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# On the prediction and the formation of the sigma phase in CrMnCoFeNi<sub>x</sub> high entropy alloys

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## Abstract

The almost limitless variations in potential compositions of high entropy alloys necessitates the use of computational methods when attempting to optimise for any given application. However, the accuracy of the current thermodynamic approaches commonly being used for this purpose remains under debate, as relatively few validity studies have been performed. Within the CrMnFeCoNi family of alloys, the formation of the  $\sigma$  phase and how it is influenced by compositional variations is of particular interest for elevated temperature structural applications. Here, the role of Ni on the formation of the  $\sigma$  phase has been studied through a systematic series of CrMnFeCoNi<sub>x</sub> alloys,  $0 \leq x \leq 1.5$ , following 1000 hour exposures at temperatures typically found to promote  $\sigma$  formation. Ni was found to have a significant effect on the phase stability of these alloys, suppressing the  $\sigma$  phase such that a single solid solution phase was the only stable phase in the CrMnFeCoNi<sub>1.5</sub> alloy, whilst the CrMnFeCo alloy formed the  $\sigma$  phase during solidification. The corresponding thermodynamic predictions varied dramatically from the experimentally observed microstructures, indicating that the underlying databases require further optimisation. Interestingly, it was found that a relatively simple electronic structure based approach, New PhaComp, provided much more accurate predictions of the observed  $\sigma$  phase formation in the CrMnFeCoNi<sub>x</sub> and CrMn<sub>x</sub>FeCoNi systems and could be manipulated to obtain  $\sigma$  formation temperatures. As such, this method could be extremely useful to those wanting to design CrMnFeCoNi high entropy alloys that are free from the  $\sigma$  phase.

Keywords: High Entropy Alloys; phase diagrams; phase transitions; thermodynamic modeling

## Introduction

The CrMnFeCoNi high entropy alloy (HEA), first proposed and discussed by Cantor *et al.* [1], has formed the basis of a number of studies on HEAs, many of which focus on understanding the phase equilibria and precipitation behaviour of the alloy [2-10]. It has now been conclusively established that a single, stable *fcc* (A1) solid solution phase exists at temperatures above 800°C and up to the alloy melting temperature. However, phase equilibria studies at temperatures below 800°C have revealed the formation of additional phases in the microstructure. Long duration exposures at 700°C (up to 12000 hours), resulted in the formation of the topologically close packed (TCP) sigma ( $\sigma$ ) phase [2-4], which is believed to be stable in the 480 – 800°C temperature range. Exposures at 500°C, identified the formation of the tetragonal (L1<sub>0</sub>) NiMn phase, an ordered B2 FeCo phase, as well as the precipitation of a second solid solution phase based on Cr with a *bcc* (A2) structure [3,4]. Furthermore, the formation of carbides and ceramic inclusions has also been reported at various temperatures, which are believed to be a result of impurities incorporated into the material during processing [2,11,12].

In the context of evaluating HEAs for engineering applications at elevated temperatures, the nature of the precipitating phases is of critical importance to the viability of the alloys. In particular, TCP phases such as the  $\sigma$  phase, cannot be tolerated in the microstructure as they lead to a deterioration of the mechanical properties of the material [13-16]. Therefore, understanding the origins of  $\sigma$  phase precipitation is key in order to achieve the required balance of properties necessary for engineering applications [17]. However, this endeavour is further complicated by the inherent nature of the  $\sigma$  phase. Whilst the phase is routinely observed in structural metallic systems incorporating Cr additions, such as Ni-based superalloys and stainless steels, its chemical composition is known to vary from system-to-system

[13,18,19]. Crystallographically, the phase consists of close packed atomic layers separated from each other by large interatomic distances and arranged in a body centred tetragonal structure, which is responsible for the embrittling nature of the phase [13,18,19]. Generally, it is believed to form with the  $A_xB_y$  formula, in which  $x$  and  $y$  are approximately equal [18,19]. In structural metallics, Cr has been identified as the major  $\sigma$  forming element, and is found in the phase in concentrations of  $\sim 50$  at.%. Refractory elements, such as Mo and W are also known to play a key role on the stability of the phase [13].

One of the great challenges for the high entropy alloy community is to identify optimal alloy compositions for a certain set of targets. Given that these materials already have a high degree of compositional complexity, which becomes greatly exacerbated when considering variations away from equiatomic ratios, it's clear that computer guided design methods are essential [17,20-22]. In this regard, thermodynamic calculations based on the CALPHAD approach seem highly suitable, as large swathes of compositional space can be evaluated in a relatively short period of time. Indeed, several down-selection strategies based on this approach have been reported in the literature [17,20-22]. However, all of these methodologies rely on the accuracy of the thermodynamic data in the underlying databases and the efficacy of the energy minimisation processes. Consequently, for these approaches to be viable, their predictions must accurately reflect experimental observations and thus, it is critically important to compare these two types of data. It is also essential that validity activities are not solely limited to higher temperatures, e.g.  $> 1000^\circ\text{C}$ , where solid solution phases are favoured.

