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Notes

# Welfare-improving ambiguity in insurance markets with asymmetric information

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## Abstract

We consider a model of competitive insurance markets involving both asymmetric information and ambiguity about the accident probability. We show that there can exist a full-insurance pooling equilibrium. We also present an example where an increase in ambiguity leads to a strict Pareto improvement. Higher ambiguity relaxes high-risks' incentive compatibility constraint and allows low risks to buy more insurance. Higher ambiguity also deteriorates low risks' expected utility from holding an uncertain prospect. If the former effect dominates, the expected utility of low risks increases and given that high risks' utility remains unaffected, the increase in ambiguity implies a strict Pareto improvement.

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## 1. Introduction

There is a growing body of literature uncovering the negative effects of ambiguity such as: limited participation and reduced liquidity in the market, adverse effects on risk sharing, uncertainty premium in equilibrium prices of financial assets, market inefficiency (see [1] and [2] for recent surveys on effects of ambiguity in financial markets). Furthermore, Snow [10] shows that information which reduces ambiguity has a positive value under symmetric information. The

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intuition behind these results is that individuals dislike ambiguity – a phenomenon that has been explored extensively in psychological and experimental literature (see [3] and [5] for surveys).

In markets with asymmetric information, however, agents' welfare may not fall with the degree of ambiguity. An increase in the degree of ambiguity can have a positive effect on the welfare of a type of agents if this is sufficiently smaller than that of the other type. We extend the Rothschild and Stiglitz [9] model of competitive insurance markets with asymmetric information by introducing ambiguity about the accident probability. Insurees fail to estimate accurately their own accident probabilities and make decisions based on *intervals* of possible probabilities according to the maxmin expected utility of Gilboa and Schmeidler [4].

In this paper we make two points. Firstly, we show that there can exist a full-insurance pooling equilibrium. Intuitively, ambiguity aversion increases the utility cost of under- or overinsurance. In particular, if insurees with the lower true accident probability face a sufficiently higher degree of ambiguity than insurees with higher accident probability, the utility cost of under- or overinsurance strictly dominates the monetary benefit of the lower per-unit premium. As a result, the low risks prefer to buy full insurance at a high (pooling) per-unit premium than under- or overinsurance at a lower per-unit premium. That is, the high degree of ambiguity makes the cost of separation prohibitively high for the low risks.

Secondly, we present an example where an increase in the degree of ambiguity for both types of insurees leads to a Pareto improvement. Intuitively, an increase in the degree of ambiguity faced by the high-risk insurees relaxes their incentive compatibility constraint and allows low risks to buy more insurance and move closer to full insurance. At the same time, the increased level of ambiguity for low risks deteriorates their expected utility from holding an uncertain prospect. If the former effect dominates, the expected utility of low risks increases and given that the high risks always buy full insurance and so their utility remains unaffected, the increase in ambiguity implies a Pareto improvement.

## 2. The model

We consider the basic framework introduced by Rothschild and Stiglitz [9]. There is a continuum of insurees and a single consumption good. All insurees have the same twice continuously differentiable utility function  $U : \mathbb{R} \rightarrow \mathbb{R}$  with  $U' > 0$  and  $U'' < 0$  and the same wealth level,  $W > 0$ . There are two possible states of nature: good and bad. In the bad state the individual suffers a gross loss of  $d \in (0, W)$ . There are two types of insurees: high risks (Hs) and low risks (Ls) which differ in the probability of having the accident:  $1 > p_H > p_L > 0$ . Let  $\lambda \in (0, 1)$  be the fraction of the Ls in the economy and this fraction is common knowledge.

Each insuree may insure himself against the accident by accepting an insurance contract  $A = (\alpha_1, \alpha_2)$ , where  $\alpha_1 \geq 0$  is the insurance premium and  $\alpha_2 \geq 0$  is the coverage. We can represent the insurance contract as  $A = (w_G, w_B)$ , where  $w_B = W - d - \alpha_1 + \alpha_2$  and  $w_G = W - \alpha_1$  are wealth levels of the insuree in the bad and good states respectively. In this environment, if insuree of type  $i$  knows precisely his own accident probability  $p_i$ , then his expected utility is<sup>2</sup>:

$$EU((w_G, w_B), p_i) = p_i U(w_B) + (1 - p_i) U(w_G), \quad i = H, L.$$

We extend the above setting by introducing aversion to ambiguity. We assume that insurees do not know precisely their accident probabilities. Their beliefs consist of a set of priors about

