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Al-musawi, H., Figueiredo, F.P., Bernal, S.A. [orcid.org/0000-0002-9647-3106](https://orcid.org/0000-0002-9647-3106) et al. (2 more authors) (2019) Performance of rapid hardening recycled clean steel fibre materials. *Construction and Building Materials*, 195. pp. 483-496. ISSN 0950-0618

<https://doi.org/10.1016/j.conbuildmat.2018.11.026>

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# 1 Performance of rapid hardening recycled clean steel fibre materials

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## 9 HIGHLIGHTS

- 10 • Mixes achieve 90% of their one year flexural strength at the age of one day.
- 11 • RCSF enhances flexural strength and toughness resulting in hardening behaviour.
- 12 • Constitutive equations based on the RILEM and MC 2010 recommendations overestimate loading  
13 capacity.
- 14 • FEA analysis using multilinear  $\sigma - \epsilon$  tensile curves obtained by inverse analysis can capture well the  
15 post cracking strength and cracking pattern.

## 16 Abstract

17 To minimise disruption due to repairs of concrete pavements, rapid hardening and tough materials need  
18 to be used. This paper investigates the flexural performance of rapid hardening mortar mixes made with  
19 two commercial cement types, calcium sulfo-aluminate cement and calcium aluminate cement, for thin  
20 concrete repair applications. Three-point bending tests are performed on plain and steel fibre reinforced

21 concrete specimens containing  $45 \text{ kg/m}^3$  of recycled clean steel fibres to characterise the flexural  
22 performance of notched and unnotched prisms at different ages, ranging from one hour up to one year.  
23 The recycled fibers are shown to enhance both the flexural strength and toughness of FRC prisms,  
24 leading to hardening behaviour. Constitutive equations based on the RILEM and Model Code 2010  
25 recommendations are found to overestimate the loading capacity of the bending tests. FE analyses using  
26 multilinear  $\sigma - \epsilon$  tensile curves obtained by employing inverse analysis can capture better the post  
27 cracking strength and cracking pattern of the tested prisms.

28 **Key words:** SFRC, recycled clean steel fibres, rapid hardening cements, mechanical properties, FEA

## 29 **1. Introduction**

30 Progressive deterioration of infrastructure, particularly pavements, occurs due to increasing vehicular  
31 axle loads, worsening environmental conditions (due to climate change) and higher traffic volumes.  
32 Excessive deterioration can lead to serious service disruptions and higher costs for infrastructure owners  
33 and road users. Conventional ordinary Portland cement (OPC) based repair materials attain their  
34 strength rather slowly and need between 12h to 24h to develop sufficient strength before roads can be  
35 back in service, adding to delays and disruption during maintenance. To minimise disruption, rapid  
36 hardening cements can be used in repairs. There are several special rapid hardening Portland-free  
37 cements available in the market; such as calcium sulfo-aluminate (CSA) cement and calcium aluminate  
38 (CA) cement. CSA can achieve early rapid strength development even in cold environments and can have  
39 expansive properties. It is reported to have good durability in aggressive environments, particularly  
40 when exposed to sulfates [2]. Furthermore, this cement requires less energy for its production  
41 compared to OPC [1], thus it is considered to be environmentally friendly. However, despite its lower  
42 energy demand, it is still more expensive due to the cost of its raw materials.

43 CA cements are characterised by high early strength development and high resistance to elevated  
44 temperatures, depending on their aluminum content. An important aspect for the rapid strength  
45 development of this cement is the substantial amount of heat of hydration which can result in high heat  
46 generation [3]. Self-heating may be a concern in sections thicker than 100 mm [3], but not necessarily  
47 for thinner repair layers. Despite the high temperature rise during hydration, CA concretes do not seem  
48 to be overly susceptible to thermal cracking. This may be due to creep relaxation of thermally induced  
49 strains, facilitated by a conversion reaction, during which some metastable phases of this cement  
50 convert to stable phases of lower volume [3, 4]. As porosity increases, the densification due to  
51 conversion causes loss of strength [3]. Hence, when used for repairs, the key concern to be addressed is  
52 cracking due to restrained shrinkage.

53 Restrained shrinkage is one of the main factors that govern the serviceability and durability of concrete  
54 repairs [5,6]. Shrinkage in concrete results due to moisture diffusion from the new concrete to the  
55 environment and to the concrete substrate [7] if not adequately saturated. However, shrinkage  
56 deformation (of the new layer) is restrained by the substrate layer leading to the development of tensile  
57 and interfacial shear stresses. If these stresses exceed the material capacity at any time, cracking will  
58 develop in the repair material and/or debonding along the interface between the repair material and  
59 the substrate. Micro-cracks induced by shrinkage can propagate and coalesce into macro-cracks under  
60 the effect of applied loads.

61 Cracks beyond a certain width can adversely affect the durability of repair materials by creating easy  
62 access for deleterious agents leading to early saturation, freeze–thaw damage, scaling, and steel  
63 corrosion, which promote further internal and external cracking and accelerate the rate of deterioration  
64 [8]. This issue can be worsen with rapid hardening (non-expansive) materials due to the rapid hydration  
65 rate which accelerates shrinkage development. Furthermore, due to the rapid stiffness development  
66 and decrease in creep compliance of rapid hardening cements [9], their ability to redistribute stresses

67 may be affected, thereby increasing cracking potential. To address this issue, fibres can be added to  
68 control crack widths [10] as well as increase the tensile strength and fatigue resistance [11], thus  
69 resulting in more durable layers. To reduce the environmental impact of manufactured steel fibres  
70 (MSF), recycled clean steel fibres (RCSF) can be used as alternative fibre reinforcement.

71 During the manufacture of tyres, parallel steel cords are embedded in continuous thin rubber belts.  
72 After being cut to shape, these are placed in overlapping layers to provide flexible reinforcement within  
73 the tread and side walls of the tyre. The complex configuration of each layer generates significant levels  
74 of waste (approximately 5% by mass). The available amount of waste steel cord is therefore around  
75 100,000 tonnes per year worldwide. The steel reinforcement used in tyre manufacture typically consists  
76 of parallel filaments of very fine wire (0.1-0.4 mm dia.) twisted together to form a cord about 0.5-1.0  
77 mm in diameter [12]. Recycled clean steel fibre (RCSF) filaments extracted from pre-vulcanised rubber  
78 belt offcuts have become available recently and were adopted in this study. However, knowledge on  
79 their use in concrete is scarce and it is limited to research at the University of Sheffield [13]. Knowledge  
80 of the effect of industrial fibres on CSA and CA matrices is also rather limited [9,14-16] and no published  
81 data exist regarding the effect of RCSF. A study on the effect of CSA matrix on pullout performance of  
82 steel fibres [9] suggests that the synergetic effect of a stiff matrix like ettringite and high modulus steel  
83 fibres can increase crack propagation in the composite material, evidenced by an increase in debonding  
84 energy density.

85 Since cracking is the main concern for repairs, understanding the effect of fibres in controlling crack  
86 widths under mechanical and hygral loads, as well as the complex interaction of shrinkage, stiffness and  
87 tensile strength evolution are of paramount importance. For this purpose, finite element analysis can be  
88 a useful tool. However, appropriate material parameters need to be determined experimentally and the  
89 tensile  $\sigma$ - $\epsilon$  curves of the repair materials need to be derived from direct tension or bending results.  
90 Although there are several procedures in the literature to derive the  $\sigma$ - $\epsilon$  of SFRC in tension [17-20], they

91 may not be entirely suitable for modelling mortars reinforced with RCSF due to the different fracture  
92 energies of the two concretes. In numerical studies performed by [20, 21], it was found that RILEM  
93 proposed  $\sigma$ - $\epsilon$  equations overestimate the predicted capacity of FRC. As a result, a simplified  $\sigma$ - $\epsilon$  model  
94 was suggested to overcome issues in the other methods and to include the post-consumer tyres steel  
95 fibres (RTSF) effect.

96 This paper presents experimental and numerical work on the flexural performance of RCSF on rapid  
97 hardening mortars produced using CSA or CA as sole cementitious materials. Constitutive relationships  
98 derived based on code recommendations and by others [19, 20] are used to predict flexural behaviour  
99 and the results are compared with predictions obtained from inverse analysis.

## 100 **2. Experimental details and methodology**

### 101 **2.1. Materials**

102 Two commercial cement types were used in this study; calcium sulfoaluminate cement<sup>1</sup> (CSA) and rapid  
103 setting calcium aluminate cement<sup>2</sup> (RSC). According to the manufacturer, RSC consists of hydrated  
104 alumina, oxides of iron and titanium, with small amounts of silica. For production of mortars, fine  
105 aggregates, medium grade river washed sand (0-5mm sourced from Shardlow in Derbyshire, UK,  
106 SG=2.65, A = 0.5, FM = 2.64), were used. Recycled clean steel fibres (RCSF) were obtained from tyre  
107 cords extracted from un-vulcanised rubber belts (see Figure 1). The length of the RSCF used in this study  
108 was 21 mm and the diameter 0.2 mm. The strength of these fibres is reported to exceed 2600 MPa [13].  
109 Superplasticiser<sup>3</sup> was added to enhance the workability and adjust the setting time.

110



111

112

**Figure 1.** Photograph of the RCSF used in this study

113

## **2.2. Mortar mix design**

114

A total of  $600 \text{ kg/m}^3$  of cement was used with low w/c ratios to obtain high early strength. For durability

115

requirements, w/c should be kept lower than 0.4. However, as CSA cement consumes more water to

116

form hydration products than ordinary Portland cement [22], this limit can be relaxed slightly for this

117

cement. As a result, two different w/c ratios and SP dosages were tested. The w/c ratios for mixes with

118

CSA cement were 0.4 and 0.41, and 0.35 and 0.36 for RSC mixes. The water content and superplasticiser

119

(SP) were carefully selected to achieve a workable mix with setting time of no longer than 15 minutes.