In the CrMnFeCoNi system, thermodynamic predictions have been used to investigate the stability of the single *fcc* solid solution phase, particularly with respect to the  $\sigma$  phase. He *et al.* [9] used the CALPHAD approach to identify the limits of the stability of single phase solid solutions based on the CrFeCoNi system. These results, which were verified using experimental data of alloys exposed at  $900^\circ\text{C}$  for 72 hours, illustrated that the concentration of Cr that may be tolerated in the alloys is lower than any of the other constituent elements. This point was further strengthened by Laurent-Brocq *et al.* [10] and Bracq *et al.* [8], in similar studies using CrMnFeCoNi as the base system. The addition of Mn has also been shown to play a key role in destabilising the single phase solid solution in favour of the  $\sigma$  phase. CALPHAD calculations have shown an expansion of the  $\sigma$  phase field with increasing concentration of Mn as well as an increase in the  $\sigma$  phase solvus temperature. Experimental studies of CrMnFeCoNi with varying Mn contents examined at temperatures between  $500 - 1000^\circ\text{C}$  validated these general trends, but found that the  $\sigma$  phase stability was under predicted by the thermodynamic models [8,11].

Whilst the effects of Cr and Mn on promoting the formation of the  $\sigma$  phase are now well established, understanding the role that Ni, Co and Fe have on the stability of the single phase *fcc* solid solution is equally important in providing a holistic view of the CrMnFeCoNi system. He *et al.* [9], postulated that additions of Fe, Co and Ni of up to 40 at. % would stabilise the single *fcc* solid solution phase at  $900^\circ\text{C}$ . In contrast, Bracq *et al.* [8] suggested that the stability of the *fcc* solid solution would increase with Ni and Co additions, but that increased concentrations of Fe destabilised the alloy towards the formation of intermetallic phases. Laurent-Brocq *et al.* [10], have shown experimentally that even relatively low concentrations of Ni promotes the formation of a single *fcc* phase in the CrMnFeCoNi<sub>x</sub> system. However, the phase formation was only considered following 6 hours at  $1100^\circ\text{C}$  and no experimental data exists for thermal exposures at lower temperatures or longer durations, where the formation of the  $\sigma$  phase is more likely.

Consequently, further investigations are required to experimentally establish the role of Ni, Co and Fe on the stability of the single *fcc* phase in the CrMnFeCoNi system and understand the nature of the  $\sigma$  phase. To this end, herein we present a detailed microstructural analysis of three alloys within the CrMnFeCoNi<sub>x</sub> system, with  $x=0, 0.5$  and  $1.5$ , which were homogenised just below their solidus temperatures prior to 1000 hour exposures at  $700$  and  $900^\circ\text{C}$ . The experimental results were compared to thermodynamic predictions performed using ThermoCalc to assess the fidelity of the TCHEA1 database.

## Experimental

Three alloys with nominal compositions of  $\text{CrMnFeCoNi}_x$ ,  $x = 0, 0.5, 1.5$ , Table 1, were manufactured by arc melting under an Ar atmosphere from raw elements of at least 99.5 % purity. The ingots were inverted and re-melted five times in order to increase the macroscopic homogeneity of the cast material. Differential scanning calorimetry (DSC) measurements of the as-cast material were performed on a Netzsch DSC 404 instrument operated at heating/cooling rates of  $10^\circ\text{C}\cdot\text{min}^{-1}$ . The results obtained were used to ensure that the homogenisation heat treatment temperatures chosen were below the alloy solidus temperatures. Alloys  $\text{CrMnFeCoNi}_{0.5}$  and  $\text{CrMnFeCoNi}_{1.5}$  were both heat treated at  $1200^\circ\text{C}$  for 100 hours whereas, due to the presence of two melting events, alloy  $\text{CrMnFeCo}$  was heat treated at  $1050^\circ\text{C}$  for 100 hours. Following the homogenisation heat treatments, samples from each alloy were thermally exposed at  $700$  and  $900^\circ\text{C}$  for 1000 hours, as these temperatures were believed to bracket the  $\sigma$  solvus temperature. All heat treatments were performed in Ar-backfilled quartz tubes to limit the effects of oxidation and environmental attack on the material.

Specimens from each heat-treated condition were prepared for microstructural characterisation by grinding with progressive grades of SiC paper and polishing to a  $0.06\ \mu\text{m}$  finish using colloidal silica. Imaging and compositional characterisation was performed using scanning electron microscopy (SEM) coupled with energy dispersive X-ray spectroscopy (EDX) on an FEI Nova NanoSEM equipped with a Bruker XFlash 6 solid-state detector. Additional phase characterisation of each heat-treated condition was performed using X-ray Diffraction (XRD) on a Bruker D8 Advance diffractometer using Ni-filtered  $\text{Cu K}\alpha$  radiation. Patterns were recorded in the  $20$ - $100^\circ$   $2\theta$  range at  $0.02^\circ$  increments and were analysed using the full-pattern Pawley fitting procedure [23] in TOPAS-academic.

## Results

### *As-cast & Homogenised material*

Understanding the transition temperatures, and in particular the liquidus and solidus temperatures of the alloys, was critical in determining suitable homogenisation heat treatments for each alloy. **Therefore, the as-cast material was heated to  $1400^\circ\text{C}$  in a DSC and the corresponding data are shown in Figure 1 and the key results summarised in Table 2.** The liquidus temperature of the alloys studied was found to decrease from  $1370^\circ\text{C}$  for the quaternary  $\text{CrMnFeCo}$  alloy to  $1340^\circ\text{C}$  for both alloys containing Ni. Similarly, the solidus temperature of the Ni-containing alloys was found to marginally decrease for the alloy with a higher Ni concentration. The quaternary alloy,  $\text{CrMnFeCo}$ , displayed two melting events one beginning at  $\sim 1125^\circ\text{C}$  and another one at  $1310^\circ\text{C}$ .