<sup>2</sup> Since insurees of the same type have identical utility, we index them by their types without loss of generality.

the true probability  $p_i, i = H, L$ , and this set is described by an interval  $[p_i, \bar{p}_i]$ . The length of this interval is the measure of the degree of ambiguity faced by the insuree. We assume that the true probability  $p_i \in [p_i, \bar{p}_i], i = H, L$ .<sup>3</sup>

Insurees have the maxmin expected utility preferences of Gilboa and Schmeidler [4]:

$$\begin{aligned}
 MEU_i(w_G, w_B) &= \min_{p_i \in [p_i, \bar{p}_i]} EU((w_G, w_B), p_i) \\
 &= \begin{cases} EU((w_G, w_B), p_i), & w_G \geq w_B, \\ EU((w_G, w_B), \bar{p}_i), & w_G < w_B, \end{cases} \quad i = H, L.
 \end{aligned}$$

That is, for each contract, insurees compute the worst outcome with respect to  $p_i$  and then maximize this worst-case utility with respect to  $(w_G, w_B)$ . The indifference curve of type  $i, MEU_i = \{(w_G, w_B) : MEU((w_G, w_B), p_i) = k\}$ , has a kink at the intersection with the full-insurance locus  $\{(w_G, w_B) : w_G = w_B\}$  (45° line) as its slope (marginal rate of substitution between incomes in the good and bad states) in the overinsurance region  $\{(w_G, w_B) : w_G < w_B\}$  is  $-(1 - \underline{p})/\underline{p}$  and in the underinsurance region  $\{(w_G, w_B) : w_G > w_B\}$  it is equal to  $-(1 - \bar{p})/\bar{p}$ .

There are  $S, |S| \geq 2$ , risk neutral insurance companies involved in Bertrand competition. Insurers cannot observe the type of insurees but they know  $\lambda$ . They also know the insurees' utility function and the probability interval for each type. We assume that insurers are ambiguity neutral.<sup>4</sup> They use reference accident probabilities, one for each type, which, for simplicity, coincide with the true probabilities  $p_i$ .<sup>5</sup> This assumption is justified by the insurers' capacity to collect large data sets and obtain accurate estimates of the true probabilities.

The expected profit of an insurer offering contract  $A = (\alpha_1, \alpha_2)$ , given this contract is chosen by type  $i$ , is  $\pi(A|i) = \alpha_1 - p_i\alpha_2 = w_B - \frac{W - dp_i}{p_i} + \frac{1 - p_i}{p_i} w_G$ . The expected profit of the insurer is

$$\pi(A) = \begin{cases} \pi(A|i), & \text{if contract } A \text{ is taken by only insuree } i, \\ \lambda\pi(A|L) + (1 - \lambda)\pi(A|H), & \text{if contract } A \text{ is taken by both types.} \end{cases}$$

Insurance companies and insurees play the standard two-stage screening game:

**Stage 1:** Each insurer  $s \in S$  simultaneously offers one (but possibly an infinite number of copies) contract  $A^s$ . Let  $\Sigma = \{A^s \mid s \in S\} \in 2^{\mathbb{R}_+^2}$  be the set of contracts offered by insurers.

**Stage 2:** Each insuree may apply for (at most) one contract from  $\Sigma$ . The choice of an insuree of type  $i \in \{H, L\}$  can be described by a function  $I_i : 2^{\mathbb{R}_+^2} \rightarrow \mathbb{R}_+^2$  such that  $I_i(\Sigma) \in \Sigma$  for any  $\Sigma$ .

We only consider pure-strategy sub-game Perfect Nash equilibria of the above game. A pair  $\{\Sigma, I_i(\Sigma)\}$ , where  $\Sigma \subseteq \mathbb{R}_+^2$  is a set of contracts offered by insurers and  $I_i(\Sigma)$  is the contract chosen by an insuree of type  $i$ , is an *equilibrium* if the following conditions hold:

- E1. Insurees maximize their maxmin expected utility given the set of contracts  $\Sigma$  offered:  
 $I_i(\Sigma) = \arg \max_{A \in \Sigma} MEU_i(A)$  for  $i \in \{H, L\}$ .

<sup>3</sup> We make this assumption to distinguish the effects of ambiguity from over-optimism or over-pessimism.

<sup>4</sup> If insurers are ambiguity averse, most of the results are qualitatively similar except two main differences: First, ambiguity averse insurers charge higher per-unit price (ambiguity premium). Second, if the insurers' degree of ambiguity is sufficiently higher than that of insurees, the insurees are not willing to pay the high ambiguity premium the insurers charge and the market collapses (no trade).

