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1 Performance of rapid hardening recycled clean steel fibre materials

2 Hajir Al-musawi ^{a,*}, Fabio P. Figueiredo ^a, Susan A. Bernal ^b, Maurizio Guadagnini ^a, Kypros

3 Pilakoutas ^a

4 ^a *Department of Civil and Structural Engineering, The University of Sheffield, Sir Frederick Mappin Building, Mappin*
5 *Street, S1 3JD Sheffield, UK.*

6 ^b *School of Civil Engineering, University of Leeds, Woodhouse Lane, Leeds, LS2 9JT, United Kingdom.*

7 * Corresponding author's email: Haal-musawi1@sheffield.ac.uk Tel: +44 (0) 114 222 5729, Fax: +44 (0) 114
8 2225700

9 HIGHLIGHTS

- 10 • Mixes achieve 90% of their one year flexural strength at the age of one day.
- 11 • RCSF enhances flexural strength and toughness resulting in hardening behaviour.
- 12 • Constitutive equations based on the RILEM and MC 2010 recommendations overestimate loading
13 capacity.
- 14 • FEA analysis using multilinear $\sigma - \epsilon$ tensile curves obtained by inverse analysis can capture well the
15 post cracking strength and cracking pattern.

16 Abstract

17 To minimise disruption due to repairs of concrete pavements, rapid hardening and tough materials need
18 to be used. This paper investigates the flexural performance of rapid hardening mortar mixes made with
19 two commercial cement types, calcium sulfo-aluminate cement and calcium aluminate cement, for thin
20 concrete repair applications. Three-point bending tests are performed on plain and steel fibre reinforced

21 concrete specimens containing 45 kg/m^3 of recycled clean steel fibres to characterise the flexural
22 performance of notched and unnotched prisms at different ages, ranging from one hour up to one year.
23 The recycled fibers are shown to enhance both the flexural strength and toughness of FRC prisms,
24 leading to hardening behaviour. Constitutive equations based on the RILEM and Model Code 2010
25 recommendations are found to overestimate the loading capacity of the bending tests. FE analyses using
26 multilinear $\sigma - \epsilon$ tensile curves obtained by employing inverse analysis can capture better the post
27 cracking strength and cracking pattern of the tested prisms.

28 **Key words:** SFRC, recycled clean steel fibres, rapid hardening cements, mechanical properties, FEA

29 **1. Introduction**

30 Progressive deterioration of infrastructure, particularly pavements, occurs due to increasing vehicular
31 axle loads, worsening environmental conditions (due to climate change) and higher traffic volumes.
32 Excessive deterioration can lead to serious service disruptions and higher costs for infrastructure owners
33 and road users. Conventional ordinary Portland cement (OPC) based repair materials attain their
34 strength rather slowly and need between 12h to 24h to develop sufficient strength before roads can be
35 back in service, adding to delays and disruption during maintenance. To minimise disruption, rapid
36 hardening cements can be used in repairs. There are several special rapid hardening Portland-free
37 cements available in the market; such as calcium sulfo-aluminate (CSA) cement and calcium aluminate
38 (CA) cement. CSA can achieve early rapid strength development even in cold environments and can have
39 expansive properties. It is reported to have good durability in aggressive environments, particularly
40 when exposed to sulfates [2]. Furthermore, this cement requires less energy for its production
41 compared to OPC [1], thus it is considered to be environmentally friendly. However, despite its lower
42 energy demand, it is still more expensive due to the cost of its raw materials.

43 CA cements are characterised by high early strength development and high resistance to elevated
44 temperatures, depending on their aluminum content. An important aspect for the rapid strength
45 development of this cement is the substantial amount of heat of hydration which can result in high heat
46 generation [3]. Self-heating may be a concern in sections thicker than 100 mm [3], but not necessarily
47 for thinner repair layers. Despite the high temperature rise during hydration, CA concretes do not seem
48 to be overly susceptible to thermal cracking. This may be due to creep relaxation of thermally induced
49 strains, facilitated by a conversion reaction, during which some metastable phases of this cement
50 convert to stable phases of lower volume [3, 4]. As porosity increases, the densification due to
51 conversion causes loss of strength [3]. Hence, when used for repairs, the key concern to be addressed is
52 cracking due to restrained shrinkage.

53 Restrained shrinkage is one of the main factors that govern the serviceability and durability of concrete
54 repairs [5,6]. Shrinkage in concrete results due to moisture diffusion from the new concrete to the
55 environment and to the concrete substrate [7] if not adequately saturated. However, shrinkage
56 deformation (of the new layer) is restrained by the substrate layer leading to the development of tensile
57 and interfacial shear stresses. If these stresses exceed the material capacity at any time, cracking will
58 develop in the repair material and/or debonding along the interface between the repair material and
59 the substrate. Micro-cracks induced by shrinkage can propagate and coalesce into macro-cracks under
60 the effect of applied loads.

61 Cracks beyond a certain width can adversely affect the durability of repair materials by creating easy
62 access for deleterious agents leading to early saturation, freeze–thaw damage, scaling, and steel
63 corrosion, which promote further internal and external cracking and accelerate the rate of deterioration
64 [8]. This issue can be worsen with rapid hardening (non-expansive) materials due to the rapid hydration
65 rate which accelerates shrinkage development. Furthermore, due to the rapid stiffness development
66 and decrease in creep compliance of rapid hardening cements [9], their ability to redistribute stresses

67 may be affected, thereby increasing cracking potential. To address this issue, fibres can be added to
68 control crack widths [10] as well as increase the tensile strength and fatigue resistance [11], thus
69 resulting in more durable layers. To reduce the environmental impact of manufactured steel fibres
70 (MSF), recycled clean steel fibres (RCSF) can be used as alternative fibre reinforcement.

71 During the manufacture of tyres, parallel steel cords are embedded in continuous thin rubber belts.
72 After being cut to shape, these are placed in overlapping layers to provide flexible reinforcement within
73 the tread and side walls of the tyre. The complex configuration of each layer generates significant levels
74 of waste (approximately 5% by mass). The available amount of waste steel cord is therefore around
75 100,000 tonnes per year worldwide. The steel reinforcement used in tyre manufacture typically consists
76 of parallel filaments of very fine wire (0.1-0.4 mm dia.) twisted together to form a cord about 0.5-1.0
77 mm in diameter [12]. Recycled clean steel fibre (RCSF) filaments extracted from pre-vulcanised rubber
78 belt offcuts have become available recently and were adopted in this study. However, knowledge on
79 their use in concrete is scarce and it is limited to research at the University of Sheffield [13]. Knowledge
80 of the effect of industrial fibres on CSA and CA matrices is also rather limited [9,14-16] and no published
81 data exist regarding the effect of RCSF. A study on the effect of CSA matrix on pullout performance of
82 steel fibres [9] suggests that the synergetic effect of a stiff matrix like ettringite and high modulus steel
83 fibres can increase crack propagation in the composite material, evidenced by an increase in debonding
84 energy density.

85 Since cracking is the main concern for repairs, understanding the effect of fibres in controlling crack
86 widths under mechanical and hygral loads, as well as the complex interaction of shrinkage, stiffness and
87 tensile strength evolution are of paramount importance. For this purpose, finite element analysis can be
88 a useful tool. However, appropriate material parameters need to be determined experimentally and the
89 tensile σ - ϵ curves of the repair materials need to be derived from direct tension or bending results.
90 Although there are several procedures in the literature to derive the σ - ϵ of SFRC in tension [17-20], they

91 may not be entirely suitable for modelling mortars reinforced with RCSF due to the different fracture
92 energies of the two concretes. In numerical studies performed by [20, 21], it was found that RILEM
93 proposed σ - ϵ equations overestimate the predicted capacity of FRC. As a result, a simplified σ - ϵ model
94 was suggested to overcome issues in the other methods and to include the post-consumer tyres steel
95 fibres (RTSF) effect.

96 This paper presents experimental and numerical work on the flexural performance of RCSF on rapid
97 hardening mortars produced using CSA or CA as sole cementitious materials. Constitutive relationships
98 derived based on code recommendations and by others [19, 20] are used to predict flexural behaviour
99 and the results are compared with predictions obtained from inverse analysis.

100 **2. Experimental details and methodology**

101 **2.1. Materials**

102 Two commercial cement types were used in this study; calcium sulfoaluminate cement¹ (CSA) and rapid
103 setting calcium aluminate cement² (RSC). According to the manufacturer, RSC consists of hydrated
104 alumina, oxides of iron and titanium, with small amounts of silica. For production of mortars, fine
105 aggregates, medium grade river washed sand (0-5mm sourced from Shardlow in Derbyshire, UK,
106 SG=2.65, A = 0.5, FM = 2.64), were used. Recycled clean steel fibres (RCSF) were obtained from tyre
107 cords extracted from un-vulcanised rubber belts (see Figure 1). The length of the RSCF used in this study
108 was 21 mm and the diameter 0.2 mm. The strength of these fibres is reported to exceed 2600 MPa [13].
109 Superplasticiser³ was added to enhance the workability and adjust the setting time.

110



111

112

Figure 1. Photograph of the RCSF used in this study

113

2.2. Mortar mix design

114

A total of 600 kg/m^3 of cement was used with low w/c ratios to obtain high early strength. For durability

115

requirements, w/c should be kept lower than 0.4. However, as CSA cement consumes more water to

116

form hydration products than ordinary Portland cement [22], this limit can be relaxed slightly for this

117

cement. As a result, two different w/c ratios and SP dosages were tested. The w/c ratios for mixes with

118

CSA cement were 0.4 and 0.41, and 0.35 and 0.36 for RSC mixes. The water content and superplasticiser

119

(SP) were carefully selected to achieve a workable mix with setting time of no longer than 15 minutes.

