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# Adaptive Low Computational Cost Optimisation Method for Performance-Based Seismic Design of Friction Dampers

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## ABSTRACT

This study aims to improve the computational efficiency and convergence rate of an already existing optimisation method for friction-based dampers based on the concept of Uniform Distribution of Deformation (UDD) and demonstrate the reliability of the results compared to Heuristic optimisation methods such as GA. In the proposed approach, the computational cost is considerably reduced by using a convergence factor that is modified in proportion to the level of performance violation. To investigate the efficiency of the proposed method, 3, 5 and 10-storey RC frames with friction dampers are optimised using adaptive UDD method, Genetic Algorithm (GA) and a coupled UDD-GA approach. The results indicate that the adaptive UDD method can lead to optimum design solutions with significantly lower computational costs (up to 300 times lower number of non-linear dynamic analyses) compared to both GA and coupled UDD-GA methods. It is shown that frames optimised under a under a single spectrum-compatible earthquake can efficiently satisfy the predefined performance targets under a set of synthetic earthquakes representing the design spectrum. Therefore, the proposed method should provide a reliable approach for more efficient design of friction-based dampers.

***Keywords:*** *Adaptive optimisation; Uniform Distribution of Deformation; Genetic Algorithm; Friction damper; Performance-based Design*

# 1. INTRODUCTION

Friction and metallic dampers dissipate considerable amounts of seismic input energy through yielding metal and friction between two or more solid bodies, respectively. Both dampers, which are referred as hysteretic dampers, exhibit hysteretic behaviour that can be idealised by an elastic perfectly plastic load-displacement relationship [1]. Due to their high energy dissipation capacity, the hysteretic dampers are considered as potentially efficient passive control systems to enhance seismic performance of substandard structures. Optimum design of energy dissipation devices for earthquake resistant structures can be a challenging task due to the complexity and high nonlinearity of these systems under earthquake excitations (e.g. activation/deactivation of the slip mechanism and development of plastic hinges in structural elements).

In general, lateral displacements have been considered as appropriate parameters to measure structural and non-structural damage under earthquake excitations. Therefore, in most current seismic performance-based design guidelines, such as FEMA 356 [2] and ASCE/SEI 41-17 [3], the maximum plastic rotation of structural elements and/or maximum inter-storey drift displacements are restricted using predefined target values to satisfy a desired performance level under a design earthquake. The concept of performance-based seismic design has been successfully utilised for optimisation of structures with supplemental dampers. For example, performance-based optimal design methodologies were developed by Liu et al. [4], Lavan and Levy [5] and Lavan and Amir [6] to obtain the best sizing and allocation of viscous dampers in regular and irregular building structures. Lavan [7] used the same concept to develop a performance-based approach for optimum design of nonlinear structures with viscous dampers aimed at limiting the maximum drift and acceleration of all storey levels as well as reducing the seismic forces applied to the structure. Similarly, a displacement-based design procedure was developed by Kim and Choi [8] to obtain an optimum number of velocity-dependent dampers for existing steel structures while satisfying a pre-defined performance limit state.

Yeow et al. [9] demonstrated the cost effectiveness of using friction beam-column joint connections over more traditional variants (e.g. extended bolted-end-plate connections) using seismic loss estimation. Latour et al. [10] assessed the seismic response of steel beam-to-column assemblies equipped with two types of friction dampers experimentally and analytically. In a follow up study conducted by Latour et al. [11], the connection structural detail and the friction pad material were modified to achieve a better performance. Their results showed that using pads coated with thermally sprayed aluminium as the friction interface, can result in higher friction coefficient compared to the other metallic or rubber materials. Using a similar approach, Piluso et al. [12]

utilised friction dampers in beam-to-column and column-to-base connections to reduce damage under earthquake excitations and prevent the yielding of first storey columns at their base.

Fiorino et al. [13] and De Matteis et al. [14] investigated the efficiency of using gypsum sheathing and aluminium shear panels for seismic upgrading of cold-formed steel and RC buildings, respectively. Similarly, Nastri et al. [15] evaluated the seismic performance of a precast reinforced concrete building retrofitted using cladding walls rigidly connected to internal columns, while adopting metallic hysteretic dampers at the base of the panels. Based on their results, the proposed system was able to develop a significant over-strength and ductility and reduce the lateral drifts under serviceability limit states. More recently, Di Lauro et al. [16] evaluated the partial safety factors and the overstrength coefficient needed to account for the influence of random material variability in the seismic design of dissipative connections equipped with friction dampers. They clearly showed the influence of different sources of uncertainty such as friction coefficient, yielding strength of steel and bolt preloading due to the tightening.

In one of the early attempts, Mohammadi et al. [17] used the concept of uniform deformation of displacement demands to determine the optimal distribution of stiffness in shear-buildings by using a basic iterative procedure. In follow-up studies, Moghaddam and Hajirasouliha [18, 19] and Hajirasouliha and Moghaddam [20] proposed an empirical equation to improve the efficiency of the optimisation process for performance-based design of shear-building structures subjected to seismic excitations. The concept of Uniform Distribution of Defoemration (UDD) (or Fully Stressed Design (FSD)) has been previously adopted for optimal design of different passive control systems such as viscous dampers [21, 22], tuned mass dampers [23], metallic yielding dampers [17] and friction dampers [24, 25] Using the same approach, Wilkinson and Lavan [26] presented a Practical Modal Pushover Design (PMPD) method for displacement-based seismic design of RC wall structures.

More recently, Nabid et al. [25] utilised the UDD concept to develop a practical optimisation methodology for seismic design of friction dampers in RC structures. However, their proposed method is based on redistributing a constant total slip load value (sum of slip loads in all dampers), and hence cannot be directly used to achieve a specific target for performance-based seismic design purposes. During the optimisation process using UDD approach, previous studies showed that the convergence factor is one of the key parameters to determine the speed of the optimisation. The value of this factor depends on the type of structure, number of storeys and adopted optimisation algorithm. It should be mentioned that all the above mentioned studies on UDD optimisation are based on a constant predefined convergence parameter, which may potentially cause instability in the optimisation process or result in a low convergence rate if it is not properly selected. In this study, for the first time, an adaptive equation is proposed for convergence parameter to improve the efficiency of standard UDD optimisation methods. The convergence rate of the proposed adaptive UDD optimisation is then compared with those obtained for the previously proposed standard UDD optimisation algorithms using constant values of the convergence factor.

Several optimisation methods have been adopted for optimum design of energy dissipation devices such as Linear Quadratic Regulator (LQR) [27, 28] Simulated Annealing (SA) [29], Gradient-based Optimisation [30]; [31, 32, 33], Genetic Algorithm (GA) techniques [34, 35, 36], Energy Based Design [37, 38], Backtracking Search Algorithm (BSA) [39], Search Group Algorithm (SGA) [40]. Criteria-based and sensitivity-based design algorithms were also introduced by Takewaki [41] and Murakami et al. [42] for optimal quantity and placement of passive energy dissipation devices, where displacement, acceleration, and earthquake input energy were regarded as the main performance-based design indices. Pollini et al. [43] dealt with a problem of optimal placement of nonlinear viscous dampers by using the adjoint sensitivity analysis method. Shiomi et al. [44] and Akehashi and Takewaki [45] investigated a problem of optimal placement of hysteretic and viscous dampers for elastic-plastic MDOF structures under the double impulse as a representative of near-fault ground motions, respectively. It should be noted that, in general, most of the above mentioned optimisation methods are computationally expensive and/or require complex mathematical calculations, and therefore, may not be suitable for optimisation of large non-linear systems.

Genetic Algorithm (GA) is a directed population-based random search, based on a biological evolution mechanism and Darwin's survival-of-the-fittest theory for solving complex problems where the number of parameters is large and the analytical solutions are difficult to obtain [46, 47]. Unlike to most classical optimisation methods, GA produces multiple optima, rather than a single local optimum, with no need to gradient information that makes GA a powerful tool for global optimisation [48]. Due to the high accuracy and reliability, standard GA and its improved versions are widely adopted in optimisation of different control systems. In an early attempt, Hadi and Arfiadi [49] employed the genetic algorithm method to find the optimum mass value of TMD dampers. In a research conducted by Singh and Moreschi [50], GA was utilised for optimal design of size and location of viscous and viscoelastic dampers by considering a desired level of reduction in the performance index. Moreschi and Singh [34] also used GA for optimum height-wise placement of friction dampers in steel braced frames when satisfying a predefined performance objective. A simultaneous optimisation design method using GA was presented by Park et al. [32] for a visco-elastically damped structural system by considering the structure and the damper as an integrated system. In their proposed method, the size of structural members, the amount and the location of viscoelastic dampers was considered as design variables while the life-cycle cost was minimised. Using a similar approach, Miguel et al. [51] applied the GA technique for multi-objective optimisation of friction dampers in shear-buildings subjected to seismic loading. It should be noted that, in general, GA techniques cannot be practically used for optimum seismic design of large non-linear systems under dynamic excitations due to their high computational costs (e.g. required number of nonlinear dynamic analyses). In this study, a standard GA is adopted to find the global optimum design solutions for selected RC frames with friction dampers as a bench mark for comparison purposes.