<sup>5</sup> Our results would be similar if the reference probabilities are different from the true ones. However, if the reference probabilities are lower than the true ones, the insurers should have initial capital to fulfill their promises.

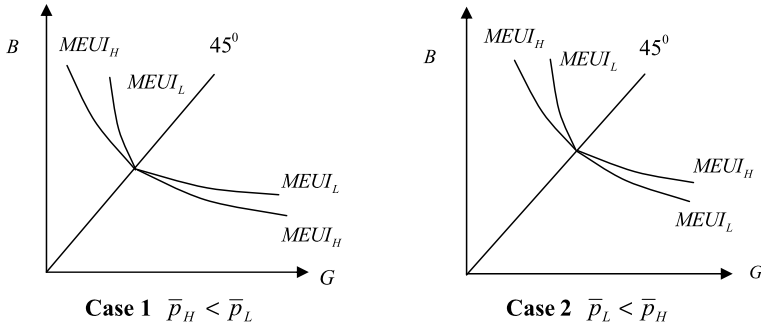


Fig. 1. Relative slopes and shapes of insurees’ indifference curves for different degrees of ambiguity faced by the two types.

- E2. No contract in the equilibrium allocation  $\{I_H(\Sigma), I_L(\Sigma)\}$  makes negative expected profit:  $\pi(A) \geq 0 \forall A \in I_H(\Sigma) \cup I_L(\Sigma)$  (individual rationality constraint).
- E3. The equilibrium allocation  $\{I_H(\Sigma), I_L(\Sigma)\}$  is incentive compatible:  $\forall A \in I_H(\Sigma)$  and  $\forall B \in I_L(\Sigma)$ :  $MEU_L(A) \geq MEU_L(B)$  and  $MEU_H(B) \geq MEU_H(A)$ .
- E4. No other contract  $C \notin I_H(\Sigma) \cup I_L(\Sigma)$  introduced alongside  $I_H(\Sigma) \cup I_L(\Sigma)$  would increase insurers’ expected profits:  $\forall C \notin I_H(\Sigma) \cup I_L(\Sigma)$ :  $\pi(C) \leq 0$ .

**3. Analysis**

We begin by examining how degrees of ambiguity faced by the two types of insurees affect the relative slopes and shapes of their indifference curves. This is important as relative slopes and shapes of the indifference curves determine the nature of the equilibrium (pooling or separating). For simplicity we make the following assumption:

**Assumption 1.**  $\underline{p}_L < \underline{p}_H$ .<sup>6</sup>

There are two cases to consider which are:

**Case 1:**  $\bar{p}_H < \bar{p}_L$ , (1)

**Case 2:**  $\bar{p}_L < \bar{p}_H$ . (2)

In Case 1 the indifference curves of the both types intersect twice (single-crossing condition fails) whereas in Case 2 they cross only once (single-crossing condition is satisfied). Fig. 1 illustrates these cases.

Let  $(w_{iG}, w_{iB}) := I_i(\Sigma)$  denote the contract chosen by insuree of type  $i$  in equilibrium.

**Lemma 1.** *In any equilibrium allocation of our game the Hs takes full insurance:  $w_{HB} = w_{HG}$ .*

**Proof.** Due to Bertrand competition any equilibrium contract must lie on the corresponding zero-profit line:  $\pi(A|i) = 0 \forall A \in I_i(\Sigma)$  and  $i \in \{H, L\}$ . Consider an incentive compatible and zero-profit allocation (contracts  $A_H = (w_{HG}, w_{HB})$  and  $A_L = (w_{LG}, w_{LB})$  in Fig. 2) where

<sup>6</sup> This assumption implies that there cannot exist equilibria where some insurees choose overinsurance.