120

Fibre dosage of 45 kg/m^3 ($V_f = 0.57\%$) was investigated as is commonly used in European practice for

121

structural applications. The plain and fibre reinforced mortar mixes for each cement type are almost

122

identical, to reliably investigate the effect of fibres on the mechanical properties. The details of the

123

optimised mortar mixes are summarised in Table 1.

124

The specimens were cured for one hour before demoulding and exposure to standard laboratory

125

conditions.

126

Table 1

¹ provided by Kershin International Co., Ltd

² sourced from Instarmac

³ Sika Viscoflow 2000

127 Mortar mix composition

mix	Cement (kg/m ³)	w/c	Sand (kg/m ³)	SP ^a	Fibre dosage (kg/m ³)
CSA	600	0.40	1420	0.60	0
FCSA	600	0.41	1420	0.61	45
RSC ^b	600	0.35	1300	0.20	0
FRSC	600	0.36	1300	0.21	45

128 ^a % by cement mass. ^b mixes containing CA cement are called RSC in this study.

129

130 **2.3. Fresh state properties**

131 **2.3.1. Vicat test**

132 The setting time of cement pastes was assessed using an automatic Vicat apparatus according to ASTM
133 C191 (2013) [23]. As the cements used in this study are fast setting, the instrument was set to take
134 measurements every 30 seconds.

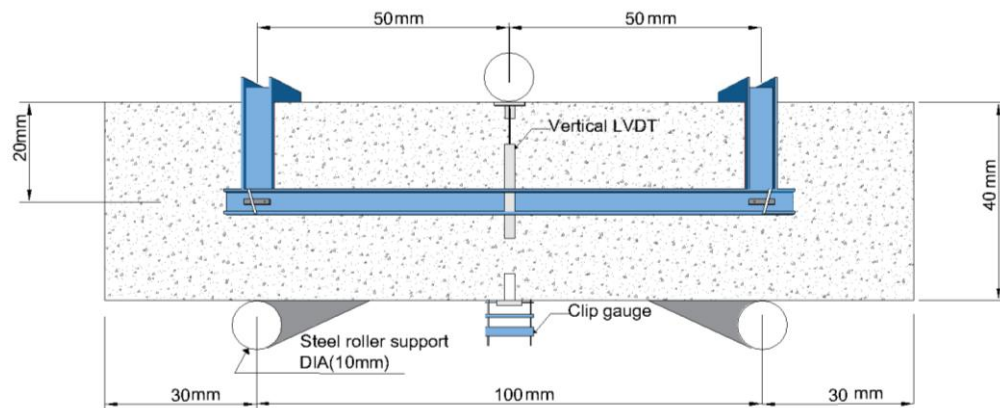
135 **2.3.2. Semi-adiabatic calorimetry**

136 The semi-adiabatic calorimeter records the temperature evolution and key temperature related
137 properties for a tested mix, such as time to peak heat, peak heat, and cumulative heat [24]. Since the
138 mortar mixes are designed for thin repairs, heat loss due to dissipation is expected to take place and
139 hence, the semi-adiabatic test could reveal a temperature evolution that is close to practical
140 applications. After mixing the required quantity for each mix, the mortar was directly placed in an
141 insulated thermal flask cylinder of 0.5 l and a thermocouple was inserted inside the mortar to record the
142 temperature.

143 **2.4. Flexural tests**

144 To characterise the flexural performance, mortar prisms of 40 × 40 × 160 mm were tested according to
145 BS EN 13892-2 [25]. To obtain the load deflection curve after the peak load, displacement control was
146 adopted rather than load control as required by the standard. The rate of loading was 0.25 mm/min

147 until 1 mm deflection, and 1 mm/min after that. To eliminate errors due to machine stiffness, spurious
148 support displacements and local concrete crushing, a specially designed aluminum yoke (based on the
149 Japanese standard JSCE-SF4 [26]) was mounted on the specimens. To assess the flexural behaviour over
150 time, the prisms were tested at one hour, three hours, one day, seven days, 28 days and 365 days. The
151 test was also performed on notched prisms (the notch depths range from 3.57 to 4.94 mm) to assess
152 crack development. The Crack Mouth Opening Displacement (CMOD) was measured at mid span with a
153 12.5 mm clip gauge (mounted across the bottom part of the notch, Figure 2). For practical reasons, this
154 test was performed at 2 days (at the earliest age) and up to one year.



155

Figure 2. Flexural test set up

156

157 **2.5. Compressive strength**

158 Directly after flexural testing, the halves of the fractured prisms were tested in uniaxial compression
159 according to BS EN 13892-2 [25]. Only the one-hour compressive strength of FRC specimens was
160 examined separately due to practical time constraints.

161 **3. Experimental Results and Discussion**

162 **3.1. Fresh state properties of rapid hardening materials**

163 The water content and SP dosage were optimised for each mix to achieve a workable mix with setting

164 time of no longer than 15 minutes. As shown in Table 2, the CSA cement had a relatively shorter setting
 165 time compared to the RSC cement. Slightly higher water content and SP dosages for the fibre reinforced
 166 mixes lead to a slight increase of the setting time for these mixes.

167 **Table 2**

168 Setting time and maximum temperature (T_{peak}) for different mixes

Mixes	Vicat setting time (min.)		T_{peak} (°C)
	Initial	Final	
CSA	9.5	10.5	68
FCSA	9.5	11.0	68
RSC	12.0	14.5	91
FRSC	12.5	15.0	88

169

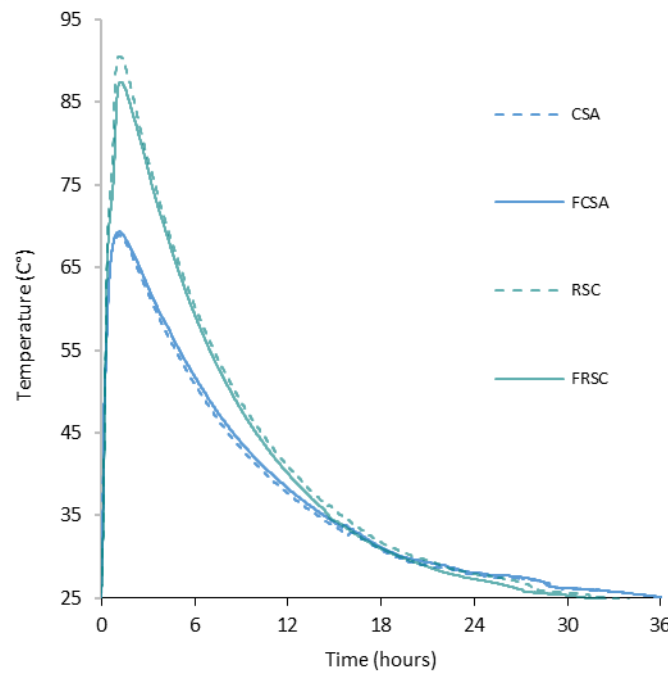
170 The results of the semi-adiabatic calorimetry test (for the first 36 hours) are shown in Figure 3. For mixes
 171 with CSA cement, the peak temperature (T_{peak}) was about 68° C (see Table 2) occurring during the first
 172 hour regardless of fibre content. The temperature rise in RSC mixes was much higher than in mixes with
 173 CSA cement, with T_{peak} at 91° and 88° C for RSC and FRSC, respectively. The time half way to the peak
 174 ($T_{1/2 peak}$) can be taken as an indication of the initial setting time of cementitious mixes [27]. For CSA and
 175 FCSA, $T_{1/2 peak}$ was achieved at around 11 minutes, whilst for RSC and FRSC, it was recorded at around 16
 176 minutes. These results agree well with the results of the vicat test. The temperature achieved for these
 177 cements upon hydration dropped to laboratory temperature in less than 24 hours. Heat dissipation is
 178 expected to occur faster onsite than in the semi-adiabatic test and, therefore, no major thermal cracking
 179 is expected for thin repairs, especially when curing is applied during the first two hours when T_{peak}
 180 occurs.

181 **3.2. Mechanical performance of rapid hardening mortars**

182 **3.2.1. Compressive strength**

183 The average compressive strength f_{cu} (from six specimens) and standard deviation developed over time

184 is shown in Figure 4. At one hour, FCSA achieved the highest compressive strength of 26.1 MPa while
185 RSC achieved 17.2 MPa. This behavior changes at later ages as RSC achieves a higher strength than FCSA
186 by approximately 6% after one-year. The fibres seem to have a positive effect on the compressive
187 strength of both mortars, with the highest strength increase noticed at one hour (24% increase in f_{cu}). At
188 later ages, this increase ranges from 10% to 17%.



189

190

Figure 3. Temperature rise for mixes in semi-adiabatic test

191 There is no consensus in literature on the effect of fibers on compressive strength. While some
192 researchers [28-30] report a strength enhancement of up to 20% for Portland cement-based specimens
193 containing recycled fibres with dosages less than 50 kg/m^3 , others [31-33] found only a marginal effect
194 due to air entrainment.

