Simple conditions for sampled-data stabilization by using artificial delay

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Abstract: It is well known that in some systems a stabilizing feedback that depends on the output and its derivative can be replaced by *delay*-dependent feedback where the derivative is approximated by a finite difference. We study sampled-data implementation of such delay-dependent feedback. The analysis is based on the Taylor representation of the delayed signal with the remainder in the integral form, which is then compensated by appropriate Lyapunov-Krasovskii functional. This allows to obtain simple LMI-based conditions guaranteeing a desired decay rate of convergence. Using these conditions, we prove that if the system can be stabilized by continuous-time derivative-dependent feedback then it can be stabilized by sampled-data delay-dependent feedback with small enough sampling and delay. Finally, we introduce the event-triggering mechanism that allows to reduce the amount of transmitted signals at the cost of larger memory used.

1. INTRODUCTION

It has been shown for some classes of systems that if a system is stable under a linear feedback depending on output derivatives (e.g., $u = K_1 y + K_2 \dot{y}$), it remains stable if the derivatives are replaced by their finite difference approximations (e.g., $u = K_1 y(t) + K_2[y(t) - y(t-h)]/h$). That is, the delay-induced stability is guaranteed if the derivative-dependent control stabilizes the system. Such results have been obtained, e.g., for a chain of integrators in Niculescu and Michiels (2004) and minimum-phase systems of relative degree two in Ilchmann and Sangwin (2004). The latter work was then extended to nonlinear systems with an arbitrary relative degree and disturbances (see Karafyllis (2008), French et al. (2009)).

A simple constructive approach to the stabilization by using artificial delays has been proposed in Fridman and Shaikhet (2016, 2017). The idea is to use the Taylor expansion of the delayed output with the remainder in the integral form, which is compensated by appropriate Lyapunov-Krasovskii functional. This method leads to LMI-based stability conditions that are feasible if the output-derivative-dependent controller stabilizes the system and the delays are small enough.

In Section 2 we extend the results of Fridman and Shaikhet (2016) to arbitrary linear systems with relative degree two and sampled output. Data sampling simplifies the implementation of the delayed control, since it requires to store a finite number of sampled measurements $y(t_k), y(t_{k-1}), \dots y(t_{k-q})$ instead of a history function $y(t-\theta)$ with $\theta \in [0, h]$. Moreover, it allows to study delay-induced stabilization for networked control systems, where only sampled signals can be transmitted through communication channels. Sampled-data control with stabilizing delay has been analysed in Liu and Fridman (2012) via complete Lyapunov-Krasovskii functionals with a Wirtinger-based term and in Seuret and Briat (2015) via impulse representation and looped-functionals. Both methods lead to complicated LMIs with many decision variables. Differently from Liu and Fridman (2012) and Seuret and Briat (2015), we obtain simple LMIs with much smaller number of decision variables that lead to reasonable sampling intervals. In addition, our results allow to guarantee a desired decay rate of convergence.

In networked control systems, where sampled measurements are transmitted through communication channels, it is often beneficial to reduce the amount of transmitted signals in order to save communicational and computational resources. To achieve this goal, we study event-triggered control in Section 3. The key idea is to transmit only those measurements whose relative change is large enough (Åström and Bernhardsson (1999); Tabuada (2007)). For sampled-data delayed control this approach allows to reduce the amount of transmitted signals by increasing the number of stored sampled measurements $y(t_k), y(t_{k-1}), \dots, y(t_{k-q})$ (see Example 2).

Notations. $P > 0$ indicates that $P \in \mathbb{R}^{n\times n}$ is positive-definite, $*$ stands for symmetric terms, $\mathbb{N}_0$ are nonnegative integers, $I_m \in \mathbb{R}^{m\times m}$ is the identity matrix.