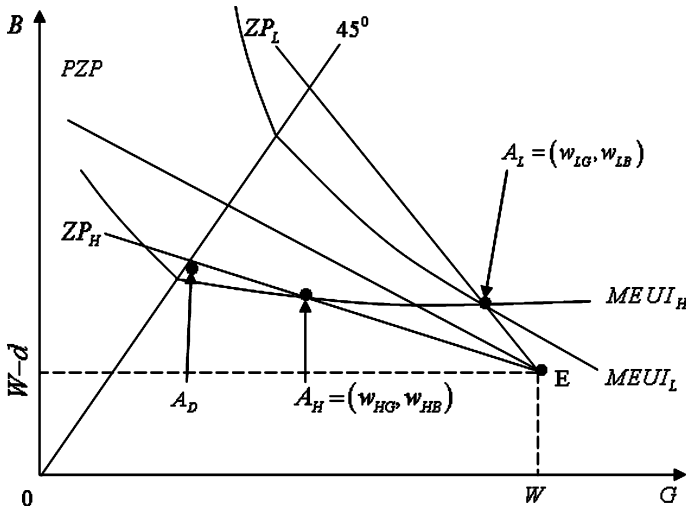


Fig. 2. The H-type buys full insurance.

both types of insureds are underinsured:  $w_{iG} > w_{iB}$  for  $i \in \{H, L\}$ .<sup>7</sup> Denote by  $(w_G^*, w_B^*)$  the intersection of the zero-profit line  $ZP_H = \{(w_G, w_B): w_B = \frac{W - p_H d}{p_H} - \frac{(1 - p_H)}{p_H} w_G\}$  and the indifference curve  $MEUI_H$  which goes through the point  $A_L = (w_{LG}, w_{LB})$  and is described by the relation  $(1 - \bar{p}_H)u(w_{LG}) + \bar{p}_H u(w_{LB}) = (1 - \bar{p}_H)u(w_G) + \bar{p}_H u(w_B)$ . Conditions E1 and E4 in the definition of equilibrium imply that  $(w_G^*, w_B^*) = (w_{HG}, w_{HB})$ . The slope of the indifference curve  $MEUI_H$  at point  $(w_{HG}, w_{HB})$  is smaller in absolute value than its slope at the intersection with the 45° line which, in turn, is smaller in absolute value than the slope of the zero-profit line  $ZP_H: (1 - \bar{p}_H)/\bar{p}_H < (1 - p_H)/p_H$ . Therefore, there exists a new contract  $A_D = (w_{DG}, w_{DB})$  such that  $MEU_H(A_D) > MEU_H(A_H)$  and  $w_{DB} < (W - p_H d)/p_H - (1 - p_H)w_{DG}/p_H$ . Thus, a new entrant can offer  $A_D$  profitably attracting the Hs, so allocation  $(A_H, A_L)$  cannot be an equilibrium (as the Ls strictly prefer  $A_L$  to  $A_D$ ). Thus, we get a contradiction. So, in any equilibrium the Hs can only buy full insurance. □

**Proposition 1.** (i) If Ls' degree of ambiguity is sufficiently higher than that of the Hs so that (1) is satisfied then the pooling allocation  $A_P = (w_P, w_P) = I_H(\Sigma) = I_L(\Sigma)$  where both types buy full insurance is the unique sub-game Perfect Nash equilibrium allocation.

(ii) If (2) is satisfied, then there cannot exist a pooling equilibrium. Furthermore, if the proportion  $\lambda$  of the Hs in the population is sufficiently low so that there exists a separating allocation  $(A_H, A_L)$  that is not Pareto-dominated by any other feasible allocation, then this separating allocation is the unique sub-game Perfect Nash equilibrium allocation.

**Proof.** (i) Let  $A_P = (w_P, w_P)$  be a contract defined by the intersection of the full-insurance locus and the pooling zero-profit line  $\{(w_G, w_B): \pi(w_G, w_B) = 0\}$ . Consider two indifference curves  $MEUI_L$  and  $MEUI_H$  that go through point  $A_P$ . Since  $(1 - \bar{p}_L)/\bar{p}_L < (1 - \bar{p}_H)/\bar{p}_H$  and  $(1 - \underline{p}_L)/\underline{p}_L > (1 - \underline{p}_H)/\underline{p}_H$  (by Assumption 1),  $MEUI_L$  is flatter than  $MEUI_H$  on  $\{(w_G, w_B): w_G > w_B\}$  and steeper on  $\{(w_G, w_B): w_G < w_B\}$ . This implies

<sup>7</sup> A similar argument applies if the H-type is offered overinsurance off-the-equilibrium path.

$$\begin{aligned} & \{(w_G, w_B): MEU((w_G, w_B), p_L) \geq U(w_P)\} \\ & \subset \{(w_G, w_B): MEU((w_G, w_B), p_H) \geq U(w_P)\}. \end{aligned} \tag{3}$$

