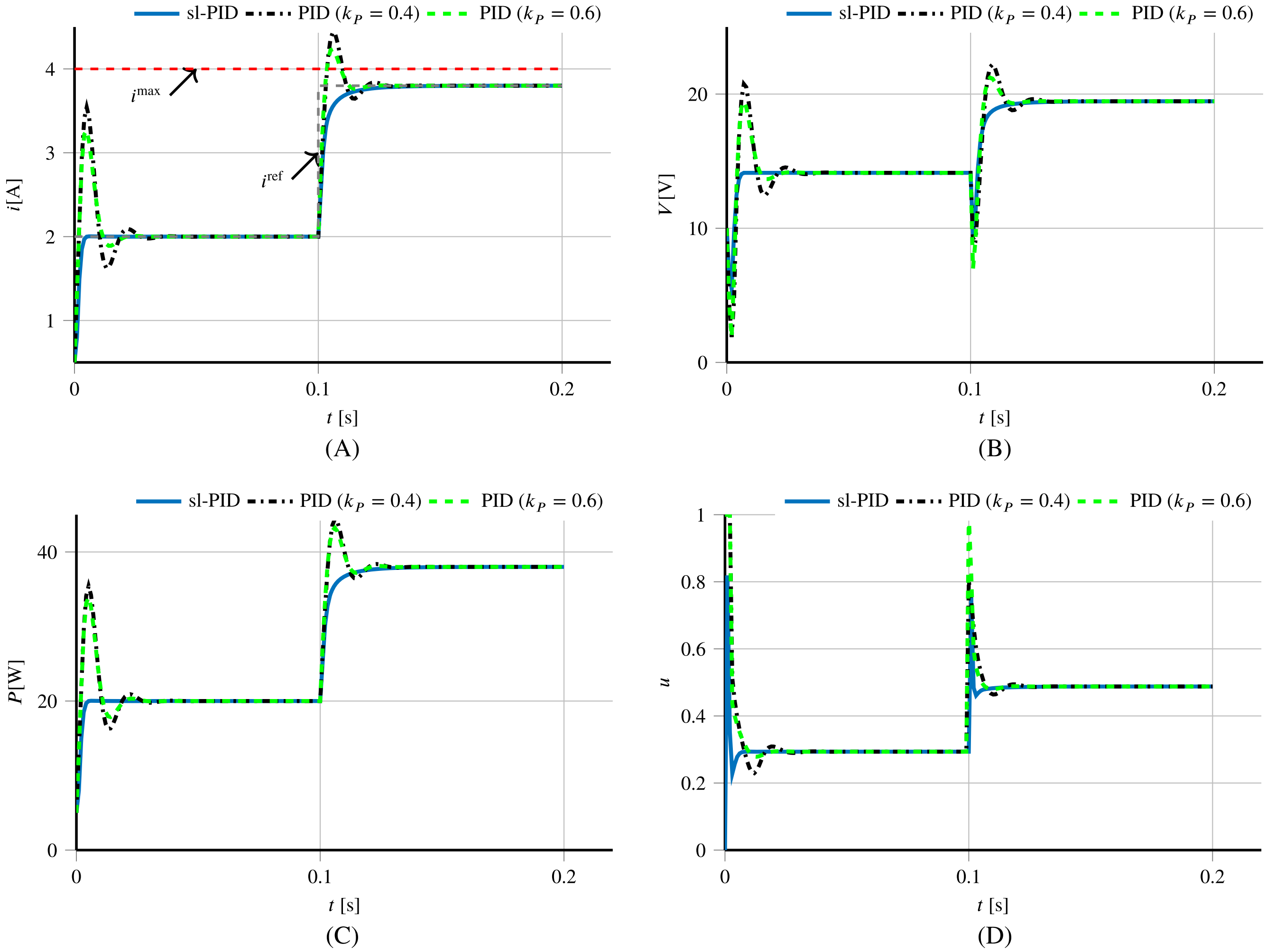

FIGURE 4 Simulation results of a dc/dc power converter using the conventional PID and the proposed sl-PID control. A, Converter current; B, Load voltage; C, Load power; D, Duty-ratio input u [Colour figure can be viewed at wileyonlinelibrary.com]

manage to regulate the converter power to any desired value. The converter voltage reaches the expected steady-state value $v_e = \sqrt{Ri^{ref}(V_{in} - ri^{ref})}$, as demonstrated in Figure 4C.

However, when the conventional PID controller is applied, the desired state constraint $|i| \leq i^{max}$ is not guaranteed at all times because during the transient response, current i violates the desired maximum value (Figure 4B). On the other hand, as expected from the theoretical analysis, the proposed nonlinear sl-PID controller leads the converter current to the desired regulation without violating the maximum bound. It should be highlighted that for a different choice of the proportional and integral gains, it is possible that the conventional PID controller can maintain the current below the maximum value for the given regulation scenario. However, there is no analytic method for calculating the gains and guaranteeing that $|i| \leq i^{max}$, $\forall t \geq 0$, for different values of i^{ref} or different load cases, opposed to the proposed approach, which guarantees the desired state constraint at all times. Furthermore, from the duty-ratio input performance shown in Figure 4D, it is clear that for a larger value of k_P for the conventional PID controller, the input value will exceed the value of 1, which represents the physical limit of the converter. As a result, the proposed nonlinear PI controller offers a superior performance during transients and guarantees the desired state limitation for the generic nonlinear model, whereas the proposed analysis offers a rigorous methodology for the selection of the proportional and integral gains.

6 | CONCLUSIONS

A novel nonlinear sl-PID controller was proposed in this paper to guarantee accurate output regulation and state constraint satisfaction for a wide class of nonlinear systems with constant uncertainties. Asymptotic stability of the desired equilibrium point and a given upper bound for the desired system states were analytically proven. A design procedure for the controller gains was also presented. The superiority of the proposed sl-PID controller compared with the conventional approach was demonstrated in two practical examples consisting of a motor with a constant power load and a dc/dc power electronic converter.

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