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Can sustainable withdrawal rates be enhanced by trend following?

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Abstract

We examine the consequences of alternative popular investment strategies for the decumulation of funds invested for retirement through a defined contribution pension scheme. We examine in detail the viability of specific 'safe' withdrawal rates including the '4%-rule' of Bengen (1994). We find two powerful conclusions. First that smoothing the returns on individual assets by simple trend following techniques is a potent tool to enhance withdrawal rates. Second, we show that while diversification across asset classes does lead to higher withdrawal rates than simple equity/bond portfolios, "smoothing" returns in itself is far more powerful a tool for raising withdrawal rates. In fact, smoothing the popular equity/bond portfolios (such as the 60/40 portfolio) is in itself an excellent and simple solution to constructing a retirement portfolio. Alternatively, trend following enables portfolios to contain more risky assets, and the greater upside they offer, for the same level of overall risk compared to standard portfolios.

Keywords: Sequence Risk; Perfect Withdrawal Rate; Decumulation; Trend Following.

JEL Classification: G10, G11, G22.

In this paper we examine alternative popular investment strategies for the decumulation of funds invested for retirement. Usually these are funds created in a defined contribution pension scheme. We examine in detail the viability of specific 'safe' withdrawal rates (e.g. see the '4%-rule' of Bengen, 1994). To anticipate our empirical findings, we find two powerful conclusions:

(i) Smoothing the returns on individual assets by simple trend following techniques (or similar) is a potent tool to enhance withdrawal rates.

(ii) While diversification across asset classes does lead to higher withdrawal rates than simple equity/bond portfolios, "smoothing" returns is a far more powerful tool for raising withdrawal rates; in fact, smoothing the popular equity/bond portfolios (such as the 60/40 portfolio) is an excellent and simple solution to constructing a retirement portfolio.[1]

The move away from defined benefit (DB) towards defined contribution (DC) and personal savings for pensions is well underway for a wide variety of reasons, comprehensively described in the OECD Pensions Outlook, 2016. This, of course, means that both investment and longevity risk rest with the individual. OECD data on assets and members in DB and DC plans from 2000 to 2015 confirm the increasing prominence of DC plans in many OECD countries and, to the extent new schemes have been introduced in recent decades, they have almost entirely been DC schemes, though the exact arrangements differ between countries, (OECD, 2016). However, assets in occupational DC plans together with those in personal plans exceeded assets in DB plans in most reporting countries. In the United States, around half of private sector employees have no pension saving, only 2% have DB plans with 33% having DC; around 11% have both DB and DC. In the United Kingdom, the number of members of occupational DC schemes rose from about 2.5m to nearly 7.0m between 2013 and 2015. At the

[1] Chris Dillow, FT Money, p10, 22/4/2017, points out that if one believes that equity markets are expensive measured by the CAPE ratio, and bond markets are expensive after a 30-year bull market, and that both could well fall together following a shock, then there is no benefit in this diversification. But cash, or switching to cash, offers powerful protection.

same time, membership of occupational DB schemes drifted down from more than 11.5m to less than 11.0m.

The decline in importance of DB saving and the increasing domination of DC means that individuals need to concern themselves with two rather important questions:

i) how much should I save through my working life, and what sort of investment portfolio should I use,

and

ii) how do I hold my assets in retirement (assuming I will not buy an annuity) such that I can withdraw a 'suitable' regular amount to live on?

The construction of investment portfolios for both phases has been relatively neglected in the study of retirement planning leading to them being described as the 'known unknowns' (Merton, 2014). Indeed the study of long term accumulation and decumulation usually treats the two processes as completely separate phenomena. For the former, the emphasis is on changing the riskiness of portfolios as retirement beckons. This is usually de-risking in the form of glidepath or target-date investing, by raising the proportion in bonds and reducing the percentage in equities, (eg see Blanchett et al, 2016, Estrada, 2017). For the latter, the issue is what percentage of wealth can be withdrawn for consumption each year in a world with uncertain life expectancy and stochastic returns (see Bengen, 1994, Blanchett et al, 2016). Sometimes, of course, the glidepath glides through the retirement date and becomes the decumulation portfolio, though most discussions distinguish between the two for investing purposes. In this paper we analyse the decumulation phase following retirement.

Withdrawals can be either fixed or variable, nominal or real. So far most attention has been given to fixed, real withdrawals since Bengen (1994) shows that an initial withdrawal rate of

4%, with annual withdrawals subsequently adjusted by inflation, was 'safe' in the sense that, historically, this strategy never depleted a portfolio in the US in less than 30 years. The chosen portfolio was 50% US equity and 50% bonds.

Subsequent research (Estrada, 2018) shows that over the 115 years between 1900 and 2014, a 60/40 portfolio of U.S. stocks and bonds had a failure rate of 4.7%, and portfolios with at least 70% in U.S. stocks had an even lower (3.5%) failure rate. However, in other markets much higher failure rates would have been experienced with the 4% rule. Estrada (2017) shows failures rates between 1.2% for Canada and 70.9% for Italy with the UK at 22.1% over the same period with the same 60/40 portfolios. He shows lower failure rates for portfolios with higher proportions of stocks. The world average failure rate of 23.3% for 60/40 portfolios is reduced to 14.0% for equity-only portfolios. Blanchett et al (2016) suggest that 20th century US returns' experience is unusual and that most countries would not have generated returns sufficient to provide a 4% drawdown rule: for example, the UK would only have managed 2.8% per annum (see, also, FT Money, 4-2-2017). In addition, some researchers are troubled by current market conditions that suggest lower than historic expected returns for stocks and bonds going forward (Crook, 2013) and therefore increased risk of failure. The debate on the pros and cons of the 4% rule, and on fixed real withdrawals more generally, is alive and well.

Variable withdrawals encompass a broad set of strategies in which withdrawals are adjusted based on changing life expectancy (Dus et al, 2005), changing market conditions (Estrada, 2016), or both (Stout and Mitchell, 2006). Withdrawals depending on market conditions, in particular, are the subject of extensive study (see references in Suarez et al, 2015 and Clare et al, 2017), for example. A further development would be to consider optimal withdrawal rates based on a utility function as in the treatment of longevity risk by Milevsky and Huang (2011).

There are of course both pros and cons for fixed and variable withdrawals. Fixed withdrawals are generally easier to understand and, in the case of fixed real withdrawals, they preserve purchasing power. However, they do not adjust to changing market conditions or life expectancy: this may lead to depletion of a retirement portfolio earlier than desired, with potentially calamitous results. Meanwhile variable withdrawals do adjust to changing conditions and hence reduce or eliminate the risk of a very bad outcome. However, they typically are more difficult to understand and implement (see, for example, Stout, 2008) and may require a retiree to reduce their real consumption at some point. An optimising retiree might choose to do this depending on their view of longevity risk, as discussed by Milevsky and Huang (2011).

The maximum withdrawal rate, as used in this paper, is defined in Section 2 and belongs to the category of fixed *real* withdrawals and hence keeps real purchasing power constant throughout the decumulation period. However, in practice, if the safe (or maximum withdrawal) rate is updated periodically with new information on (actual returns) during the retirement period, then it will most likely lead to variable withdrawals in both nominal and real terms.

However, there is a crucial common thread to both accumulation and decumulation life-cycle phases which is often overlooked in considering investment performance of a portfolio in the context of saving or dissaving *regular* amounts; namely the threat presented by poor returns occurring at the 'wrong' time, otherwise known as *'sequence risk'*. For accumulation, it is particularly bad news if large negative returns occur shortly before retirement, while for decumulation it is just as disastrous if they occur just after the start of drawing down from the savings pot (see Figure 1). Clare et al (2017) explore sequence risk in the context of a 100%

US equity portfolio for 20-year decumulation periods and show that there is no simple statistic to measure the phenomenon, (which is why it may well be largely neglected by researchers), but that simple smoothing using trend following techniques, which removes the large drawdowns associated with major market falls, such as occurred in 2000 and 2008, allows a much better withdrawal rate experience.