Backscattered electron images (BSE) of the three alloys examined following their respective homogenisation heat treatments are shown in Figure 2. Alloys  $\text{CrMnFeCoNi}_{0.5}$  and  $\text{CrMnFeCoNi}_{1.5}$  both displayed homogeneous microstructures consisting of large, single phase grains, as shown in Figure 2b & 2c. In contrast, in the quaternary alloy  $\text{CrMnFeCo}$ , Figure 2a, at least two phases were visible in the microstructure. This observation, in conjunction with the DSC results of the as-cast material, indicated that it was not possible to heat treat, and hence homogenise, this alloy in a single-phase field. The bulk compositions of the alloys obtained by averaging at least five large area EDX scans ( $500\ \mu\text{m} \times 500\ \mu\text{m}$ ) are given in Table 1. Alloys  $\text{CrMnFeCoNi}_{0.5}$  and  $\text{CrMnFeCoNi}_{1.5}$  were found to exhibit compositions with only  $\sim 1.0$  at. % deviation from the nominal for each element, whereas, alloy  $\text{CrMnFeCo}$  was found to deviate by  $\sim 2.0$  at. % on the concentration of Co, which was compensated across the remaining four elements. XRD patterns collected from the homogenised alloys are shown in Figure 3, along with the corresponding Pawley refinements. These results confirmed that alloys  $\text{CrMnFeCoNi}_{0.5}$  and  $\text{CrMnFeCoNi}_{1.5}$  consisted of a single phase *fcc* solid solution following homogenisation with a lattice parameter of  $3.60 \pm 0.01\ \text{\AA}$  for both alloys. Similarly, the XRD results obtained from alloy  $\text{CrMnFeCo}$  confirmed the presence of at least two phases; an *fcc* solid solution and a  $\sigma$  phase. A third phase was also believed to be present in the pattern, although identification of the phase could not be reliably achieved from the diffraction peaks available as a result of the pattern being dominated by signal from the *fcc* and  $\sigma$  phases.

### Long duration exposures

The propensity of the CrMnFeCoNi<sub>x</sub> alloys to form the  $\sigma$  phase was assessed following prolonged exposures at 700 and 900°C using SEM/EDX analysis. This was supplemented by XRD for phase identification and determination of the associated lattice parameters. Figure 4 shows the collective SEM/EDX results from all alloys examined following 1000 hours at 900°C. The CrMnFeCo alloy, Figure 4, displayed a microstructure consisting of a Co-Fe based matrix and bright, Cr-rich islands. Analysis of the XRD patterns, Figure 5, confirmed the presence of an *fcc* phase with a refined lattice parameter of  $3.59 \pm 0.01 \text{ \AA}$ , and the  $\sigma$  phase, with refined lattice parameters of  $a = 8.79 \pm 0.01 \text{ \AA}$  and  $c = 4.56 \pm 0.01 \text{ \AA}$ . In addition to the *fcc* matrix (dark contrast) and the  $\sigma$  phase (globular morphology, bright contrast), a fine, lenticular phase was also observed within the Co-Fe based matrix. Further examination of the BSE images and rigorous analysis of the XRD data, shown in Figure 6, indicated that this was the *hcp* allotrope of Co, with lattice parameters of  $a = 2.54 \pm 0.01 \text{ \AA}$  and  $c = 4.10 \pm 0.01 \text{ \AA}$ .

The CrMnFeCoNi<sub>0.5</sub> alloy exhibited large matrix grains and a bright Cr-rich phase precipitating both along the grain boundaries and intragranularly, Figure 4. The associated XRD patterns, Figure 5, indicated the presence of an *fcc* phase with a lattice parameter of  $3.60 \pm 0.01 \text{ \AA}$ , as well as peaks typical of the  $\sigma$  phase in the  $40 - 60^\circ 2\theta$  range, giving refined lattice parameters of  $a = 8.81 \pm 0.01 \text{ \AA}$  and  $c = 4.55 \pm 0.01 \text{ \AA}$ . Compositional data obtained via EDX from the Cr-rich phase are given in Table 3, and are typical of the  $\sigma$  phase in this system [11]. Hence, the inter- and intragranular precipitates are the  $\sigma$  phase, which reside in an *fcc* multi-component matrix.

In contrast, no evidence of precipitates were observed either within the grains or along the grain boundaries of the CrMnFeCoNi<sub>1.5</sub> alloy Figure 4. Similarly, there were no variations in the corresponding elemental distribution maps, suggesting the alloy existed as a single solid solution phase. The XRD data contained reflections consistent with a single *fcc* structure that had a refined lattice parameter of  $3.60 \pm 0.01 \text{ \AA}$ . Consequently, at this temperature, this alloy was believed to be stable as a single solid solution phase.

Samples exposed at 700°C for 1000 hours were examined in a similar manner, with the SEM/EDX data presented in Figure 7, and the XRD data shown in Figure 8. As with the sample exposed at 900°C, the microstructure of the CrMnFeCo alloy following exposure at 700°C contained a Cr-rich phase in an FeCo-based matrix, Figure 7. Fine, lenticular features were once again observed, identical to those shown in Figure 6. The XRD data indicated three phases corresponding to an *fcc* phase ( $a = 3.59 \pm 0.01 \text{ \AA}$ ), an *hcp* phase ( $a = 2.55 \pm 0.01 \text{ \AA}$  and  $c = 4.11 \pm 0.01 \text{ \AA}$ ) and the  $\sigma$  phase ( $a = 8.79 \pm 0.01 \text{ \AA}$  and  $c = 4.55 \pm 0.01 \text{ \AA}$ ).