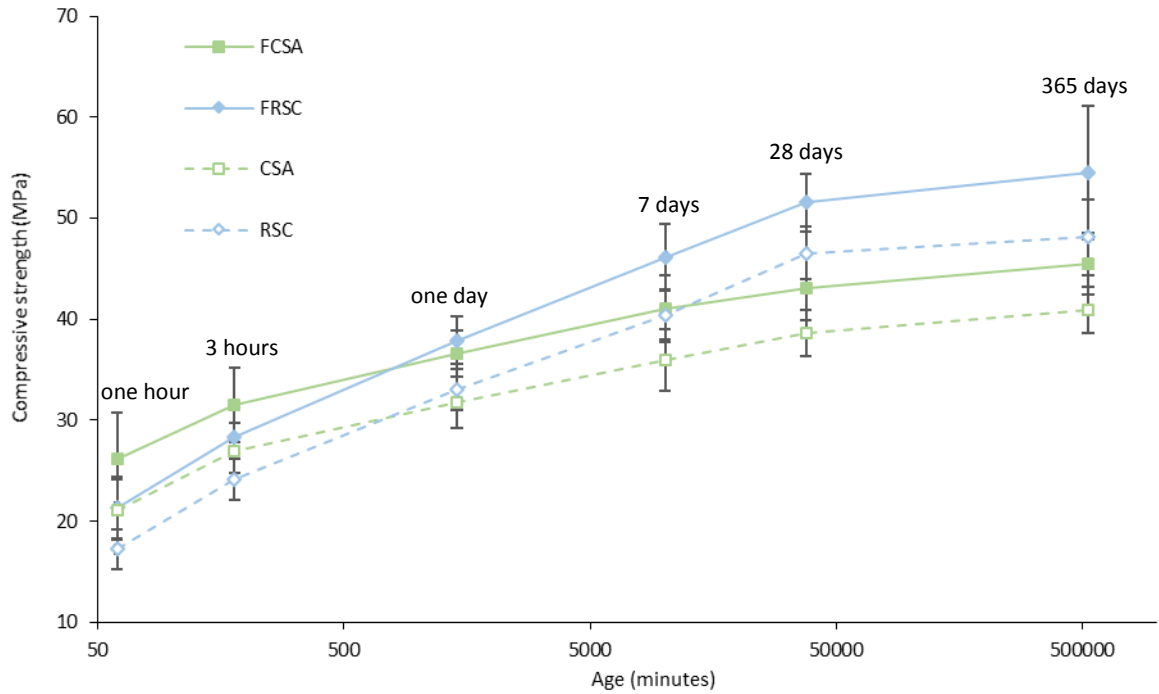
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196

197

No strength reduction has been observed for any of the mixes at the age of one-year, indicating that
there were no significant conversion issues. It should be noted that for fully cured rapid hardening CSA
mortar-based samples (tested at 28 days), a compressive strength of 31.4 – 52.6 MPa for w/c ratios 0.4

198 – 0.5 was reported in literature [34] and this agrees well with the results of this study.



199

200

Figure 4. Development of f_{cu} as a function of time

201

To describe the compressive strength development with time, the $\beta_{cc}(t)$ function that describes the

202

strength development with time used in Model Code 2010 [18] is followed.

$$\beta_{cc} = \exp\left\{s \cdot \left[1 - \left(\frac{28}{t}\right)^{0.5}\right]\right\} \quad \text{equation 1}$$

203

where, t is the concrete age in days, s is a coefficient that depends on the class of cement which ranges

204

from 0.2 – 0.38 for $f_{cm} \leq 60$ MPa. As the cements used in this study are rapid hardening, a 0.2 value for s

205

was adopted. To obtain the strength at various ages, $\beta_{cc}(t)$ is multiplied by the mean compressive

206

strength at the age of 28 days (f_{cm}). The estimated compressive strength at various ages is shown against

207

the experimental results in Figure 5. As expected, the function underestimates the strength at the early

208

ages by approximately 100% for the different rapid hardening mixes. As the strength evolves very

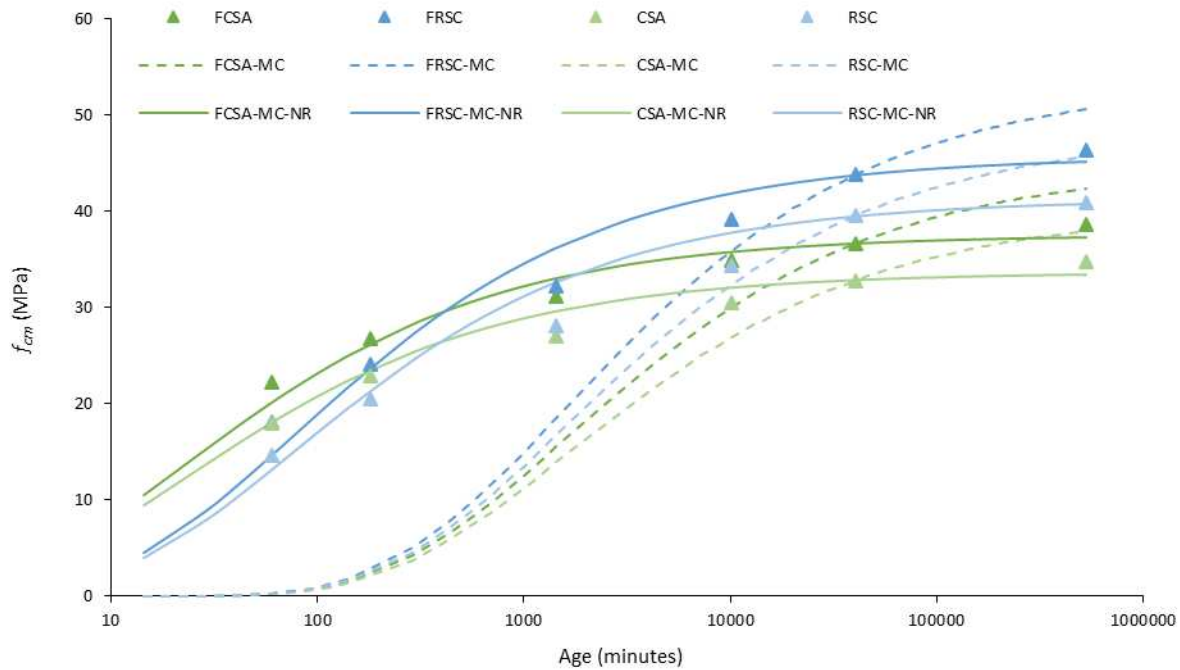
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rapidly at the early ages and then it slows down, smaller s values could offer a better representation for

210

strength development with time. The s values of 0.024 and 0.044 for mixes with CSA and RSC cements,

211 respectively, were found by regression analysis to represent well the strength evolution with time (see
 212 Figure 5).

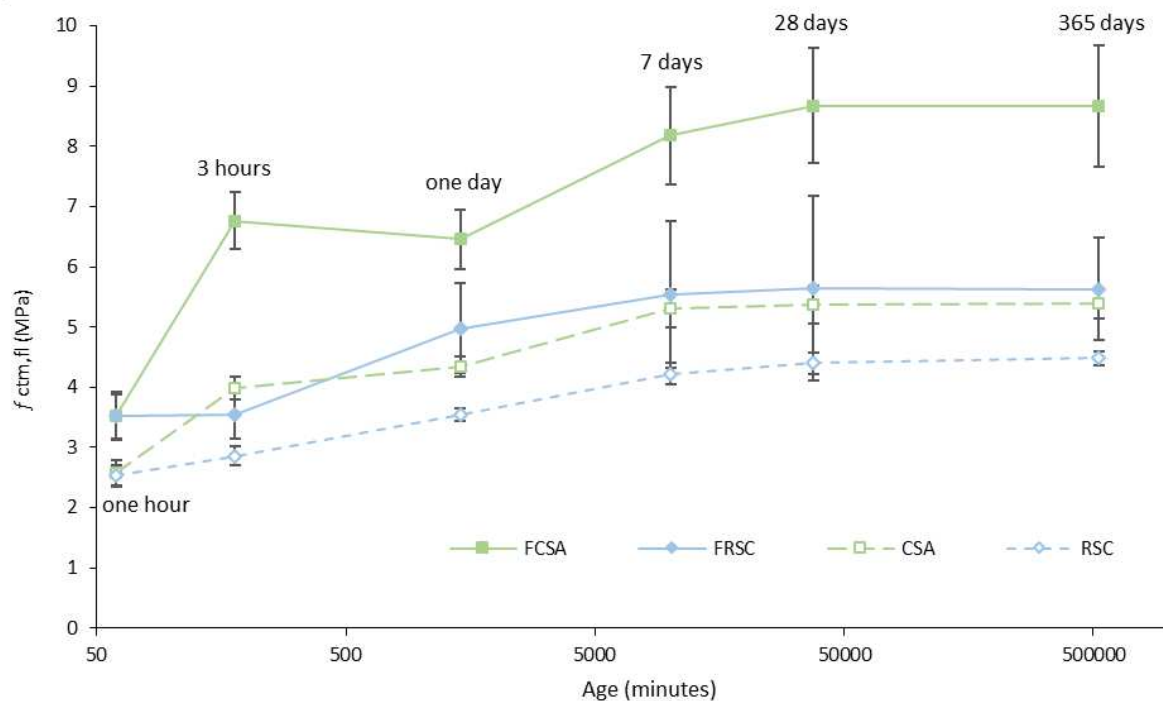


213
 214 **Figure 5.** Development of experimental and estimated f_{cm} as a function of time using $s = 0.2$ (dashed lines) and
 215 suggested s values (solid lines-NR)

216 **3.2.2. Flexural behaviour**

217 The average flexural strength development over time (and standard deviation) is illustrated in Figure
 218 6. The reported values represent the limit of proportionality (LOP), or first cracking strength ($f_{ctm,fl}$),
 219 determined according to BS EN 14651:2005 [35]. It is noted that strength develops very fast and both
 220 plain and fibre reinforced specimens achieved 90% of their one-year strength in one day. The specimens
 221 made with CSA cement showed higher flexural strength than those with CA cement tested at the same
 222 age, probably due to the rigid dense crystal microstructure of the CSA cement [9]. RSC mixes have lower
 223 w/c ratio, hence, their compressive strength is expected to be higher in the long term. Due to high
 224 shrinkage in RSC mixes, their flexural strength is reduced. The effect of RCSF on the flexural strength

225 enhancement of the mixes is evident at all ages. Compared to their plain counterparts, FCSA and FRSC
 226 mixes showed a flexural strength increase of approximately 36% to 70% and 24% to 41%, respectively.
 227 This agrees well with Hu et al. [33], who reported an increase of 45% - 70% in $f_{ctm,fl}$ of concrete
 228 reinforced with blends of manufactured and post-consumer recycled fibres.