Lemma 1. (Jensen's inequality). Let $f\colon [a,b] \to [0,\infty)$ and $x\colon [a,b] \to \mathbb{R}^n$ be such that the integration concerned is well-defined. Then for any $0 < Q \in \mathbb{R}^{n\times n}$,

$$\left[\int_a^b f(s)x(s)\,ds\right]^T Q \left[\int_a^b f(s)x(s)\,ds\right] \leq \int_a^b f(s)\,ds \int_a^b f(s)x^T(s)Qx(s)\,ds.$$

Proof is given in Solomon and Fridman (2013).

Lemma 2. (Wirtinger's inequality). Let $0 \leq W \in \mathbb{R}^{n\times n}$ and $f\colon [a,b] \to \mathbb{R}^n$ be an absolutely continuous function with a square integrable first derivative such that $f(a) = 0$ or $f(b) = 0$. Then for any $\alpha \in \mathbb{R}$,

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$$\int_a^b e^{2\alpha t} f^T(s) W f(s)\, ds \le e^{2|\alpha|(b-a)} \frac{4(b-a)^2}{\pi^2} \int_a^b e^{2\alpha s} \dot f^T(s) W \dot f(s)\, ds.$$

Proof is given in Selivanov and Fridman (2016). It combines ideas from Gelig and Churilov (1998) and Liu et al. (2010).

2. SYSTEM DESCRIPTION AND STABILITY CONDITIONS

Consider a linear system

$$\begin{aligned} \dot x &= Ax + Bu, \\ y &= Cx, \end{aligned} \qquad x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^l, \tag{1}$$

such that $CB = 0$. Let there exist $\bar K_1, \bar K_2$ such that the control signal $u = \bar K_1 y + \bar K_2 \dot y$ stabilizes the system (1). Since $CB = 0$, substituting u into (1), we obtain

$$\begin{aligned} \dot x &= Ax + B[\bar K_1 Cx + \bar K_2 C\dot x] \\ &= Ax + B[\bar K_1 Cx + \bar K_2 CAx] = \bar D x, \end{aligned} \tag{2}$$

where $\bar D = A + B\bar K_1 C + B\bar K_2 CA$ is Hurwitz.

Remark 1. One can always find appropriate $\bar K_1$ and $\bar K_2$ if (1) is a square ($m = l$) minimum-phase system with $\det CAB \neq 0$ (Ilchmann and Sangwin (2004)).

Consider now a sampled-data controller

$$u(t) = K_1 y(t_k) + K_2 y(t_{k-q}), \quad t \in [t_k, t_{k+1}) \tag{3}$$

with a sampling $t_k = kh$ ($k \in \mathbb{N}_0$ throughout the paper) and delay $q \in \mathbb{N}$. For the sake of notation convenience, we set $y(t_{k-q}) = 0$ for $k - q < 0$. Defining

$$\begin{aligned} \tau(t) &= t - t_k \le h, \\ v(t) &= -\frac{1}{\tau(t)} \int_{t_k}^t \dot y(s)\, ds, \end{aligned} \qquad t \in [t_k, t_{k+1}),$$

we have

$$y(t_k) = y(t) - \int_{t_k}^t \dot y(s)\, ds = y(t) + \tau(t) v(t). \tag{4}$$

Using Taylor's expansion for $y(t - qh)$ with the remainder in the integral form, we obtain

$$\begin{aligned} y(t_{k-q}) &= [y(t_{k-q}) - y(t - qh)] + y(t - qh) \\ &= \delta(t) + y(t) - \dot y(t) qh + r(t), \end{aligned} \tag{5}$$

where $y(t) = 0$ for $t < 0$ and

$$\begin{aligned} \delta(t) &= y(t_{k-q}) - y(t - qh), \\ r(t) &= \int_{t-qh}^t (s - t + qh) \ddot y(s)\, ds, \end{aligned} \qquad t \in [t_k, t_{k+1}).$$