Given (3), there exists no allocation which is preferred to  $A_P$  by the Ls and not by the Hs as for any  $A$  with  $MEU(A, p_L) \geq U(w_P)$  we have  $MEU(A, p_H) \geq U(w_P)$ . Moreover, because of the relation  $(1 - \bar{p}_L)/\bar{p}_L < (1 - \hat{p})/\hat{p} < (1 - \underline{p}_L)/\underline{p}_L$  with  $\hat{p} = \lambda p_L + (1 - \lambda)p_H$ , we have that

$$\{(w_G, w_B): MEU((w_G, w_B), p_L) \geq U(w^P)\} \subset \{(w_G, w_B): \pi(w_G, w_B) \leq 0\},$$

that is, no insurer can profitably attract the Ls (or both types) which implies that  $A_P$  is an equilibrium. Furthermore, according to Lemma 1, in any equilibrium allocation the Hs buy full insurance which means that in any pooling equilibrium both types buy full insurance. Thus, according to E2,  $A_P$  is the unique pooling equilibrium. Finally, (3) implies that a separating equilibrium cannot exist. Therefore, the pooling allocation  $A_P$  where both types buy full insurance is the unique sub-game Perfect Nash equilibrium allocation.

(ii) By Lemma 1, in any equilibrium the Hs buy full insurance. Thus, the only candidate for a pooling equilibrium is the pooling allocation involving full insurance for both types. However, condition (2) implies that the indifference curve  $MEUI_L$  through the candidate pooling equilibrium  $A_P = (w_P, w_P)$  will lie below of the indifference curve  $MEUI_H$  in the underinsurance region (since its slope  $-(1 - \bar{p}_L)/\bar{p}_L < -(1 - \bar{p}_H)/\bar{p}_H$ ) and above of  $MEUI_H$  in the overinsurance region by Assumption 1 (see Fig. 3, panel A). That is, the set  $\Omega_1 \equiv \{(w_G, w_B): MEU((w_G, w_B), p_L) < U(w_P)\} \cap \{(w_G, w_B): MEU((w_G, w_B), p_H) > U(w_P)\} \neq \emptyset$  (single-crossing condition holds). Given that  $A_P \in \Omega_2 \equiv \{(w_G, w_B): w_B < (W - p_L d)/p_L - (1 - p_L)w_G/p_L\}$  ( $A_P$  lies below the zero-profit line  $ZP_L$  corresponding to the Ls), there exists  $A_D \in \Omega_1 \cap \Omega_2$ . If an insurer offers the pooling contract  $A_P$ , a deviant insurer can profitably attract the Ls by offering the contract  $A_D$ . Since the contract  $A_D \in \Omega_2$ , it implies strictly positive profits for the deviant insurer:  $\pi(A_D) = \pi(A_D|L) > 0$ . As a result, the contract  $A_P$  cannot be an equilibrium and therefore no pooling equilibrium can exist.

In order to prove the second part of the statement, define the allocation  $(A_H, A_L)$  as follows. Let  $A_H = (w_H, w_H)$  be the intersection of zero-profit line  $ZP_H$  with the full-insurance locus, i.e.,  $w_H = W - p_H d$  and let the contract  $A_L = (w_{LG}, w_{LB})$  be the intersection of  $ZP_L$  with  $MEUI_H = \{(w_G, w_B): MEU((w_G, w_B), p_H) = U(w_H)\}$ . Suppose now that  $\lambda$  is sufficiently low so that the allocation  $(A_H, A_L)$  is not Pareto-dominated by any other feasible allocation. This implies that for any  $(w_G, w_B) \in MEUI_L = \{(w_G, w_B): MEU((w_G, w_B), p_L) = MEU((w_{LG}, w_{LB}), p_L)\}$  we have that  $w_B > (W - \hat{p}d)/\hat{p} - (1 - \hat{p})w_G/\hat{p}$  with  $\hat{p} = \lambda p_L + (1 - \lambda)p_H$  (that is, the indifference curve of the Ls which is going through their separating contract  $A_L$  also passes above the pooling zero-profit line  $PZP$ , see panel B of Fig. 3). Otherwise, there would be a contract  $(w_G, w_B) \in \{(w_G, w_B): MEU((w_G, w_B), p_L) > MEU((w_{LG}, w_{LB}), p_L)\} \cap \{(w_G, w_B): w_B < \frac{W - \hat{p}d}{\hat{p}} - \frac{(1 - \hat{p})}{\hat{p}} w_G\}$  Pareto dominating  $(A_H, A_L)$ .