This study aims to improve the computational efficiency of the previously developed optimisation method based on the concept of Uniform Distribution of Deformation (UDD) by using an adaptive convergence parameter, which is a function of performance violation level. The computational efficiency of the proposed method is then demonstrated through optimisation of 3, 5 and 10-storey frames with friction dampers and comparison with the results obtained from standard UDD optimisation with constant convergence factors as well as a Genetic Algorithm (GA) and a coupled UDD-GA approach.

# 2. MODELLING AND DESIGN ASSUMPTIONS

## *2.1. Design Assumptions*

The benchmark structures used in this study consist of 3, 5 and 10-storey RC frames equipped with friction dampers with the typical geometry shown in Fig. 1 (a). The details of the friction damper are shown in Fig. 1 (b). The employed assembly comprises a concrete wall panel, a friction device at the top, horizontal supports at the bottom, and vertical supports at the sides. The lateral connections can prevent transferring extra shear forces to the adjacent columns and the floor beams by using appropriate slot direction. The vertical support for the concrete panel is provided by using panel-to-column connections with horizontal slots, while the panel is connected to the lower floor by horizontally fixed connections with vertical slots. This arrangement ensures that the displacement of the friction device at the top of the panel is equal to the inter-storey drift at each level. The bottom of the concrete panels at ground level is fixed to the base to transfer the imposed loads directly to the foundation system, and therefore, reduce the maximum column axial loads at the base. A Slotted Bolted Connection (SBC) is used as the friction device, and the friction mechanism is provided by the relative movement between the two external steel plates attached to the wall, and a T-shape central steel plate attached to the beam with brass plates inserted at the interfaces. By using over-sized holes in the central steel plate of the adopted friction device, the main friction forces are developed between the T-shape central steel plate and the brass plates (or friction interface) as shown in Fig. 1 (b). Experimental tests conducted by Grigorian et al. [52] showed that Slotted Bolted Connections with brass on steel frictional surfaces exhibit an idealised Coulomb behaviour with a reasonably constant slip force under seismic excitations. It should be noted that Nabid et al. [53] showed that a small variation in the friction force does not considerably affect the seismic performance of the optimum design friction dampers. More detailed information about the adopted system can be found in Nabid et al. [53]. The adopted friction wall damper should be equally efficient in controlling the seismic performance of other structural systems such as steel moment resisting frames. However, the efficiency of this system may be limited for very stiff structural systems such as RC frames with strong shear-walls.

The studied RC frames were assumed to be located on a soil type C of Eurocode 8 (EC8) [54] category and were designed based on low-to-medium seismicity regions using PGA of 0.2g. The uniformly distributed live and dead loads were considered to be 1.0 kN/m$^2$ and 5.3 kN/m$^2$ for the roof level; and 2.5 kN/m$^2$ and 5.5 kN/m$^2$ for all the other floors. The reference frames were initially designed in accordance with EC8 [54] and Eurocode 2 (EC2) [55] for moment-resisting RC frames with medium ductility (DCM). The concrete compressive strength ($f_c'$) and the yield strength of steel reinforcement bars ($f_y$) were assumed to be 35 MPa and 400 MPa, respectively.

### *2.2. Analytical Modelling*

In this study, the OpenSees software [56, 57] was used for modelling and conducting nonlinear time-history analyses. To model the concrete and reinforcing steel bars, a uniaxial constitutive material with linear tension softening (Concrete02) and a Giuffre–Menegotto–Pinto model (Steel02) with 1% isotropic strain hardening were used, respectively. Beam and column members were divided into three elements and modelled using displacement-based nonlinear beam-column elements with fibre sections while four Gauss–Lobatto integration points were considered for each element. P-Delta effects were taken into account and the Rayleigh damping model with a constant damping ratio of 0.05 was assigned to the first mode and to the mode at which the cumulative mass participation exceeds 95%. In this study, it was assumed that the strength of the concrete wall panels (15 cm thickness) is always higher than the maximum loads transferred from the friction device, and therefore, they were modelled using equivalent elastic elements. A nonlinear spring with an elastic-perfectly plastic uniaxial material, representing an ideal Coulomb friction hysteretic behaviour, was used to model the friction device. The beam-to-column connections were assumed to be fully rigid with no shear failure in the panel zones. A computer code in MATLAB [58] platform was developed and linked to the OpenSees [56, 57] program to analyse the output data.

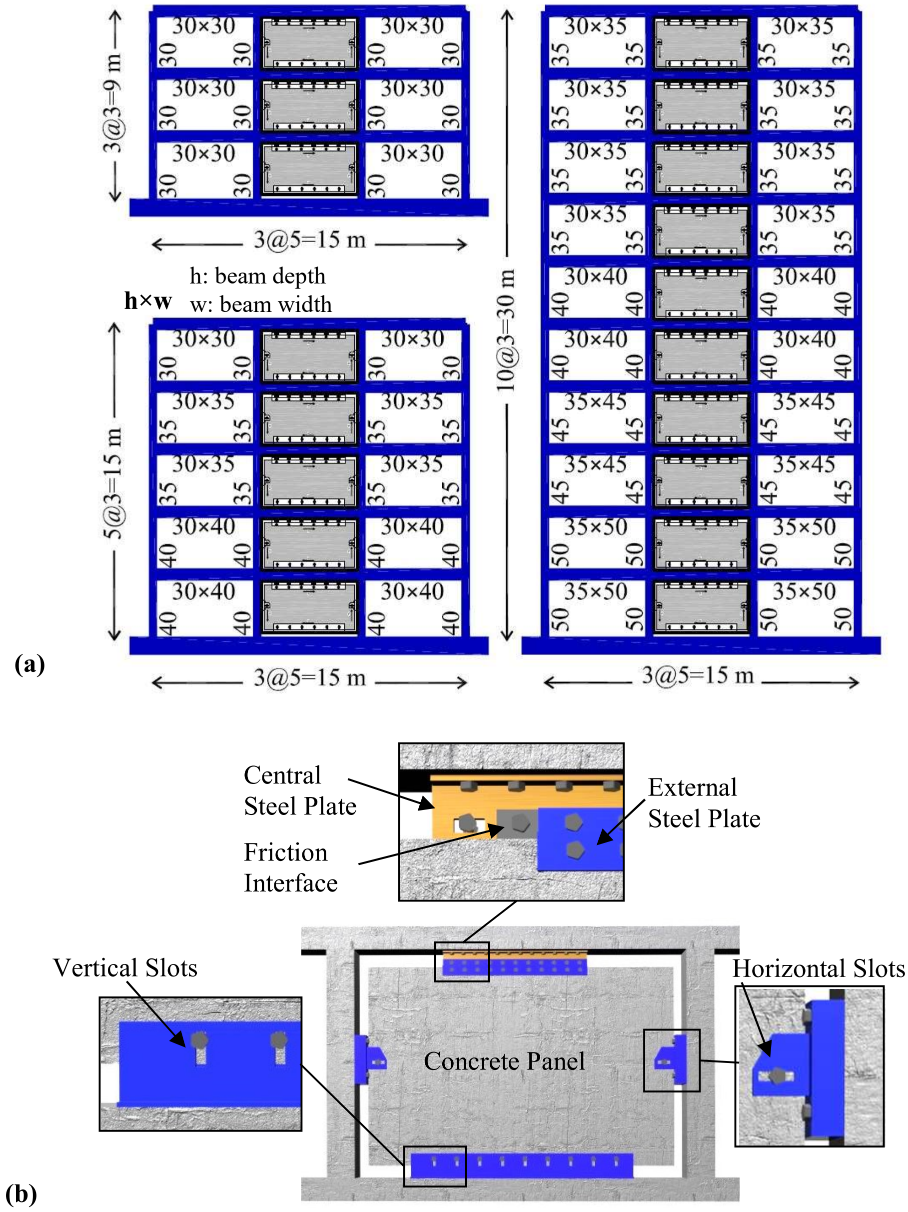


**Fig. 1.** (a) Geometry of the reference RC frames equipped with friction wall dampers, (b) schematic view of the friction wall damper (adopted from Nabid et al. [53])

## 3. SYNTHETIC EARTHQUAKE RECORD

Previous research studies (e.g. [59, 25]) have suggested that the earthquake uncertainty, in terms of acceleration response spectra, can be efficiently managed by using the optimum design corresponding to a synthetic earthquake generated to match a response spectrum obtained as an average of the response spectra of a selected set of natural earthquakes. It should be noted that most seismic performance-based design guidelines (e.g. [2, 3]) aim to control the seismic response of the buildings under two different earthquake levels: (a) Design Basis Earthquake (DBE) with 10% probability of exceedance in 50 years, and (b) Maximum Considered Earthquake (MCE) with 2% probability of exceedance in 50 years. In this study, the optimisation process is conducted under DBE level and the results are then controlled to satisfy MCE level requirements. This implies that the structure is optimised under an earthquake event with higher probability of occurrence and then is controlled under a less frequent earthquake scenario. A similar approach has been adopted by Hajirasouliha et al. [59] for optimum seismic design of RC frames.