Therefore, the control (3) takes the form

$$\begin{aligned} u(t) = (K_1 + K_2) y(t) - K_2 qh \dot y(t) \\ + K_1 \tau(t) v(t) + K_2(\delta(t) + r(t)) \end{aligned}$$

and the closed-loop system (1), (3) is given by

$$\dot x = Dx + BK_1 \tau v + BK_2(\delta + r), \tag{6}$$

with

$$D = A + B(K_1 + K_2)C - qhBK_2 CA.$$

Note that for

$$K_1 = \bar K_1 + \frac{\bar K_2}{qh}, \quad K_2 = -\frac{\bar K_2}{qh}$$

we have $D = \bar D$. That is, if (1) can be stabilized by $u = \bar K_1 y + \bar K_2 \dot y$, it can be stabilized by (3) provided τv, δ, r are small enough. The latter values can be reduced by reducing the sampling period h. The next proposition gives a bound for the sampling period in terms of LMIs for which the system (6) (that is (1), (3)) is exponentially stable.

Proposition 1. For given sampling period h and decay rate α let there exist $n \times n$ matrices $P > 0$, P_2, P_3 and $l \times l$ nonnegative matrices Q, W, U such that [1]

$$\Phi(0) \le 0 \quad \text{and} \quad \Phi(h) \le 0, \tag{7}$$

where $\Phi(\tau) = \{\Phi_{ij}(\tau)\}$ is composed of

$$\begin{aligned} \Phi_{11} &= 2\alpha P + P_2^T D + D^T P_2, \\ \Phi_{12} &= P - P_2^T + D^T P_3, \\ \Phi_{13} &= \Phi_{14} = P_2^T BK_2, \\ \Phi_{15} &= \tau P_2^T BK_1, \\ \Phi_{22} &= -P_3 - P_3^T + (h - \tau) C^T UC \\ &\quad + (qh)^2 A^T C^T QCA + h^2 e^{2\alpha h} C^T WC, \\ \Phi_{23} &= \Phi_{24} = P_3^T BK_2, \\ \Phi_{25} &= \tau P_3^T BK_1, \\ \Phi_{33} &= -\frac{4}{(qh)^2} e^{-2\alpha qh} Q, \\ \Phi_{44} &= -\frac{\pi^2}{4} e^{-2\alpha qh} W, \\ \Phi_{55} &= -\tau e^{-2\alpha h} U, \end{aligned}$$

other blocks are zero. Then the system (1), (3) is exponentially stable with the decay rate α.

Proof. For $t \ge h$ consider the functional

$$V = V_0 + V_v + V_\delta + V_r, \tag{8}$$

where

$$\begin{aligned} V_0 &= x^T P x, \\ V_v &= (t_{k+1} - t) \int_{t_k}^t e^{-2\alpha(t-s)} \dot y^T(s) U \dot y(s)\, ds, \\ V_\delta &= h^2 e^{2\alpha h} \int_{t_k - qh}^t e^{-2\alpha(t-s)} \dot y^T(s) W \dot y(s)\, ds \\ &\quad - \frac{\pi^2}{4} \int_{t_k - qh}^{t - qh} e^{-2\alpha(t-s)} [y(s) - y(t_k - qh)]^T \times \\ &\qquad\qquad W[y(s) - y(t_k - qh)]\, ds, \\ V_r &= \int_{t-qh}^t e^{-2\alpha(t-s)} (s - t + qh)^2 \ddot y^T(s) Q \ddot y(s)\, ds \end{aligned}$$

for $t \in [t_k, t_{k+1})$. The term V_v, taken from Fridman (2010), compensates the error due to sampling $v(t)$. The term V_δ compensates another error due to sampling $\delta(t)$ and follows the constructions of Liu and Fridman (2012); Selivanov and Fridman (2016). Note that $V_\delta \ge 0$ due to Lemma 2. The term V_r compensates the remainder $r(t)$ in the Taylor expansion (5). In Fridman and Shaikhet (2016) the corresponding term has the form $\bar V_r = \int_{t-qh}^t (s - t + qh) \ddot y^T(s) Q \ddot y(s)\, ds$, which leads to more conservative results (see Remark 3.1 in Fridman and Shaikhet (2017)).