In this paper, we address two closely related issues:

i) while the literature focusses exclusively on bonds and equities, diversification to other asset classes such as commodities has been shown to dramatically improve the risk-return possibilities for investors; we introduce commodities, real estate, and credit (see Clare et al, 2016) and compare the decumulation possibilities with equity/bond portfolios; unsurprisingly the former offer a better withdrawal rate experience in general. Note that Authers (August 2016, FT) sees such diversification as part of the pensions 'solution' in the face of malfunctioning target date funds.

ii) second, rather than worrying whether particular percentages of equity and bond portfolios will give suitable drawdown experience, are there any more general desirable features of decumulation portfolios about which we should be aware? The answer lies in a detailed understanding of the nature of sequence risk. We then explore the withdrawal experience of a range of popular retirement portfolios over the period 1971-2015 for the UK retiree and show how a smoothed investment solution gives a far better withdrawal rate experience: it turns out that smoothing even just a two asset-bond and equity-portfolio leads to superior results which are similar to those for a far more diversified portfolio.

In other words, smoothing returns dominates diversification when it comes to improving the decumulation experience. This results in the 4% rule being a lower bound for post-retirement experience even in an environment where some researchers think that only 2-3% pa is feasible, e.g., the Blanchett et al (2016) calculations for the UK, above.

The paper is constructed as follows: in Section 2, we introduce sequence risk and the perfect withdrawal rate as tools for evaluating alternative portfolio strategies. In section 3, we then look more closely at the '4% rule' which has established this concept as a reference point in the world of retirement planning. Section 4 compares the performance of several popular decumulation strategies, both smoothed and 'raw'. Section 5 concludes with the practical observation that advisers should look closely at sequence risk and its implications for investors who are making regular contributions or withdrawals to or from a pot of wealth. Choosing a strategy with little chance of large drawdowns (i.e. 'smoothed') should raise both the end-value of regular contributions and the withdrawal possibilities in retirement.

2. Sequence risk: measurement and importance

The major risks identified here in both the accumulation and decumulation phases are examples of sequence risk, the risk that investment returns occur in (rather unfortunately) the wrong order. Sequence risk can have a disastrous impact on savings as it does on withdrawals, though you could argue that with savings' losses one can simply work and save longer (as many will have done following the recent financial crisis). Suarez et al (2015) introduced the idea of Perfect Withdrawal Rates (PWR), to compare investing strategies. These PWRs are the maximum withdrawal rate possible over a fixed period of time if one had perfect foresight of investment returns, and are a useful metric for comparing investment strategies in the context of withdrawals in retirement (or any other period).

This concept of sequence risk is of particular interest to the decumulation industry. Okusanya (2015) and Chiappinelli and Thirukkonda (2015) point out the basic importance of 'path dependency' of investment returns (i.e. the order in which returns occur). This concept is at least as important to the outcomes of the retirement journey as the total return earned by the investment. Yet portfolio construction, both academic and practical, has typically focussed on total return and volatility, thus we estimate Sharpe ratios or similar performance statistics as a way of comparing strategies. Using simple arithmetic examples, studies typically show that higher withdrawal rates are always possible when the worst investment years occur later in the decumulation period (for *any given set* of returns). The natural reaction to sequence risk has therefore been to de-risk a portfolio as one approaches 'retirement' along the lines of 'glidepath' or similar strategies. As pointed out by Estrada (2017), many of the empirical exercises in this area focus on varying investment returns and individual longevity but assume the returns are *constant* over the decumulation period: this, of course, assumes away sequence risk and all the associated real world problems.

The concept of PWR is a relatively new one to create withdrawal strategies from retirement portfolios and is based not on heuristics and/or empirical testing but on analytics. Suarez et al (2015) and also Blanchett et al (2012) construct a probability distribution for the PWR and apply it sequentially, deriving a new measure of sequence risk in the process. We use these ideas to show that a particular class of investment strategies (both simple and transparent) can offer superior (Perfect) Withdrawal Rates across virtually the whole range of return environments. This smoothing of returns leads to a better decumulation experience across virtually all investing time frames.

Here we assume that in the decumulation phase annual withdrawals from the pot of wealth are made on the first day of each year and annual investment returns accrue on the last day of the year: there are no taxes or transactions costs. Then for any given series of annual returns there is one and only one constant withdrawal amount that will leave the desired final balance on the account after n years (the planning horizon). The final balance could well be a bequest or indeed zero. We can think of this as equivalent to finding the fixed-amount payment that will fully pay off a variable-rate loan after n years. It involves withdrawing the same amount every year, giving the desired final balance with no variation in the income stream, no failure and no surplus[2].

The change in account balances in consecutive periods is:

$$K_{i+1} = \left(K_i - w\right).\left(1 + r_i\right) \tag{1}$$

where K_i is the balance at the beginning of year i, w is the yearly withdrawal amount, and r_i is the rate of return in year i in annual percent. Applying equation (1) chain-wise over the entire planning horizon (n years), we obtain the relation between the starting balance K_S (or K_i) and the end balance K_E (or K_n):

$$K_E = \left(\left\{\left[\left(K_S - w\right)\ \left(1 + r_1\right)\ - w\right]\ \left(1 + r_2\right)\ - w\right\}\ \left(1 + r_3\right)\ \ldots\ - w\right).\left(1 + r_n\right) \tag{2}$$

and we solve equation (2) for w to get:

$$w = [K_S \prod_{i=1}^{n} \left(1 + r_i\right) - K_E] / \sum_{i=1}^{n} \prod_{j=i}^{n} \left(1 + r_j\right) \tag{3}$$

Equation (3) provides the constant amount that will draw the account down to the desired final balance if the investment account provides, for example, a 5% return in the first year, 3% in the second year, even minus 6% in the third year, etc., or any other particular sequence of annual returns. This figure, w, is called the Perfect Withdrawal Amount (PWA). Quite simply, if one knew in advance the sequence of returns that would come up in the planning horizon,

[2] Note that Blanchett et al (2012) present a measure similar to PWA called Sustainable Spending Rate (SSR). Suarez et al (2015) point out that the PWA is a generalization of SSR, with SSR being the PWA when the starting balance is $1 and the desired ending balance is zero.

one would compute the PWA, withdraw that amount each year, and reach the desired final balance exactly and just in time.

Note that the analysis offers a number of useful insights into sequence risk measurement. First, equation (3) can be restated in a particularly useful way since the term $\prod_{i=1}^{n}(1+r_i)$ in the numerator is simply the cumulative return over the entire retirement period, (call it R_n). The *denominator*, in turn, can be interpreted as a measure of sequencing risk:

$$\begin{aligned}\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_i) &= (1+r_1)(1+r_2)(1+r_3)...(1+r_n)+(1+r_2)(1+r_3)...(1+r_n)+ \\ &(1+r_3)(1+r_4)...(1+r_n)+...+(1+r_n-1)(1+r_n)+(1+r_n)\end{aligned} \tag{4}$$

The interpretation of this is straightforward: for any given set of returns equation (4) is *smaller* if the *larger* returns occur *early* in the retirement period and *lower* rates occur at the *end*. This is because the later rates appear more often in the expression. Suarez et al (2015) suggest the use of the reciprocal of equation (4) to capture the effect of sequencing: so let $S_n = 1/\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_i)$. This rises as the sequence becomes more favourable, and even though one set of returns appearing in two different orders will have the same total return (i.e. R_n with different S_n values), so the PWA rates will be different.

Given this definition of S_n, we can show, using (3), that the PWA can be written as: $w = (R_n K_s - K_E)S_n$. The PWR then follows, as $w/K_s = R_n S_n - S_n(K_E/K_s)$. So every sequence of returns is characterised by a particular PWA value and hence the retirement withdrawal question is really a matter of “guessing” what the PWA will turn out to be (eventually) for each retiree’s portfolio and objectives. Therefore, the problem now becomes how to estimate the probability distribution of PWAs from the probability distribution of the returns on the assets held in the retirement account.

We emphasised earlier that whereas in most finance contexts *total* return is the key variable, in both accumulation and decumulation the *order* of returns also matters. An example will make this clearer. Suppose we have three sets of returns for a stylised decumulation period of 5 years. As can be seen in Table 1, clearly the mean, volatility and Sharpe (and indeed Maximum Drawdown) are the same, but the returns' sequence differ as is evidenced by the different values of Sequence Risk $(1/S_n)$ with higher values of this metric, which result from poor returns in the first years of decumulation, associated with lower PWA and PWRs.

So in the real world of unforecastable asset returns and the consequent failure of tactical asset allocation culminating in the empirical (and costly) failure of de-risking in general and glidepath investing in particular, what is the solution to the dangers of sequence risk if risk-free investing in inflation linked government securities, annuities or similar is not attractive or not available as an option?