Six synthetic DBE level earthquakes compatible with the EC8 design response spectrum were generated using the TARSCTHS [60] software, assuming a high seismicity region (i.e. PGA=0.4g) and soil class C. Fig. 2 demonstrates the elastic response spectra of the generated synthetic earthquake records and the EC8 design spectrum. It should be mentioned that although the generated synthetic earthquakes are all compatible with a same design response spectrum, they have random acceleration vibration specifications. In this study, SIM01 is assumed as the DBE event to be used during the performance-based optimisation process, while the other earthquake records are then used to evaluate the sensitivity of the optimum design solutions. The MCE records were obtained by scaling the generated records to have a PGA= 0.6g.

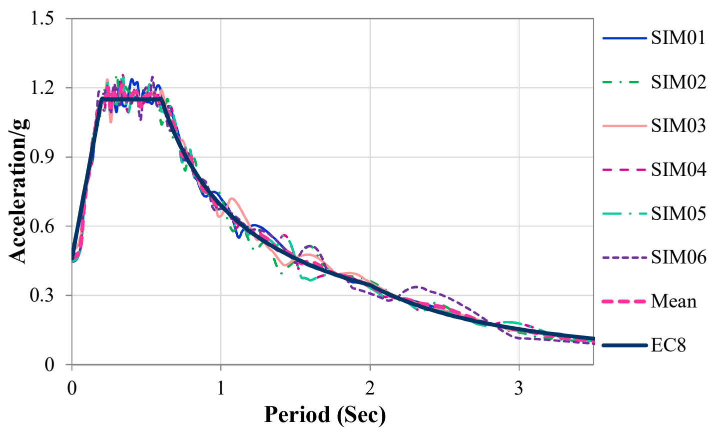


**Fig. 2.** Elastic acceleration response spectra of the synthetic earthquake records and the EC8 design spectrum, 5% damping ratio

# 4. ADAPTIVE UDD OPTIMISATION ALGORITHM

This section presents the details of the proposed adaptive UDD optimisation algorithm for optimal configuration of friction dampers in RC frame structures under a design earthquake. In this study, the structures are assumed to satisfy the Life Safety (LS) and Collapse Prevention (CP) performance levels under DBE and MCE events, respectively. Therefore, in the proposed iterative optimisation method, the slip load values of the friction devices are gradually modified until the pre-defined performance targets are satisfied under the representative design earthquakes. Structural and non-structural damage measures are usually considered to be related to maximum and residual inter-storey drifts in building structures [61]. As a result, maximum inter-storey drift has been widely used as a damage index in previous studies on performance-based design and optimisation (e.g. [59, 24, 62]). In this study maximum inter-storey drift is considered as the main design parameter to control the required performance limits. However, the proposed optimisation algorithm is general and other performance parameters such as maximum plastic rotation, energy dissipation capacity and cumulative damage index can be easily adopted.

Unlike the previously adopted UDD optimisation algorithms with constant values of convergence parameters, the proposed method employs an adaptive equation in which the convergence factor is modified in proportion to the level of performance violation. The suggested optimisation algorithm comprises the following iterative steps:

1) A uniform slip load distribution with identical slip load values for all the storey levels is assumed for the initial design of the friction devices. It should be noted that the final optimum design solution is independent of the initial slip load values as shown in a previous study performed by Nabid et al. [25], however, the optimisation rate can be affected by the initial design.

2) The benchmark structure is then subjected to the selected spectrum-compatible DBE record and the value of maximum inter-storey drift is obtained at each storey level and compared with the performance target value (e.g. LS). For the initial designs with very high or very low slip load values, the maximum drifts may be far below or far above the performance target, which increases the number of iterations in the optimisation procedure.

3) During the optimisation process, the slip loads are changed so that all of the relative displacements reach the predefined performance-based design objective. To satisfy this, the slip load is increased in the storeys

where the inter-storey drift exceeded the predefined performance target, and reduced in the storeys with inter-storey drifts below the target value. The process continues until all inter-storey drifts are close to the performance target within a predefined tolerance, where the structure is considered to be practically optimum. The following equation was used to obtain the optimum distribution of slip loads:

$$\left(F_{s,i}\right)_{n+1} = \left(F_{s,i}\right)_{n} \times \left(\frac{\Delta_i}{\Delta_{target}}\right)_{n}^{\alpha} \tag{1}$$

where $\Delta_i$ and $\Delta_{target}$ are maximum and target inter-storey drifts of $i^{th}$ storey for $n^{th}$ iteration, respectively; $F_{s,i}$ is defined as the slip load of the friction device at the $i^{th}$storey; $\alpha$ is the convergence parameter, which has a prominent effect on the speed of the optimisation process. The value of this factor depends on several parameters such as the type of structure, number of storeys and optimisation algorithm. Previous studies have proposed different ranges for the convergence parameter including: 0.1 to 0.2 proposed by Hajirasouliha et al. [59] for shear-building structures, 0.4 to 0.8 suggested by Mohammadi et al. [63] for steel frames with metallic-yielding dampers, 0.2 to 0.5 suggested by Nabid et al. [25] for RC frames with friction dampers, and 0.5 recommended by Lavan [7] for nonlinear structures with viscous dampers.

To provide the best convergence rates, in this study an adaptive equation (Equation 2) is proposed for the convergence factor used in the UDD optimisation algorithm. The value of the convergence factor depends on the relative displacement obtained from the non-linear dynamic analysis in each step and the constant, predefined target displacement. The aim is to accelerate the optimisation by increasing the α value where the difference between the maximum drift and the performance target is smaller, and decreasing the α value where the ratio between the drift and the target displacement is larger. This is to avoid significant changes in the slip loads of the storeys with large drift to target displacement ratio. However, for faster convergence, there is still more alteration of the slip loads in storeys with higher ratio of drift to target displacement compared to those with smaller ratios. The proposed equation is expected to achieve a good convergence during the optimisation process irrespective to the size of the selected frame.

$$\alpha_i = Abs\left(Ln\left(\frac{\Delta_i}{\Delta_{target}}\right)\right)^{-0.25} \tag{2}$$

It will be shown in the following sections that the adaptive convergence parameter, in general, leads to the highest convergence rate compared to the UDD optimisation with constant values, while the final design solution is unique.

In every optimisation iteration, the coefficient of variation of the inter-storey drifts ($\mathrm{COV}_{\Delta}$) is also calculated. The optimisation algorithm continues from step 2 until an acceptable level of $\mathrm{COV}_{\Delta}$ is achieved (e.g. less than 0.1). The final design solution is then subjected to the MCE record and the maximum inter-storey drifts are controlled to ensure CP level is satisfied. If the performance criteria are violated at any storey level, the corresponding slip load is adjusted using a simple iteration process. Since the initial structure is designed for gravity and seismic loads based on a seismic design code (here EC8), at some storey levels the target inter-storey drift may be satisfied without using friction dampers. Therefore, it is not usually possible to reach a very uniform inter-storey drift distribution (i.e. very low $\mathrm{COV}_{\Delta}$), especially when the effect of gravity loads is dominant. However, as will be discussed in the following sections, the proposed algorithm is capable of removing the unnecessary dampers during the optimisation process.