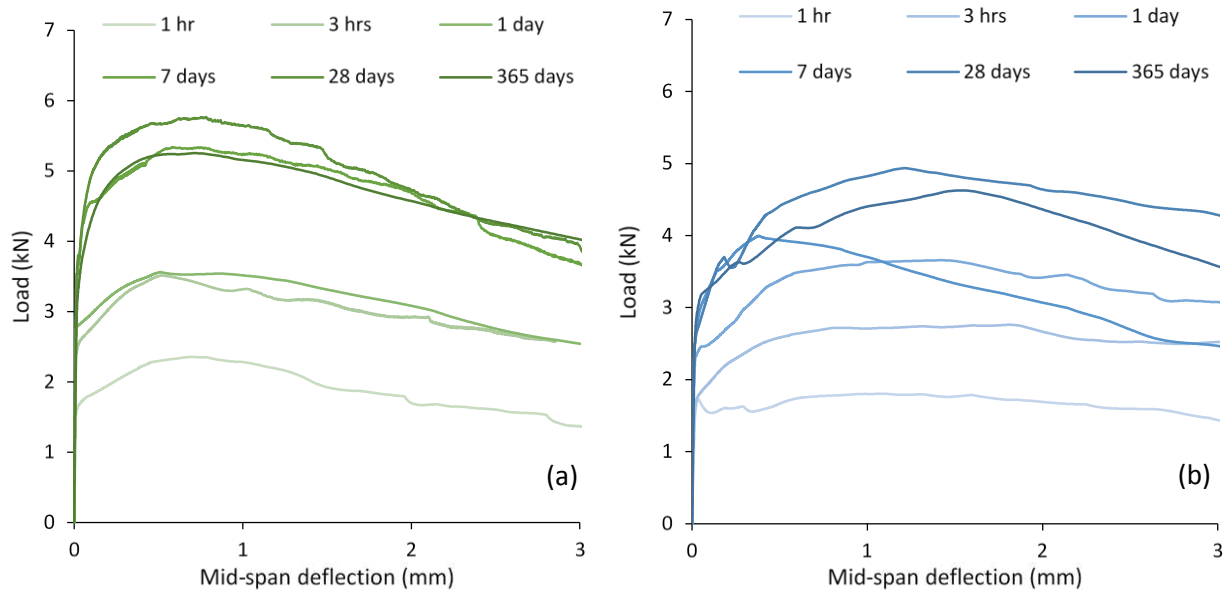
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230 **Figure 6.** The flexural strength $f_{ctm,fl}$ development as a function of time

231 The load-deflection curves for FCSA and FRSC prisms are shown in Figure 7. The behaviour of the
 232 specimens made with the unreinforced mixes is not shown as they failed suddenly after peak load
 233 without any post cracking strength, highlighting the poor toughness of plain mortars in tension. The
 234 deflection hardening shown by reinforced mixes can be attributed to the high number of fibres spanning
 235 the cracked section and the excellent bond between steel fibres and dense matrix systems, like the CSA
 236 cement. This hypothesis is supported by the fact that in the current study, many specimens developed
 237 more than one principal crack, confirming the excellent load transfer by the RSCF. It should be noted
 238 that the preferential alignment of the fibres in the direction of stress due to the small mould size ($40 \times$

239 40 × 160 mm) may have contributed to this. Deflection hardening was also reported in a study by
 240 Bordelon [36] for concrete specimens cut from prisms of 150 × 150 × 450 mm and tested using a 50 mm
 241 beam depth (to simulate a thin overlay). Deflection hardening performance for notched concrete prisms
 242 reinforced with 45kg/m³ of blends of recycled post-consumer and manufactured steel fibres was also
 243 reported in a recent study published by Hu et al. [33].

244 At large deflections (greater than 2 mm), the FCSA specimens show a slight reduction in load resistance
 245 compared to FRSC specimens, possibly due to the inherent brittleness of the CSA cement. However, in
 246 most repair applications, it is not expected that the mortar will reach such high level of deformation and
 247 as a result, minimal cracking is expected.



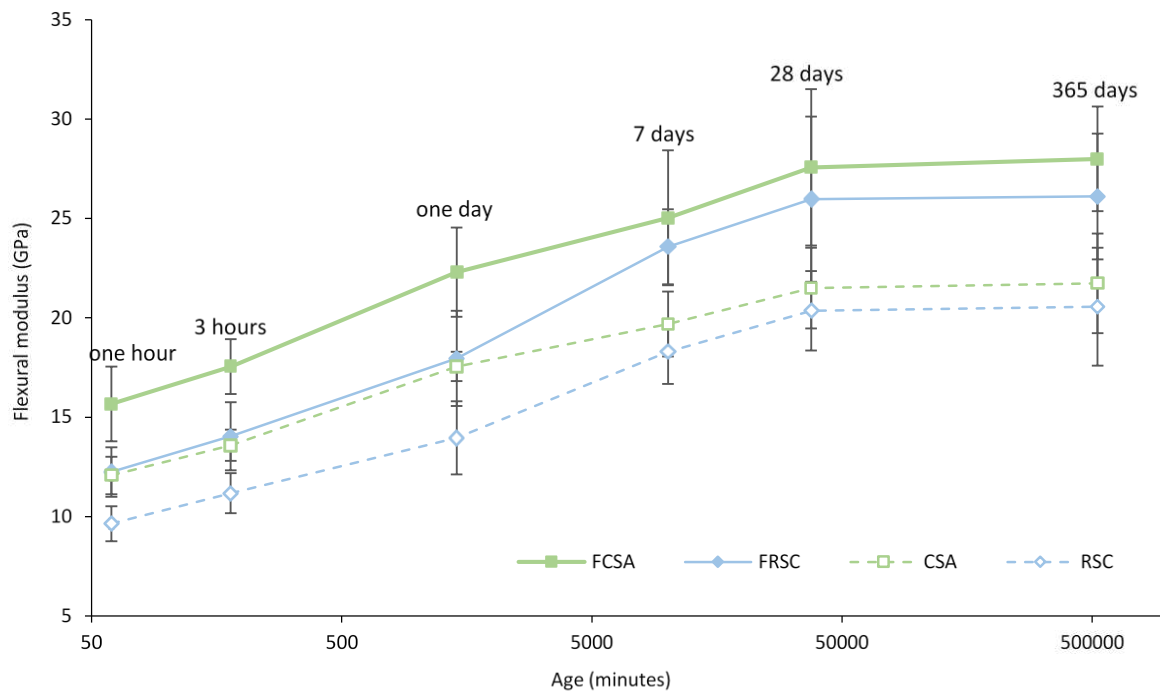
248
 249 **Figure 7.** Load-deflection response of rapid hardening fibre reinforced mortars tested at different ages: (a) FCSA;
 250 (b) FRSC

251 3.2.3. Flexural modulus of elasticity (E_{fm})

252 The flexural modulus of elasticity (E_{fm}) was determined from load-deflection curves using elastic analysis
 253 and ignoring shear deformations. E_{fm} is the maximum flexural modulus between 30 – 60% of the peak

254 load (P_{peak}) [37]. Figure 8 shows the development of E_{fm} and related standard deviations over time for all
 255 mixes. The plain mortar mixes are shown in dotted lines. As with flexural strength, the stiffness of the
 256 mixes develops quickly and reaches around 90% of the one year modulus within 7 days.

257 The fibres have a remarkable effect on the modulus of elasticity. FCSA and FRSC have higher E_{fm}
 258 compared to CSA and RSC mixes respectively with the highest noticeable increase (29.7%) for FCSA
 259 occurring at one-hour of age. This behaviour was not reported in [33] and [38] who only noticed a
 260 marginal effect on the modulus of concrete with fibre addition. The remarkable increase in modulus of
 261 elasticity, though also reflected in the flexural strength, is beyond what is expected from a perfect
 262 composite. This may be partially due to fibre alignment, but also to the slightly longer mixing time that
 263 was necessary to integrate the fibres. An increase of approximately 36% in the modulus of elasticity of
 264 OPC based mortars reinforced with 2% (by volume) industrial steel fibres was reported in literature [39].

