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RESEARCH ARTICLE

Optimal insurance under adverse selection and ambiguity aversion

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Abstract We consider a model of competitive insurance markets under asymmetric information with ambiguity-averse agents who maximize their maxmin expected utility. The interaction between asymmetric information and ambiguity aversion gives rise to some interesting results. First, for some parameter values, there exists a unique pooling equilibrium where both types of insurees buy full insurance. Second, in separating equilibria where the low risks are underinsured, their equilibrium contract involves more coverage than under standard expected utility. Finally, due to the endogeneity of commitment to the menus offered by insurers, our model has always an equilibrium which is unique (in terms of allocation) and interim incentive efficient (second-best).

Keywords Adverse selection · Ambiguity aversion · Endogenous commitment

JEL Classification D82 · G22

1 Introduction

Most theoretical models of competitive insurance markets under asymmetric information assume that individuals' preferences admit the standard von Neumann–Morgenstern expected utility representation. The classical model rules out the situation where insurees are uncertain about the likelihood of a state of the world occurring and

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cannot assess precisely their own probability of events. This assumption might be too restrictive in reality. Individuals, contrary to insurance companies, may not have perfect confidence on the perceived probability measure simply due to the lack of experience or data at their disposal. With imprecise information, individuals may consider several probability measures without knowing which of these measures is the correct one. This lack of knowledge about the underlying probabilities is referred to as Knightian uncertainty or ambiguity and was defined by Knight (1921).[1] A decision criterion which is compatible with this pattern of preferences is the maxmin (or multiple-prior) expected utility model which has been given axiomatic foundations by Gilboa and Schmeidler (1989).[2] Under this model, an individual has a set of probability beliefs (priors) instead of a single one, and evaluates an action according to the minimum expected utility over this set of priors. Such a behavior is often referred to as ambiguity aversion, for it indicates the dislike of uncertainty associated with unknown or ambiguous odds.

This paper considers a model with two key features: First, it introduces ambiguity (in the form of Gilboa–Schmeidler preferences), in a competitive insurance model with asymmetric information. Second, it employs an optimal mechanism to study the effects of the interaction between asymmetric information and aversion to ambiguity on the equilibrium allocations.[3] The introduction of ambiguity aversion into an asymmetric information framework allows us to derive some interesting results which cannot obtain in the expected utility setting.

First, for some parameter values, there exists a unique pooling equilibrium where both types of insurees buy full insurance. If the degree of ambiguity aversion of low-risk insurees is high, the utility cost of under- or overinsurance becomes excessively high and ensures the existence of this pooling equilibrium. Under the expected utility preferences, in a neighborhood of full insurance, this utility cost is not sufficiently high to support this pooling allocation as an equilibrium. Furthermore, the existence of the full-insurance pooling equilibrium we establish here is not driven by the indemnity (no-overinsurance) principle.

Second, under ambiguity aversion, the equilibrium contract of the low risks is closer to the first-best one than under the standard expected utility. In fact, ambiguity aversion relaxes the (binding) incentive compatibility constraint of high risks. As a result, the low risks buy more insurance (while still revealing their type) and move closer to the first-best allocation.

Finally, because the mechanism we employ in this paper is optimal, the results discussed above are driven by ambiguity aversion and not by the sub-optimality of the mechanism used for the allocation of resources or their interaction. To the best of our knowledge, none of the existing papers in the ambiguity aversion literature

[1] There is a large body of experimental literature documenting ambiguity-averse preferences among individuals. See Etner et al. (2012) and Gilboa and Marinacci (2013) for two recent comprehensive surveys.

[2] Maccheroni et al. (2006) have generalized maxmin preferences to variational preferences. Also, Siniscalchi (2006) has provided a behavioral foundation for such preferences.