To demonstrate the efficiency of the proposed performance-based adaptive optimisation algorithm, the 3, 5, and 10-storey RC frames with friction dampers were optimised. In this study, LS and CP performance limits were considered to be 2% and 4% maximum inter-storey drift ratio under DBE and MCE representative spectrum compatible earthquakes, respectively. Fig. 3 demonstrates the variation of (a) convergence parameters and (b) slip load values in different storey levels of the 3, 5, and 10-storey frames as the iterations proceed. As expected from Equation 2, the storey levels with higher maximum inter-storey drifts, and in turn higher values of slip loads, exhibited more fluctuations in the convergence parameter. According to the results, in general, the slip loads reached their final optimum values in less than 25 steps. As illustrated in Fig. 3 (b), the slip loads of the first floor and the top two floors tend to be zero as they have already satisfied the predefined performance levels without using friction dampers. The reason for the low lateral displacement of the first floor is the fixed connections at the base.

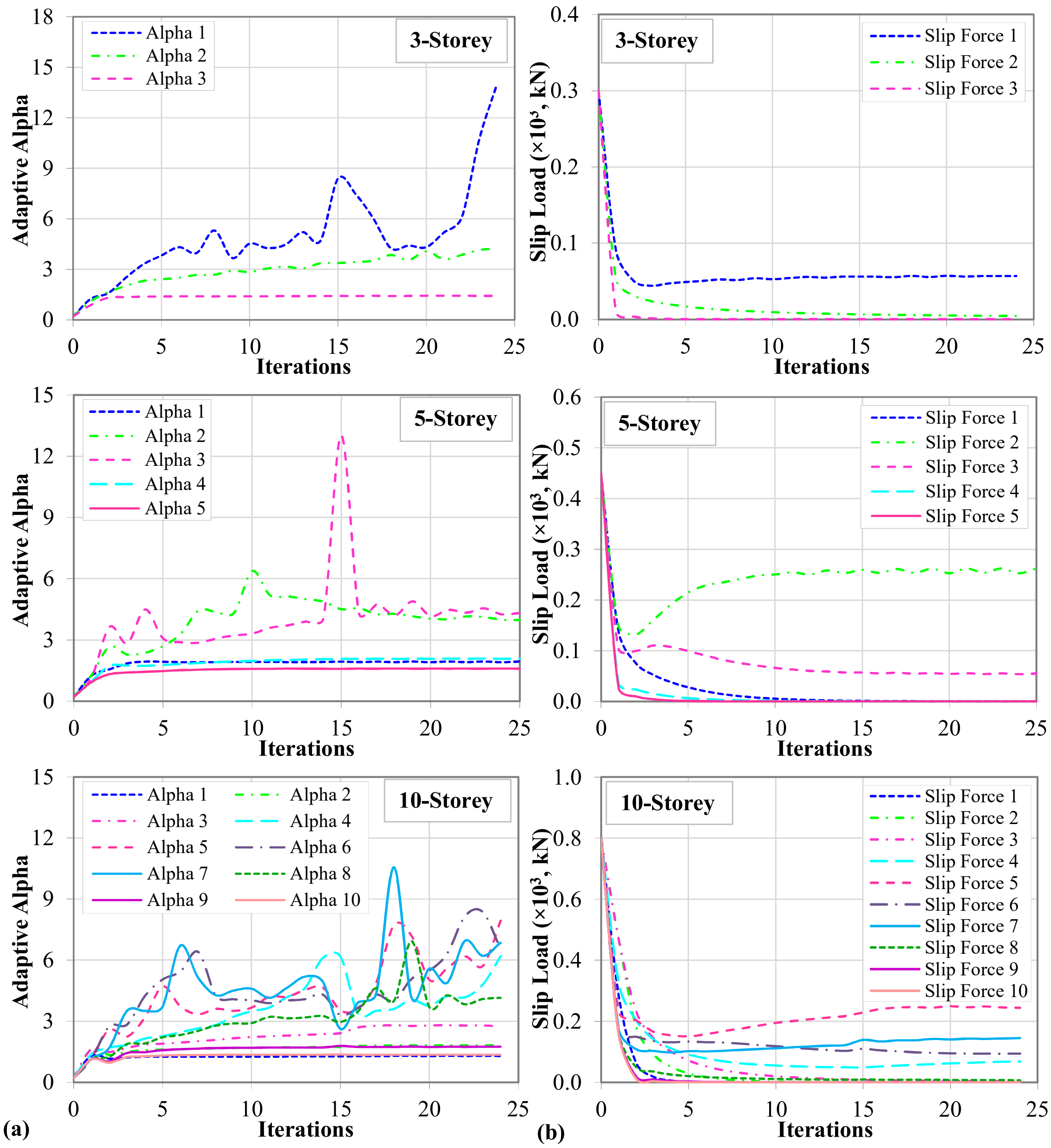


**Fig. 3.** Variations of (a) convergence parameter and (b) slip load value at each storey level for 3, 5, and 10-storey frames, DBE event

## 5. COMPARISON BETWEEN ADAPTIVE AND STANDARD UDD OPTIMISATION METHODS

According to previous research studies (e.g. [63, 25, 59]), the range for the efficient convergence rate varies for different optimisation problems. In addition, while this parameter is affected by the type and size of the structure, it may not be efficient to use a constant value for frame structures with different number of storeys. In this section, the efficiency of the proposed adaptive UDD optimisation algorithm is compared to the standard UDD method with constant values of the convergence factor for the selected 3, 5, and 10-storey frames under the representative DBE event. In case of standard UDD optimisation, the convergence factors are selected between 0 and a value which leads to the fluctuation of the results and divergence of the iterations. Fig. 4 compares the convergence rate of the adaptive and standard UDD in terms of maximum drift ratio (drift scaled to storey height) and total slip load value required to satisfy LS performance target under the DBE representative earthquake.

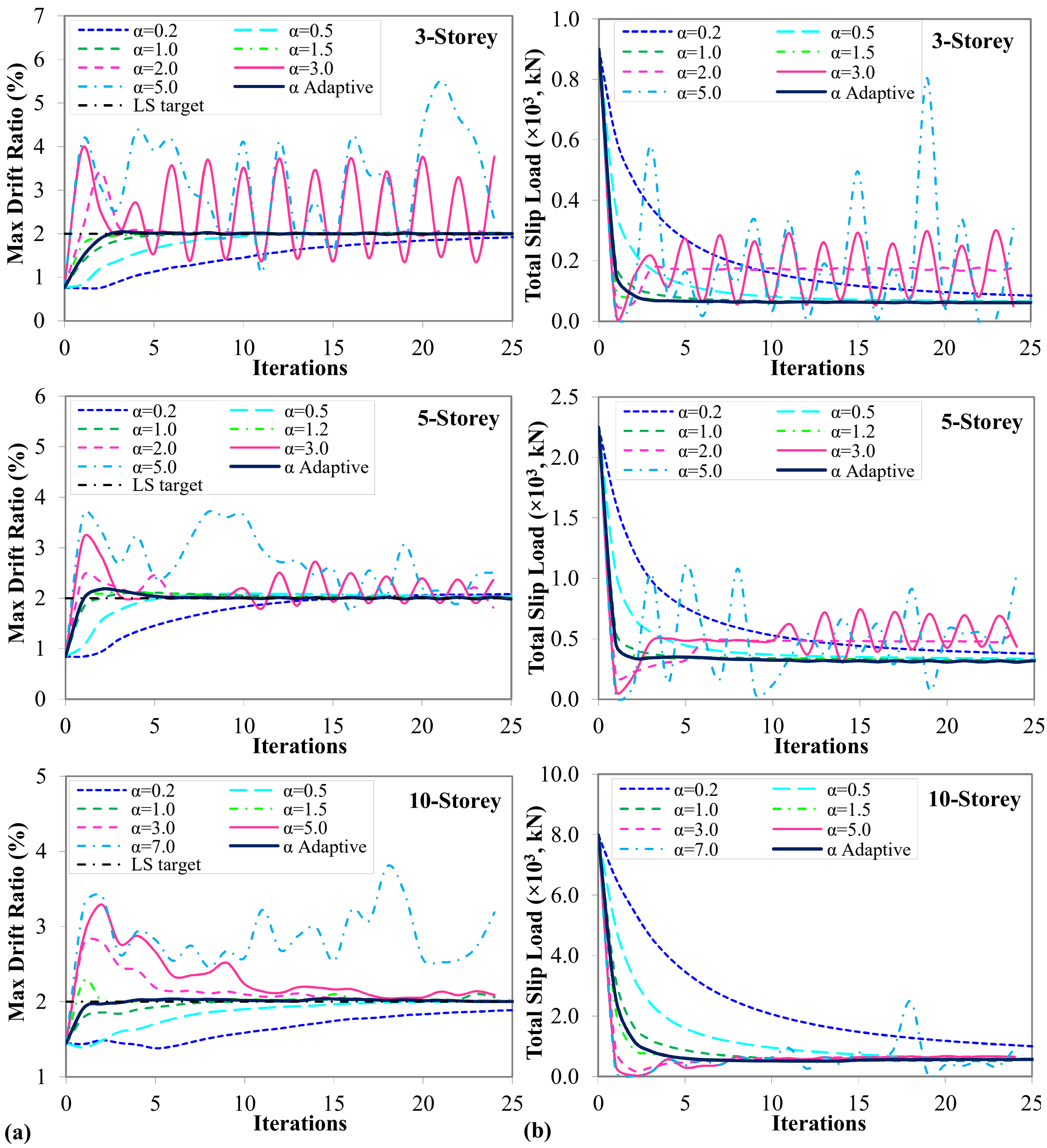


**Fig. 4.** Variations of (a) maximum inter-storey drift ratio (scaled to the storey height) and (b) total slip load value for 3, 5, and 10-storey frames, DBE event