The microstructure of the CrMnFeCoNi<sub>0.5</sub> alloy, varied a little from that observed following the exposure at 900°C. Cr-rich precipitates were observed to have formed along the grain boundaries, but there was no evidence of intragranular precipitation. The XRD data, shown in Figure 8, contained reflections consistent with an *fcc* phase ( $a = 3.59 \pm 0.01 \text{ \AA}$ ) and the  $\sigma$  phase ( $a = 8.80 \pm 0.01 \text{ \AA}$  and  $c = 4.55 \pm 0.01 \text{ \AA}$ ). EDX data obtained from the Cr-rich phase indicated that it had a composition that was nearly identical to that obtained from the Cr-rich phase obtained from the sample exposed at 900°C, Table 3.

Cr-rich precipitates were also observed decorating the grain boundaries of the CrMnFeCoNi<sub>1.5</sub> alloy following 1000 hours at 700°C, Figure 8. However, these precipitates gave a darker BSE contrast in comparison to the matrix, unlike those observed in the CrMnFeCoNi<sub>0.5</sub> alloy, which gave a brighter contrast. This would suggest that these two phases are different; an observation supported by the notably different quantified compositions given in Table 3. The precipitates formed in the CrMnFeCoNi<sub>1.5</sub> alloy are significantly enriched in Cr and depleted in Mn, Fe and Co when compared to the  $\sigma$  phase observed in the CrMnFeCoNi<sub>0.5</sub> alloy. These elemental variations are not consistent with the expected composition of the  $\sigma$  phase and are far more in line with those of a carbide phase, as outlined in reference [11]. The XRD data for the CrMnFeCoNi<sub>1.5</sub> alloy, Figure 8, were also devoid of reflections typical of the  $\sigma$  phase but contained lower angle peaks, which aligned well with those expected from the cubic M<sub>23</sub>C<sub>6</sub> phase. Again, this is in good agreement with previous studies in this system [2,3,11], however, additional site-specific characterisation would be required to conclusively establish the phase crystallography. It is worth noting that the peak present in some of the XRD patterns at

an angle of  $\sim 29^\circ$  was found to be an artefact from the diffractometer rather than being related to the samples being studied.

## Discussion

### *The role of Ni on phase stability*

Using the data presented above and results previously published in the literature for the equiatomic alloy CrMnFeCoNi [3,24], the role of Ni on the stability of the single phase *fcc* solid solution can be established. Ni is clearly a key stabilising element for the *fcc* phase, as demonstrated by the presence of multiple phases at all temperatures below the solidus in the quaternary CrMnFeCo alloy. Raising the Ni content to an atomic fraction of 0.5, or above, increases the relative stability of the *fcc* solid solution phase such that a single phase region exists below the solidus. The extent of this single phase field increases as a function of the Ni concentration with the formation of the  $\sigma$  phase depressed to progressively lower temperatures. For example, in the present work, the  $\sigma$  phase was observed at  $900^\circ\text{C}$  in the CrMnFeCoNi<sub>0.5</sub> alloy, whereas in the equiatomic CrMnFeCoNi alloy the material retained a single *fcc* structure [3,24]. Similarly, the  $\sigma$  phase was observed in the equiatomic CrMnFeCoNi alloy following exposure at  $700^\circ\text{C}$ , but the *fcc* phase of the CrMnFeCoNi<sub>1.5</sub> alloy remained stable. In an attempt to identify the  $\sigma$  solvus for each composition more accurately, samples of the exposed material that contained  $\sigma$  precipitates were studied using calorimetric methods. However, this proved unsuccessful as, with the exception of the quaternary CrMnFeCo alloy, the heat flow associated with the  $\sigma$  phase dissolution was too small for the solvus temperature to be reliably determined. This was most likely due to the low volume fraction of the  $\sigma$  phase in these alloys. For the quaternary alloy, where a clear signal was obtained, the onset of the  $\sigma$  phase dissolution was found to be  $\sim 1125^\circ\text{C}$  and corresponded to the incipient melting temperature.

### *Fidelity of thermodynamic predictions*

A section of the pseudo-binary phase diagram between the quaternary CrMnFeCo alloy and Ni, predicted using the TCHEA1 database and ThermoCalc, is shown in Figure 9. These predictions suggest that there is a large single *fcc* phase field below the solidus, which is the only thermodynamically stable phase at temperatures above  $500^\circ\text{C}$  when the concentration of Ni is greater than 32 at.%. With concentrations of Ni less than  $\sim 10$  at.%, the thermodynamic predictions indicate that a *bcc* phase should also be present, which from the predicted molar fraction should constitute a significant proportion of the microstructure. The  $\sigma$  phase is also predicted to form at temperatures below  $765^\circ\text{C}$  in the quaternary alloy and the formation temperature decreases progressively as the Ni content in the alloy increases.

When compared to the experimentally observed phases, the fidelity of these predictions is questionable. In the present study, no evidence of a *bcc* phase was found in any of the alloys, neither was any reported in the study by Bracq *et al.* [8]. However, in the quaternary alloy, the hexagonal  $\epsilon$  allotrope of Co was identified in the experimental microstructures but was not predicted to form by the thermodynamic modelling. The predicted stability of the  $\sigma$  phase also deviated dramatically from what was observed experimentally. In the quaternary alloy, the  $\sigma$  phase was found to be present at all temperatures below the solidus. With increasing Ni content the formation temperature of the  $\sigma$  was observed to decrease but in all cases these temperatures were consistently higher than those predicted by ThermoCalc. To illustrate this point, the colour coded experimental data points have been overlaid on the phase diagram shown in Figure 9. The square green markers indicate a microstructure that is free from the  $\sigma$  phase and, therefore, barring impurity driven carbides, consist of a single solid solution phase. In contrast, the circular red markers indicate the presence of the  $\sigma$  phase, irrespective of whether it was observed intra- or intergranularly. The circular red points clearly demonstrate that the  $\sigma$  phase is much more prevalent than the thermodynamic simulations predict. This is consistent with previous similar comparisons of experimental and predicted phase stability in the CrMnFeCoNi system [8,9,11].