120

Fibre dosage of  $45 \text{ kg/m}^3$  ( $V_f = 0.57\%$ ) was investigated as is commonly used in European practice for

121

structural applications. The plain and fibre reinforced mortar mixes for each cement type are almost

122

identical, to reliably investigate the effect of fibres on the mechanical properties. The details of the

123

optimised mortar mixes are summarised in Table 1.

124

The specimens were cured for one hour before demoulding and exposure to standard laboratory

125

conditions.

126

### **Table 1**

---

<sup>1</sup> provided by Kershin International Co., Ltd

<sup>2</sup> sourced from Instarmac

<sup>3</sup> Sika Viscoflow 2000

127 Mortar mix composition

mix	Cement (kg/m <sup>3</sup> )	w/c	Sand (kg/m <sup>3</sup> )	SP <sup>a</sup>	Fibre dosage (kg/m <sup>3</sup> )
CSA	600	0.40	1420	0.60	0
FCSA	600	0.41	1420	0.61	45
RSC <sup>b</sup>	600	0.35	1300	0.20	0
FRSC	600	0.36	1300	0.21	45

128 <sup>a</sup> % by cement mass. <sup>b</sup> mixes containing CA cement are called RSC in this study.

129

## 130 **2.3. Fresh state properties**

### 131 **2.3.1. Vicat test**

132 The setting time of cement pastes was assessed using an automatic Vicat apparatus according to ASTM  
133 C191 (2013) [23]. As the cements used in this study are fast setting, the instrument was set to take  
134 measurements every 30 seconds.

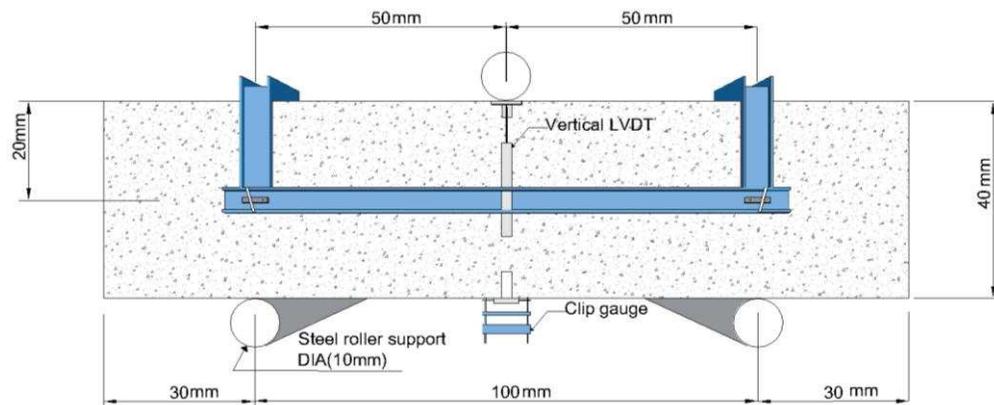
### 135 **2.3.2. Semi-adiabatic calorimetry**

136 The semi-adiabatic calorimeter records the temperature evolution and key temperature related  
137 properties for a tested mix, such as time to peak heat, peak heat, and cumulative heat [24]. Since the  
138 mortar mixes are designed for thin repairs, heat loss due to dissipation is expected to take place and  
139 hence, the semi-adiabatic test could reveal a temperature evolution that is close to practical  
140 applications. After mixing the required quantity for each mix, the mortar was directly placed in an  
141 insulated thermal flask cylinder of 0.5 l and a thermocouple was inserted inside the mortar to record the  
142 temperature.

## 143 **2.4. Flexural tests**

144 To characterise the flexural performance, mortar prisms of 40 × 40 × 160 mm were tested according to  
145 BS EN 13892-2 [25]. To obtain the load deflection curve after the peak load, displacement control was  
146 adopted rather than load control as required by the standard. The rate of loading was 0.25 mm/min

147 until 1 mm deflection, and 1 mm/min after that. To eliminate errors due to machine stiffness, spurious  
148 support displacements and local concrete crushing, a specially designed aluminum yoke (based on the  
149 Japanese standard JSCE-SF4 [26]) was mounted on the specimens. To assess the flexural behaviour over  
150 time, the prisms were tested at one hour, three hours, one day, seven days, 28 days and 365 days. The  
151 test was also performed on notched prisms (the notch depths range from 3.57 to 4.94 mm) to assess  
152 crack development. The Crack Mouth Opening Displacement (CMOD) was measured at mid span with a  
153 12.5 mm clip gauge (mounted across the bottom part of the notch, Figure 2). For practical reasons, this  
154 test was performed at 2 days (at the earliest age) and up to one year.



155

156

**Figure 2.** Flexural test set up

157

## **2.5. Compressive strength**

158 Directly after flexural testing, the halves of the fractured prisms were tested in uniaxial compression  
159 according to BS EN 13892-2 [25]. Only the one-hour compressive strength of FRC specimens was  
160 examined separately due to practical time constraints.

161

## **3. Experimental Results and Discussion**

162

### **3.1. Fresh state properties of rapid hardening materials**

163

The water content and SP dosage were optimised for each mix to achieve a workable mix with setting

164 time of no longer than 15 minutes. As shown in Table 2, the CSA cement had a relatively shorter setting  
 165 time compared to the RSC cement. Slightly higher water content and SP dosages for the fibre reinforced  
 166 mixes lead to a slight increase of the setting time for these mixes.

167 **Table 2**

168 Setting time and maximum temperature ( $T_{peak}$ ) for different mixes

Mixes	Vicat setting time (min.)		$T_{peak}$ ( $^{\circ}$ C)
	Initial	Final	
CSA	9.5	10.5	68
FCSA	9.5	11.0	68
RSC	12.0	14.5	91
FRSC	12.5	15.0	88

169

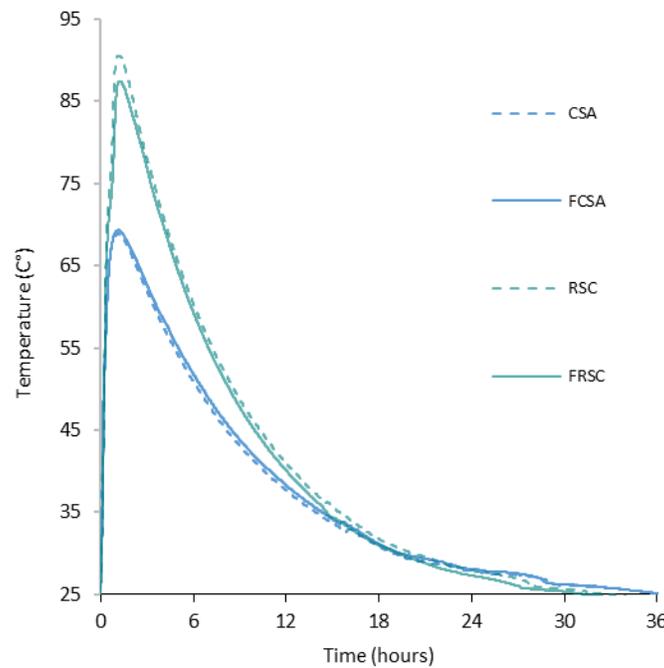
170 The results of the semi-adiabatic calorimetry test (for the first 36 hours) are shown in Figure 3. For mixes  
 171 with CSA cement, the peak temperature ( $T_{peak}$ ) was about 68° C (see Table 2) occurring during the first  
 172 hour regardless of fibre content. The temperature rise in RSC mixes was much higher than in mixes with  
 173 CSA cement, with  $T_{peak}$  at 91° and 88° C for RSC and FRSC, respectively. The time half way to the peak  
 174 ( $T_{1/2 peak}$ ) can be taken as an indication of the initial setting time of cementitious mixes [27]. For CSA and  
 175 FCSA,  $T_{1/2 peak}$  was achieved at around 11 minutes, whilst for RSC and FRSC, it was recorded at around 16  
 176 minutes. These results agree well with the results of the vicat test. The temperature achieved for these  
 177 cements upon hydration dropped to laboratory temperature in less than 24 hours. Heat dissipation is  
 178 expected to occur faster onsite than in the semi-adiabatic test and, therefore, no major thermal cracking  
 179 is expected for thin repairs, especially when curing is applied during the first two hours when  $T_{peak}$   
 180 occurs.

181 **3.2. Mechanical performance of rapid hardening mortars**

182 **3.2.1. Compressive strength**

183 The average compressive strength  $f_{cu}$  (from six specimens) and standard deviation developed over time

184 is shown in Figure 4. At one hour, FCSA achieved the highest compressive strength of 26.1 MPa while  
185 RSC achieved 17.2 MPa. This behavior changes at later ages as RSC achieves a higher strength than FCSA  
186 by approximately 6% after one-year. The fibres seem to have a positive effect on the compressive  
187 strength of both mortars, with the highest strength increase noticed at one hour (24% increase in  $f_{cu}$ ). At  
188 later ages, this increase ranges from 10% to 17%.



189

190

**Figure 3.** Temperature rise for mixes in semi-adiabatic test

191 There is no consensus in literature on the effect of fibers on compressive strength. While some  
192 researchers [28-30] report a strength enhancement of up to 20% for Portland cement-based specimens  
193 containing recycled fibres with dosages less than  $50 \text{ kg/m}^3$ , others [31-33] found only a marginal effect  
194 due to air entrainment.