We have

$$\dot V_0 + 2\alpha V_0 = 2x^T P \dot x + 2\alpha x^T P x,$$

[1] MATLAB codes for solving the LMIs are available at `https://github.com/AntonSelivanov/IFAC17a`

Table 1. Sampling intervals and the amounts of decision variables

	Sampling h	# Variables
(Seuret and Briat, 2015, Theorem 3.3)	$[10^{-5}, 0.788]$	$37n^2 + 5n + \frac{(n+1)n}{2}$
(Liu and Fridman, 2012, $N = 2$)	$[10^{-5}, 0.499]$	$10n^2 + 4n + \frac{(n+1)n}{2}$
(Liu and Fridman, 2012, $N = 1$)	$[10^{-5}, 0.380]$	$7n^2 + 3n + \frac{(n+1)n}{2}$
Proposition 1 and Remark 2	$[2 \times 10^{-5}, 0.222]$	$2n^2 + \frac{(n+1)n}{2} + 3\frac{(l+1)l}{2}$

$$\begin{aligned}\dot V_\delta + 2\alpha V_\delta &= h^2 e^{2\alpha h}\dot x^T(t)C^TWC\dot x(t)\\ &\quad - \frac{\pi^2}{4}e^{-2\alpha qh}\delta^T(t)W\delta(t).\end{aligned}$$

Using Lemma 1 with $f(s) \equiv 1$, we obtain

$$\begin{aligned}&\dot V_v + 2\alpha V_v = (h-\tau(t))\dot y^T(t)U\dot y(t)\\ &\quad - \int_{t_k}^t e^{-2\alpha(t-s)}\dot y^T(s)U\dot y(s)\,ds \le (h-\tau(t))\times\\ &\quad \dot x^T(t)C^TUC\dot x(t) - e^{-2\alpha h}\tau(t)v^T(t)Uv(t).\end{aligned}$$

Using Lemma 1 and $CB = 0$, we derive

$$\begin{aligned}&\dot V_r + 2\alpha V_r = (qh)^2\ddot y^T(t)Q\ddot y(t)\\ &\quad - 2\int_{t-qh}^t e^{-2\alpha(t-s)}(s-t+qh)\ddot y^T(s)Q\ddot y(s)\,ds \le\\ &\quad (qh)^2\dot x^T(t)A^TC^TQCA\dot x(t) - \frac{4e^{-2\alpha qh}}{(qh)^2}r^T(t)Qr(t).\end{aligned}$$

We use the following descriptor representation of (6):

$$\begin{aligned}0 = 2[x^TP_2^T + \dot x^TP_3^T]&\times\\ &[-\dot x + Dx + BK_1\tau v + BK_2(\delta + r)]. \quad (9)\end{aligned}$$

Therefore,

$$\dot V + 2\alpha V + (9) \le \varphi^T\Phi(\tau)\varphi,$$

where $\varphi = \operatorname{col}\{x, \dot x, r, \delta, v\}$. Since $\Phi(\tau)$ is affine in $\tau \in [0, h]$, the condition (7) guarantees $\dot V \le -2\alpha V$ and, therefore, exponential stability with the decay rate α. □

Corollary 1. (Stabilizability for small h). Let there exist $\bar K_1$ and $\bar K_2$ such that the system (1) is stable under the continuous-time controller $u = \bar K_1 y + \bar K_2\dot y$. Then it can be stabilized by sampled-data controller (3), where

$$K_1 = \bar K_1 + \frac{\bar K_2}{qh}, \quad K_2 = -\frac{\bar K_2}{qh}$$

with small enough sampling period h and appropriate q.