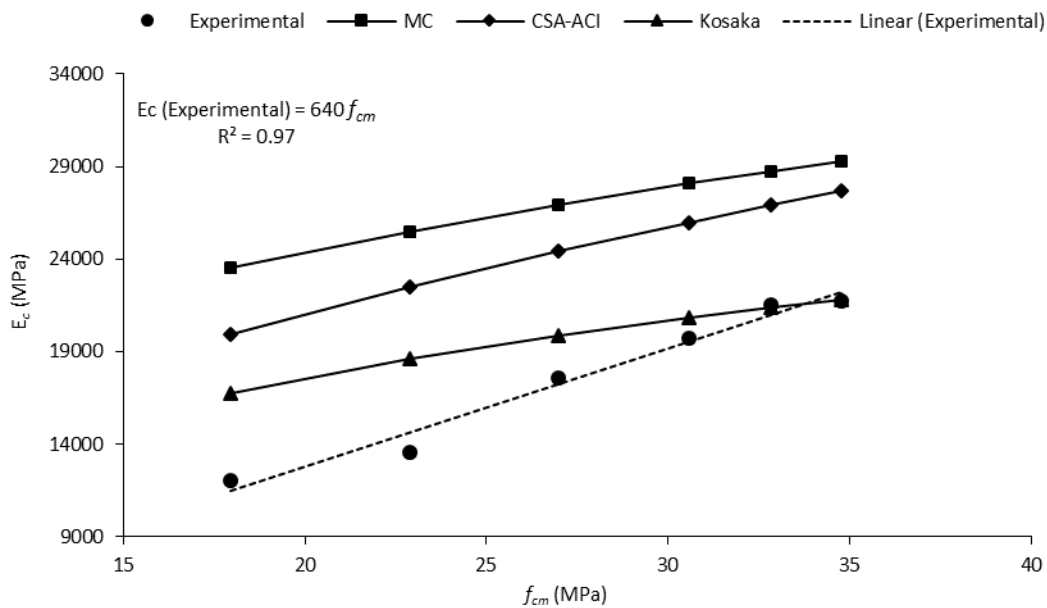


265

266

Figure 8. Flexural modulus (E_{fm}) of fast setting fibre reinforced mortars as a function of time

267 To estimate the modulus of elasticity of the mixes, based on compressive strength, equations from
 268 Model code [17], ACI 318-05 [40] and Kosaka et al. [41] were used. The latter equation was developed
 269 specifically for mortars. The estimated modulus of elasticity (E_c) for CSA (using the above equations) is
 270 presented in Figure 9. As shown, the equations overestimate E_c for CSA mix, especially at the early ages.
 271 It should be noted that both Model code and ACI code adopt equations that use the 1/3 and 1/2 power
 272 of f_{cm} respectively. However, the results show that for these mortars, the linear relationship is more
 273 appropriate and the constant values of 720, 580, 640 and 520 were determined by regression analysis
 274 for FCSA, FRSC, CSA and RSC mixes respectively.



275
 276 **Figure 9.** The relationship between f_{cm} and E_c using different equations for CSA mix

277 **3.2.4. Relationship between measured deflection and CMOD values**

278 A linear relationship between CMOD and average deflection is suggested in BS EN 14651:2005 [35], as
 279 given below,

280 Average deflection (mm) = $k \times \text{CMOD (mm)} + 0.04 \text{ mm}$, $k = 0.85$

281 This linearity has also been confirmed for FCSA and FRSC at all ages tested with coefficients of
282 determination $R^2 > 0.99$, but as expected with lower K values, between 0.55 and 0.65, due to the
283 different geometry of the testing arrangement. It should be noted that the CMOD measured by the clip
284 gauge is corrected for the position of the clip gauge using the BS EN 14651:2005 [35].

285 A relationship between deflection and CMOD can facilitate the testing of such materials by using clip
286 gauges only to measure the CMOD as accurate measurement of deflection requires the use of a special
287 frame (yoke) to obtain net deflection. It also provides a benchmark for comparisons.

288 **3.2.5. Residual flexural tensile strength (f_R)**

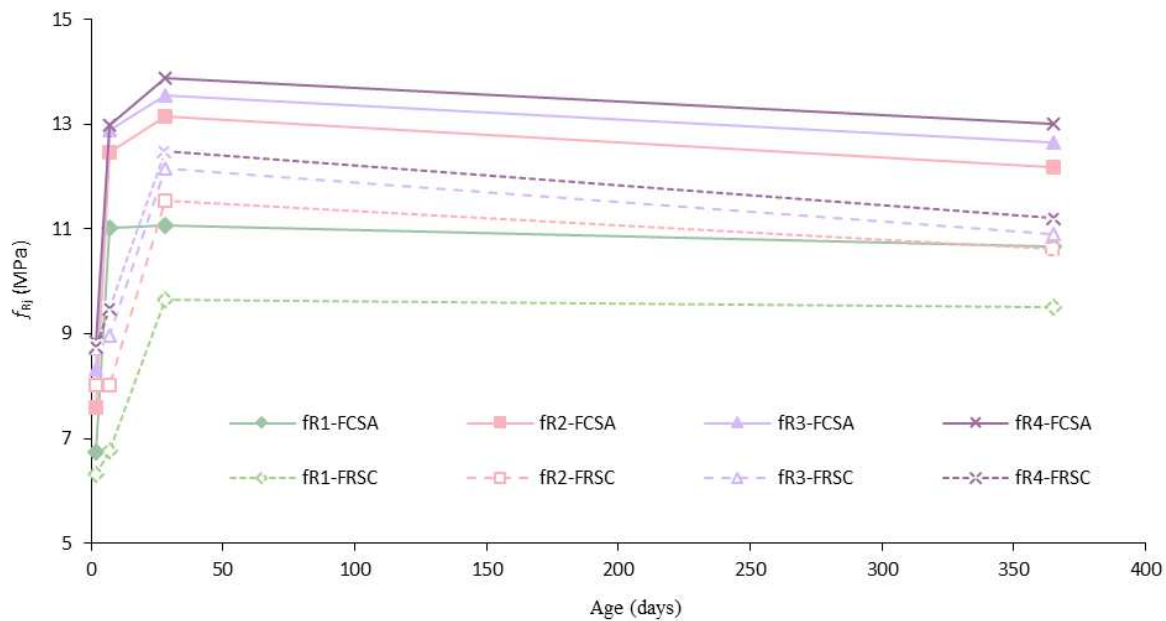
289 RILEM TC 162-TDF [42] presents a methodology to calculate the residual flexural tensile strength of SFRC
290 prisms, which was later adopted by BS EN 14651:2005 [33]. Residual flexural stresses (f_{R1} , f_{R2} , f_{R3} and
291 f_{R4}) are calculated from the load-CMOD curves at 0.5, 1.5, 2.5 and 3.5 mm of CMOD, respectively.
292 However, these CMODs are suggested for concrete prisms of 500 mm span length. For this study, the
293 residual stresses are calculated at CMOD equal to 1/5 of those used for 500 mm span specimens; i.e.
294 0.1, 0.3, 0.5 and 0.7.

295 Figure 10 shows the f_{Ri} values of all FCSA and FRSC mixes tested at different ages. The f_R values for
296 FCSAs are shown in solid lines while FRSCs are shown in dashed lines. It is noticed that for both mixes
297 the f_R values continue to increase from CMOD 0.1 mm to 0.7 mm which shows the high efficiency of the
298 RCSF in carrying the loads across cracks. This is also evidenced by the multiple cracks that form in some
299 samples at, or more than, seven days of age. The residual strengths of FCSA are higher than those of
300 FRSC for the same crack width, which implies better bond strength for RCSF in FCSA matrices.

301 The f_R values continue to increase with time for both FRC mixes and reach their peak values at 28 days.
302 However, there is a slight strength reduction at one year compared to 28 days. This could be attributed

303 to the effect of the conversion reaction occurring in the RSC cement. This is unlikely, however, as there
 304 was no reduction in compression strength at one-year of age. Another possible explanation is the effect
 305 of shrinkage on the bond strength of RCSF. This reduction in f_R is more obvious at higher CMOD levels
 306 (for f_{R2} to f_{R4}), which means that the frictional resistance along the fibres reduces slightly at one year.

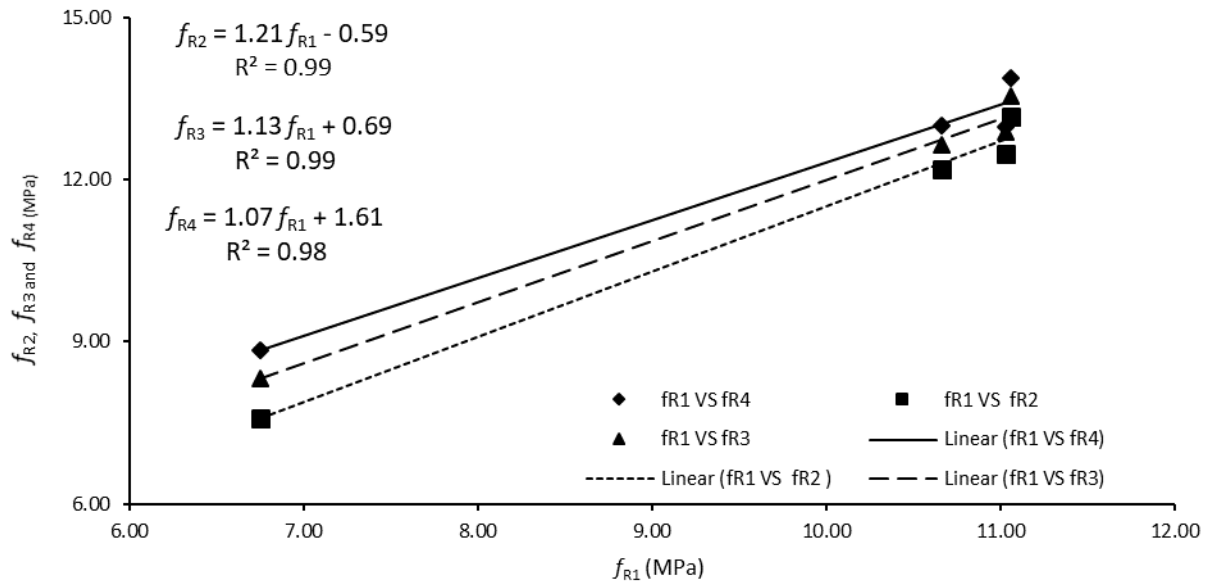
307 Figure 11 and Figure 12 show the relationship of f_{R1} vs f_{R2} , f_{R1} vs f_{R3} and f_{R1} vs f_{R4} for FCSA and FRSC,
 308 respectively. The values of f_{R2} , f_{R3} and f_{R4} correlate very well with f_{R1} for FCSA prisms with $R^2 \geq 0.98$. A
 309 similar trend was also found for FRSC prisms, however, with a relatively smaller coefficient of
 310 determination ($R^2 \geq 0.92$). A linear relationship between f_{R1} vs f_{R3} , f_{R1} vs f_{R4} were also reported by
 311 Zamanzadeh et al. [43] for unclassified RTSF. The strong correlation between the f_R values can lead to
 312 simpler design guidelines.