The separating allocation  $(A_H, A_L)$  is an equilibrium. Indeed, Bertrand competition implies that in any separating equilibrium each contract must lie on the corresponding zero-profit line. Given that  $(A_H, A_L)$  is offered, there is no other zero-profit contract which is preferred to  $A_H$  by the Hs. Also, there is no contract below Ls' zero-profit line which is preferred by the Ls and not by the Hs. Finally, there is no pooling contract below  $PZP$  that is preferred by the Ls to  $A_L$ . Therefore, there is no profitable deviation and  $(A_H, A_L)$  is an equilibrium.

Furthermore, no other separating allocation can be equilibrium. Indeed, the incentive compatibility constraint of the Ls is not binding (the Ls strictly prefer their own contract to Hs' contract).

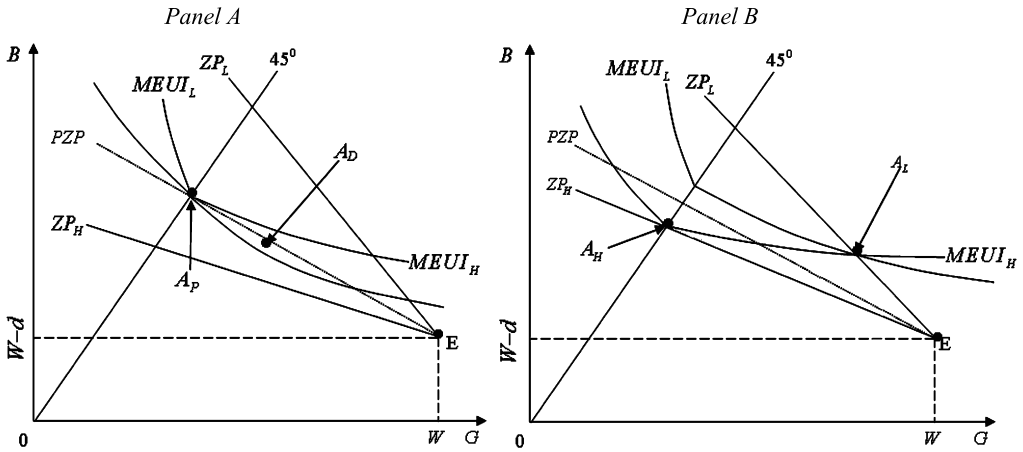


Fig. 3. Non-existence of a pooling equilibrium (panel A) and separating equilibrium (panel B) under condition  $\bar{p}_L < \bar{p}_H$ .

**Lemma 1** implies that no zero-profit contract but  $A_H$  (full insurance) can be an equilibrium contract for the Hs. Given that, the Ls will be offered  $A_L$  and therefore only  $(A_H, A_L)$  can be an equilibrium. □

Intuitively, ambiguity aversion increases the utility cost of under- or overinsurance. In particular, if the Ls face sufficiently higher degree of ambiguity than the Hs, the utility cost of under- or overinsurance strictly dominates the monetary benefit of the lower per-unit premium. As a result, the Ls prefer to purchase full insurance at a high (pooling) per-unit premium than under- or overinsurance at a lower per-unit premium. That is, the high degree of ambiguity makes the cost of separation prohibitively high for the Ls. In contrast, if the degree of ambiguity faced by the Ls is not sufficiently higher than that of the Hs, the utility cost of underinsurance is lower than the monetary benefit of the lower per-unit premium. Hence, the Ls prefer underinsurance to full insurance and a separating equilibrium arises.

Three points should be made here: First, the pooling equilibrium with full insurance is due to ambiguity aversion and cannot be obtained in the standard expected utility framework. Second, failure of the single-crossing condition is necessary but not sufficient for the existence of the full-insurance pooling equilibrium. The necessary and sufficient condition specifies that the indifference curve of the Ls is steeper than that of the Hs in the overinsurance region and flatter in the underinsurance one. Third, this result does not depend on the maxmin formulation of ambiguity aversion we have adopted in this paper. It can be also achieved under rank-dependent expected utility model of Quiggin [8], more general variational preferences of Maccheroni et al. [7] and for some parameter values under the smooth representation of ambiguity aversion of Klibanoff et al. [6]. In the latter case, it happens when the parameters are such that the indifference curves of the two types are tangent on the 45° line and the transformation functions are such that the indifference curve of the Ls lies inside of the Hs. This obtains when the expectations of the accident probability for two types are the same.