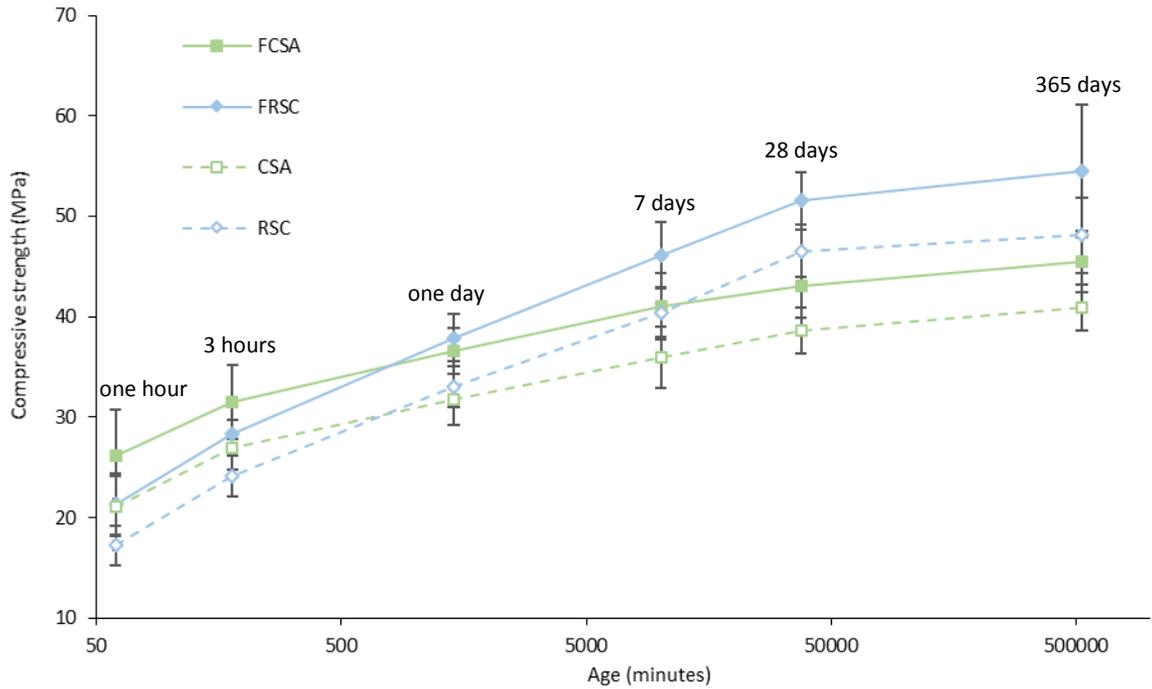
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197

No strength reduction has been observed for any of the mixes at the age of one-year, indicating that  
there were no significant conversion issues. It should be noted that for fully cured rapid hardening CSA  
mortar-based samples (tested at 28 days), a compressive strength of 31.4 – 52.6 MPa for w/c ratios 0.4

198 – 0.5 was reported in literature [34] and this agrees well with the results of this study.



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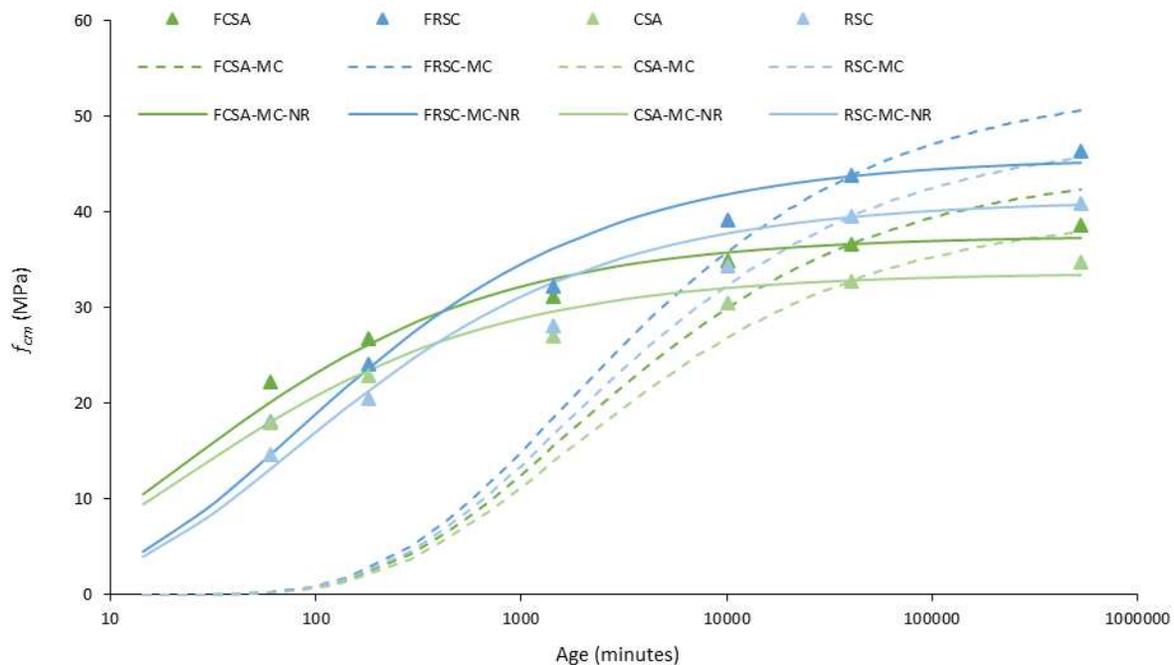
200 **Figure 4.** Development of  $f_{cu}$  as a function of time

201 To describe the compressive strength development with time, the  $\beta_{cc}(t)$  function that describes the  
 202 strength development with time used in Model Code 2010 [18] is followed.

$$\beta_{cc} = \exp\left\{s \cdot \left[1 - \left(\frac{28}{t}\right)^{0.5}\right]\right\} \quad \text{equation 1}$$

203 where,  $t$  is the concrete age in days,  $s$  is a coefficient that depends on the class of cement which ranges  
 204 from 0.2 – 0.38 for  $f_{cm} \leq 60$  MPa. As the cements used in this study are rapid hardening, a 0.2 value for  $s$   
 205 was adopted. To obtain the strength at various ages,  $\beta_{cc}(t)$  is multiplied by the mean compressive  
 206 strength at the age of 28 days ( $f_{cm}$ ). The estimated compressive strength at various ages is shown against  
 207 the experimental results in Figure 5. As expected, the function underestimates the strength at the early  
 208 ages by approximately 100% for the different rapid hardening mixes. As the strength evolves very  
 209 rapidly at the early ages and then it slows down, smaller  $s$  values could offer a better representation for  
 210 strength development with time. The  $s$  values of 0.024 and 0.044 for mixes with CSA and RSC cements,

211 respectively, were found by regression analysis to represent well the strength evolution with time (see  
 212 Figure 5).

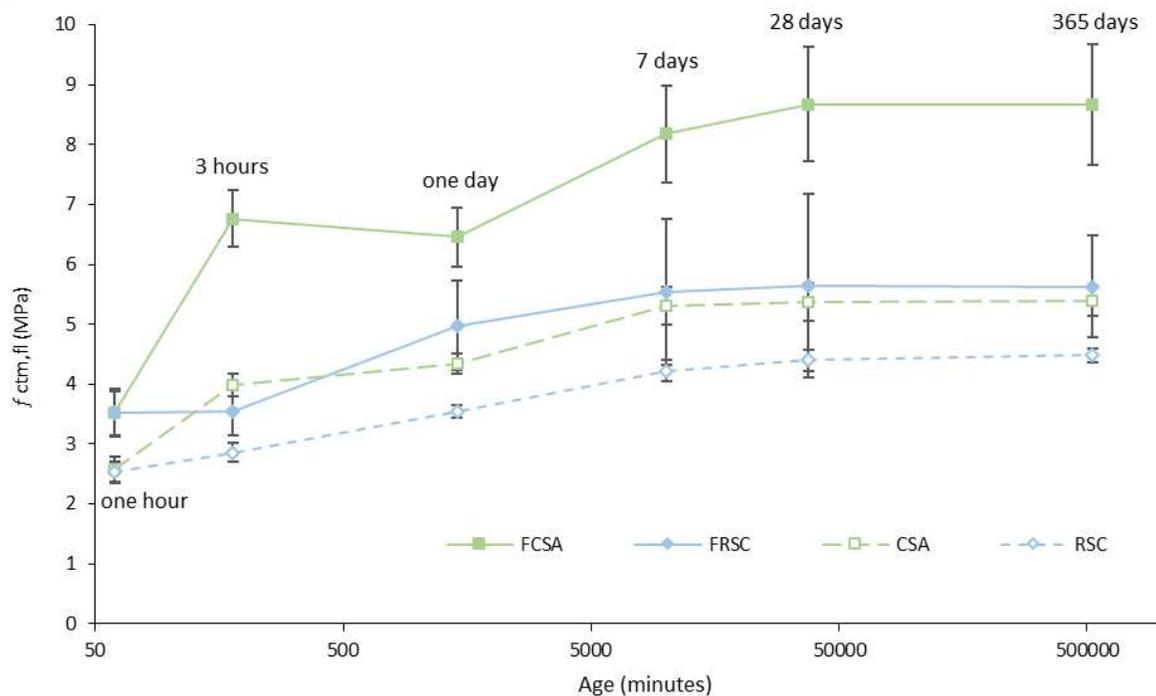


213  
 214 **Figure 5.** Development of experimental and estimated  $f_{cm}$  as a function of time using  $s = 0.2$  (dashed lines) and  
 215 suggested  $s$  values (solid lines-NR)

216 **3.2.2. Flexural behaviour**

217 The average flexural strength development over time (and standard deviation) is illustrated in Figure  
 218 6. The reported values represent the limit of proportionality (LOP), or first cracking strength ( $f_{ctm,fl}$ ),  
 219 determined according to BS EN 14651:2005 [35]. It is noted that strength develops very fast and both  
 220 plain and fibre reinforced specimens achieved 90% of their one-year strength in one day. The specimens  
 221 made with CSA cement showed higher flexural strength than those with CA cement tested at the same  
 222 age, probably due to the rigid dense crystal microstructure of the CSA cement [9]. RSC mixes have lower  
 223 w/c ratio, hence, their compressive strength is expected to be higher in the long term. Due to high  
 224 shrinkage in RSC mixes, their flexural strength is reduced. The effect of RCSF on the flexural strength

225 enhancement of the mixes is evident at all ages. Compared to their plain counterparts, FCSA and FRSC  
 226 mixes showed a flexural strength increase of approximately 36% to 70% and 24% to 41%, respectively.  
 227 This agrees well with Hu et al. [33], who reported an increase of 45% - 70% in  $f_{ctm,fl}$  of concrete  
 228 reinforced with blends of manufactured and post-consumer recycled fibres.