313

314

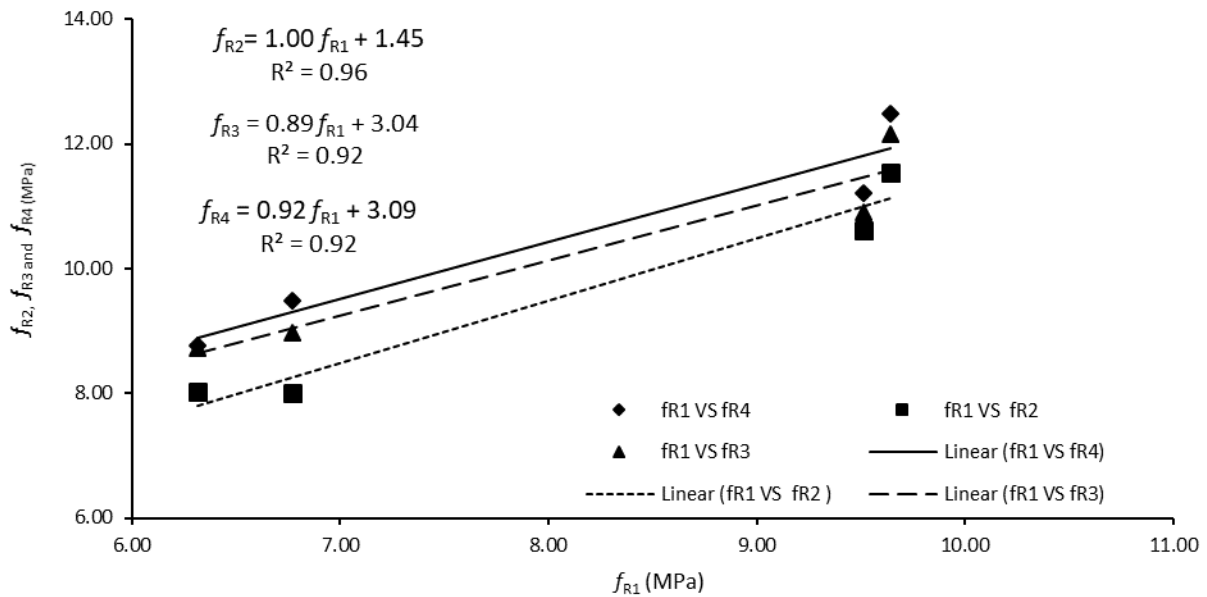
Figure 10. f_R values of FCSA and FRSC prisms (in MPa) development with age



315

316

Figure 11. Correlation between f_{R1} and f_{R2} , f_{R1} and f_{R3} , f_{R1} and f_{R4} of FCSA prisms



317

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Figure 12. Correlation between f_{R1} and f_{R2} , f_{R1} and f_{R3} , f_{R1} and f_{R4} of FRSC prisms

319 **4. Numerical study**

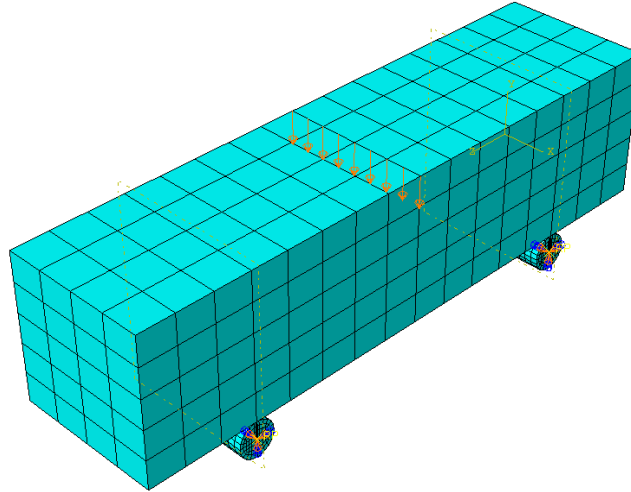
320 **4.1. FE modelling**

321 To model the flexural performance of these materials, the FE package ABAQUS is used, which offers

322 three material models for concrete simulation; Concrete Smeared Cracking (CSC), Brittle Cracking (BC)

323 and Concrete Damaged Plasticity (CDP) [44]. It was found that, for this application, CSC is prone to
324 numerical instabilities soon after crack development. Similar issues were also reported in [45] when
325 modelling SFRC prisms using CSC. Although the BC model was applied successfully to model FRSC [46], it
326 was considered unsuitable for the current study as it assumes that the concrete remains elastic in
327 compression. Since, due to the high flexural strength of the mortars, in this study, the material is
328 expected to become non-linear in compression. Therefore, the analysis was performed by using the
329 concrete damage plasticity (CDP) model for which the user can define the tensile and compression
330 behavior of concrete in as many steps as required. In CDP, the ratio of biaxial to uniaxial compressive
331 strength (σ_{b0}/σ_{c0}) and the ratio of the second stress invariant on tensile meridian to that on the
332 compressive meridian (K_c) characterise the failure surface of concrete. The dilation angle (ψ) and flow
333 potential eccentricity (ϵ) are used to define the flow rule [44]. σ_{b0}/σ_{c0} was taken as 1.2 (slightly higher
334 than the value usually assigned for plain concrete due to presence of fibres), K_c was 0.667, ψ was 31°
335 and after a sensitivity analysis for ϵ , the default value of 0.1 was adopted. The CDP model can be
336 regularised by using viscoplasticity to assist in overcoming convergence issues, that occur in materials
337 exhibiting softening behaviour in implicit analysis computations, by permitting the stress to be outside
338 the yield surface. Since high values of viscosity (μ) compared to characteristic time increment can
339 compromise the results, a value of zero was adopted.

340 Unnotched beams under 3-point bending were modelled in Abaqus with the same dimensions as tested.
341 The mesh was kept constant at 10 mm size (Figure 13) and a 3D 20-noded quadratic brick element with
342 reduced integration (C3D20R) was chosen, as second-order elements are very effective in bending-
343 dominated problems [44]. Uniform displacement control loading was applied to minimise convergence
344 problems and to better simulate the experimental loading conditions.



345

346

Figure 13. Prism assembly in Abaqus

347

4.2. Evaluation of tensile constitutive equations

348

RILEM TC 162-TDF (RILEM) [17], MODEL CODE 2010 (MC) [18], Barros et al. (Barros) [19] and Hu et al.

349

(Hu) [20] procedures were selected to derive the tensile constitutive equations. Although MC allows the

350

use of stress-crack width relationship, RILEM, Barros and Hu models all use stress-strain relationships,

351

and since stress-crack width relationship also leads to mesh dependency in CDP, it was decided to the

352

use stress-strain approach in modelling, to be able to make a direct comparison between different

353

models. The derived tensile σ - ε relationships (see Table 3) using the aforementioned procedures were

354

implemented in Abaqus to determine the load-deflection response of FCSA and FRSC prisms (at 28

355

days). MC requires the maximum value of crack width (w_u) to calculate the stress at ultimate strain. The

356

value 0.5 mm was used for the max crack width as it corresponds to $CMOD_3$.