The answer lies in removing the chance of large drawdowns: in our related paper (Clare et al, 2017) we discuss how to address sequence risk in the US using the S&P500 index return as our representative portfolio. If we start by accepting that returns are inherently unpredictable and hence we cannot tactically switch investments to avoid large losses at key times - witness the lack of success of multi-asset funds say around 2009-10 - then a better option might be to use an investing strategy with lower sequence risk. In Clare et al, (2017) it is shown that one simple strategy called Trend Following leads to smoother equity investment outcomes in the US but with dramatically improved decumulation experiences (see also Faber, 2007 and Capone and Akant, 2016).

As an illustration, the first column of Table 2 compares the raw performance of the MSCI UK total equity market and the performance of a trend following strategy applied to the same index. The trend following rule that we apply works as follows: if the current price is above the 10-month moving average then a position is taken in stocks otherwise, the portfolio is held in cash receiving the 3-month Treasury bill interest rate. In Clare et al (2013) different trend rules and explored and the results are found to be remarkably insensitive to the choice of trend filter. The results show that the trend following filter: improves real returns by around 0.6% per annum; reduces volatility by one third; reduces maximum drawdown from around 74% to 29%; and produces a higher single factor alpha which we find to be statistically significant at the 95% level of confidence. The other columns in the table also show improvements in the risk return characteristics of these other asset classes with the application of the trend following rule.

The key question in the current context is whether smoothing via trend following improves the PWR experience. We will show this is the case for a range of popular investment portfolios in Section 4 below. But first of all we need to examine the evidence surrounding the fabled '4% Rule': in particular how replicable is it outside the US data environment and can simple trend following as explained above give a better outcome even sticking with the narrow 2-asset portfolios of the literature?

3. Decumulation and the 4% rule

As we have indicated, there is a growing body of literature on safe withdrawal rates for retirees; however most of this research is based on the historical returns of assets used by investors in the United States. While there has been some more recent research using projected returns for the United States (Blanchett et al, 2015) its applicability to the UK and other countries is questionable. Research by Bengen (1994), among others, suggests an initial safe withdrawal rate from a portfolio is 4% of the assets, where the initial withdrawal amount would subsequently be increased annually by inflation and assumed to last for 30 years (which is the expected duration of retirement). This finding led to the creation of the "4% Rule," a concept that is often incorrectly applied (see Blanchett, et al, 2016). They point out that, firstly, the "4%" value only applies to the first year of retirement, with subsequent withdrawals assumed to be based on that original amount, increased by inflation. Secondly, a retirement period of 30 years may be too short or too long based on the unique attributes of that retiree household, and thirdly that the analysis was based entirely on historical U.S. returns, which may not be applicable for international retirees today. A number of studies have introduced 'adaptive' rules: Guyton and Klinger (2006) manipulate the inflationary adjustment when rates of return are too low, modifying the withdrawal amount, while Frank, Mitchell, and Blanchett (2011) use adjustment rules dependant on how much the return deviates from the historical averages. Similarly, Zolt (2013) suggests curtailing the inflationary adjustment to the withdrawal amount in order to increase the portfolio's survival rate where appropriate. The principle being that withdrawal rates 'adapt' to changing circumstances.

Based on a 50% Bonds / 50% Equity portfolio, the Bengen (1994) safe withdrawal rate bottoms out at 4% around the 1970's. However, there are a number of problems extrapolating these results to non-US retirees: firstly, the analysis assumes retirement lasts 30 years, whereas of course this should vary by retiree while in reality the expected duration of retirement, thus respective modelling period, should vary by retiree. It is certainly the case that for a 65-year old man in the US or UK, expectancy in retirement is just over 20 years, so 30 years is a 'safe' option for simulation. In our analysis below we will describe both 20- and 30-year outcomes, with a strong preference for a 20-year decumulation period concurrent with a 20-year deferred annuity, (see Chen, et al, 2016). Secondly, this problem ignores the experience of retirees in other countries. Blanchett et al (2016) calculate safe withdrawal rates for a range of countries, and for the UK a rate of 2.5% pa is proposed as the safe level, with the early years of the 20^{th} century especially challenging. Finally, the Bengen analysis assumes that past returns are a reasonable basis for retirees to use today. While the past indeed provides some window into the future, the markets today are in a different place than historical long-term averages, and perhaps this needs to be taken into account when advising a retiree on a safe initial withdrawal rate?[3]

The true safe withdrawal rate varies significantly by country and, of course, target success rate. Based on a comprehensive analysis across 19 countries, Drew and Walk (2014) argue that the 4% rule does present us with an opportunity to form a baseline, which can dramatically improve the expectations of what is possible in retirement but is not a silver bullet approach to retirement withdrawal decisions. Estrada (2018) examines the Maximum Withdrawal Rate' in the context of failure rates and bequest possibilities for 21 countries and 115 years and for 11 asset allocations, i.e. different percentages of equities and government bonds. He concludes, unsurprisingly, that these rates vary substantially across countries and over time.

The study by Blanchett et al (2016) provides a consistent set of comparisons. For example, using the historical returns in Japan, a 95% target success rate would yield an initial safe withdrawal rate of .2%, while for the UK a 95% target success rate would yield an initial safe withdrawal rate of 2.8%, (Blanchett et al, 2016, p4). Perhaps unsurprisingly US returns have yielded the highest initial safe withdrawal rates across the 20 countries historically. This suggests initial safe withdrawal rates based on historical US returns may be overly optimistic

[3] We acknowledge the diverse and varied conclusions on the sustainability of the 4 per cent 'golden rule' as proposed by Bengen (2004). Whilst some studies firmly support this withdrawal rate (see Pye, 2000, Guyton, 2004, Guyton and Klinger, 2006), recent literature questions the sustainability of the 4 per cent 'safe withdrawal rate' and its ability to provide retirement portfolios. Spitzer, Strieter, and Singh (2008) suggest that the 4 per cent rule may be an oversimplification while studies by Sharpe, Scott, and Watson (2007) believe the rule is inefficient. Other studies which oppose the 4% percent rule include Pfau (2011), and Drew and Walk (2014).

on a global basis. For example, based on the results in Exhibit 3 of Blanchett et al (2016), using the US returns and targeting a 90% success rate yields an initial safe withdrawal rate of 3.6% (just edging out Denmark at 3.5%). This is the highest initial safe withdrawal rate among the 20 countries and is considerably higher than the 20-country average, which is 2.30%. UK-based investors have experienced returns that are broadly in-line with other countries. The real equity return of 5.23% ranks the UK 9th of 20, the real bond return of 1.54% ranks 13th, and the 50/50 portfolio real return of 3.72% ranks 11th highest. These relatively average returns result in historical safe initial withdrawal rates that are slightly higher (approximately 0.5% higher, on average) than the international averages across different target probabilities of success but lower than the historical US initial withdrawal rates (approximately 0.5% lower, on average).

Blanchett et al (2016) provide a relatively comprehensive overview of safe withdrawal rates for retirees based on both historical returns and forward-looking returns. Overall, these findings suggest that financial advisers and retirees in the United Kingdom should use lower initial safe withdrawal rates than noted in prior research - the lower end of the range now starts towards 2.5% or 3.0% and not the previous 4.0%. The generous capital market returns of the prior century that bolstered a comfortable and long-lasting retirement portfolio may give 21st-century retirees a false sense of security.

So what happens if we 'smooth' UK equity and bond returns with our simple trend following rule in terms of the retirement experience? Can we do better than the 4% rule? Table 3 shows the 20- and 30-year PWRs available from the 50/50 equity/bond portfolio to compare with Bengen (1994) using Monte Carlo simulations. The empirical data from 1971 – 2005 is resampled with replacement across assets to create 20 and 30-year retirement periods. This method retains the cross-correlation of returns in the various assets. We can see clearly that for both withdrawal periods, the bad left tail experiences are far less likely to occur with the smoothed portfolios. While there is a 1% chance of experiencing less than 2.30% pa over 20-years with the simple portfolio, this rises to 4.46% pa with the smoother strategy. Similar improvement can be seen over the 30-year horizon. It is certainly the case that better withdrawal experiences sometimes occur with the simple portfolios in the right hand tail but these are of far less importance to the decumulation experience.

Whereas all the focus is on the decumulation experience based on equity and bond portfolios, diversification is possibly the most widely accepted concept in portfolio construction. Why have researchers not added other asset classes? One can only assume that the popularity of the 60/40 portfolio in the US along with the paucity of historical data on other asset classes has led to this focus.

In the next section, we examine what happens to withdrawal rates if we add new asset classes and ask whether trend following still has a role in improving the PWA experience.

4. Portfolio diversification, PWRs and popular retirement strategies

In this section, we compare PWRs for different portfolio strategies in the UK.