Proof. To proof the corollary, we show that LMIs (7) are feasible for small enough h and appropriate q. Since $u = \bar K_1 y + \bar K_2\dot y$ stabilizes (1), the matrix $\bar D$ from (2) must be stable. For chosen K_1, K_2, we have $D = \bar D$, therefore, there exists $P > 0$ such that $PD + D^TP < 0$, where D is from (6). Then, there exist P_2, P_3 such that

$$\Lambda = \begin{bmatrix} P_2^TD + D^TP_2 & P - P_2^T + D^TP_3\\ * & -P_3 - P_3^T\end{bmatrix} < 0.$$

To see this, one can take $P_2 = P$, $P_3 = \varepsilon I$ and apply Schur complement to obtain

$$PD + D^TP + \frac{\varepsilon}{2}D^TD < 0,$$

which is true for small $\varepsilon > 0$.

Consider now $q = q(h) = O(1/\sqrt{h})$ for $h \to 0$. Then $qh = O(\sqrt{h})$ and $K_1 = O(1/\sqrt{h})$, $K_2 = O(1/\sqrt{h})$. By taking $U = O(1/\sqrt{h})$, $Q = O(1/\sqrt{h})$, $W = O(1/(h\sqrt{h}))$ and applying Schur complement to $\Phi(\tau)$ with $\alpha = 0$ and $\tau = 0, h$, we obtain that (7) holds if

$$\Lambda + O(\sqrt{h}) < 0,$$

which is true for small enough h. Obviously, (7) remains true for small $\alpha > 0$. □

Remark 2. (Polytopic uncertainty). The results of Proposition 1 are applicable to polytopic-type uncertain A. Indeed, by applying Schur complement to the term $(qh)^2A^TC^TQCA$, we obtain that (7) is equivalent to [2]

$$\Xi(0) \le 0 \quad \text{and} \quad \Xi(h) \le 0, \qquad (10)$$

where

$$\Xi(\tau) = \left[\begin{array}{ccccc|c} & & & & & 0\\ & & & & & qhA^TC^TQ\\ & & \bar\Phi(\tau) & & & 0\\ & & & & & 0\\ & & & & & 0\\ \hline * & * & * & * & * & -Q\end{array}\right]$$

and $\bar\Phi(\tau)$ coincides with $\Phi(\tau)$ except for the block

$$\bar\Phi_{22}(\tau) = -P_3 - P_3^T + (h-\tau)C^TUC + h^2e^{2\alpha h}C^TWC.$$

The matrix Ξ is affine in A, therefore, if A resides in the uncertain polytope

$$A = \sum_{j=1}^M \mu_j A^{(j)}, \quad 0 \le \mu_j \le 1, \quad \sum_{j=1}^M \mu_j = 1,$$

one needs to solve the LMIs (10) simultaneously for the M vertices $A^{(j)}$, applying the same decision matrices P_2, P_3, Q, W, U.

Example 1 (Liu and Fridman (2012)). Consider the system

$$\dot x(t) = \begin{bmatrix}0 & 1\\ g & 0\end{bmatrix}x(t) + \begin{bmatrix}0\\1\end{bmatrix}u(t), \quad y(t) = [1\ 0]\,x(t)$$

with the uncertainty $g \in [-0.1, 0.1]$. This system is not stabilizable by sampled-data controller $u(t) = Ky(t_k)$, $t \in [t_k, t_{k+1})$. However, it can be stabilized by

$$u(t) = -0.35y(t_k) + 0.1y(t_{k-3}), \quad t \in [t_k, t_{k+1}).$$

Table 1 shows intervals for the sampling $h = t_{k+1} - t_k$ that preserves the stability, obtained via different methods. As one can see, Proposition 1 leads to a reasonable sampling interval while the number of decision variables is significantly smaller: 14 instead of 37 as in Liu and Fridman (2012) with $N = 1$.

Differently from Liu and Fridman (2012), Seuret and Briat (2015), our results can easily guarantee a decay rate of exponential convergence α. E.g., for $\alpha = 10^{-3}$, Remark 2 gives $h \in [0.04, 0.185]$.