357

The predicted numerical load-deflection curves are compared against the experimental results for FCSA

358

in Figure 14. It can be seen that all the approaches fail to model the full behaviour of the prisms and for

359

most of them the analysis does not converge beyond 0.6 mm (even after using high values of μ). At 0.2

360

mm deflection, RILEM, MC and Barros overestimate the loading capacity by 29.44%, 16.65% and 7.11%

361

while Hu underestimates the loading by 14.88% respectively. Barros's model, however, can capture the

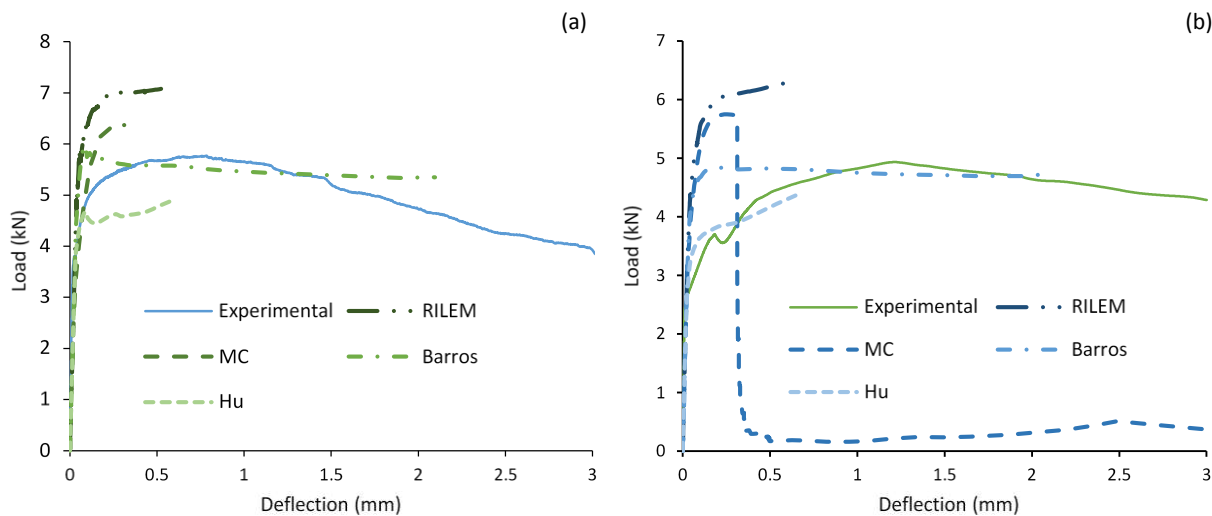
362 post-cracking behaviour of FCSA up to a certain extent. The models are even less effective in predicting
 363 the flexural behaviour of FRSC (see Figure 14). Overall, none of the above models seem to be able to
 364 capture the complete load-deflection behaviour of the tested specimens.

365 **Table 3**

366 σ - ϵ relationships for FCSA and FRSC at 28 days using different approaches

Mixes	RILEM		MC		Barros		Hu	
	σ	ϵ	σ	ϵ	σ	ϵ	σ	ϵ
FCSA	9.473	0	2.980	0	7.037	0	4.771	0
	4.977	0.000263	3.311	0.000030	3.981	0.001056	2.986	0.001892
	5.140	0.024814	4.977	0.002319	3.751	0.103864	3.929	0.024857
	0.095	0.025000	4.561	0.012335	0.080	0.104000	0.050	0.025000
	0.090	0.500000	0.030	0.012500	0.074	0.500000	0.048	0.500000
			0.029	0.500000				
FRSC	6.165	0	3.354	0	4.580	0	3.105	0
	4.340	0.00017	3.727	0.000006	3.472	0.001066	2.604	0.002019
	4.619	0.024822	4.340	0.002333	3.370	0.103870	3.523	0.024864
	0.070	0.025000	4.145	0.012340	0.050	0.104000	0.040	0.025000
	0.065	0.500000	0.040	0.012600	0.046	0.500000	0.035	0.500000
			0.035	0.500000				

367



368

369 **Figure 14.** Comparison between experimental and numerical load-deflection curves at 28 days for: (a) FCSA; (b)

370

FRSC

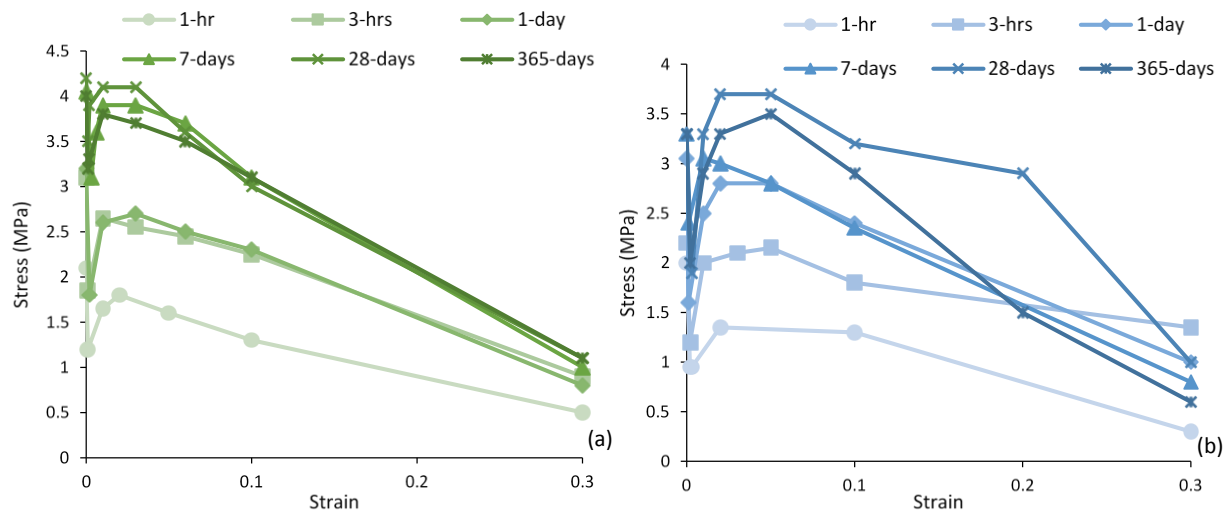
371 4.3. Numerical approach using inverse analysis

372 Inverse analysis was adopted to determine the post-cracking $\sigma - \epsilon$ relationships for the different SFRC
373 mixes and obtain a better prediction of the flexural performance of the tested specimens. The tensile
374 properties are defined by using multilinear $\sigma - \epsilon$ curves. The analysis is repeated while adjusting the
375 tensile parameters until the numerical load-deflection curve matches the experimental response in
376 capacity and energy dissipation within 2%.

377 The determined tensile $\sigma - \epsilon$ curves shown in Figure 15 are then used to predict the structural behaviour
378 of the FRC tested specimens. To better capture the flexural performance at larger displacements, the
379 strain at failure should be accurately determined. The failure strain is calculated by dividing the ultimate
380 width of crack (which is considered to be equal to half of the fibre length (l_f)) by the characteristic
381 length. It was shown in a previous study on SFRC [45] that using a characteristic length of $h_{sp}/2$ (the
382 depth of a notched prism divided by 2) gives good results when converting displacements into
383 equivalent strains. Thus, for this study, a value of 0.5 was adopted as a strain failure which is fairly close
384 to $l_f/2$ divided by half of the prism depth. It should be noted though that most tests were stopped at 5
385 mm deflection as not to damage the LVDTs and thus, complete failure was never reached. For design
386 purposes, a max strain of 0.025 is deemed sufficient so as to prevent the development of large crack
387 widths.

388 The predicted curves are shown together with the experimental results in Figure 16 through Figure 16.
389 As expected, the predictions match well the results.

390 The results for FCSA at 28 days was further analysed (using the same material model for the 10mm mesh
391 size) with two mesh sizes; 16.6 mm and 5 mm to examine the effect of mesh size. The results (Figure 17)
392 confirm that there is a slight mesh dependence when using this approach.



393

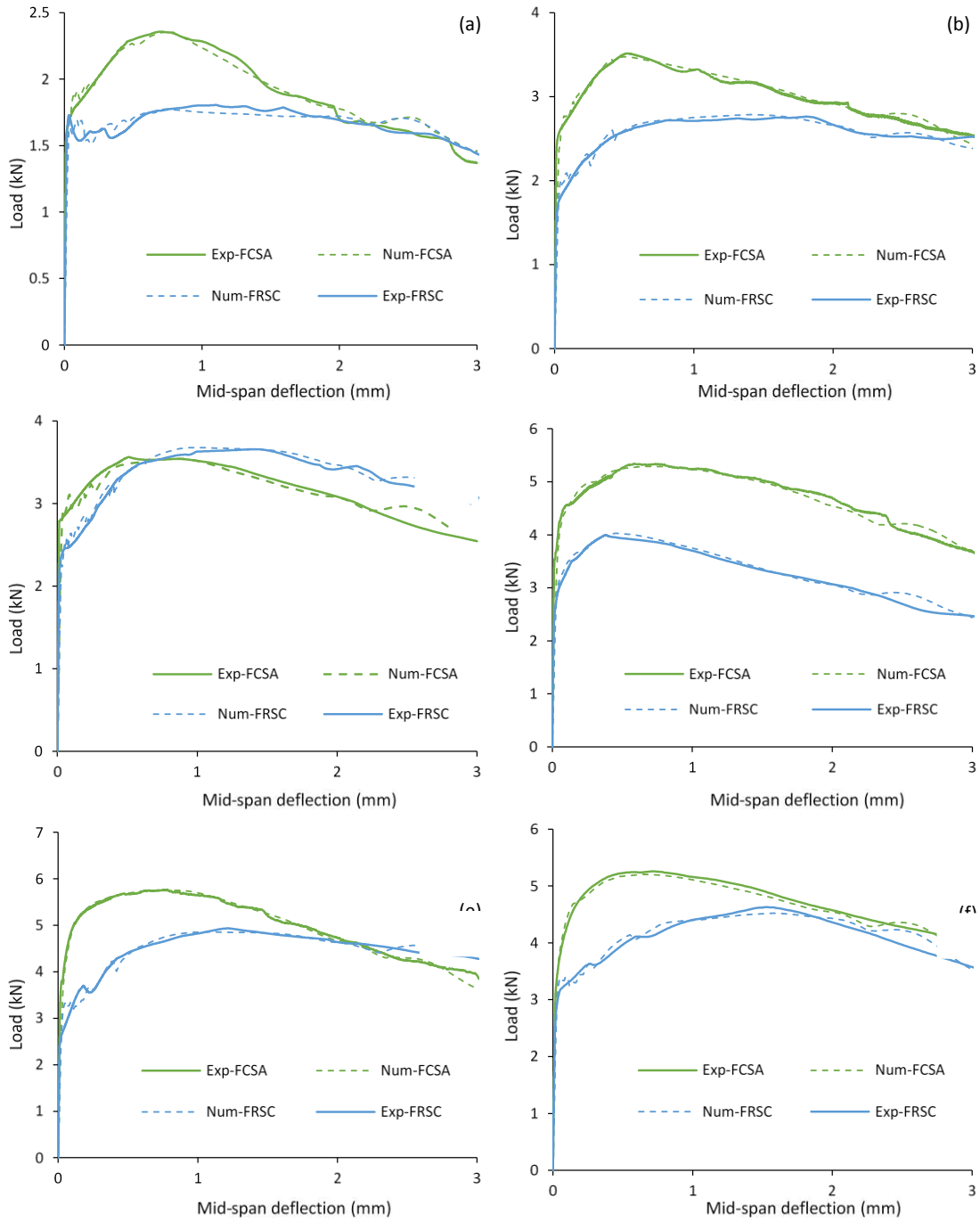
394

Figure 15. Tensile $\sigma - \epsilon$ curves for mixes at different ages for: (a) FCSA; (b) FRSC