[3] We have also examined the case of ambiguity-seeking agents. The main difference between the ambiguity aversion and the ambiguity-seeking cases is that in the latter case, in any separating equilibrium, no insuree (regardless of his risk type) buys full insurance. These results are available upon request.

is concerned with the optimality of the mechanism used. As a result, it is not clear which of the results are only driven by ambiguity aversion and which by the (possible) sub-optimality of the mechanism.

2 Related literature

There is a growing body of literature studying the effects of ambiguity in financial markets. Ambiguity is known to lead to limited participation and reduced liquidity in the market, adverse effects on risk sharing, uncertainty premium in equilibrium prices of financial assets, market inefficiency (see Bossaerts et al. 2010; Epstein and Schneider 2010 for recent surveys on effects of ambiguity in financial markets).

Although the effects of ambiguity on its own have been studied extensively, the effects of the interaction between ambiguity aversion and asymmetric information are relatively unexplored. Tallon (1998) and Condie and Ganguli (2011a, b) illustrate how ambiguity aversion helps to resolve the Grossman–Stiglitz paradox and demonstrates that an agent facing ambiguity might be willing to pay to acquire information which is already contained in the equilibrium price. Kajii and Ui (2009) and Martins-da-Rocha (2010) characterize weakly interim efficient allocations under ambiguity using the notion of compatible priors.

Jeleva and Villeneuve (2004) characterize optimal insurance contracts with imprecise probabilities (rank-dependent utility) and adverse selection. They have also obtained a pooling equilibrium for some parameter values. However, there are three important differences between their paper and ours: First, they consider a monopolistic insurer instead of competition. Second, their pooling equilibrium involves partial insurance, whereas in our case the pooling involves full insurance. Third, in Jeleva and Villeneuve, the pooling equilibrium is, in general, inefficient, whereas in our paper it is always (first-best) efficient.

The two papers more closely related to ours are De Castro and Yannelis (2012) and Koufopoulos and Kozhan (2014). Both of them also consider an economy with ex ante asymmetric information (adverse selection) and ambiguity-averse agents. The aim of the De Castro and Yannelis (2012) paper is to determine the restrictions on preferences such that efficient (first-best) allocations are always incentive compatible. Koufopoulos and Kozhan (2014) focus on two interesting equilibria which do not involve cross-subsidization across contracts. In contrast, the objective of this paper is to characterize the whole set of interim efficient (second-best) equilibria of the game under maxmin preferences.

3 The model

We consider the basic framework introduced by Rothschild and Stiglitz (1976). There is a continuum of individuals (insurees) and a single consumption good. All individuals have the same twice continuously differentiable von Neumann–Morgenstern utility function $U : \mathbb{R} \to \mathbb{R}$ with $U'(x) > 0$ and $U'' < 0$ for all $x \in \mathbb{R}$ and the same wealth level, W. We denote by $\Phi(x)$ the inverse of $U(x)(\Phi(x) \equiv U^{-1}(x))$. There are two possible states of nature: good and bad. In the good state, there is no loss, whereas in the bad state the individual suffers a gross loss of $d \in (0, W)$. Individuals differ with respect to the probability of having the bad state (accident), p. There are two types of individuals: high risks (Hs) and low risks (Ls) with $1 > p_{\mathrm{H}} > p_{\mathrm{L}} > 0$. Let $\lambda \in (0, 1)$ be the fraction of the Ls in the economy.