(a) Data

The multi-asset class data used in this section spans the period from 1970 to 2015 inclusive with all observations being monthly sterling total returns[4]. We use the first year of data for various calculations and so all results are reported from the beginning of 1971. Equity indices throughout are gross values from MSCI; gilts returns are derived from the FTSE Actuaries All Stocks Index from 1976 onwards and 20-year gilts prior to this date; commodity returns are

[4] As they only consider two assets, Blanchett et al (2016) use the ABN Amro/CS/LBS international equity and bond data back to 1900, to look at 4% Safe Withdrawal Rates for the 30-year decumulation period.

proxied by the S&P GSCI index and UK Property returns are proxied by the FTSE Property index for quoted real estate companies until 1990 and the FTSE EPRA UK REIT index, thereafter. Where cash rates refer to 3-month UK Treasury Bills. Throughout the paper, all returns quoted are real and are relative to the UK Retail Price Index. All values are in British Pounds.

(b) Results

The main input into the retirement portfolio choice problem and the consequent PWRs is the investment performance of the chosen portfolio. We first summarise the range of portfolios that we consider. Throughout this section, we compare standard buy-and-hold investing with trend following where risk assets are only held if the particular index is in an uptrend. Specifically, we use the rule advocated by Faber (2007). This rule says that if the index is trading above its 10-month moving average then a long position is taken in that asset class, if not then a cash position is taken.

Table 2 shows summary statistics for returns for the five asset classes both with and without the trend following overlay. We observe that for standard investments, equities have the highest returns while gilts and commodities have the lowest. Gilts have the lowest volatility. While property and commodities both have volatility above 20%. The application of the trend following rule to each asset class leads to slightly higher returns but considerably lower volatility. The real maximum drawdowns shown in Table 2 are particularly large, albeit somewhat lower for trend following. During the 1970s, the United Kingdom experienced a severe bout of inflation with the general price level increasing at double-digit annual rates for much of the decade. Such an environment made it difficult to maintain the purchasing power of portfolios.

Table 4 reports the returns of three different portfolio strategies: a conventional domestic 60-40 portfolio, i.e. 60% in UK equities and 40% in gilts, a more risk averse 30-70 portfolio of the same assets and finally an equally-weighted, multi-asset portfolio of the five instruments shown in Table 2. Each of the portfolios is rebalanced monthly. The bottom panel in the table shows the performance statistics of these strategies with the addition of the same trend following filter used to produce the results in Table 2. For the strategies without the trend following rule the equally-weighted strategy produced a return of around 4.6% annually; the 60-40 portfolio produced a return of 4.5%; and the 30-70 portfolio produced a return of 3.7% pa. The latter also had the lowest annualized volatility at just above 10%. The reward to risk ratios of these three strategies range from 0.35 to 0.39, and we find none of the estimated one factor alphas to be statistically significant at conventional confidence levels. The application of trend following sees returns increase by between around 0.2% to 0.3% annually for the 60-40 and 30-70 portfolios and by nearly 0.8% for the equally-weighted multi-asset version. Volatility also falls markedly, while drawdowns decrease by at least a half. The reward to risk ratios now range from 0.51 to 0.62, and we find all three single factor alphas to be higher and statistically significant at at least the 95% level of confidence.

We alluded above to the unusual economic climate in the United Kingdom during the 1970s. Figure 2 shows the annual returns for the multi asset portfolio both with and without trend following. The years 1973 and 1974 saw large negative returns for the standard portfolio with a big turnaround in 1975. The trend following version sees much smaller losses in the first two years but it also loses money in the subsequent year. We highlight this period because it represents one of the best examples of sequence risk. An investor commencing decumulation at the start of 1973 would have sharply reduced the value of their pension pot without the ability to replenish it. By contrast the period of the 1980s and 1990s was relatively benign and sequence risk played a much smaller role.

To examine the characteristics of each of the portfolios in a retirement context we use the PWR. Whilst this value can only be known with the benefit of hindsight but it provides a good measure for comparing investment strategies. In this paper we assume the decumulation period will be 20 years. One of the issues facing retirees is the uncertainty of longevity. This tail-risk can be insured through the purchase of a deferred annuity (or sometimes called longevity annuity). We assume that one of these is bought at the start of the retirement period and thus the aim is to decumulate the remaining investment pot to zero at the end of 20 years (also assuming no bequests), (see Chen et al, 2016). We therefore examine only the investment of this remaining pot of investable funds.

Figure 3 shows the PWRs for the 6 portfolios over the period of study based on the year that decumulation commenced. Withdrawal rates are low at the beginning due to the poor sequence of returns in the early 1970s highlighted in Figure 2 and then spike higher. After the early 1980s, the PWRs of the portfolios remain in a narrow band of 7% - 10%. The 60-40 portfolio has the highest PWR after the initial blip for much of the period until near the end when the multi-asset class, trend following portfolio achieves higher PWRs. In general, the trend following portfolios show less variation in PWR over time compared to their standard counterparts. At this point, we should note that although the PWRs look very high in 1975, unless a retiree was extremely fortuitous, they would have been invested during the preceding years. Thus, whilst they can take a higher *rate* of withdrawal relative to pot size at the beginning of decumulation compared to someone retiring two years earlier, it seems highly likely that investment pots in absolute terms in 1975 would be much lower than 1973, i.e. a higher rate of a smaller pot or vice versa. The main advantage that a 1975 retiree could potentially have had would have been the option to perhaps defer retirement by a year or two to enable the investment account to recover.

To further assess the decumulation possibilities of the portfolios we next run Monte Carlo simulations. Random draws with replacement using the annual returns of each strategy are made to create new 20-year return strings from which the PWR is individually calculated. By using annual returns of the portfolios, we maintain the integrity of the correlations between asset classes. For each portfolio, we run 20,000 simulations. Figure 4 shows the PWR frequency for the six portfolios. The modes of the distributions are very similar in the 6.5% to 7.5% range, but their shapes are quite different. Each of the standard portfolios has much larger tails than their trend following counterparts. Retirees would be particularly concerned about the 'bad' outcomes in the left tail. Trend following substantially reduces the probability of these for only a small reduction in probability of the 'good' outcomes in the right tail. The smaller amount of dispersion in the trend following distributions also makes it easier to target sustainable withdrawal rates.

Table 5 shows the PWR distributions generated by the strategies presented in Table 4, by percentile, while Figure 5 plots this as a cumulative frequency chart. Comparing the different strategies, we firstly observe that in the lowest percentiles the PWRs are higher for the multi-asset portfolios relative to the 60-40 and 30-70 counterparts, presumably as a result of the greater diversification. Furthermore, the application of trend following reduces the probability of achieving a very low PWR. In the case of 30-70, every percentile has a higher PWR using trend following and for multi asset one has to go to the 90th percentile to find a higher withdrawal rate. The 60-40 portfolio is the only one that has a higher median value without trend following and is the combination that has the possibility, albeit small, of a high PWR.

The takeaway from this analysis is that multi-asset portfolios are preferable to domestic stock-bond portfolios and that trend following substantially reduces the probability of low withdrawal rates without much loss of unusually positive outcomes.

We next consider two alternative portfolios that typically reside at opposite ends of the risk spectrum. Risk parity is a method whereby, in its basic form, assets are weighted according to the inverse of their volatility. In this case we calculate the volatility of each of the five asset classes in Table 2 over the preceding year, take the reciprocal and then weight relatively based on this. Historically, this has led to higher weightings to bonds and smaller weights to equities, commodities, etc. Indeed over the period of study bonds accounts for approximately three-eighths of the total portfolio on average. This would generally be considered to be a less risky proposition than the equal weight multi asset portfolio described earlier. At the other end of the spectrum, we create a portfolio that is comprised 100% of equity. Specifically the portfolio has a 20% allocation to each of the UK, Europe ex-UK, North America, Japan and Pacific ex-Japan. This is well diversified globally but is now concentrated to only stocks. We refer to this the Regional Equity portfolio.

Table 6 reports the summary statistics for the two new portfolios both with and without trend following. The standard risk parity portfolio does indeed have a lower volatility than any of the comparable standard portfolios in Table 4 at just 9.6%, but the return of 4.5% is only slightly lower than the 60-40 and equal-weight multi asset portfolios. By contrast, the regional equity portfolio has an annual return of nearly 5.9% and a volatility in excess of 15%. As before, we note that the application of trend following considerably reduces the volatility of the portfolios without any loss of return; in fact there is a small improvement in returns. We should also note that the trend following versions of these strategies also produce one factor alphas that are statistically significant at the 99% level of confidence.