### *Alternative methods of predicting $\sigma$ formation*

Aside from thermodynamic calculations, electronic structure based methods, such as the Valence Electron Concentration (VEC) approach, have been proposed as potential alternatives for predicting phase stability [25]. This approach uses experimental observations to define certain empirical VEC ranges where different combinations of phases are stable. Whilst this approach does not provide reliable correlations over a wide range of different alloy

systems, it can be informative within a single alloy family [26]. This concept has been extended specifically to predict the formation of the  $\sigma$  phase [27], where it has been suggested that if an alloy has a VEC value between 6.88 and 7.84 then it will be prone to  $\sigma$  phase formation at 700°C.

The VEC numbers for the alloys in the CrMnFeCoNi<sub>x</sub> family, where  $x = 0, 0.5, 1, 1.5$ , are 7.50, 7.78, 8.00 and 8.18 respectively. As such, at 700°C the VEC approach would predict that the alloys with Ni<sub>0</sub> and Ni<sub>0.5</sub> are prone to the formation of  $\sigma$ , whilst the equiatomic alloy and Ni<sub>1.5</sub> would be stable. This is in reasonable agreement with the experimental data, although the  $\sigma$  phase has been observed at this temperature in the equiatomic alloy [2,3]. However, the application of this approach is somewhat limited, as the critical VEC range when  $\sigma$  is likely to form will have a temperature dependence and hence a significant number of experimental data points would be required to define these critical ranges at multiple temperatures.

Another electronically based approach to predicting phase stability is the calculation of average d-orbital energies. These averaged energies correlate well with the atomic misfit and electronegativity of the alloy and, therefore, are a parametric representation of the Hume-Rothery rules [28-32]. The average d-orbital energy,  $\bar{M}_d$  is calculated using the following expression where  $X_i$  is the atomic concentration and  $M_{d,i}$  the d-orbital energy of the  $i$ th element.

$$\bar{M}_d = \sum_{i=1}^n X_i M_{d,i}$$

The use of average d-orbital energy has previously been used to consider TCP formation in HEAs [33,34]. When considering the wide range of different alloys assessed, the magnitude of the average d-orbital energy at which the  $\sigma$  phase was observed varied quite significantly [33,34]. Thus, it is not possible to define a general criterion and the crystal structure of the solid phase may also have an effect [34]. However, in nearly all of the different systems studied, the  $\sigma$  phase was found to form at higher average d-orbital energies. Again, as with the VEC ranges discussed above, these critical average d-orbital energies are likely to exhibit a temperature dependence, which will need to be taken into account if more reliable predictions are to be made.

An alternative way of utilising average d-orbital energies in *fcc* crystals is the New PhaComp method [28], which was originally developed to predict the occurrence of TCP phases in Ni-based superalloys [28,35,36]. Since the majority of the HEAs in the CrMnFeCoNi<sub>x</sub> family exist as a single *fcc* solid solution phase at temperatures close to the solidus, the New PhaComp approach may be suitable for predicting the formation of the  $\sigma$  phase in these alloys.

In contrast to the methods outlined above, the New PhaComp method incorporates a temperature dependence to the critical average d-orbital energy above which the formation of the  $\sigma$  phase is likely to occur. The formulation of this expression is shown below, where  $T$  is the absolute temperature and the two numerical constants are those empirically determined for Ni-based superalloys.

$$\bar{M}_{d,crit} = 6.25 \times 10^{-5}T + 0.834$$

Critically, this expression allows the formation temperature of the  $\sigma$  phase to be assessed as a function of composition by calculating the temperature at which  $\bar{M}_{d,crit} = \bar{M}_d$ . Following this approach for the current set of alloys, the variation in the  $\sigma$  formation temperature as a function of composition has been overlaid as a dotted line on the pseudo-binary phase diagram shown in Figure 9. As can be seen, this line correctly describes the presence of the  $\sigma$  phase in all but one condition, the CrMnFeCoNi alloy at 700°C. However, it should be noted that the construction of the pseudo-binary phase diagram necessitates the use of nominal compositions, where Cr, Mn, Fe and Co must be present in equiatomic concentrations. Clearly, this is unlikely to be the case in real alloys. Consequently, if the  $\sigma$  formation temperature is recalculated using the actual compositions then the experimentally observed behaviour in the CrMnFeCoNi<sub>x</sub> system is accurately captured.

A similar outcome is reached if the phase equilibria of alloys with varying Mn content, i.e. CrMn<sub>x</sub>FeCoNi, are considered in the same way. Experimental data has shown that whilst the equiatomic alloy is prone to the formation of the  $\sigma$  phase,

decreasing the Mn content stabilises the *fcc* solid solution phase [11]. Thermodynamic calculations predict the formation of the  $\sigma$  phase at 600°C for Mn concentrations between 0 and ~85 at.%, which does not align with the experimental observations. In contrast, New PhaComp approach correctly predicts the change in  $\sigma$  formation behaviour. This suggests, that within the CrMnFeCoNi system, this simple method may enable a rapid assessment of the likelihood of  $\sigma$  phase formation.