Figs. 4 (a) and (b) show that the small values of the convergence factor lead to very slow convergence rates of both maximum drift and slip load, while the higher values of this factor may result in divergence of the iterations. The proposed adaptive α factor, however, leads to a convergence in only a few steps (i.e. less than 10 steps). Previous studies conducted by Hajirasouliha et al. [59] and Nabid et al. [25] showed that their optimisation iterations did not converge for convergence factors higher than 0.5 and 1, respectively. However, as shown in Fig. 4 (a), for the current optimisation problem, only α factors greater than 2, 3 and 5 lead to fluctuation of the results for the 3, 5, and 10-storey frames, respectively. Therefore, the efficient range of convergence factor is not limited to a specific domain for all optimisation problems even if similar structural systems are used.

# 6. OPTIMISATION USING A GENETIC ALGORITHM (GA)

The genetic optimisation algorithm (GA) was first introduced by Holland [46] to simulate the natural evolution for generating the best or the fittest species. In general, GA aims at increasing the average fitness of the population at each generation by combining the more fit individuals to converge the solution to an optimal point. The evolution of the design solutions at each iteration is an iterative process, which is usually initiated from a population of randomly generated individuals. The next generation of population is produced using three dominant operators including selection, crossover and mutation. Each individual or design solution has a set of properties such as chromosomes which can be mutated. The solutions can be represented in binary as strings of 0s and 1s. In each generation, the fitness (according to the objective function) of every individual is evaluated and the individuals with high fitness have several chances for joining in the reproduction phase. Based on the individual’s fitness, a probability is allocated to each individual for being selected as a parent. The next generation is then developed from selected parent strings and by using explorative operators such as crossover and mutation. The crossover operator creates variations in the population by dividing the selected parent string into parts and exchanging some of these parts with corresponding parts of another parent string. Mutation operator is usually used as an insurance policy [47], and is responsible for reintroducing random changes and diversity in a solution population. Mutation enables children to have features that are non-existing in their parents’ strings. Without using the mutation operator, some regions of the search space may never be discovered. The termination criteria of the algorithm are to satisfy either a maximum number of generations or a desired fitness level for the population [64].

Conventionally, genetic algorithms are developed to solve unconstrained optimisation problems. For constrained optimisation problems, either GA operations should be modified or the problem should be transformed to an unconstrained problem before the GA can be adopted [65, 41]. One of the most popular methods to handle constrained optimisation problems in GA is to use penalty functions where infeasible solutions are penalised by reducing their fitness values proportional to the degrees of constraint violation [66]. Different penalty functions have been proposed to solve the constraint problems such as Death Penalty, Static Penalties, Dynamic Penalties, Annealing Penalties and Adaptive Penalties [67]. In most penalty schemes, constant coefficients are specified at the beginning of the optimisation process. However, it is challenging to estimate appropriate values as there is no clear physical meaning for these coefficients [68]. While a large penalty value can prevent a search in an infeasible region, a small penalty will cause the algorithm to spend too much time on searching for the best solutions in an infeasible region. In the first case, GA will converge to a feasible solution very quickly even if it is far from the optimum, while in the latter case, GA would converge to an infeasible answer [67].

In this study, the following GA optimisation procedure is employed for optimum design of the 3, 5 and 10-storey frames with friction dampers, using a MATLAB [58] code developed for this purpose:

1) The initial population of 50 individuals is randomly selected from a wide range of slip loads starting from 0 to a relatively high value (almost equal to the corresponding mean storey shear strength).

2) Non-linear time-history dynamic analysis is performed using the representative earthquake design event as input, and the maximum inter-storey drift of all the storeys is determined.

3) The maximum inter-storey drift is compared with the predefined performance target (here LS performance level under DBE) and a penalty is applied to the slip loads of the storeys which exceed that value. Then, the objective function (defined as a summation of the slip loads required for all the friction devices) and the penalty function is calculated. Exploration of the optimal result is then performed according to the fitness of the objective function. Equations 3 and 4 are used for calculating the penalty function *(PF)* and objective function *(OF)*, respectively:

$$PF = a \times \left( \frac{\Delta_{\max}}{\Delta_{target}} - 1 \right) \tag{3}$$

$$OF = \sum_{i=1}^{N} F_{s,i} + PF \tag{4}$$

where $a$ is an empirical scale factor considered to be equal to 1000, 1500 and 3000 for the 3, 5 and 10-storey frames, respectively. These numbers were found to be sufficient to scale the penalty value to the same order with the objective function. $\Delta_{max}$ and $\Delta_{target}$ are the maximum and target inter-storey drifts, and $N$ is the number of storeys. The proposed penalty function which was obtained based on several trial and errors, adds an appropriate (not very large nor very small) expense to the objective function based on the maximum inter-storey displacement and its exceedance of the selected performance target. For an analysis with maximum drift less than the target value, penalty function is considered as 0. In this algorithm, Steps 2 and 3 are repeated for the entire population of samples.

4) Using the above mentioned GA operators, a new population is generated from the best individuals of the previous generation.

5) The algorithm is then repeated from step 2 to 4 for each generation until the mean value of the objective function converges to the best fitness function in the same generation or the maximum number of generations is achieved. Subsequently, the final design solution is subjected to the MCE representative record and the maximum inter-storey drifts are compared with the predefined CP performance level target. If the inter-storey drift exceeds the target at any storey level, the corresponding slip load is increased using a simple iteration process.

In this study, stochastic uniform approach is chosen as the selection function, and Uniform and Heuristic methods are considered as the mutation and crossover operators, respectively, during the GA optimisation process. In uniform mutation strategy, the value of the chosen gene is replaced with a uniform random value selected between the user-specified upper and lower bounds for that gene [69]. In heuristic crossover, the fitness values of two parent chromosomes are utilised to ascertain the direction of the search moving from the parent with the worse fitness value to the parent with the better fitness value [70]. Fig. 5 illustrates the variation of the mean and best values of the fitness function (objective function, Equation 4) for the 3, 5, and 10-storey frames subjected to the representative DBE event as the generations proceed during the GA optimisation. For better comparison, the optimisation was repeated three times for each frame using different sets of random initial populations (slip load values) and the answer with the lowest fitness values was retained as the optimum solution. In all cases, the global optimum answer was reasonably achieved with a small standard deviation.

Based on the results, the optimisation procedures converged to the fittest values after about 40 generations for the 3, 5 storey frames and about 60 generations for the 10-storey frame. In this study, the GA optimisation is considered to converge to an optimum design solution when there is no/negligible (i.e. less than 1%) fluctuation in the best fitness values for several consecutive generations (e.g. 10 generations).