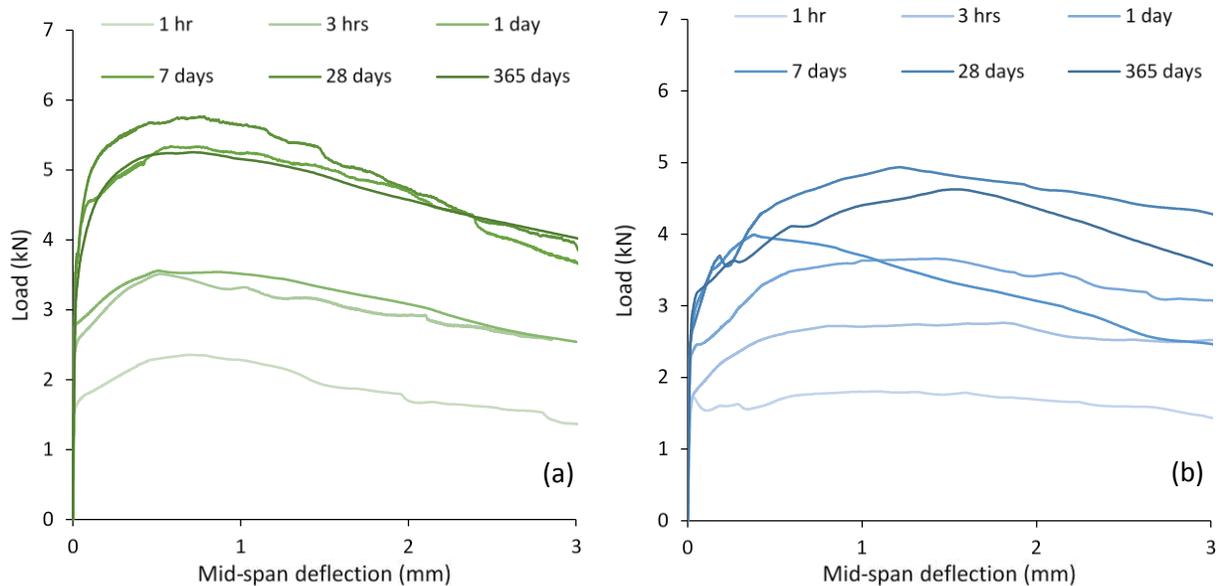
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230 **Figure 6.** The flexural strength  $f_{ctm,fl}$  development as a function of time

231 The load-deflection curves for FCSA and FRSC prisms are shown in Figure 7. The behaviour of the  
 232 specimens made with the unreinforced mixes is not shown as they failed suddenly after peak load  
 233 without any post cracking strength, highlighting the poor toughness of plain mortars in tension. The  
 234 deflection hardening shown by reinforced mixes can be attributed to the high number of fibres spanning  
 235 the cracked section and the excellent bond between steel fibres and dense matrix systems, like the CSA  
 236 cement. This hypothesis is supported by the fact that in the current study, many specimens developed  
 237 more than one principal crack, confirming the excellent load transfer by the RSCF. It should be noted  
 238 that the preferential alignment of the fibres in the direction of stress due to the small mould size ( $40 \times$

239 40 × 160 mm) may have contributed to this. Deflection hardening was also reported in a study by  
 240 Bordelon [36] for concrete specimens cut from prisms of 150 × 150 × 450 mm and tested using a 50 mm  
 241 beam depth (to simulate a thin overlay). Deflection hardening performance for notched concrete prisms  
 242 reinforced with 45kg/m<sup>3</sup> of blends of recycled post-consumer and manufactured steel fibres was also  
 243 reported in a recent study published by Hu et al. [33].

244 At large deflections (greater than 2 mm), the FCSA specimens show a slight reduction in load resistance  
 245 compared to FRSC specimens, possibly due to the inherent brittleness of the CSA cement. However, in  
 246 most repair applications, it is not expected that the mortar will reach such high level of deformation and  
 247 as a result, minimal cracking is expected.



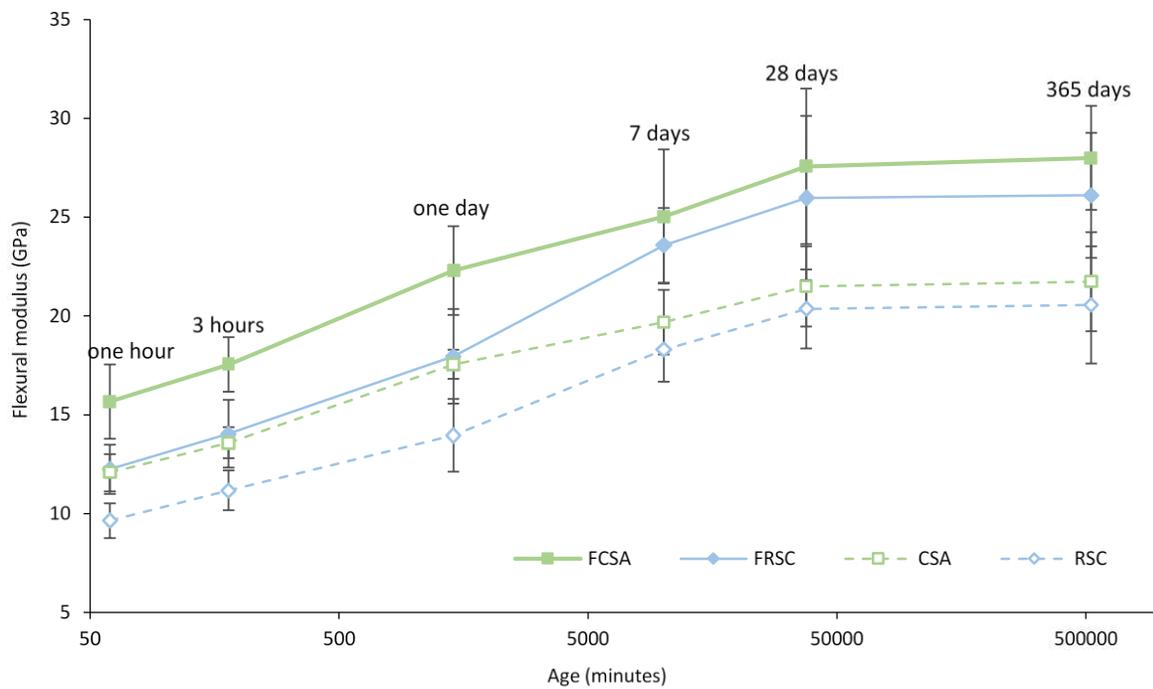
248  
 249 **Figure 7.** Load-deflection response of rapid hardening fibre reinforced mortars tested at different ages: (a) FCSA;  
 250 (b) FRSC

### 251 3.2.3. Flexural modulus of elasticity ( $E_{fm}$ )

252 The flexural modulus of elasticity ( $E_{fm}$ ) was determined from load-deflection curves using elastic analysis  
 253 and ignoring shear deformations.  $E_{fm}$  is the maximum flexural modulus between 30 – 60% of the peak

254 load ( $P_{peak}$ ) [37]. Figure 8 shows the development of  $E_{fm}$  and related standard deviations over time for all  
 255 mixes. The plain mortar mixes are shown in dotted lines. As with flexural strength, the stiffness of the  
 256 mixes develops quickly and reaches around 90% of the one year modulus within 7 days.

257 The fibres have a remarkable effect on the modulus of elasticity. FCSA and FRSC have higher  $E_{fm}$   
 258 compared to CSA and RSC mixes respectively with the highest noticeable increase (29.7%) for FCSA  
 259 occurring at one-hour of age. This behaviour was not reported in [33] and [38] who only noticed a  
 260 marginal effect on the modulus of concrete with fibre addition. The remarkable increase in modulus of  
 261 elasticity, though also reflected in the flexural strength, is beyond what is expected from a perfect  
 262 composite. This may be partially due to fibre alignment, but also to the slightly longer mixing time that  
 263 was necessary to integrate the fibres. An increase of approximately 36% in the modulus of elasticity of  
 264 OPC based mortars reinforced with 2% (by volume) industrial steel fibres was reported in literature [39].