395 **4.4. Cracking**

396 In the CDP model, cracking can be assumed to initiate at points where the tensile equivalent plastic
 397 strain is greater than zero and the maximum principal plastic strain is positive. The direction of the
 398 vector normal to the crack plane is assumed to be parallel to the direction of the maximum principal
 399 plastic strain [44]. Figure 18 shows maximum principal strain contours for FCSA prism at 28 days. It is
 400 clear that the failure of the prisms is characterised by tensile cracking at the midspan of the beam as
 401 occurred in the experiments.

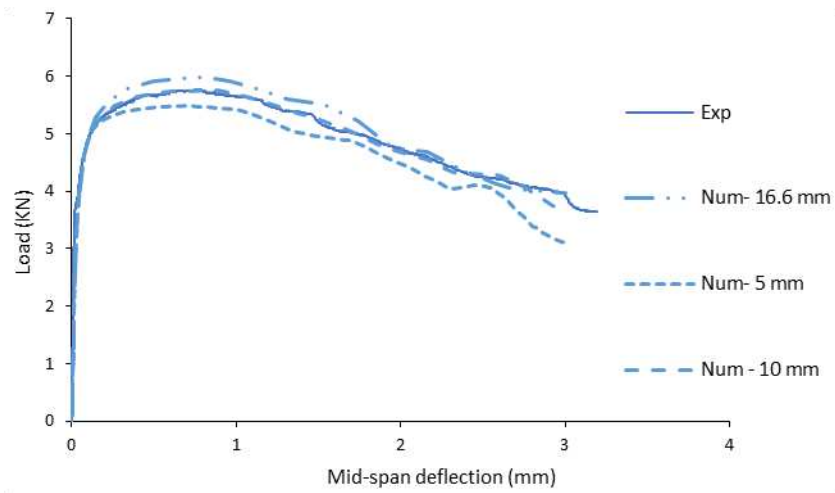
402 The crack width at the bottom of the specimens can be determined from the analysis by examining the
 403 spreading of the beam using the horizontal deformation (U_3) as shown in Figure 19. The crack width
 404 determined at 3 mm of deflection are compared with CMOD values measured by the clip gauge in Table
 405 4. The predicted values are slightly lower than the experimental values with the biggest error of 14.66%
 406 (presented in brackets) for FCSA at 28 days. This confirms that the numerical models were not only
 407 successful in predicting the flexural capacity, but also the crack widths of the tested prisms and as a
 408 result, they could be used for further studies on repair layers.



409

410 **Figure 16.** Experimental load-deflection versus numerical curves of FCSA and FRSC prisms at age of: (a) one-hour;

411 (b) three hours; (c) one-day; (d) seven days; (e) 28 days; (f) 365 days



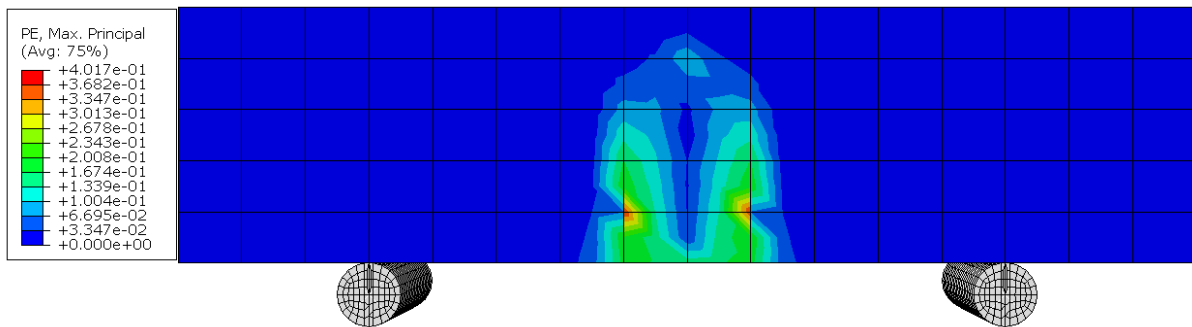
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Figure 17. Experimental load-deflection curve of FCSA at 28 days versus numerical curves using three different

414

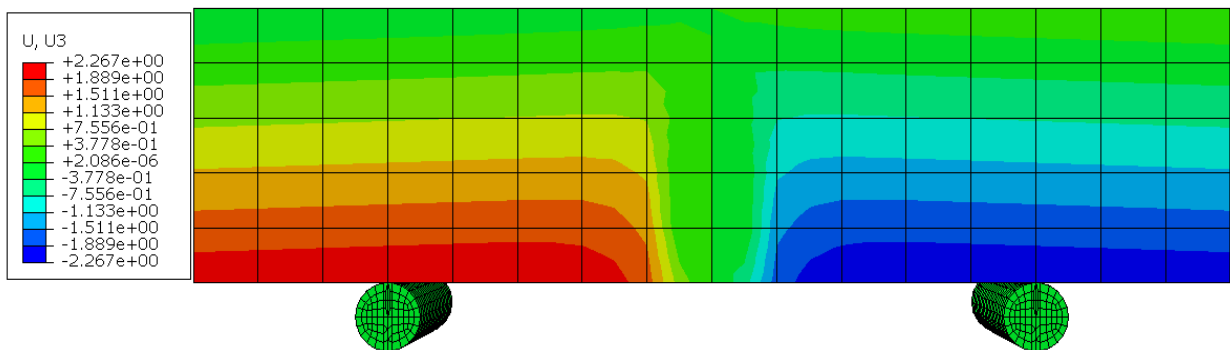
mesh sizes



415

416

Figure 18. Max principal strain contour for FCSA prisms at 28 days at the end of analysis



417

418

Figure 19. Horizontal displacement (U_3) contour for FCSA prisms at 28 days at the end of analysis

419

420 **Table 4**

421 The measured and predicted crack widths for fibre reinforced mixes

Mix	Age	1hour	3 hours	1 day	7 days	28 days	365 days
FCSA	Numerical	4.68 (4.10)	4.64 (5.60)	4.68 (5.45)	4.52 (8.87)	4.54 (14.66)	4.64 (9.02)
	Experimental	4.88	4.903	4.953	4.96	5.32	5.10
FRSC	Numerical	4.37 (12.07)	4.60 (8.18)	4.59 (9.82)	4.71 (9.25)	4.57 (12.45)	4.6 (13.21)
	Experimental	4.97	5.01	5.09	5.19	5.22	5.3

422 Note: Values in brackets represent the error (%) between experimental and numerical crack width

423 **5. Conclusions**

424 Experimental and numerical investigations were performed on plain and fibre reinforced rapid
425 hardening mortars. The main findings of this study are:

- 426 • Flexural strength evolves rapidly and both plain and fibre reinforced specimens achieved 90% of
427 their one-year strength in one day. The specimens made with CSA cement showed higher flexural
428 strength than those made with RSC cement tested at the same age due to the rigid dense crystal
429 microstructure of the CSA cement.
- 430 • The fibres have a remarkable effect on the strength and modulus of elasticity of prisms. FCSA and
431 FRSC mixes showed a flexural strength increase of approximately 36% to 70% and 24% to 41%
432 respectively. For E_{fm} , an increase of 29.7% was found for FCSA at the age of one-hour. For
433 compressive strength, the highest strength increase of around 24% was observed at one hour. No
434 compressive strength reduction was noticed for any of the mixes tested in this study up to the age
435 of one-year.

- 436 • The flexural residual strength for both FCSA and FRSC specimens continued to increase up to 0.7
437 mm, which corresponds to $CMOD_4$. FCSA prisms show higher f_R than FRSC prisms for the same crack
438 width. The values of f_R continue to increase with time for both FRC mixes and reach their peak
439 values at 28 days. However, there is a slight strength reduction at one year compared to 28 days.
- 440 • Strong correlations exist between f_{R1} and f_{R2} , f_{R1} and f_{R3} , f_{R1} and f_{R4} with $R^2 \geq 0.98$ and $R^2 \geq 0.92$ for
441 FCSA and FRSC, respectively.
- 442 • FE-predictions using CDP overestimate the loading capacity of FCSA and FRSC when using the tensile
443 constitutive laws based on RILEM TC 162-TDF, CEB FIB MODEL CODE 2010, Barros et al. Conversely,
444 the use of the models proposed by Hu et al. leads to underestimation.
- 445 • Inverse analysis was used successfully to obtain multilinear $\sigma - \epsilon$ tensile curves and model the global
446 load-displacement behaviour.
- 447 • Numerical analyses using the refined $\sigma - \epsilon$ curves were successful in capturing the cracking widths of
448 FRC tested prisms.

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