Note that the results of Liu and Fridman (2012), Seuret and Briat (2015) are applicable to time-varying $g(t)$ while we consider constant g. Our approach would be applicable

[2] MATLAB codes for solving the LMIs are available at `https://github.com/AntonSelivanov/IFAC17a`

to time-varying uncertainty $g(t)$ if the matrix P in (10) was the same for all the M vertices $A^{(j)}$. However, such conditions are not feasible for the considered example.

3. EVENT-TRIGGERED CONTROL

In this section we use event-triggered control to reduce the amount of transmitted signals. The basic idea is to transmit only those measurements whose relative change is large enough. Namely, the control is given by

$$u(t) = K_1\hat{y}(t_k) + K_2\hat{y}(t_{k-q}), \quad t \in [t_k, t_{k+1}), \tag{11}$$

where $\hat{y}(t_k)$ is the last sent measurement at time t_k:

$$\begin{aligned} &\hat{y}(t_0) = y(t_0), \\ &\hat{y}(t_k) = \begin{cases} \hat{y}(t_{k-1}), & (13) \text{ is true,} \\ y(t_k), & \text{otherwise,} \end{cases} \end{aligned} \tag{12}$$

with the event-triggering rule

$$\begin{aligned} (\hat{y}(t_{k-1}) - y(t_k))^T\Omega(\hat{y}(t_{k-1}) - y(t_k)) \le \sigma y^T(t_k)\Omega y(t_k), \\ 0 < \Omega \in \mathbb{R}^{l\times l}, \quad 0 \le \sigma \in \mathbb{R}. \end{aligned} \tag{13}$$

For convenience, we set $y(t_{k-q}) = \hat{y}(t_{k-q}) = 0$ for $k-q<0$.

Proposition 2. For given sampling period h, decay rate α, event-triggering parameter σ, and tuning parameter $\nu > 0$ let there exist $n \times n$ matrices $P > 0$, P_2, P_3, $l \times l$ nonnegative matrices Q, W, U, and $0 < \Omega \in \mathbb{R}^{l\times l}$ such that [3]

$$\left.\begin{bmatrix} \Phi(\tau) & \Psi(\tau) \\ * & \Upsilon \end{bmatrix}\right|_{\tau=0,h} \le 0, \tag{14}$$

where $\Phi(\tau)$ is defined in Proposition 1,

$$\Psi(\tau) = \begin{bmatrix} P_2^TBK_1 & P_2^TBK_2 & \sigma C^T\Omega & \sigma C^T\Omega \\ P_3^TBK_1 & P_3^TBK_2 & 0 & -\sigma qhC^T\Omega \\ 0 & 0 & 0 & \sigma\Omega \\ 0 & 0 & 0 & \sigma\Omega \\ 0 & 0 & \sigma\tau\Omega & 0 \end{bmatrix},$$

$$\Upsilon = \operatorname{diag}\{-\Omega, -\nu\Omega, -\sigma\Omega, -\sigma\Omega/\nu\}.$$

Then the system (1) under the event-triggered control (11)–(13) is exponentially stable with the decay rate α.

Proof. Introduce the event-triggering error $e_k = \hat{y}(t_k) - y(t_k)$. Then the control signal (11) takes the form

$$u(t) = K_1y(t_k)+K_2y(t_{k-q})+K_1e_k+K_2e_{k-q},\ t \in [t_k, t_{k+1}).$$

Using (4), (5), we obtain the following descriptor representation of the system (1), (11)–(13) on $t \in [t_k, t_{k+1})$:

$$\begin{aligned} 0 = 2[x^TP_2^T + \dot{x}^TP_3^T]\times \\ [-\dot{x} + Dx + BK_1(\tau v + e_k) + BK_2(\delta + r + e_{k-q})]. \end{aligned} \tag{15}$$