Each individual may insure himself against the accident by accepting an insurance contract $A = (\alpha_1, \alpha_2)$, where $\alpha_1 \geq 0$ is the insurance premium, and $\alpha_2 \geq 0$ is the coverage (gross payout in the event of loss). We can represent the insurance contract equivalently as $A = (w_{\mathrm{G}}, w_{\mathrm{B}})$, where $w_{\mathrm{B}} = W - d - \alpha_1 + \alpha_2$ is the wealth of the agent in the bad state, $w_{\mathrm{G}} = W - \alpha_1$ is the agent's wealth in the good state. Let $C = \{(w_{\mathrm{G}}, w_{\mathrm{B}}) \in \mathbb{R}^2_+\}$, $C^{\mathrm{U}} = \{(w_{\mathrm{G}}, w_{\mathrm{B}}) \in C | w_{\mathrm{G}} > w_{\mathrm{B}}\}$ and $C^{\mathrm{O}} = \{(w_{\mathrm{G}}, w_{\mathrm{B}}) \in C | w_{\mathrm{G}} < w_{\mathrm{B}}\}$ be the sets of all contracts, underinsurance and overinsurance regions, respectively. Also, denote, $\overline{C}^{\mathrm{U}} = \{(w_{\mathrm{G}}, w_{\mathrm{G}}) \in C | w_{\mathrm{G}} \geq w_{\mathrm{B}}\}$ and $\overline{C}^{\mathrm{O}} = \{(w_{\mathrm{G}}, w_{\mathrm{B}}) \in C | w_{\mathrm{G}} \leq w_{\mathrm{B}}\}$. In this environment, if an agent i knows precisely his own accident probability p_i, then his expected utility is given by:

$$\mathrm{EU}(W, d, (w_{\mathrm{G}}, w_{\mathrm{B}}), p_i) = p_i U(w_{\mathrm{B}}) + (1 - p_i) U(w_{\mathrm{G}}), \quad i = H, L.$$

In this paper, we extend the above setting by introducing aversion to ambiguity. In particular, we assume that individuals do not know precisely the distribution of accident probabilities. Their beliefs consist of a set of priors about the true probability of accident p_i an individual of type i and this set is described by an interval $[\underline{p}_i, \overline{p}_i]$. If this interval shrinks to a singleton, the set of beliefs of the individual is reduced to a single probability of accident. We also assume that the true probability $p_i \in [\underline{p}_i, \overline{p}_i]$, $i = H, L$.[4]

More specifically, insurees' preferences admit the maxmin expected utility representation of Gilboa and Schmeidler (1989):

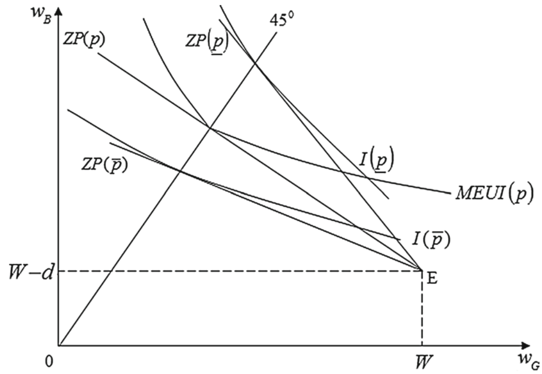
$$\mathrm{MEU}(W, d, (w_{\mathrm{G}}, w_{\mathrm{B}}), p_i) = \min_{p_i \in [\underline{p}_i, \overline{p}_i]} EU(W, d, (w_{\mathrm{G}}, w_{\mathrm{B}}), p_i), \quad i = H, L.$$

For each contract $A = (w_{\mathrm{G}}, w_{\mathrm{B}})$, an individual computes the worst outcome with respect to the accident probabilities and then maximizes the worst-case utility with respect to $(w_{\mathrm{G}}, w_{\mathrm{B}})$. The shape of this function is given in Fig. 1, where the axes represent wealth in the bad (B) and the good (G) states, respectively, and E is the endowment point.

The indifference curve $\mathrm{MEUI}(p)$ of the maxmin expected utility consists of two parts. In the underinsurance region C^{U}, it coincides with the standard expected utility indifference curve $I(\overline{p})$ based on the accident probability $\overline{p}$ (insurees act as if their true accident probability is the highest possible). In the overinsurance region C^{O},

[4] We make this assumption to distinguish the effects of ambiguity from over-optimism or over-pessimism. However, this assumption rules out the following equilibria which are driven by over-optimism: (i) separating equilibria where no incentive compatibility constraint is binding (the equilibrium allocation coincides with that under full information); (ii) equilibria where none of the two types buys full insurance; (iii) separating equilibria exhibiting negative correlation between coverage and the accident probability (see Koufopoulos 2011).