#### 4. Comparative statics: Ambiguity leads to Pareto improvement

Having established the equilibria of our game, we can consider the effect of an increase in the degree of ambiguity on welfare. Let us consider two situations, where in situation 1 Hs'



beliefs are represented by the interval  $[\underline{p}_H, \bar{p}_H^1]$  and Ls' beliefs are represented by  $[\underline{p}_L, \bar{p}_L^1]$  and in situation 2 intervals  $[\underline{p}_H, \bar{p}_H^2]$  and  $[\underline{p}_L, \bar{p}_L^2]$  represent beliefs of Hs and Ls respectively. We say that there is an *increase* in the degree of ambiguity for type  $i$  if  $\bar{p}_i^1 < \bar{p}_i^2$ .<sup>8</sup> The following proposition demonstrates that an increase in the degree of ambiguity for both types of insureds can be strictly Pareto improving.

**Proposition 2.** *Suppose that Assumption 1 and the following conditions are satisfied:*

$$\bar{p}_L^1 < \bar{p}_H^1 \quad \text{and} \quad \bar{p}_L^2 < \bar{p}_H^2. \tag{4}$$

Suppose also that  $\lambda$  is sufficiently low so that there exists a separating allocation that is not Pareto-dominated by any other feasible allocation. Then an increase in the degree of ambiguity of both types leads to a strict Pareto improvement if and only if

$$\frac{u(w_{LG}^2) - u(w_{HG}^2)}{u(w_{LG}^1) - u(w_{HG}^1)} > \frac{1 - \bar{p}_L^1/\bar{p}_H^1}{1 - \bar{p}_L^2/\bar{p}_H^2}, \tag{5}$$

where  $w_{HG}^1 = w_{HG}^2 = W - p_B d$  and  $w_{LG}^i$ ,  $i = 1, 2$ , is the solution of the following equation  $u(w_{HG}^i) = (1 - \bar{p}_H^i)u(w) + \bar{p}_H^i u(\frac{W}{p_L} - d - \frac{(1-p_L)w}{p_L})$  with respect to the variable  $w$ . Otherwise, the increase in ambiguity of both types leads to a Pareto-inferior equilibrium allocation.

**Proof.** We start by showing that conditions (4) and (5) are sufficient for a Pareto improvement. To show this, note that according to Proposition 1, condition (4) implies that the equilibria in situations 1 and 2 are separating. In both cases, the Hs have the same allocation  $(w_{HG}^1, w_{HB}^1)$  which is on the intersection of the 45° line and zero-profit line  $ZP_H = \{(w_G, w_B) : w_B = \frac{W-p_H d}{p_H} - \frac{(1-p_H)}{p_H} w_G\}$  (see Fig. 4). Therefore, an increase in the degree of ambiguity does not affect the welfare of Hs. Thus, we have to show that if condition (5) is satisfied, the Ls are strictly better off. To do this we compare the utility levels of the contracts  $A_L^1$  and  $A_L^2$  (see Fig. 4). The Ls utility level in these two allocations are given by  $EU_L^1 = (1 - \bar{p}_L^1)u(w_{LG}^1) + \bar{p}_L^1 u(w_{LB}^1)$  and  $EU_L^2 = (1 - \bar{p}_L^2)u(w_{LG}^2) + \bar{p}_L^2 u(w_{LB}^2)$  respectively, where the contract  $(w_{LG}^j, w_{LB}^j)$  is the intersection of  $ZP_L$  and the indifference curve  $MEU_H^j = \{(w_G, w_B) : u(w_{HG}^j) = (1 - \bar{p}_H^j)u(w_G) + \bar{p}_H^j u(w_B)\}$  and for  $j = 1, 2$  and hence satisfies the equation  $u(w_{HG}^j) = (1 - \bar{p}_H^j)u(w_{LG}^j) + \bar{p}_H^j u(\frac{W-p_L d}{p_L} - \frac{(1-p_L)}{p_L} w_{LG}^j)$ . The wealth level  $w_{HG}^j = W - p_H d$  is determined as the intersection of  $ZP_H$  and the full-insurance locus. Hence

$$\begin{aligned} EU_L^j &= (1 - \bar{p}_L^j)u(w_{LG}^j) + \bar{p}_L^j u(w_{LB}^j) = (1 - \bar{p}_L^j)u(w_{LG}^j) + \frac{\bar{p}_L^j}{\bar{p}_H^j} (\bar{p}_H^j u(w_{LB}^j)) \\ &= (1 - \bar{p}_L^j)u(w_{LG}^j) + \frac{\bar{p}_L^j}{\bar{p}_H^j} (u(w_{HG}^j) - (1 - \bar{p}_H^j)u(w_{LG}^j)) \\ &= u(w_{HG}^j) + \left(1 - \frac{\bar{p}_L^j}{\bar{p}_H^j}\right) (u(w_{LG}^j) - u(w_{HG}^j)). \end{aligned}$$

<sup>8</sup> Since we concentrate our analysis mainly on the underinsurance region (Assumption 1), changes in the lower bound of the probability of accident do not affect the welfare analysis (since the worst-case scenario is based on the upper probability only). Therefore, we assume them to be unchanged to simplify the exposition.