The event-triggering condition (12), (13) ensures that

$$\begin{aligned} 0 \le \sigma y^T(t_k)\Omega y(t_k) - e_k^T\Omega e_k, \\ 0 \le \sigma\nu y^T(t_{k-q})\Omega y(t_{k-q}) - \nu e_{k-q}^T\Omega e_{k-q}. \end{aligned} \tag{16}$$

For the functional V given in (8), we obtain

$$\begin{aligned} \dot{V} + 2\alpha V + (15) + (16) \le \psi^T \begin{bmatrix} \Phi(\tau) & \Psi'(\tau) \\ * & \Upsilon' \end{bmatrix} \psi \\ + \sigma y^T(t_k)\Omega y(t_k) + \sigma\nu y^T(t_{k-q})\Omega y(t_{k-q}), \end{aligned}$$

where $\psi = \operatorname{col}\{x, \dot{x}, r, \delta, v, e_k, e_{k-q}\}$, $\Psi'(\tau)$ is obtained from $\Psi(\tau)$ by eliminating the last two block-columns, and $\Upsilon' = \operatorname{diag}\{-\Omega, -\nu\Omega\}$. Substituting representations (4), (5) for $y(t_k)$, $y(t_{k-q})$ and using the Schur complement for the terms with σ, we obtain that if

$$\begin{bmatrix} \Phi(\tau) & \Psi(\tau) \\ * & \Upsilon \end{bmatrix} \le 0 \tag{17}$$

then $\dot{V} \le -2\alpha V$ and, therefore, the system is exponentially stable with the decay rate α. Since both $\Phi(\tau)$ and $\Psi(\tau)$ are affine in $\tau \in [0, h]$, the condition (14) implies (17). □

[3] MATLAB codes for solving the LMIs are available at `https://github.com/AntonSelivanov/IFAC17a`

Example 2 (French et al. (2009)). Consider the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t), \quad y(t) = [1\ 0\ 0]\,x(t)$$

under the control

$$u(t) = -17y(t_k) + 13y(t_{k-q}), \quad t \in [t_k, t_{k+1}). \tag{18}$$

To reduce the amount of transmitted signals, we would like the sampling period h to be as large as possible. Using Proposition 1, we find the maximum sampling h for each $q = 1, 2, \ldots$ The overall maximum $h = 0.0876$ corresponds to $q = 3$. This implies that within 50 seconds of the system evolution, $\lfloor \frac{50}{h} \rfloor + 1 = 571$ measurements are transmitted through a network.

Event-triggered control allows to send less measurements. Consider (11)–(13) with $\sigma = 9 \times 10^{-4}$, $q = 5$. Using Proposition 2, we find the maximum sampling $h = 0.0619$. Performing numerical simulations for 100 randomly chosen initial conditions from $B_1(0) = \{x_0 \in \mathbb{R}^n \mid \|x_0\| < 1\}$, we find that the average amount of sent measurements is 425.2, what is by 25% less than in the case of periodic sampling (18) with $q = 3$. Note that for $q = 5$ Proposition 1 gives $h = 0.0708$ leading to 707 sent measurements under periodic sampling (18). Thus, *event-triggered control allows to reduce the amount of transmitted signals at the cost of larger memory used* ($q = 5$ for event-triggered control, $q = 3$ for periodic sampling).

4. CONCLUSION

We considered LTIs of relative degree two under sampled-data feedback with artificial delay. For such systems, we derived simple LMI-based conditions ensuring stability with a desired decay rate. Moreover, we proved that if the system can be stabilized by continuous-time derivative-dependent feedback then it can be stabilized by sampled-data delay-dependent feedback with small enough sampling and delay. Finally, we introduced event-triggered control and demonstrated that it allows to reduce the amount of transmitted signals at the cost of larger memory used.

Further research will be devoted to the stabilization of LTIs with arbitrary relative degree by sampled-data feedback with several artificial delays.

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