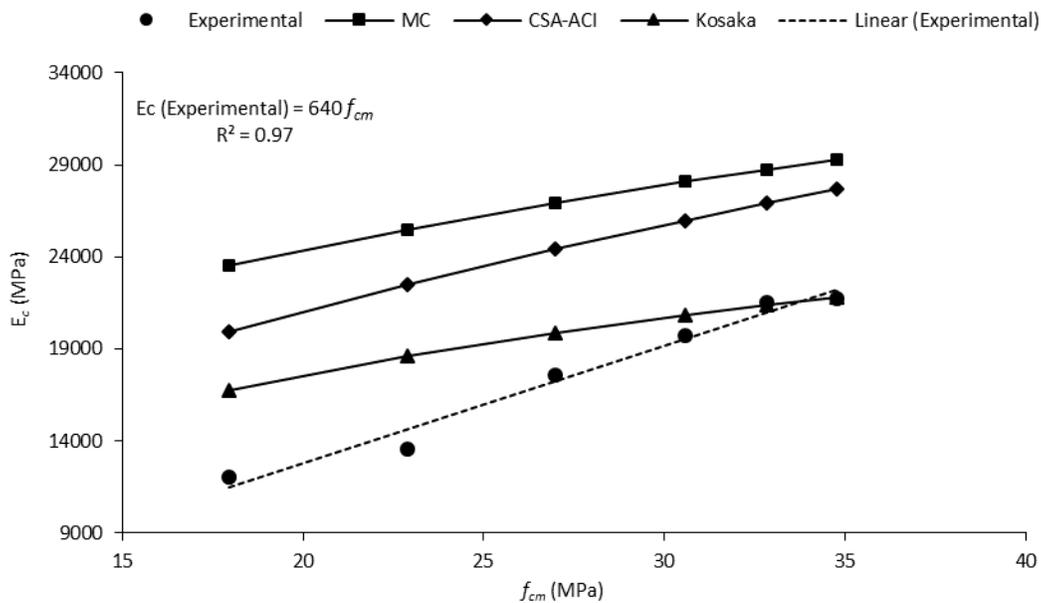


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266

**Figure 8.** Flexural modulus ( $E_{fm}$ ) of fast setting fibre reinforced mortars as a function of time

267 To estimate the modulus of elasticity of the mixes, based on compressive strength, equations from  
 268 Model code [17], ACI 318-05 [40] and Kosaka et al. [41] were used. The latter equation was developed  
 269 specifically for mortars. The estimated modulus of elasticity ( $E_c$ ) for CSA (using the above equations) is  
 270 presented in Figure 9. As shown, the equations overestimate  $E_c$  for CSA mix, especially at the early ages.  
 271 It should be noted that both Model code and ACI code adopt equations that use the 1/3 and 1/2 power  
 272 of  $f_{cm}$  respectively. However, the results show that for these mortars, the linear relationship is more  
 273 appropriate and the constant values of 720, 580, 640 and 520 were determined by regression analysis  
 274 for FCSA, FRSC, CSA and RSC mixes respectively.



275  
276 **Figure 9.** The relationship between  $f_{cm}$  and  $E_c$  using different equations for CSA mix

277 **3.2.4. Relationship between measured deflection and CMOD values**

278 A linear relationship between CMOD and average deflection is suggested in BS EN 14651:2005 [35], as  
 279 given below,

280 Average deflection (mm) =  $k \times \text{CMOD (mm)} + 0.04 \text{ mm}$ ,  $k = 0.85$

281 This linearity has also been confirmed for FCSA and FRSC at all ages tested with coefficients of  
282 determination  $R^2 > 0.99$ , but as expected with lower K values, between 0.55 and 0.65, due to the  
283 different geometry of the testing arrangement. It should be noted that the CMOD measured by the clip  
284 gauge is corrected for the position of the clip gauge using the BS EN 14651:2005 [35].

285 A relationship between deflection and CMOD can facilitate the testing of such materials by using clip  
286 gauges only to measure the CMOD as accurate measurement of deflection requires the use of a special  
287 frame (yoke) to obtain net deflection. It also provides a benchmark for comparisons.

### 288 **3.2.5. Residual flexural tensile strength ( $f_R$ )**

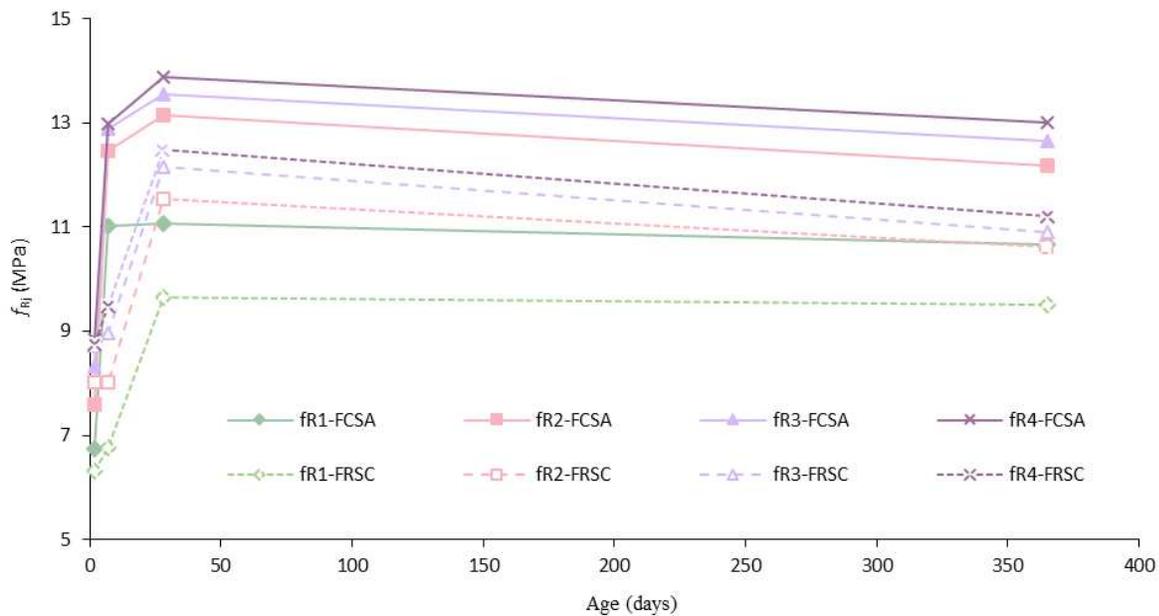
289 RILEM TC 162-TDF [42] presents a methodology to calculate the residual flexural tensile strength of SFRC  
290 prisms, which was later adopted by BS EN 14651:2005 [33]. Residual flexural stresses ( $f_{R1}$ ,  $f_{R2}$ ,  $f_{R3}$  and  
291  $f_{R4}$ ) are calculated from the load-CMOD curves at 0.5, 1.5, 2.5 and 3.5 mm of CMOD, respectively.  
292 However, these CMODs are suggested for concrete prisms of 500 mm span length. For this study, the  
293 residual stresses are calculated at CMOD equal to 1/5 of those used for 500 mm span specimens; i.e.  
294 0.1, 0.3, 0.5 and 0.7.

295 Figure 10 shows the  $f_{Ri}$  values of all FCSA and FRSC mixes tested at different ages. The  $f_R$  values for  
296 FCSAs are shown in solid lines while FRSCs are shown in dashed lines. It is noticed that for both mixes  
297 the  $f_R$  values continue to increase from CMOD 0.1 mm to 0.7 mm which shows the high efficiency of the  
298 RCSF in carrying the loads across cracks. This is also evidenced by the multiple cracks that form in some  
299 samples at, or more than, seven days of age. The residual strengths of FCSA are higher than those of  
300 FRSC for the same crack width, which implies better bond strength for RCSF in FCSA matrices.

301 The  $f_R$  values continue to increase with time for both FRC mixes and reach their peak values at 28 days.  
302 However, there is a slight strength reduction at one year compared to 28 days. This could be attributed

303 to the effect of the conversion reaction occurring in the RSC cement. This is unlikely, however, as there  
 304 was no reduction in compression strength at one-year of age. Another possible explanation is the effect  
 305 of shrinkage on the bond strength of RCSF. This reduction in  $f_R$  is more obvious at higher CMOD levels  
 306 (for  $f_{R2}$  to  $f_{R4}$ ), which means that the frictional resistance along the fibres reduces slightly at one year.

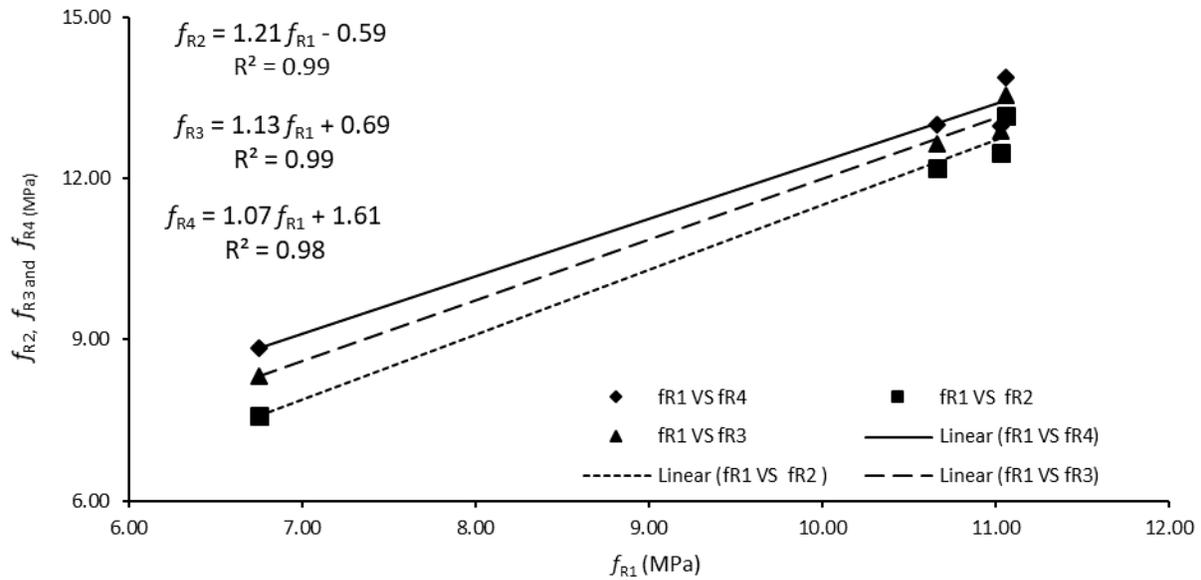
307 Figure 11 and Figure 12 show the relationship of  $f_{R1}$  vs  $f_{R2}$ ,  $f_{R1}$  vs  $f_{R3}$  and  $f_{R1}$  vs  $f_{R4}$  for FCSA and FRSC,  
 308 respectively. The values of  $f_{R2}$ ,  $f_{R3}$  and  $f_{R4}$  correlate very well with  $f_{R1}$  for FCSA prisms with  $R^2 \geq 0.98$ . A  
 309 similar trend was also found for FRSC prisms, however, with a relatively smaller coefficient of  
 310 determination ( $R^2 \geq 0.92$ ). A linear relationship between  $f_{R1}$  vs  $f_{R3}$ ,  $f_{R1}$  vs  $f_{R4}$  were also reported by  
 311 Zamanzadeh et al. [43] for unclassified RTSF. The strong correlation between the  $f_R$  values can lead to  
 312 simpler design guidelines.