Once again, we run 20,000 Monte Carlo simulations of each of the four new alternative portfolios and the PWR frequencies are plotted in Figure 6. The difference between the low and high volatility portfolios is readily apparent here, with the latter showing much greater dispersion in PWR as expected. Trend following once again reduces this, though, with Risk Parity TF showing a very substantial peak in the distribution at 7.5% and small tails.

Table 7 shows the PWR for various percentiles of the alternative portfolios and Figure 6 plots these as a cumulative frequency diagram. Once again we observe that trend following reduces the chance of a low PWR experience without losing too much of the potential upside. The probability of a PWR below 5% is small with both of the TF portfolios (5% is the PWR for a portfolio that earns a constant 0% real return of the entire decumulation period). In Figure 7 we observe that the 'crossover' between standard and trend following strategies occurs with a cumulative frequency of around 60%, i.e. approximately 60% of the time the trend following portfolio achieved a higher PWR than its standard equivalent, and, most importantly, this was at lower levels of PWR.

In order to achieve the highest PWRs one has to take additional risk and the regional equity portfolio provides this. Figure 7 shows that the crossover between the two standard portfolios occurs at around the 30th percentile but it is much lower for the trend following varieties at around the 14th percentile. Trend following thus gives greater scope for holding more risky assets. This is clearly directly relevant to the de-risking/glide-path discussion: yes, you should remain in risky assets but to take full advantage of the potential higher returns one should smooth returns. Our evidence for the period to 2015 is, therefore, that PWRs significantly higher than 4% can be achieved at significantly reduced drawdown and sequence risk through smoothing.

5. Conclusions

In this paper we examine a variety of portfolios, both conventional domestic equity and bonds and multi-asset, and look at them in the context of retirement decumulation. Using UK data, we find that multi-asset portfolios for the most part offer an improvement on 60-40 and 30-70 stock-bond investments with smaller probabilities of low safe withdrawal rates. The exception comes for those seeking the very highest PWRs, where there is little choice but to accept a large equity component.

The application of a trend following filter to the assets within each portfolio substantially improves the performance by reducing volatility and maximum drawdown without any loss of return. A result of this is much less variable PWRs, particularly through eliminating many of the lowest PWRs, but without too much reduction in the chance of unusually high outcomes. Trend following enables portfolios to contain more risky assets, and the greater upside they offer, for the same level of overall risk and significantly less maximum drawdown and sequence risk compared to standard portfolios.

One might ask: if smoothing enhances the decumulation experience so effectively, why not simply use derivatives to achieve the same outcome? This choice is resoundingly rejected in a number of recent studies including Strub (2013) and Israelov (2017), and the comments of Illmanen (2016). Options are expensive and if one used a market-timing factor such as movements in the VIX then one would tend to buy protection just when everyone else wanted it - and hence it would be especially expensive.[5] On the other hand switching to cash as part of a trend following strategy, as proposed in this paper, is ‘cheap’ to buy and hold. This is the focus of the next stage of research.

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[5] Chris Dillow, FT Money, p10, 22/4/2017

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Table 1: Example of Sequence Risk

In this table we show the impact on Sequence risk ($1/S_n$) and the Perfect Withdrawal Rate (PWR) of three series of returns which have the same arithmetic mean (Mean), standard deviation (St. Dev.) and maximum drawdown (Max Draw).

Year	Return set 1	Return set 2	Return set 3
1	10%	-10%	0%
2	5%	-5%	5%
3	0%	0%	-5%
4	-5%	5%	-10%
5	-10%	10%	10%
Mean	0.00%	0.00%	0.00%
St. Dev.	7.91%	7.91%	7.91%
Max Draw	-14.5%	-14.5%	-14.5%
$1/S_n$	4.50	5.49	5.01
PWR	21.97	17.97	19.73

Table 2: Asset Class Summary Statistics: 1971-2015

This table presents summary statistics for five broad asset classes. All statistics were generated using real monthly returns. The statistics in the upper panel are based upon unfiltered returns; the statistics in the lower panel have been generated with the application of the trend following filter. *Reward to risk* is the annualised real return, divided by the annualised real volatility. *Alpha* is estimated using a one factor model, where the factor is the return on Global Equities presented in column three of the table. Alpha values in italics, bold and bold underlined indicate that the coefficient was significant at the 90%, 95% and 99% level of confidence respectively.

	UK Equity	Global Equity	Gilts	Commodities	Property
Standard					
Annualized Real Return (%)	5.08	4.67	2.60	1.82	3.39
Annualized Real Volatility (%)	19.05	15.21	10.12	20.75	25.40
Maximum Real Drawdown (%)	73.85	51.72	59.43	77.03	80.55
Skew	0.98	-0.55	1.17	0.26	-0.07
Reward to risk	0.27	0.31	0.26	0.09	0.13
Alpha	0.17	-	*0.23*	0.19	0.25
Trend Following					
Annualized Real Return (%)	5.69	5.57	2.85	3.55	5.79
Annualized Real Volatility (%)	13.98	11.89	8.75	17.34	17.77
Maximum Real Drawdown (%)	29.10	44.01	42.87	56.82	70.81
Skew	-0.31	-0.79	2.02	0.54	-0.60
Reward to risk	0.41	0.47	0.33	0.20	0.33
Alpha	**0.32**	**0.22**	**0.26**	0.33	**0.45**

Table 3: Distribution of 50-50 Portfolio PWRs from 20,000 Monte Carlo Simulations

This table presents a summary of the distribution of the PWRs for a 50-50% equity and bond portfolio over two time frames, 20 years and 30 years, 50-50. We present an analogous summary of the distributions where Trend following has been applied, 50-50TF. We also present the skew of each distribution at the bottom of the table.

	PWR Time Frame (Years)			
	20	20	30	30
Percentile	50-50	50-50TF	50-50	50-50TF
1	2.30	4.46	1.49	3.25
5	3.48	5.16	2.45	3.89
10	4.23	5.60	3.05	4.27
25	5.67	6.37	4.25	4.97
50	7.31	7.33	5.73	5.86
75	8.91	8.39	7.37	6.87
90	10.53	9.46	8.90	7.87
95	11.57	10.12	9.91	8.50
99	13.74	11.40	11.87	9.77
Skew	0.32	0.45	0.47	0.50

Table 4: Portfolio Summary Statistics: 1971-2015

This table presents summary statistics for three investment strategies: a conventional domestic 60-40 portfolio, i.e. 60%/40% UK equities and gilts; a more risk averse 30%/70% portfolio of the same assets; and finally an equally-weighted, multi-asset portfolio of the five asset classes presented in Table 2. All statistics were generated using real monthly returns. *Reward to risk* is the annualised real return, divided by the annualised real volatility. *Alpha* is estimated using a one factor model, where the risk factor is the return on Global Equities presented in column three of the Table 2. Alpha values in italics, bold and bold underlined indicate that alpha was significant at the 90%, 95% and 99% level of confidence respectively.

	60% UK Equity - 40% Gilts	30% UK Equity - 70% Gilts	Equal Weight Multi Asset
Standard			
Annualized Real Return (%)	4.54	3.73	4.62
Annualized Real Volatility (%)	13.04	10.12	11.78
Maximum Real Drawdown (%)	67.21	62.66	45.17
Skew	1.32	1.41	-0.36
Reward to risk	0.35	0.37	0.39
Alpha	0.19	0.21	0.17
Trend Following			
Annualized Real Return (%)	4.86	3.96	5.38
Annualized Real Volatility (%)	9.45	7.81	8.62
Maximum Real Drawdown (%)	25.18	29.19	21.20
Skew	0.00	1.20	-0.52
Reward to risk	0.51	0.51	0.62
Alpha	**0.29**	<u>**0.28**</u>	<u>**0.31**</u>

Table 5: Distribution of Portfolio PWRs from 20,000 Monte Carlo Simulations

This table presents a summary of the distribution of the PWRs for three different portfolio strategies presented in Table 4: a conventional domestic 60-40 portfolio (60-40), a more risk averse 30-70 portfolio of the same assets (30-70) and finally an equally-weighted, multi-asset portfolio of the five instruments shown in Table 2 (Multi). It also shows comparable distributions of PWRs for these three strategies with a trend following filter applied: 60-40 TF, 30-70 TF and Multi TF. We also present the skew of each distribution at the bottom of the table.