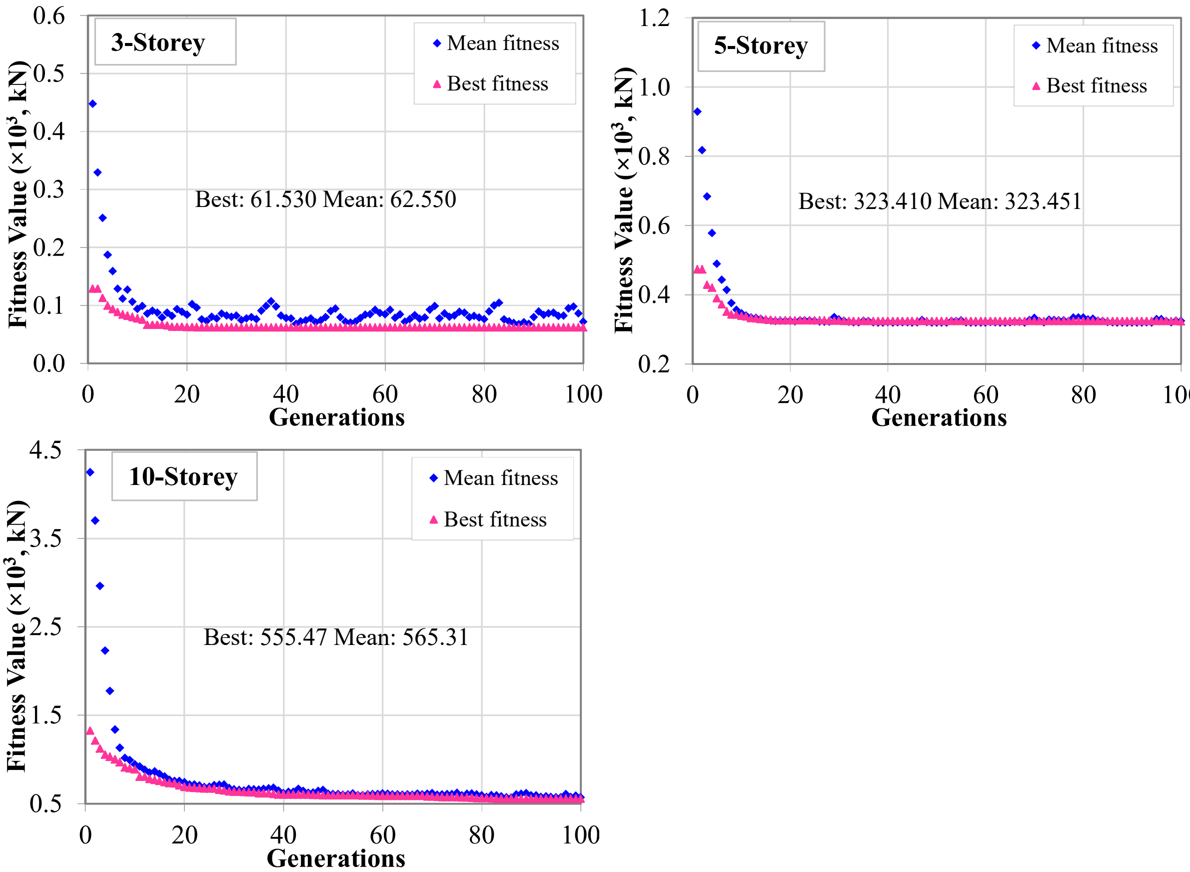


**Fig. 5.** Variations of fitness function of 3, 5, and 10-storey frames versus optimisation generation during GA optimisation, DBE event

Fig. 6 demonstrates the variations of inter-storey drift and slip load in the friction device at each storey level, the total slip load and the coefficient of variation of the inter-storey drifts ($\mathrm{COV}_{\Delta}$) for the 5-storey frame under the representative DBE event as the optimisations proceed. In the evolutionary GA optimisation, the number of nonlinear dynamic response simulations is equal to the population size in each generation (e.g. in this study, one generation is 50 nonlinear dynamic analyses). As can be observed from Fig. 6 (a), at the beginning of the optimisation, when a set of slip loads (population) is randomly selected from a wide range of upper and lower bounds of slip loads (scattered points), the resulting inter-storey drifts are considerably higher/lower than the allowable target value of 6 cm (2% inter-storey drift corresponding to LS level). As the optimisation progresses, successful individuals are mutated to create a new population (a new set of slip loads) while satisfying the performance target level with the minimum value of total slip load (Fig. 6 (c)). Fig. 6 (d) shows that the GA optimisation converged after about 2000 response simulations (i.e. 40 generations) with $\mathrm{COV}_{\Delta}$ decreasing from 105% to around 16%.

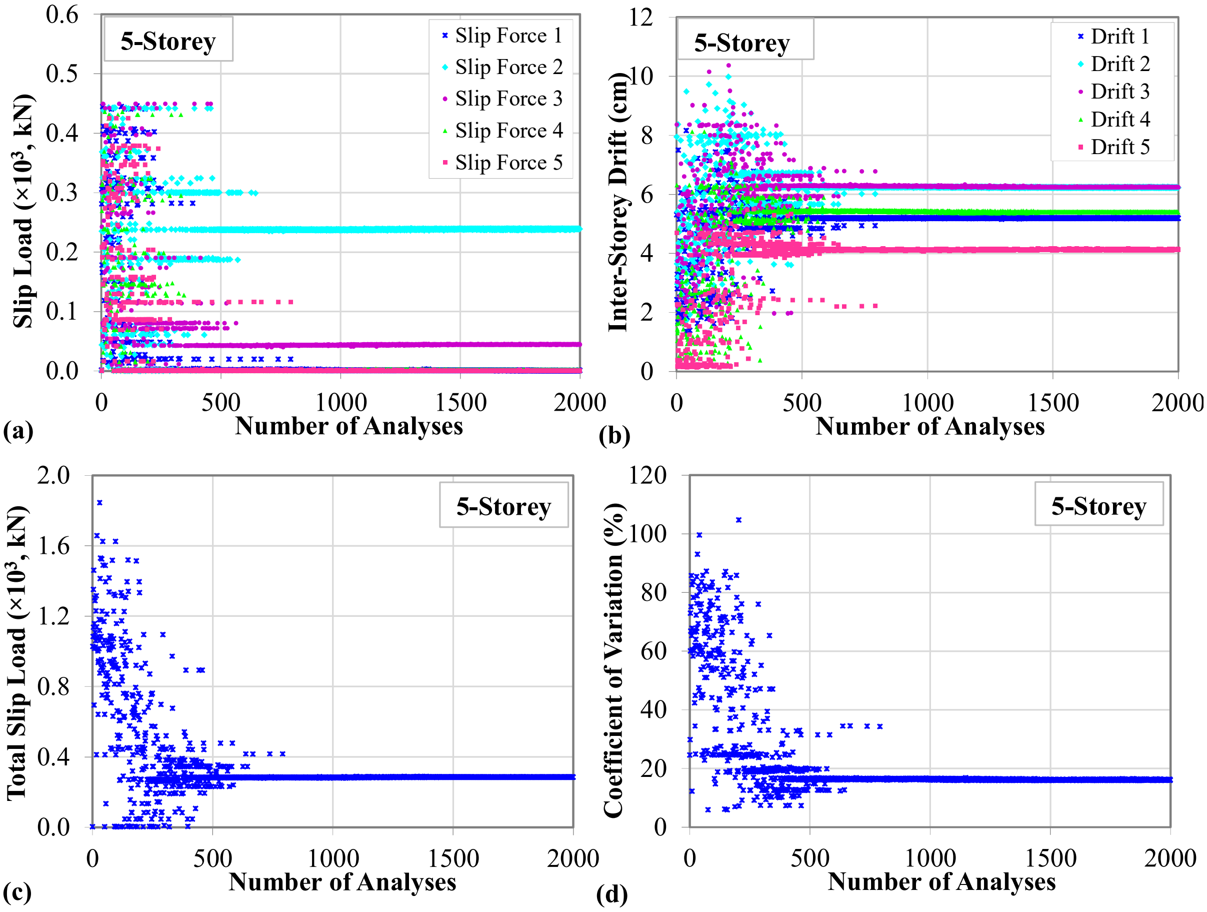


**Fig. 6.** Variations of (a) each storey slip load value, (b) each storey lateral drift, (c) total slip load and (d) COV of the drifts for 5-storey frame during GA optimisation, DBE event

## 7. COMPUTATIONAL EFFICIENCY OF ADAPTIVE UDD OPTIMISATION METHOD

To demonstrate the efficiency of the proposed adaptive optimisation method in terms of convergence speed and optimum design solutions, the results are compared with those obtained from the optimisation using the Genetic Algorithm (GA) technique. In addition to the previous optimisation results, the previous GA optimisations are repeated for the selected frames by considering the optimum slip load values obtained from the adaptive UDD algorithm as the initial slip load values (i.e. initial population) rather than using random selections.

Table 1 compares the objective functions and the total number of non-linear dynamic analyses of the 3, 5 and 10-storey frames optimised using (1) the proposed adaptive UDD method, (2) GA optimisation starting with random values (i.e. GA) and (3) GA optimisation starting with optimum values obtained from the adaptive UDD (i.e. UDD-GA) under the representative DBE event. It can be noted that the number of non-linear dynamic analyses required for the GA optimisation is significantly higher than those needed for the adaptive UDD (by up to 300 times), while these two methods lead to almost similar objective functions. It is shown that for the 3 and 10-storey frames using the GA method results in a negligible improvement (less than 2%) in the objective function of the final optimum design solutions compared to the adaptive UDD method. For the 5-storey frame, even after 2000 analyses the GA approach has not reached a better design solution than the UDD method.