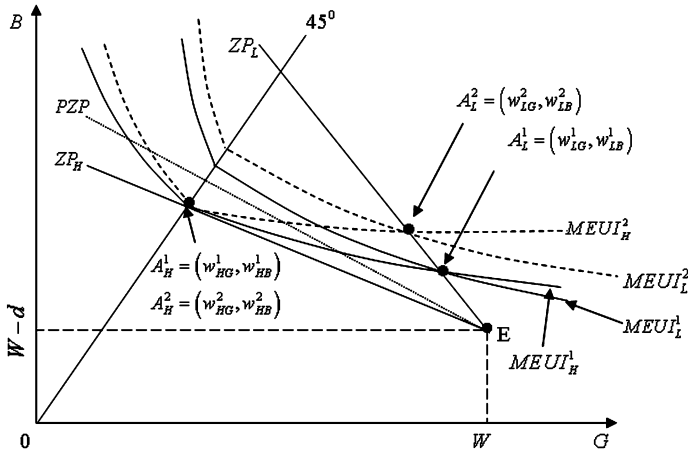


Fig. 4. An increase in ambiguity leads to a strict Pareto-improvement.

Taking into account that  $w_{HG}^1 = w_{HG}^2$  and condition (5) we obtain

$$\begin{aligned}
 EU_L^2 - EU_L^1 &= \left(1 - \frac{\bar{p}_L^2}{\bar{p}_H^2}\right) (u(w_{LG}^2) - u(w_{HG}^2)) \\
 &\quad - \left(1 - \frac{\bar{p}_L^1}{\bar{p}_H^1}\right) (u(w_{LG}^1) - u(w_{HG}^1)) > 0.
 \end{aligned}$$

In situation 2, with a higher degree of ambiguity, the Ls are strictly better off if condition (5) is satisfied. This proves the sufficiency of the conditions. On the other hand, if (5) does not hold, a Pareto improvement cannot be achieved. □

An increase in ambiguity has two opposite effects on the utility of the Ls. First, an increase in ambiguity of the Hs increases Hs’ mimicking cost (because of greater ambiguity) and allows the Ls to purchase more insurance while still separating themselves from the Hs. That is, the contract chosen by the Ls in equilibrium lies closer to the 45° line and so Ls’ welfare improves everything else given (see Fig. 4). Second, an increase in Ls’ degree of ambiguity implies a reduction in Ls’ utility at any given contract in the underinsurance region (Ls’ indifference curve becomes flatter). The final outcome depends on which of these two countervailing forces dominates. If the increase in ambiguity for the Hs is sufficiently larger than for the Ls, the first effect dominates and so the increase in ambiguity is welfare improving for the Ls. However, if the increase in ambiguity for the Ls is substantially larger than for the Hs, the second effect dominates and the Ls experience a drop in utility.

### 5. Conclusions

In this paper we develop a model with asymmetric information and present an example where an increase in ambiguity can result in a strict Pareto improvement. The increase in ambiguity has two effects: a) It relaxes the incentive compatibility constraint of the Hs. This allows the Ls to move closer to the first-best allocation and increase their utility. b) Everything else given, the increase in ambiguity reduces perceived expected utility. If the increase in ambiguity of the Hs

is sufficiently higher than that of the Ls, the first effect dominates and the increase in ambiguity leads to an increase of Ls' expected utility. Also, under certain conditions, the increase in ambiguity has no effect on Hs' welfare. As a result, an increase in ambiguity can lead to a strict Pareto improvement.

We also show that there can exist a full-insurance pooling equilibrium. This happens when the Ls face a sufficiently higher degree of ambiguity than the Hs, the utility cost of under- or overinsurance strictly dominates the monetary benefit of the lower per-unit premium. As a result, the high degree of ambiguity makes the cost of separation prohibitively high for the Ls.

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