Furthermore, it should be noted that the current predictions of the  $\sigma$  formation temperatures are based upon numerical constants that have been empirically determined for Ni-based superalloys. It is not clear that these constants are directly applicable to HEAs and, as such, it may be appropriate to optimise these values for a given HEA system if sufficient data is available as to the role of each constituent element. However, at present, insufficient data relating to the influence of each elemental species on the formation of the  $\sigma$  phase within the CrMnFeCoNi system exists to perform such an optimisation. In particular, more data is required with respect to the influence of Fe and Co on  $\sigma$  phase formation. Critically, it is clear that current, widely used, thermodynamic databases do not have sufficient accuracy in CrMnFeCoNi-based HEAs to provide predictions with the fidelity necessary for alloy design and their use in this regard requires caution.

## Conclusions

This study has experimentally investigated the role of Ni concentration on the stability of the single solid solution phase in the CrMnFeCoNi<sub>x</sub> family of HEAs and has assessed the fidelity of current thermodynamic methods for predicting the formation of the  $\sigma$  phase.

Ni has been shown to be a strong *fcc* stabiliser, supporting the stability of the single solid solution phase and suppressing the formation of the Cr-rich  $\sigma$  phase. Without Ni, the CrMnFeCo quaternary alloy formed the  $\sigma$  phase from the liquid, whereas the CrMnFeCoNi<sub>1.5</sub> alloy was found to be a single solid solution phase at temperatures above 700°C. The  $\sigma$  phase was found to form in alloys with intermediate Ni compositions but its solvus temperature progressively decreased as the Ni concentration increased.

Thermodynamic predictions of this system provide a poor description of the observed phases. This is especially true at low Ni concentrations, where a *bcc* phase is predicted to form in significant fractions but was not found in any of the experimental data. Consequently, at present, care should be taken when considering the outputs of such models in the CrMnFeCoNi HEA system.

The formation of the  $\sigma$  phase can also be predicted using methods based on the electronic structure of the material. The New PhaComp approach can be manipulated to provide a  $\sigma$  formation temperature based on two empirical constants. This method was found to accurately capture the formation of the  $\sigma$  phase in both the CrMnFeCoNi<sub>x</sub> and CrMn<sub>x</sub>FeCoNi systems, even when using the empirical constants determined for Ni-base superalloys. As such, with further optimisation, it is believed that this approach may be extremely useful as a simple predictive tool to assess the likelihood of  $\sigma$  phase formation in the CrMnFeCoNi system.

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## References

- [1] B. Cantor, I.T.H. Chang, P. Knight, A.J.B. Vincent, Microstructural development in equiatomic multicomponent alloys, *Mater Sci Eng A*. 375-377 (2004) 213–218. doi:10.1016/j.msea.2003.10.257.
- [2] E.J. Pickering, R. Muñoz-Moreno, H.J. Stone, N.G. Jones, Precipitation in the equiatomic high-entropy alloy CrMnFeCoNi, *Scr Mater*. 113 (2016) 106–109. doi:10.1016/j.scriptamat.2015.10.025.