313

314

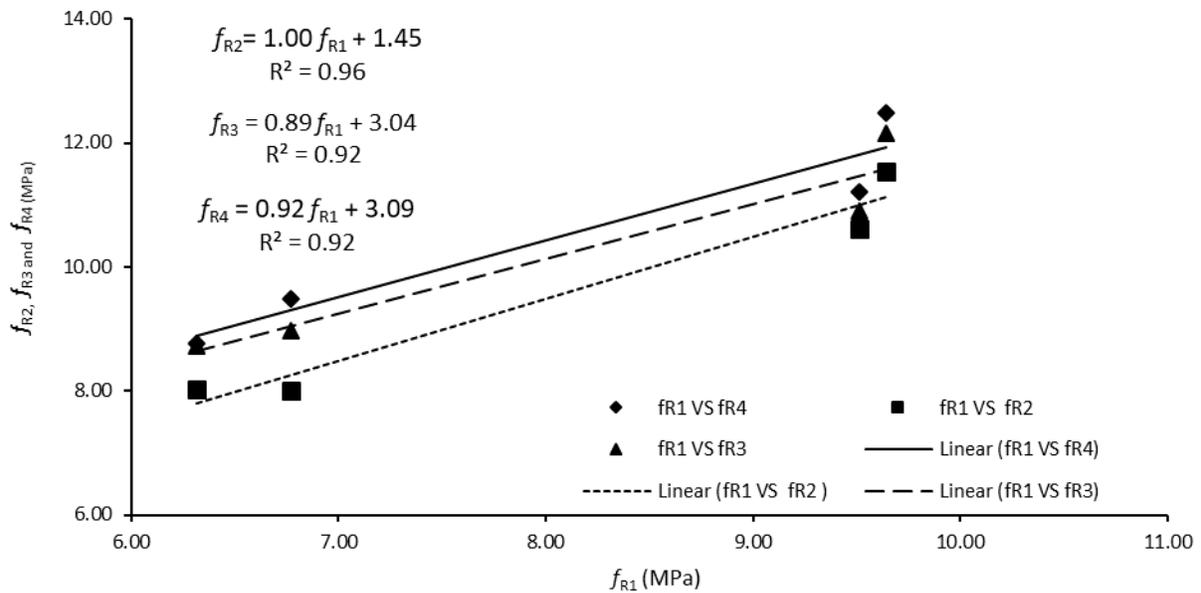
**Figure 10.**  $f_R$  values of FCSA and FRSC prisms (in MPa) development with age



315

316

**Figure 11.** Correlation between  $f_{R1}$  and  $f_{R2}$ ,  $f_{R1}$  and  $f_{R3}$ ,  $f_{R1}$  and  $f_{R4}$  of FCSA prisms



317

318

**Figure 12.** Correlation between  $f_{R1}$  and  $f_{R2}$ ,  $f_{R1}$  and  $f_{R3}$ ,  $f_{R1}$  and  $f_{R4}$  of FRSC prisms

319

#### 4. Numerical study

320

##### 4.1. FE modelling

321

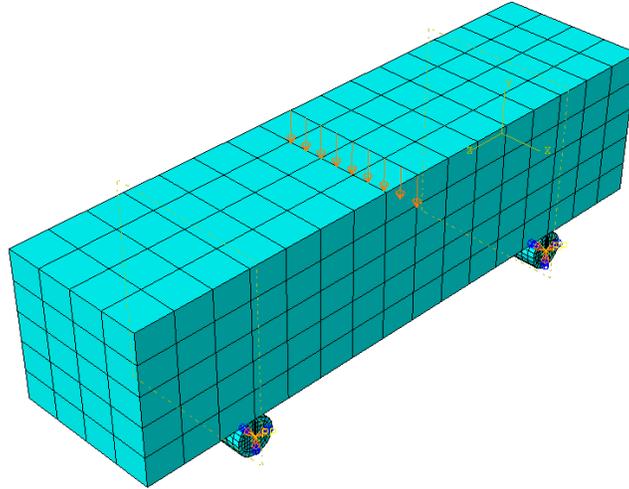
To model the flexural performance of these materials, the FE package ABAQUS is used, which offers

322

three material models for concrete simulation; Concrete Smearred Cracking (CSC), Brittle Cracking (BC)

323 and Concrete Damaged Plasticity (CDP) [44]. It was found that, for this application, CSC is prone to  
324 numerical instabilities soon after crack development. Similar issues were also reported in [45] when  
325 modelling SFRC prisms using CSC. Although the BC model was applied successfully to model FRSC [46], it  
326 was considered unsuitable for the current study as it assumes that the concrete remains elastic in  
327 compression. Since, due to the high flexural strength of the mortars, in this study, the material is  
328 expected to become non-linear in compression. Therefore, the analysis was performed by using the  
329 concrete damage plasticity (CDP) model for which the user can define the tensile and compression  
330 behavior of concrete in as many steps as required. In CDP, the ratio of biaxial to uniaxial compressive  
331 strength ( $\sigma_{b0}/\sigma_{c0}$ ) and the ratio of the second stress invariant on tensile meridian to that on the  
332 compressive meridian ( $K_c$ ) characterise the failure surface of concrete. The dilation angle ( $\psi$ ) and flow  
333 potential eccentricity ( $\epsilon$ ) are used to define the flow rule [44].  $\sigma_{b0}/\sigma_{c0}$  was taken as 1.2 (slightly higher  
334 than the value usually assigned for plain concrete due to presence of fibres),  $K_c$  was 0.667,  $\psi$  was  $31^\circ$   
335 and after a sensitivity analysis for  $\epsilon$ , the default value of 0.1 was adopted. The CDP model can be  
336 regularised by using viscoplasticity to assist in overcoming convergence issues, that occur in materials  
337 exhibiting softening behaviour in implicit analysis computations, by permitting the stress to be outside  
338 the yield surface. Since high values of viscosity ( $\mu$ ) compared to characteristic time increment can  
339 compromise the results, a value of zero was adopted.

340 Unnotched beams under 3-point bending were modelled in Abaqus with the same dimensions as tested.  
341 The mesh was kept constant at 10 mm size (Figure 13) and a 3D 20-noded quadratic brick element with  
342 reduced integration (C3D20R) was chosen, as second-order elements are very effective in bending-  
343 dominated problems [44]. Uniform displacement control loading was applied to minimise convergence  
344 problems and to better simulate the experimental loading conditions.



345

346

**Figure 13.** Prism assembly in Abaqus

347

#### **4.2. Evaluation of tensile constitutive equations**

348

RILEM TC 162-TDF (RILEM) [17], MODEL CODE 2010 (MC) [18], Barros et al. (Barros) [19] and Hu et al.

349

(Hu) [20] procedures were selected to derive the tensile constitutive equations. Although MC allows the

350

use of stress-crack width relationship, RILEM, Barros and Hu models all use stress-strain relationships,

351

and since stress-crack width relationship also leads to mesh dependency in CDP, it was decided to the

352

use stress-strain approach in modelling, to be able to make a direct comparison between different

353

models. The derived tensile  $\sigma$ - $\varepsilon$  relationships (see Table 3) using the aforementioned procedures were

354

implemented in Abaqus to determine the load-deflection response of FCSA and FRSC prisms (at 28

355

days). MC requires the maximum value of crack width ( $w_u$ ) to calculate the stress at ultimate strain. The

356

value 0.5 mm was used for the max crack width as it corresponds to  $CMOD_3$ .

357

The predicted numerical load-deflection curves are compared against the experimental results for FCSA

358

in Figure 14. It can be seen that all the approaches fail to model the full behaviour of the prisms and for

359

most of them the analysis does not converge beyond 0.6 mm (even after using high values of  $\mu$ ). At 0.2

360

mm deflection, RILEM, MC and Barros overestimate the loading capacity by 29.44%, 16.65% and 7.11%

361

while Hu underestimates the loading by 14.88% respectively. Barros's model, however, can capture the

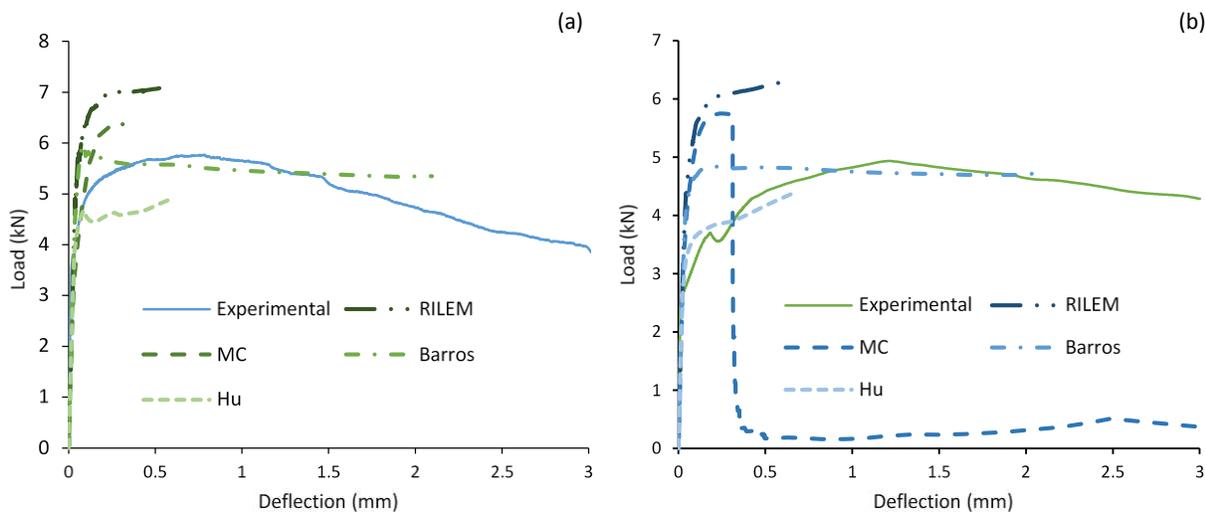
362 post-cracking behaviour of FCSA up to a certain extent. The models are even less effective in predicting  
 363 the flexural behaviour of FRSC (see Figure 14). Overall, none of the above models seem to be able to  
 364 capture the complete load-deflection behaviour of the tested specimens.