Percentile	60-40	30-70	Multi	60-40 TF	30-70 TF	Multi TF
1	2.18	2.67	4.46	4.41	3.09	5.21
5	3.39	3.73	5.20	5.04	4.15	5.84
10	4.15	4.38	5.66	5.42	4.82	6.22
25	5.68	5.59	6.46	6.13	6.00	6.93
50	7.44	7.01	7.49	7.01	7.45	7.80
75	9.23	8.41	8.64	7.97	8.95	8.77
90	10.94	9.76	9.78	8.96	10.37	9.73
95	12.05	10.55	10.53	9.59	11.22	10.32
99	14.61	12.33	11.91	10.87	12.76	11.48
Skew	0.41	0.23	0.25	0.48	0.51	0.45

Table 6: Alternative Portfolio Summary Statistics: 1971-2015

This table presents summary statistics for two investment strategies. *Multi Asset Risk Parity* is created by calculating the volatility of each of the five asset classes in Table 2 over the preceding year, the reciprocal of this value is then used to calculate the weight of each asset class. *Regional Equity* is a portfolio that is comprised of a 20% allocation to each of the UK, Europe ex-UK, North America, Japanese and Pacific ex-Japan equity markets (represented by appropriate MSCI indices). All statistics were generated using real monthly returns. *Reward to risk* is the annualised real return, divided by the annualised real volatility. *Alpha* is estimated using a one factor model, where the risk factor is the return on Global Equities presented in column three of the Table 2. Alpha values in italics, bold and bold underlined indicate that alpha was significant at the 90%, 95% and 99% level of confidence respectively.

	Multi Asset Risk Parity	Regional Equity
Standard		
Annualized Real Return (%)	4.46	5.86
Annualized Real Volatility (%)	9.64	15.32
Maximum Real Drawdown (%)	48.26	55.42
Skew	-0.11	-0.69
Reward to risk	0.46	0.38
Alpha	**0.19**	**0.12**
Trend Following		
Annualized Real Return (%)	4.93	6.41
Annualized Real Volatility (%)	7.22	10.86
Maximum Real Drawdown (%)	21.59	26.96
Skew	-0.49	-1.41
Reward to risk	0.68	0.59
Alpha	<u>**0.29**</u>	<u>**0.31**</u>

Table 7: Distribution of Alternative Portfolio PWRs from 20,000 Monte Carlo Simulations

This table presents a summary of the distribution of the PWRs generated by the strategies presented in Table 6. Multi Asset Risk Parity is created by calculating the volatility of each of the five asset classes in Table 2 over the preceding year, the reciprocal of this value is then used to calculate the weight of each asset class. Regional Equity is a portfolio that is comprised of a 20% allocation to each of the UK, Europe ex-UK, North America, Japanese and Pacific ex-Japan indices. We also present the skew of each distribution at the bottom of the table.

Percentile	Risk Parity	Regional Equity	Risk Parity TF	Regional Equity TF
1	3.23	2.49	5.19	4.50
5	4.29	3.65	5.80	5.39
10	4.95	4.43	6.15	5.96
25	6.11	5.96	6.79	6.97
50	7.39	8.02	7.55	8.31
75	8.67	10.38	8.40	9.82
90	9.83	12.67	9.20	11.39
95	10.51	14.05	9.70	12.41
99	12.04	16.53	10.73	14.34
Skew	0.11	0.48	0.37	0.61