- [3] F. Otto, A. Dlouhy, K.G. Pradeep, M. Kuběnová, D. Raabe, G. Eggeler, et al., Decomposition of the single-phase high-entropy alloy CrMnFeCoNi after prolonged anneals at intermediate temperatures, *Acta Materialia*. 112 (2016) 40–52. doi:10.1016/j.actamat.2016.04.005.
- [4] N.D. Stepanov, D.G. Shaysultanov, M.S. Ozerov, S.V. Zharebtsov, G.A. Salishchev, Second phase formation in the CoCrFeNiMn high entropy alloy after recrystallization annealing, *Materials Letters*. 185 (2016) 1–4. doi:10.1016/j.matlet.2016.08.088.
- [5] Y.L. Zhao, T. Yang, Y. Tong, J. Wang, J.H. Luan, Z.B. Jiao, et al., Heterogeneous precipitation behavior and stacking-fault-mediated deformation in a CoCrNi-based medium-entropy alloy, *Acta Materialia*. 138 (2017) 72–82. doi:10.1016/j.actamat.2017.07.029.
- [6] N. Park, B.-J. Lee, N. Tsuji, The phase stability of equiatomic CoCrFeMnNi high-entropy alloy: Comparison between experiment and calculation results, *Jalcom*. 719 (2017) 189–193. doi:10.1016/j.jallcom.2017.05.175.
- [7] B. Schuh, F. Mendez-Martin, B. Voelker, E.P. George, H. Clemens, R. Pippan, et al., Mechanical properties, microstructure and thermal stability of a nanocrystalline CoCrFeMnNi high-entropy alloy after severe plastic deformation, *Acta Materialia*. 96 (2015) 258–268. doi:10.1016/j.actamat.2015.06.025.
- [8] G. Bracq, M. Laurent-Brocq, L. Perrière, R. Pirès, J.-M. Joubert, I. Guillot, The fcc solid solution stability in the Co-Cr-Fe-Mn-Ni multi-component system, *Acta Materialia*. 128 (2017) 327–336. doi:10.1016/j.actamat.2017.02.017.
- [9] F. He, Z. Wang, Q. Wu, S. Niu, J. Li, J. Wang, et al., Solid solution island of the Co-Cr-Fe-Ni high entropy alloy system, *Scr Mater*. 131 (2017) 42–46. doi:10.1016/j.scriptamat.2016.12.033.
- [10] M. Laurent-Brocq, L. Perrière, R. Pirès, Y. Champion, From high entropy alloys to diluted multi-component alloys: Range of existence of a solid-solution, *Jmade*. 103 (2016) 84–89. doi:10.1016/j.matdes.2016.04.046.
- [11] K.A. Christofidou, E.J. Pickering, P. Orsatti, P.M. Mignanelli, T.J.A. Slater, H.J. Stone, et al., On the influence of Mn on the phase stability of the CrMnxFeCoNi high entropy alloys, *Intermetallics*. 92 (2018) 84–92. doi:10.1016/j.intermet.2017.09.011.
- [12] S. Antonov, M. Detrois, S. Tin, Design of Novel Precipitate-Strengthened Al-Co-Cr-Fe-Nb-Ni High-Entropy Superalloys, *Metall Mater Trans A*. 49 (2017) 305–320. doi:10.1007/s11661-017-4399-9.
- [13] A.S. Wilson, Formation and effect of topologically close-packed phases in nickel-base superalloys, *Mater Sci Technol*. (2016) 1–11. doi:10.1080/02670836.2016.1187335.
- [14] N.G. Jones, K.A. Christofidou, P.M. Mignanelli, J.P. Minshull, M.C. Hardy, H.J. Stone, Influence of Elevated Co and Ti Levels on Polycrystalline Powder Processed Ni-base Superalloy, *Mater Sci Technol*. 30 (2014) 1853–1861. doi:10.1179/1743284714y.0000000509.
- [15] H.M. Tawancy, On the Precipitation of Intermetallic Compounds in Selected Solid-Solution-Strengthened Ni-Base Alloys and Their Effects on Mechanical Properties, *Metallography, Microstructure, and Analysis*. 6 (2017) 200–215. doi:10.1007/s13632-017-0352-y.
- [16] C.T. Sims, N.S. Stoloff, W.C. Hagel, eds., *Superalloys II*, First, John Wiley & Sons, 1987.
- [17] D. Miracle, J. Miller, O. Senkov, C. Woodward, M. Uchic, J. Tiley, Exploration and Development of High Entropy Alloys for Structural Applications, *Entropy 2014*, Vol. 16, Pages 494-525. 16 (2014) 494–525. doi:10.3390/e16010494.
- [18] E.O. Hall, S.H. Algie, The Sigma Phase, *International Materials Reviews*. 11 (1966) 61–88. doi:10.1179/imr.1966.11.1.61.
- [19] A.H. Sully, The sigma phase in binary alloys of the transition elements, *Journal of the Institute of Metals*. 80 (1951) 173–179.
- [20] D. Miracle, B. Majumdar, K. Wertz, S. Gorsse, New strategies and tests to accelerate discovery and development of multi-principal element structural alloys, *Scr Mater*. 127 (2017) 195–200. doi:10.1016/j.scriptamat.2016.08.001.
- [21] O.N. Senkov, J.D. Miller, D.B. Miracle, C. Woodward, Accelerated exploration of multi-principal element alloys for structural applications, *Calphad: Computer Coupling of Phase Diagrams and Thermochemistry*. 50 (2015) 32–48. doi:10.1016/j.calphad.2015.04.009.
- [22] F. Tancret, I. Toda-Caraballo, E. Menou, P.E.J.R. Díaz-Del-Castillo, Designing high entropy alloys employing thermodynamics and Gaussian process statistical analysis, *Jmade*. 115 (2017) 486–497. doi:10.1016/j.matdes.2016.11.049.
- [23] G.S. Pawley, IUCr, Unit-cell refinement from powder diffraction scans, *Journal of Applied Crystallography*. 14 (1981) 357–361. doi:10.1107/S0021889881009618.
- [24] E.J. Pickering, N.G. Jones, High-entropy alloys: a critical assessment of their founding principles and future prospects, *International Materials Reviews*. 61 (2016) 183–202. doi:10.1080/09506608.2016.1180020.