365 **Table 3**

366  $\sigma$ - $\epsilon$  relationships for FCSA and FRSC at 28 days using different approaches

Mixes	RILEM		MC		Barros		Hu	
	$\sigma$	$\epsilon$	$\sigma$	$\epsilon$	$\sigma$	$\epsilon$	$\sigma$	$\epsilon$
FCSA	9.473	0	2.980	0	7.037	0	4.771	0
	4.977	0.000263	3.311	0.000030	3.981	0.001056	2.986	0.001892
	5.140	0.024814	4.977	0.002319	3.751	0.103864	3.929	0.024857
	0.095	0.025000	4.561	0.012335	0.080	0.104000	0.050	0.025000
	0.090	0.500000	0.030	0.012500	0.074	0.500000	0.048	0.500000
			0.029	0.500000				
FRSC	6.165	0	3.354	0	4.580	0	3.105	0
	4.340	0.00017	3.727	0.000006	3.472	0.001066	2.604	0.002019
	4.619	0.024822	4.340	0.002333	3.370	0.103870	3.523	0.024864
	0.070	0.025000	4.145	0.012340	0.050	0.104000	0.040	0.025000
	0.065	0.500000	0.040	0.012600	0.046	0.500000	0.035	0.500000
			0.035	0.500000				

367



368

369 **Figure 14.** Comparison between experimental and numerical load-deflection curves at 28 days for: (a) FCSA; (b)

370 FRSC

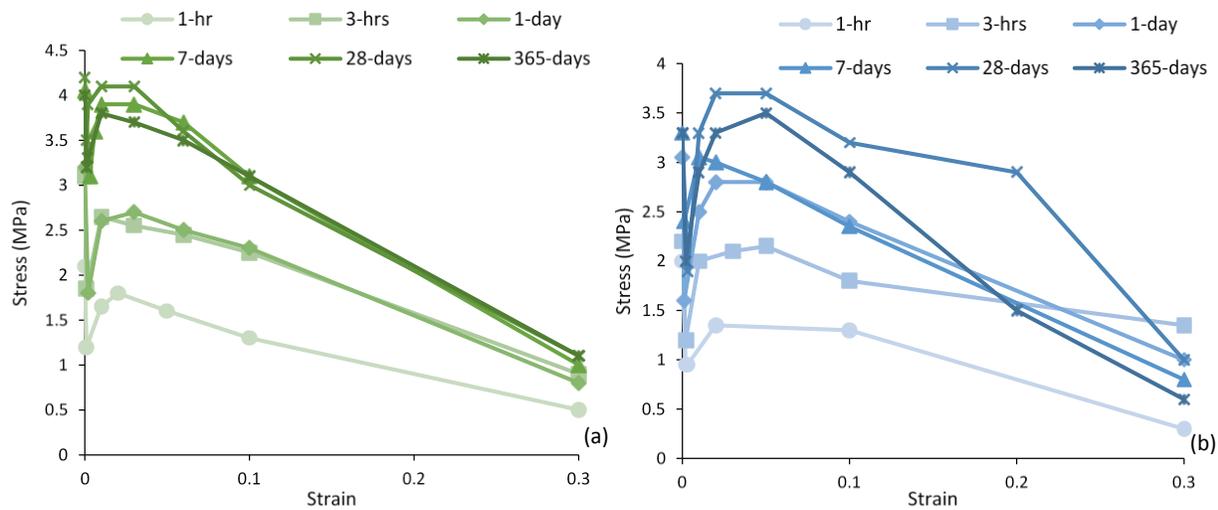
### 371 4.3. Numerical approach using inverse analysis

372 Inverse analysis was adopted to determine the post-cracking  $\sigma - \epsilon$  relationships for the different SFRC  
373 mixes and obtain a better prediction of the flexural performance of the tested specimens. The tensile  
374 properties are defined by using multilinear  $\sigma - \epsilon$  curves. The analysis is repeated while adjusting the  
375 tensile parameters until the numerical load-deflection curve matches the experimental response in  
376 capacity and energy dissipation within 2%.

377 The determined tensile  $\sigma - \epsilon$  curves shown in Figure 15 are then used to predict the structural behaviour  
378 of the FRC tested specimens. To better capture the flexural performance at larger displacements, the  
379 strain at failure should be accurately determined. The failure strain is calculated by dividing the ultimate  
380 width of crack (which is considered to be equal to half of the fibre length ( $l_f$ )) by the characteristic  
381 length. It was shown in a previous study on SFRC [45] that using a characteristic length of  $h_{sp}/2$  (the  
382 depth of a notched prism divided by 2) gives good results when converting displacements into  
383 equivalent strains. Thus, for this study, a value of 0.5 was adopted as a strain failure which is fairly close  
384 to  $l_f/2$  divided by half of the prism depth. It should be noted though that most tests were stopped at 5  
385 mm deflection as not to damage the LVDTs and thus, complete failure was never reached. For design  
386 purposes, a max strain of 0.025 is deemed sufficient so as to prevent the development of large crack  
387 widths.

388 The predicted curves are shown together with the experimental results in Figure 16 through Figure 16.  
389 As expected, the predictions match well the results.

390 The results for FCSA at 28 days was further analysed (using the same material model for the 10mm mesh  
391 size) with two mesh sizes; 16.6 mm and 5 mm to examine the effect of mesh size. The results (Figure 17)  
392 confirm that there is a slight mesh dependence when using this approach.



393

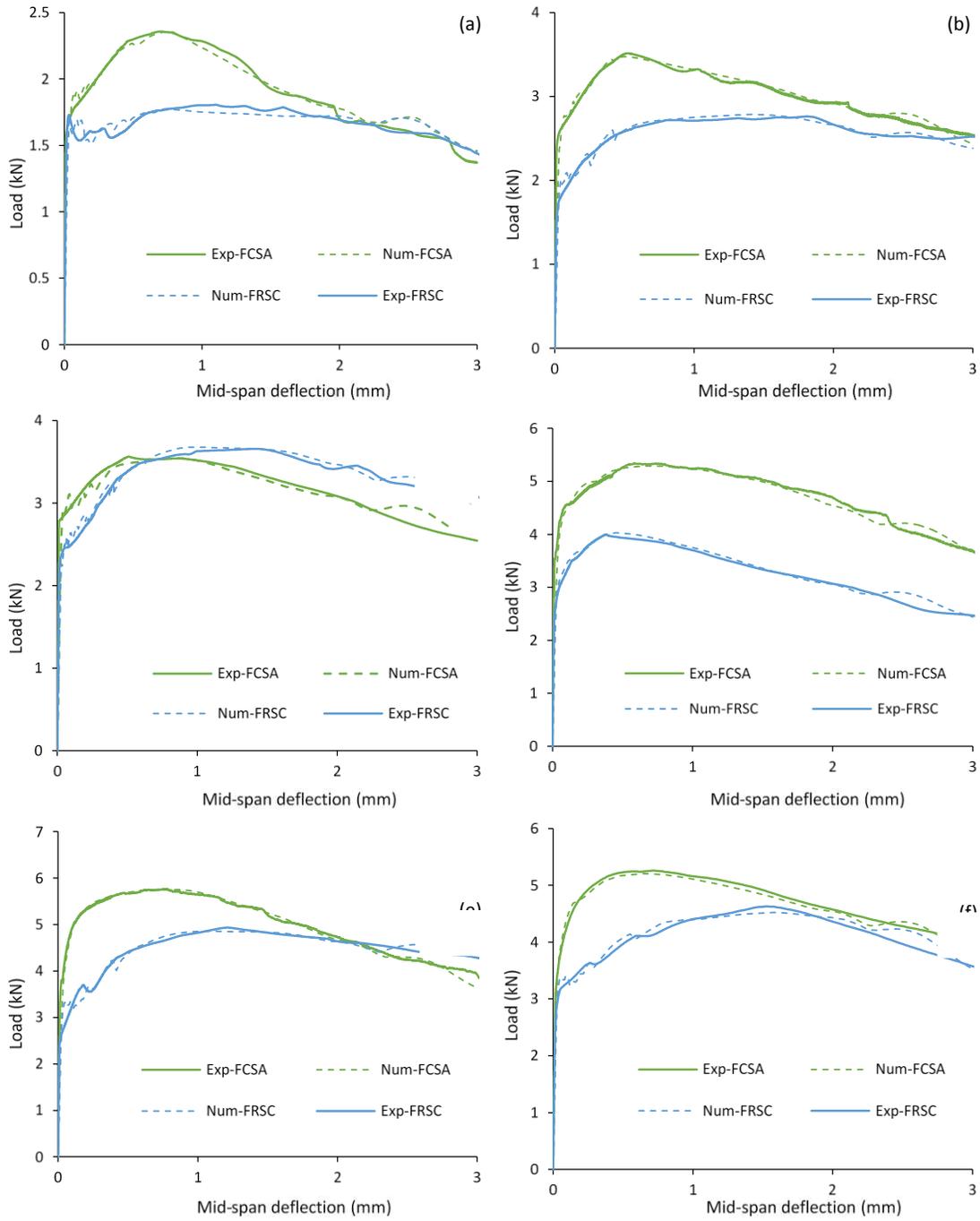
394

**Figure 15.** Tensile  $\sigma - \epsilon$  curves for mixes at different ages for: (a) FCSA; (b) FRSC

395 **4.4. Cracking**

396 In the CDP model, cracking can be assumed to initiate at points where the tensile equivalent plastic  
 397 strain is greater than zero and the maximum principal plastic strain is positive. The direction of the  
 398 vector normal to the crack plane is assumed to be parallel to the direction of the maximum principal  
 399 plastic strain [44]. Figure 18 shows maximum principal strain contours for FCSA prism at 28 days. It is  
 400 clear that the failure of the prisms is characterised by tensile cracking at the midspan of the beam as  
 401 occurred in the experiments.