Figure 1

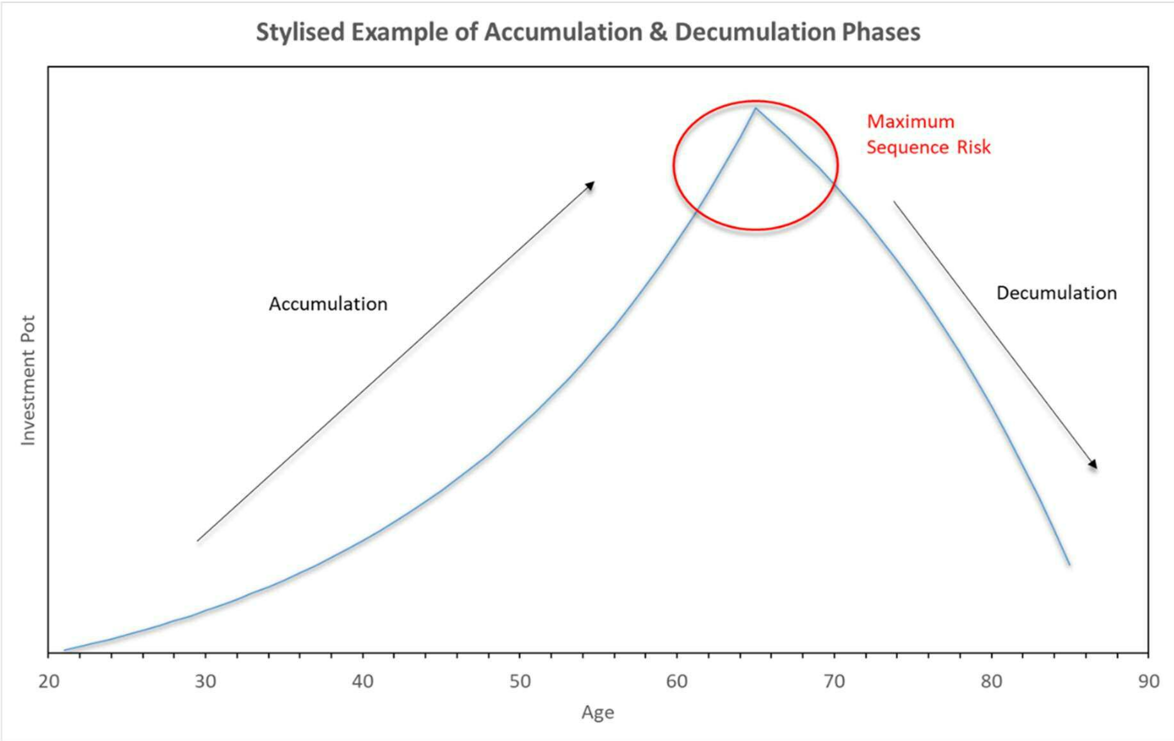


Figure 2

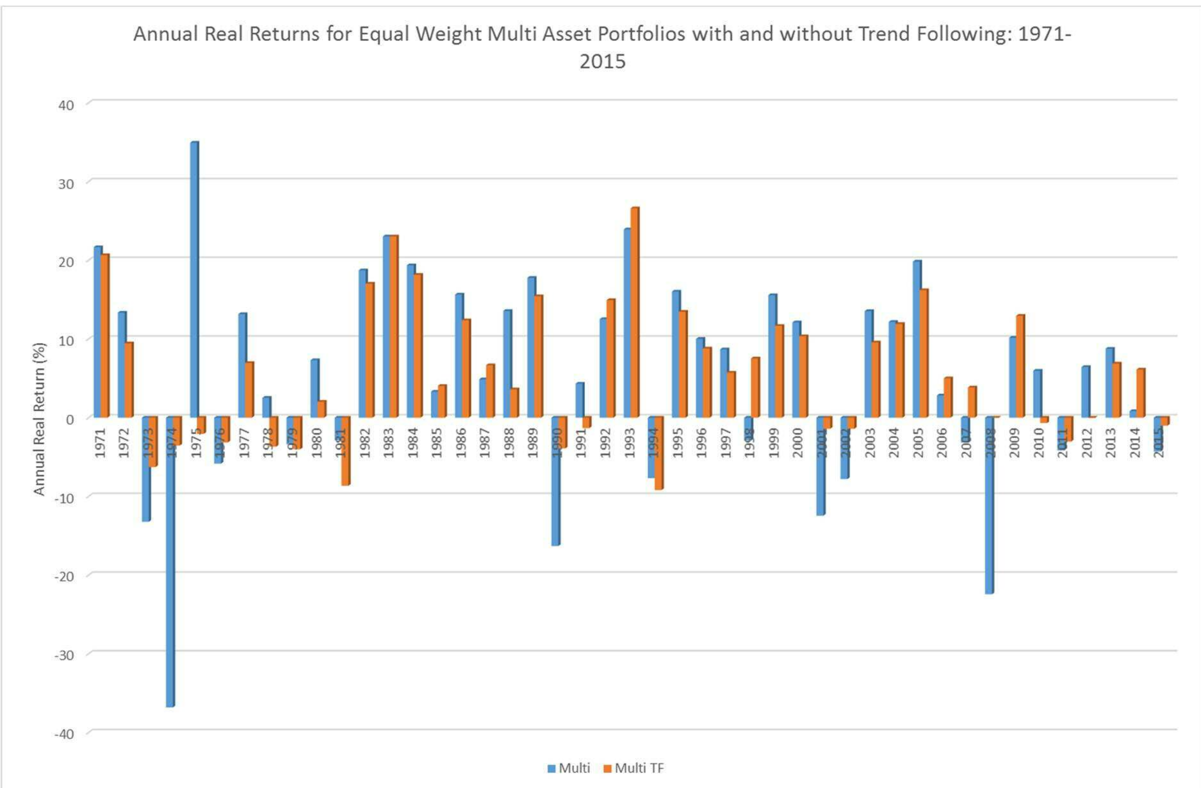


Figure 3

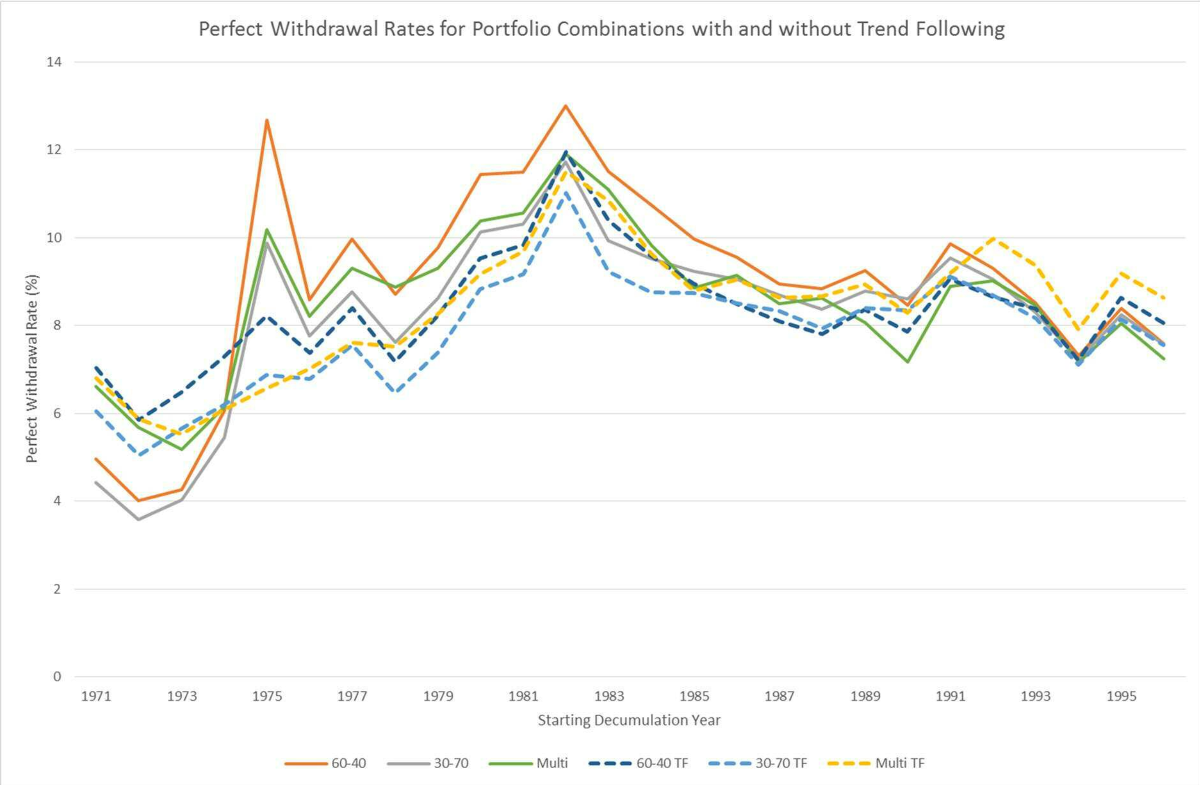


Figure 4

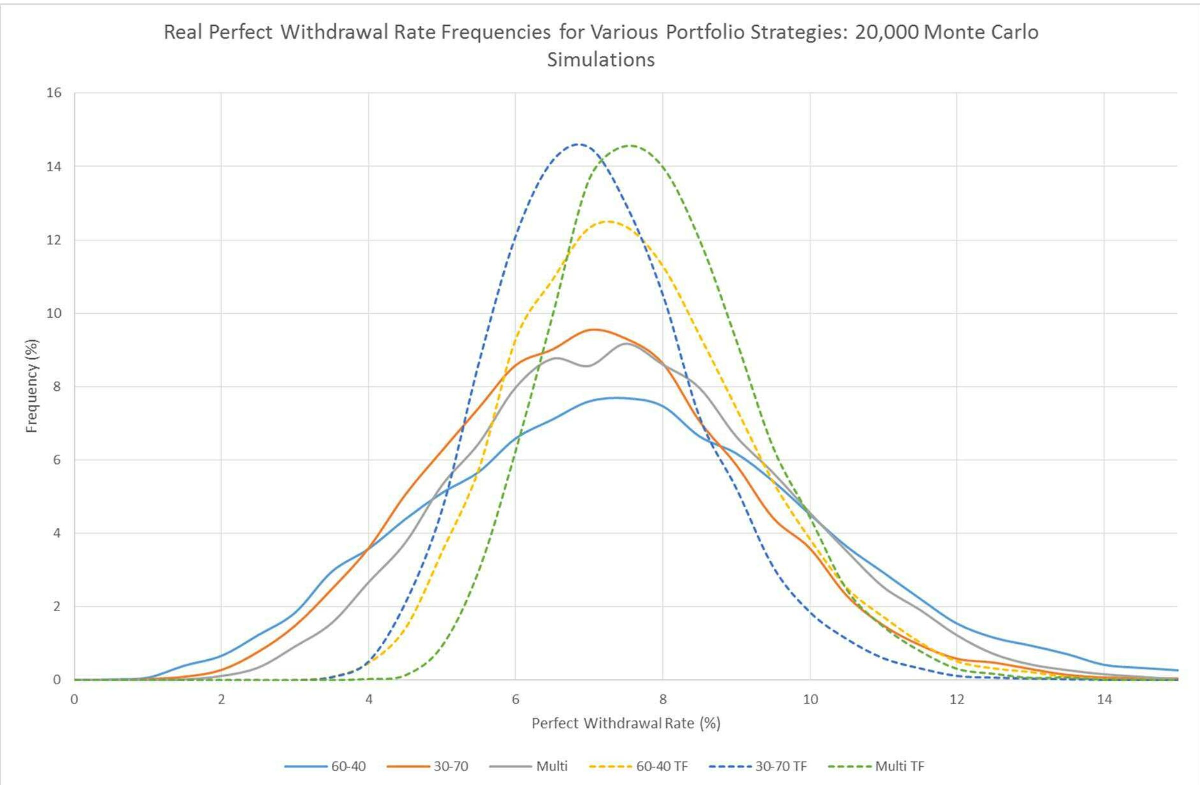


Figure 5

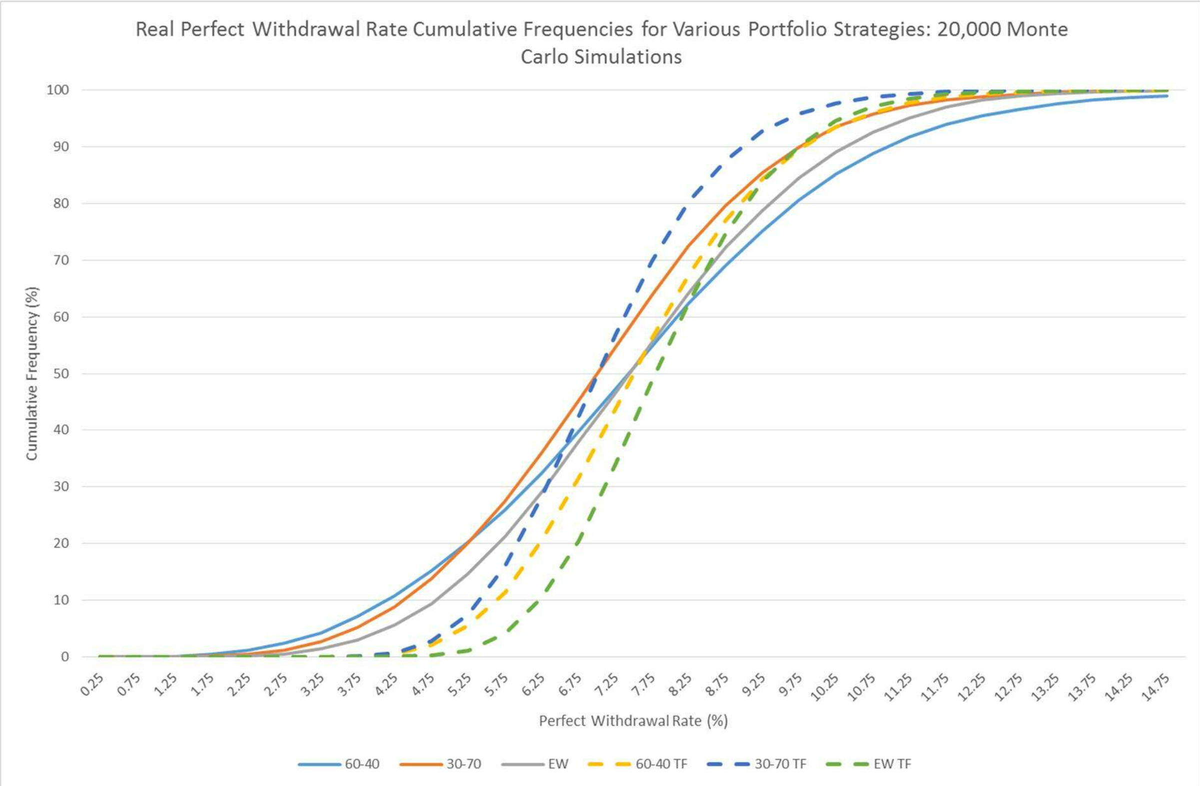


Figure 6

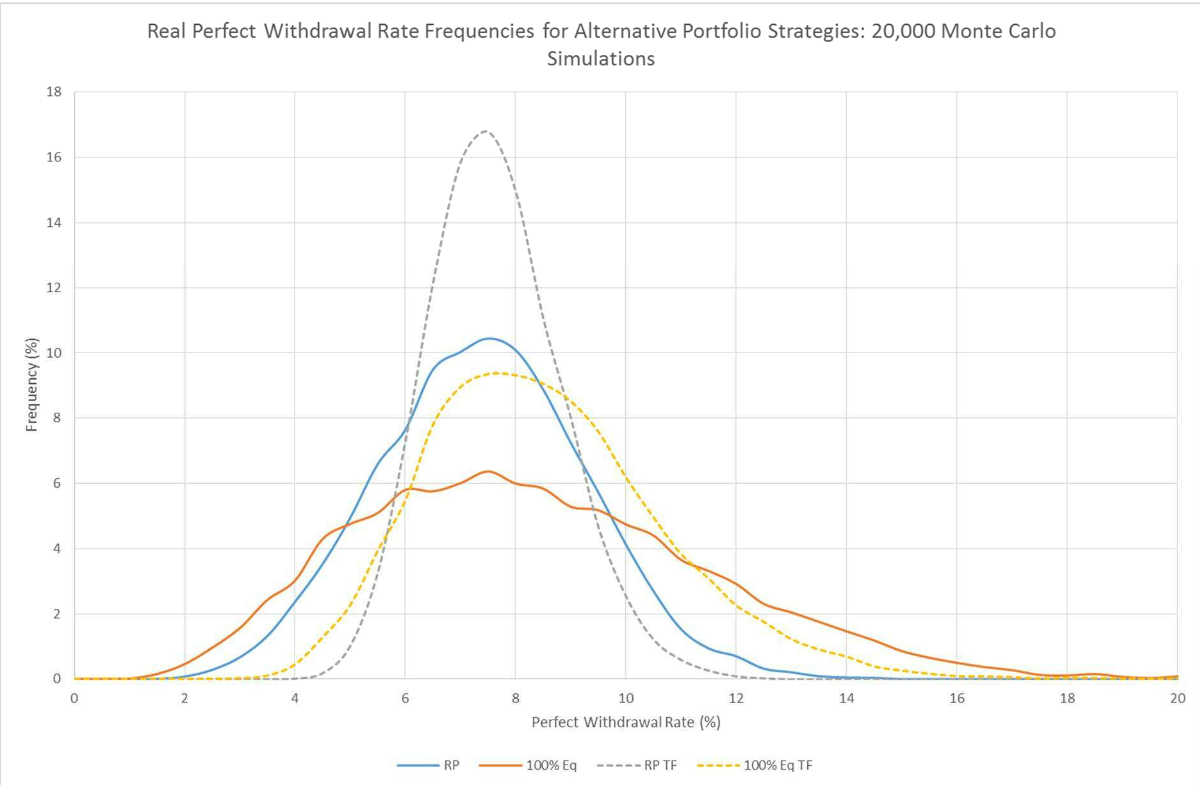
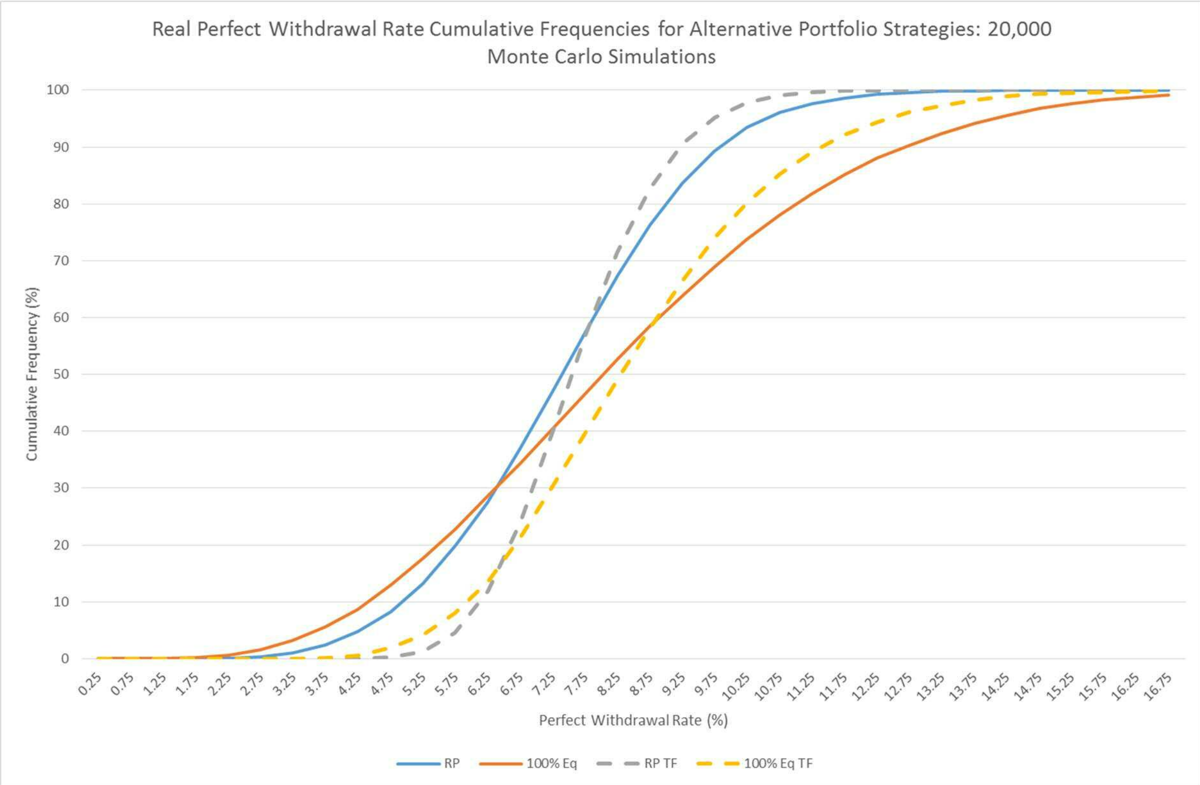


Figure 7



Real Perfect Withdrawal Rate Cumulative Frequencies for Alternative Portfolio Strategies: 20,000 Monte Carlo Simulations
Cumulative Frequency (%)
100
90
80
70
60
50
40
30
20
10
0
0.25
0.75
1.25
1.75
2.25
2.75
3.25
3.75
4.25
4.75
5.25
5.75
6.25
6.75
7.25
7.75
8.25
8.75
9.25
9.75
10.25
10.75
11.25
11.75
12.25
12.75
13.25
13.75
14.25
14.75
15.25
15.75
16.25
16.75
Perfect Withdrawal Rate (%)
RP
100% Eq
RP TF
100% Eq TF