- [25] S. Guo, C. Ng, J. Lu, C.T. Liu, Effect of valence electron concentration on stability of fcc or bcc phase in high entropy alloys, in: 2011: p. 103505. doi:10.1063/1.3587228.
- [26] D.B. Miracle, O.N. Senkov, A critical review of high entropy alloys and related concepts, *Acta Materialia*. 122 (2017) 448–511. doi:10.1016/j.actamat.2016.08.081.
- [27] M.-H. Tsai, K.-Y. Tsai, C.-W. Tsai, C. Lee, C.-C. Juan, J.-W. Yeh, Criterion for Sigma Phase Formation in Cr- and V-Containing High-Entropy Alloys, *Materials Research Letters*. 1 (2013) 207–212. doi:10.1080/21663831.2013.831382.
- [28] M. Morinaga, N. Yukawa, H. Adachi, H. Ezaki, New PHACOMP and its applications to alloy design, *Superalloys 1984*. (1984) 523–532.
- [29] B.F. Carpenter, D.L. Olson, D.K. Matlock, M.J. Cieslak, Phase stability of Fe-Al-Cr-Ni weld metal, *Mater Sci Eng A*. 147 (1991) 1–8. doi:10.1016/0921-5093(91)90799-S.
- [30] J.S. Ogborn, D.L. Olson, M.J. Cieslak, Influence of solidification on the microstructural evolution of nickel base weld metal, *Mater Sci Eng A*. 203 (1995) 134–139. doi:10.1016/0921-5093(95)09832-1.
- [31] B. Seiser, R. Drautz, D.G. Pettifor, TCP phase predictions in Ni-based superalloys: Structure maps revisited, *Acta Materialia*. 59 (2011) 749–763. doi:10.1016/j.actamat.2010.10.013.
- [32] R.C. Reed, T. Tao, N. Warnken, Alloys-By-Design: Application to nickel-based single crystal superalloys, *Acta Materialia*. 57 (2009) 5898–5913. doi:10.1016/j.actamat.2009.08.018.
- [33] Y. Lu, Y. Dong, L. Jiang, T. Wang, T. Li, Y. Zhang, A Criterion for Topological Close-Packed Phase Formation in High Entropy Alloys, *Entropy* 2014, Vol. 16, Pages 494-525. 17 (2015) 2355–2366. doi:10.3390/e17042355.
- [34] S. Sheikh, H. Mao, S. Guo, Predicting solid solubility in CoCrFeNiM<sub>x</sub> (M=4d transition metal) high-entropy alloys, *J. Appl. Phys.* 121 (2017). doi:10.1063/1.4983762.
- [35] H.J. Murphy, C.T. Sims, A.M. Beltran, PHACOMP Revisited, *Superalloys 1968*. (1968) 47–66.
- [36] N. Saunders, Z. Guo, X. Li, A.P. Miodownik, J.-P. Schille, Modelling the material properties and behaviour of Ni-based superalloys, in: K.A. Green, T.M. Pollock, H. Harada, T.E. Howson, R.C. Reed, J.J. Schirra, et al. (Eds.), *Superalloys 2004*, TMS, 1996: pp. 849–858. doi:10.7449/1996/superalloys\_1996\_101\_110.

Table 1: Nominal and actual compositions obtained through large area EDX analysis (at. %).

Alloy	Composition (at. %)	Cr	Mn	Fe	Co	Ni
CrMnFeCo	nominal	25.0	25.0	25.0	25.0	-
	actual	23.7 ± 0.5	23.2 ± 0.5	25.7 ± 0.5	27.4 ± 0.5	-
CrMnFeCoNi <sub>0.5</sub>	nominal	22.2	22.2	22.2	22.2	11.1
	actual	21.8 ± 0.2	21.5 ± 0.2	22.5 ± 0.2	23.2 ± 0.2	11.0 ± 0.2
CrMnFeCoNi <sub>1.5</sub>	nominal	18.2	18.2	18.2	18.2	27.3
	actual	18.8 ± 1.0	18.1 ± 1.0	19.0 ± 1.0	17.6 ± 3.0	26.5 ± 1.0

Table 2: DSC results of the as-cast material.

Alloy	Liquidus (°C)	Solidus (°C)
CrMnFeCo	1370	1310 / 1125
CrMnFeCoNi <sub>0.5</sub>	1340	1283
CrMnFeCoNi <sub>1.5</sub>	1340	1267

Table 3: Composition of Cr-rich phases in alloys CrMnFeCoNi<sub>0.5</sub> and CrMnFeCoNi<sub>1.5</sub> obtained through spot EDX analysis.

Alloy	Condition	Cr (at. %)	Mn (at. %)	Fe (at. %)	Co (at. %)	Ni (at. %)
CrMnFeCoNi <sub>0.5</sub>	900°C / 1000 hours	37.9 ± 0.2	17.5 ± 0.1	19.4 ± 0.1	20.0 ± 0.2	5.2 ± 0.1
	700°C / 1000 hours	38.9 ± 1.7	16.8 ± 0.6	18.6 ± 0.4	20.8 ± 0.3	4.9 ± 0.5
CrMnFeCoNi <sub>1.5</sub>	700°C / 1000 hours	72.3 ± 5.9	7.7 ± 1.2	7.3 ± 1.2	5.5 ± 1.4	7.2 ± 2.0

Figure 1: Differential Scanning Calorimeter thermograms of the as-cast alloys upon heating.

Figure 2: Backscattered electron images of the homogenised material a) CrMnFeCo following 100 hours at 1050°C, b) CrMnFeCoNi<sub>0.5</sub> and c) CrMnFeCoNi<sub>1.5</sub> following 100 hours at 1200°C.

Figure 3: X-ray diffraction patterns obtained for the three alloys tested following homogenisation heat treatments.

Figure 4: BSE images and EDX compositional maps following 1000 hours at 900°C of alloys CrMnFeCo, CrMnFeCoNi<sub>0.5</sub> and CrMnFeCoNi<sub>1.5</sub>.

Figure 5: X-ray diffraction patterns obtained for the three alloys tested following 1000 hours at 900°C.

Figure 6: BSE image and XRD analysis showing in greater detail the fine lenticular features found to precipitate within the *fcc* matrix in alloy CrMnFeCo following exposures at 900°C for 1000 hours.

Figure 7: BSE images and EDX compositional maps following 1000 hours at 700°C of alloys CrMnFeCo, CrMnFeCoNi<sub>0.5</sub> and CrMnFeCoNi<sub>1.5</sub>.

Figure 8: X-ray diffraction patterns obtained for the three alloys tested following 1000 hours at 700°C.

Figure 9: Thermodynamcially predicted pseudobinary phase diagram between CrMnFeCo and Ni, overlaid with experimental data points. Circular red data points indicate alloys that contained the  $\sigma$  phase, whilst square green data points indicate alloys that were  $\sigma$  free. The dashed line indicates the  $\sigma$  phase field boundary predicted using the New PhaComp method. It should be noted that this diagram requires the data to be plotted against nominal, rather than actual, alloy compositions.