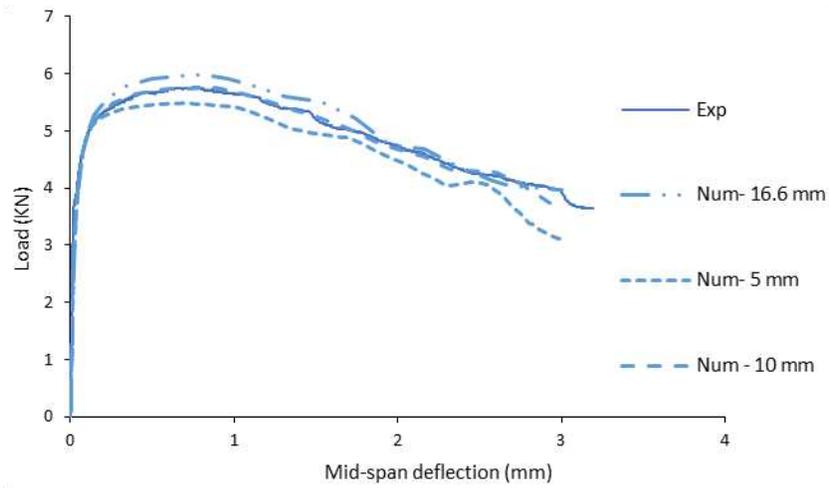
402 The crack width at the bottom of the specimens can be determined from the analysis by examining the  
 403 spreading of the beam using the horizontal deformation ( $U_3$ ) as shown in Figure 19. The crack width  
 404 determined at 3 mm of deflection are compared with CMOD values measured by the clip gauge in Table  
 405 4. The predicted values are slightly lower than the experimental values with the biggest error of 14.66%  
 406 (presented in brackets) for FCSA at 28 days. This confirms that the numerical models were not only  
 407 successful in predicting the flexural capacity, but also the crack widths of the tested prisms and as a  
 408 result, they could be used for further studies on repair layers.



409

410 **Figure 16.** Experimental load-deflection versus numerical curves of FCSA and FRSC prisms at age of: (a) one-hour;

411 (b) three hours; (c) one-day; (d) seven days; (e) 28 days; (f) 365 days



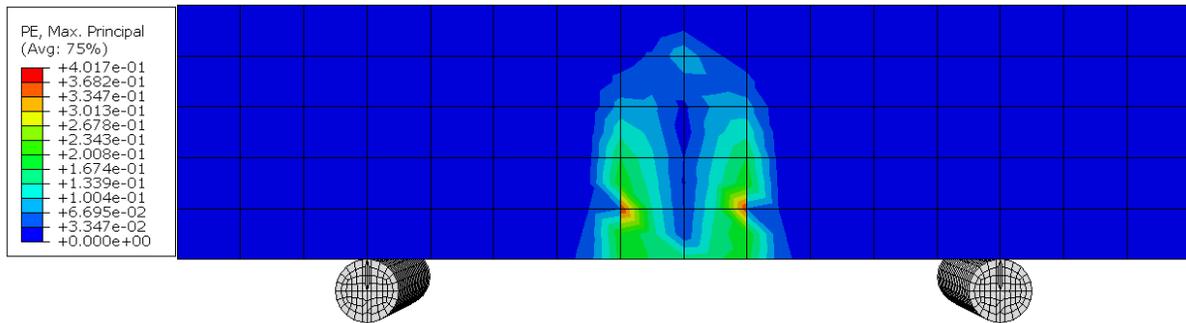
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**Figure 17.** Experimental load-deflection curve of FCSA at 28 days versus numerical curves using three different

414

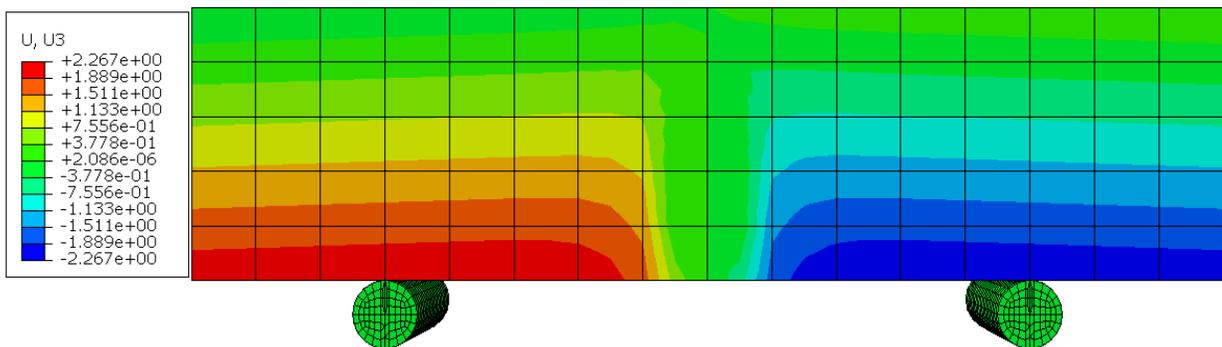
mesh sizes



415

416

**Figure 18.** Max principal strain contour for FCSA prisms at 28 days at the end of analysis



417

418

**Figure 19.** Horizontal displacement ( $U_3$ ) contour for FCSA prisms at 28 days at the end of analysis

419

420 **Table 4**

421 The measured and predicted crack widths for fibre reinforced mixes

Mix	Age	1hour	3 hours	1 day	7 days	28 days	365 days
FCSA	Numerical	4.68 (4.10)	4.64 (5.60)	4.68 (5.45)	4.52 (8.87)	4.54 (14.66)	4.64 (9.02)
	Experimental	4.88	4.903	4.953	4.96	5.32	5.10
FRSC	Numerical	4.37 (12.07)	4.60 (8.18)	4.59 (9.82)	4.71 (9.25)	4.57 (12.45)	4.6 (13.21)
	Experimental	4.97	5.01	5.09	5.19	5.22	5.3

422 Note: Values in brackets represent the error (%) between experimental and numerical crack width

423 **5. Conclusions**

424 Experimental and numerical investigations were performed on plain and fibre reinforced rapid  
425 hardening mortars. The main findings of this study are:

- 426 • Flexural strength evolves rapidly and both plain and fibre reinforced specimens achieved 90% of  
427 their one-year strength in one day. The specimens made with CSA cement showed higher flexural  
428 strength than those made with RSC cement tested at the same age due to the rigid dense crystal  
429 microstructure of the CSA cement.
- 430 • The fibres have a remarkable effect on the strength and modulus of elasticity of prisms. FCSA and  
431 FRSC mixes showed a flexural strength increase of approximately 36% to 70% and 24% to 41%  
432 respectively. For  $E_{fm}$ , an increase of 29.7% was found for FCSA at the age of one-hour. For  
433 compressive strength, the highest strength increase of around 24% was observed at one hour. No  
434 compressive strength reduction was noticed for any of the mixes tested in this study up to the age  
435 of one-year.

- 436 • The flexural residual strength for both FCSA and FRSC specimens continued to increase up to 0.7  
437 mm, which corresponds to  $CMOD_4$ . FCSA prisms show higher  $f_R$  than FRSC prisms for the same crack  
438 width. The values of  $f_R$  continue to increase with time for both FRC mixes and reach their peak  
439 values at 28 days. However, there is a slight strength reduction at one year compared to 28 days.
- 440 • Strong correlations exist between  $f_{R1}$  and  $f_{R2}$ ,  $f_{R1}$  and  $f_{R3}$ ,  $f_{R1}$  and  $f_{R4}$  with  $R^2 \geq 0.98$  and  $R^2 \geq 0.92$  for  
441 FCSA and FRSC, respectively.
- 442 • FE-predictions using CDP overestimate the loading capacity of FCSA and FRSC when using the tensile  
443 constitutive laws based on RILEM TC 162-TDF, CEB FIB MODEL CODE 2010, Barros et al. Conversely,  
444 the use of the models proposed by Hu et al. leads to underestimation.
- 445 • Inverse analysis was used successfully to obtain multilinear  $\sigma - \epsilon$  tensile curves and model the global  
446 load-displacement behaviour.
- 447 • Numerical analyses using the refined  $\sigma - \epsilon$  curves were successful in capturing the cracking widths of  
448 FRC tested prisms.

#### 449 **Acknowledgments**

450 The authors acknowledge the financial support of the Higher Committee for Education Development  
451 in Iraq (HCED-Iraq) for the PhD studies of Hajir Al-musawi. The authors also thank Twincon Ltd for  
452 material supply and in-kind contributions.

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