It is shown in Table 1 that using the optimum results from the UDD as the initial population of the GA (i.e. UDD-GA) considerably decreases the number of required analyses (by up to 65%), which means that the UDD-GA combination is a significant improvement in terms of computational costs compared to the standard GA optimisation. For example, the optimum solution of the coupled UDD-GA method for the 5-storey frame, with a total of 800 simulations, is only slightly better than the one achieved with the adaptive UDD method after only 10 simulations (313.38 kN versus 318.04 kN). This confirms the simplicity and computational efficiency of the proposed adaptive UDD method compared to the global evolutionary optimisation methods such as GA.

**Table 1.** Comparison of GA, adaptive UDD, and UDD-GA methods in terms of total number of non-linear dynamic analyses and objective function, DBE event

| | **Adaptive UDD** | | **GA** | | **UDD-GA** | |
|---|---|---|---|---|---|---|
| | No. of analyses | Objective function (kN) | No. of analyses | Objective function (kN) | No. of analyses | Objective function (kN) |
| **3-St** | 6 | 61.83 | 2000 | 61.53 | 700 | 61.43 |
| **5-St** | 10 | 318.04 | 2000 | 323.41 | 800 | 313.38 |
| **10-St** | 14 | 556.92 | 3000 | 556.60 | 1450 | 541.78 |

Fig. 7 illustrates the height-wise distribution of slip load values for the 3, 5 and 10-storey frames optimised using the proposed adaptive UDD, GA and the coupled UDD-GA optimisation methods under the representative DBE event. The slip load distributions follow almost similar patterns for all the selected optimisation approaches. The only exception is the 10-storey frame with higher number of variables, where the slip load distribution obtained from the GA optimisation after 3000 analyses is slightly different with the UDD and UDD-GA results. This implies that there are at least two different slip load distributions that lead to very similar objective functions. This can be expected as the optimisation problem in this study is non-convex.

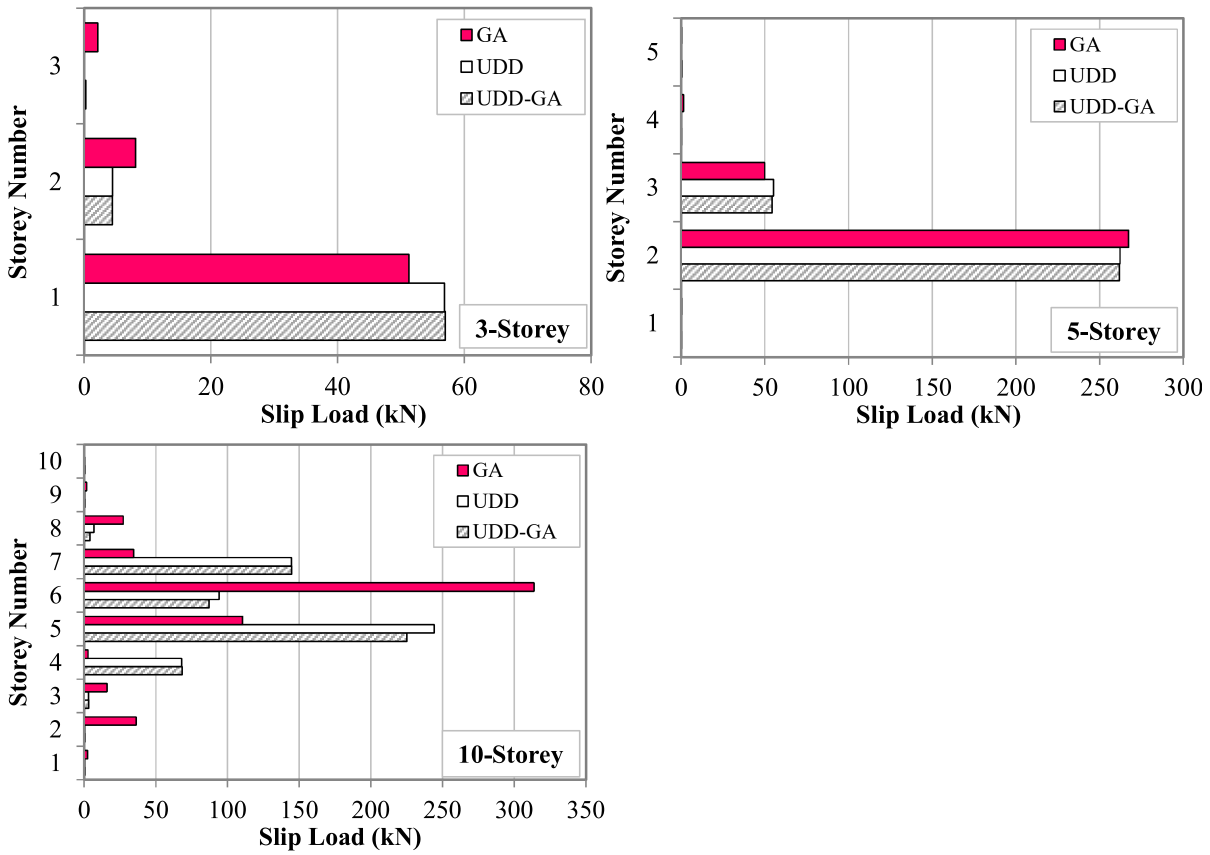


**Fig. 7.** Height-wise distribution of optimum slip loads for 3, 5 and 10-storey frames optimised using adaptive UDD, GA and UDD-GA optimisation methods, DBE event

## 8. OPTIMUM SEISMIC DESIGN FOR A SET OF SPECTRUM COMPATIBLE EARTHQUAKES

In the previous section specific distributions of slip loads were obtained for the studied frames under a synthetic spectrum compatible earthquake (i.e. SIM01). If the optimum design solution is sensitive to the selected design ground motion, it may not necessarily satisfy the predefined performance target for other earthquakes expected during the effective life of the structure. To address this issue, previous studies (e.g. [71]) showed that a better seismic design can be achieved by optimising the structure based on a synthetic earthquake representing a spectrum calculated as an average of the response spectra of several excitation records. In this section, the sensitivity of the optimum design solutions obtained from the adaptive UDD method is evaluated by using six different synthetic earthquakes with random acceleration vibration specifications previously shown in Fig. 2 (SIM01 to SIM06). As discussed before, these generated synthetic earthquakes provide a close approximation to the spectrum of the design earthquake.

The frames initially optimised for the SIM01 earthquake are subjected to the synthetic records SIM01 to SIM06 as representative DBE events, and their lateral displacements are compared to the LS performance target level. Fig. 8 shows the height-wise distribution of inter-storey drifts for the 3, 5 and 10-storey frames and their mean curves. As expected, the results show that while the optimum design frames have a more uniform distribution of deformation for SIM01 (i.e. optimum distribution), they do not perform optimally under the other synthetic earthquakes. However, the optimum design solutions could satisfy the predefined performance target (i.e. LS level) under all spectrum-compatible earthquakes. The seismic response of the 3, 5 and 10-storey optimum design frames is also investigated under representative MCE events (SIM01 to SIM06 records scaled to have a PGA= 0.6g as explained in Section 3).

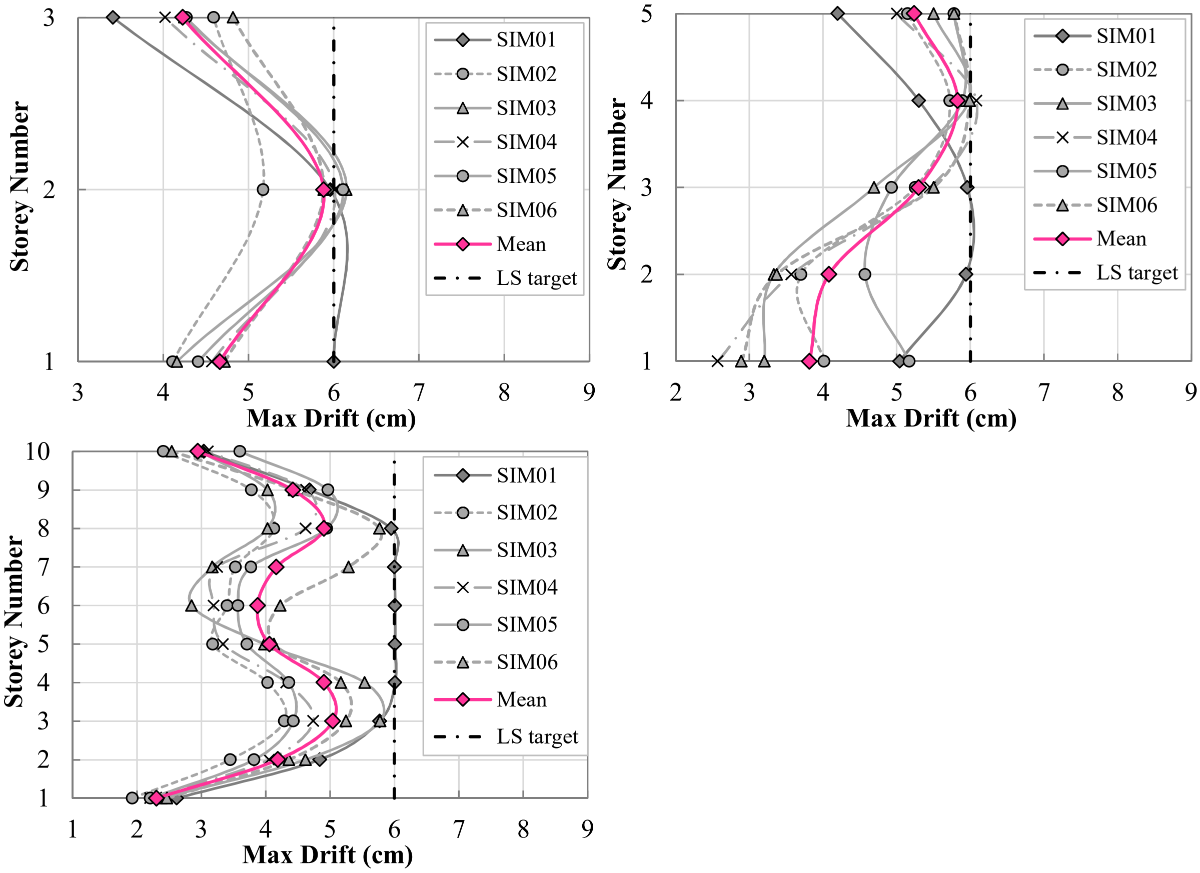


**Fig. 8.** Height-wise distribution of inter-storey drifts for 3, 5 and 10-storey optimum design frames, representative DBE events (six synthetic earthquakes)

It is shown in Fig. 9 that the optimum design solutions could efficiently satisfy the CP performance target under the set of MCE events. It can be noted that, in general, the MCE events were not as critical as the DBE records, which confirms the suitability of the adopted optimisation approach (i.e. optimising the frames under DBE and controlling their response under MCE). Therefore, the optimisation method proposed in this study should prove useful in performance-based design of RC structures with friction dampers for any code based design spectrum.

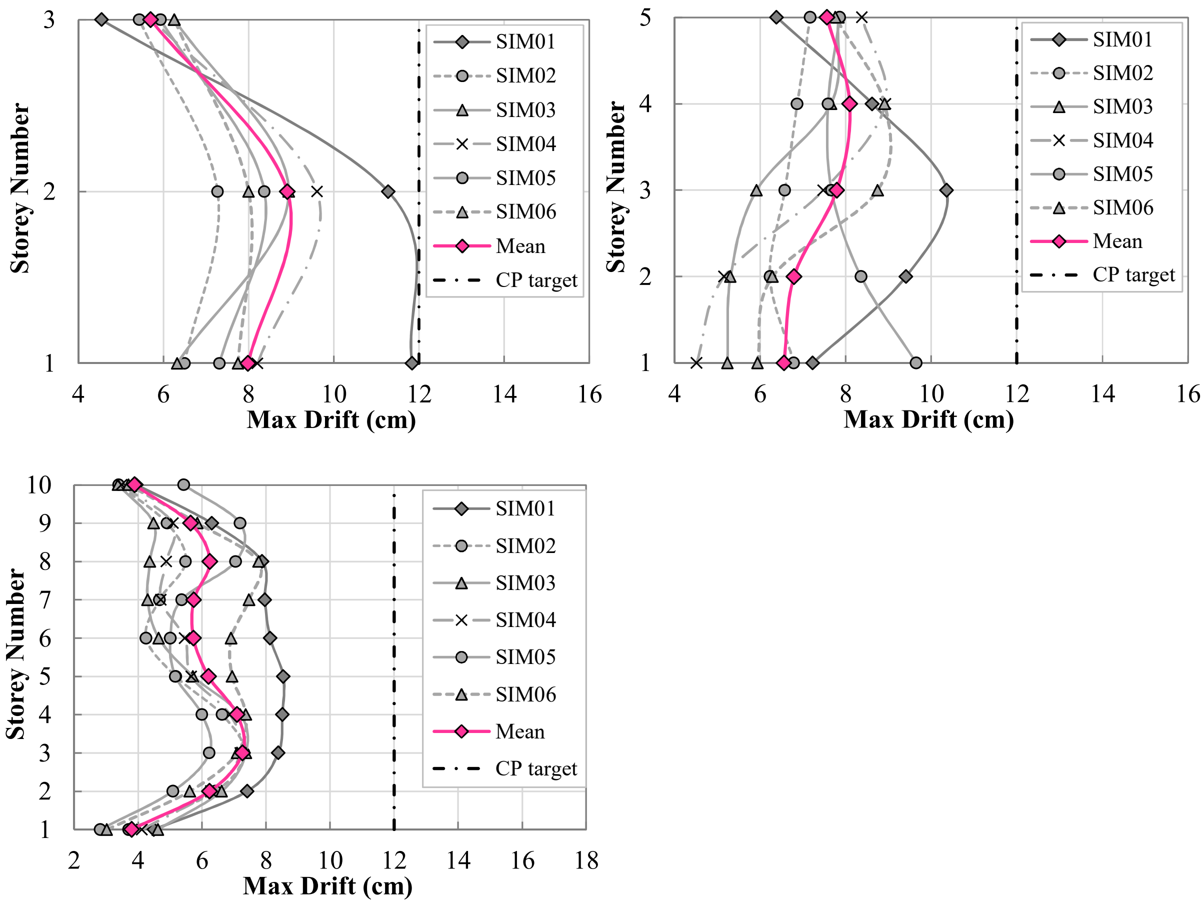


**Fig. 9.** Height-wise distribution of inter-storey drifts for 3, 5 and 10-storey optimum design frames, representative MCE events (six synthetic earthquakes)

## 9. SUMMARY AND CONCLUSIONS

An adaptive performance-based optimisation methodology based on the theory of Uniform Distribution of Deformation (UDD) was developed as a tool for optimum design of friction-based dampers in RC structures under seismic excitations by using a convergence factor that is modified in proportion to the performance violation level. The efficiency of the proposed adaptive UDD method is demonstrated through optimisation of 3, 5 and 10-storey frames with friction dampers to satisfy LS and CP performance limits under DBE and MCE representative spectrum compatible earthquakes, respectively. The results are then compared with those obtained from the standard UDD, standard GA and a combination of UDD and GA methods under a synthetic spectrum-compatible earthquake. Based on the results of this study, the following conclusions can be drawn:

- The range of efficient convergence factors for the standard UDD method can be different for different types of structures. The proposed adaptive convergence factor, which was used for three very different multi-storey buildings, was shown to be more computationally efficient than standard UDD method with constant convergence factors.

- The optimum solutions obtained from the proposed adaptive UDD after a small number of iterations (generally less than 15 nonlinear dynamic analyses) are either very close or even slightly better than those obtained from the GA approach after at least 2000 nonlinear dynamic analyses. This demonstrates a significant advantage of the proposed adaptive UDD over the GA method, in terms of computational efficiency and simplicity.

- Using optimum results obtained from the proposed adaptive UDD as a starting point of the GA considerably increases the optimisation speed (up to 3 times faster) compared to the standard GA using a random initial set. The optimum results obtained from the UDD-GA approach were only slightly better (less than 3%) than those achieved with the proposed adaptive UDD method alone. This indicates that the new adaptive UDD method is sufficiently accurate for most practical applications.

- The sensitivity of the optimisation to earthquake uncertainty was taken into consideration by optimum design of the frames using a synthetic earthquake compatible with a given design spectrum. These structures satisfied all the predefined performance targets under a set of six different synthetic earthquakes generated to represent the design spectrum.

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