Fig. 1 Maxmin expected utility



it coincides with the indifference curve $I(\underline{p})$ (insurees act as if their true accident probability is the lowest possible).

The slope of the indifference curve (the marginal rate of substitution between income in the no-accident state and income in the accident state) in the expected utility case is $[U'(W_1)(1-p)]/[U'(W_2)p]$, which is equal to $(1-p)/p$ when income in the two states is the same. The slope of $I(\underline{p})$ on the 45° line is $(1-\underline{p})/\underline{p}$, while the slope of $I(\overline{p})$ is $(1-\overline{p})/\overline{p}$. Therefore, MEUI$(p)$ has a kink on the 45° line. In Fig. 1, ZP$(\overline{p})$ and ZP$(\underline{p})$ denote the zero-profit lines corresponding to the $\overline{p}$ and $\underline{p}$ probabilities, respectively.

There are (at least) two risk-neutral insurance companies involved in Bertrand competition. Insurers cannot observe the type of insurees, but they know the proportion of the Hs and Ls in the population. They also know the utility function of insurees and the probability interval for each type. We assume that insurers are ambiguity neutral.[5] They use reference accident probabilities, one for each type, which coincide with true probabilities p_i.[6] This assumption is justified by insurers' capacity to collect large data sets and estimate true probabilities.

The insurance contract $A = (w_G, w_B)$ specifies the wealth in the good w_G and the bad w_B states. As a result, the (expected) profit of an insurer offering contract $A = (w_G, w_B)$, conditional on insurees' type, is

$$\pi_i = W - p_i d - [(1-p_i)w_G - p_i w_B], \quad i = H, L.$$

Insurers offer menus of contracts. A menu $\{(A_L, A_H), c\}$ consists of a pair of contracts (which could be the same contract) and the binary variable $c \in \{0, 1\}$. A

[5] We have also analyzed the case insurers are ambiguity averse, and most of the results are qualitatively similar except two main differences. First, ambiguity-averse insurers charge a higher per-unit price which reflects the ambiguity premium. Second, if the insurers' degree of ambiguity is sufficiently high, the insurees are not willing to pay the high ambiguity premium the insurers charge and the insurance market collapses (no trade).

[6] Our results would be qualitatively similar if the reference probabilities are different from the true ones. However, if the reference accident probabilities are lower than the true ones, the insurance companies should have some initial capital to fulfill their promises (cover their losses).

single-contract menu $\{(A, A), c\}$ is also denoted by $\{(A), c\}$. The variable c determines whether the insurer is committed to the menu or not ($c = 1$ means commitment and $c = 0$ means no commitment).[7] That is, commitment is related to the menu and not to individual contracts within the menu.[8]

4 Efficiency

Following Holmström and Myerson (1983), the allocation $(A_{\mathrm{H}}, A_{\mathrm{L}})$ with $A_{\mathrm{H}} = (w_{\mathrm{HG}}, w_{\mathrm{HB}})$ and $A_{\mathrm{L}} = (w_{\mathrm{LG}}, w_{\mathrm{LB}})$ is *interim incentive efficient* (second-best) if it solves the problem

$$\max_{(A_{\mathrm{L}}, A_{\mathrm{H}}) \in \mathbb{R}_+ \times \mathbb{R}_+} [\xi \mathrm{MEU}(W, d, A_{\mathrm{L}}, L) + (1 - \xi)\mathrm{MEU}(W, d, A_{\mathrm{H}}, H)] \tag{1}$$

for some $\xi \geq 0$ subject to the two incentive compatibility constraints

$$\begin{aligned} &\min_{p \in [\underline{p}_{\mathrm{L}}, \overline{p}_{\mathrm{L}}]} \{(1 - p)U(w_{\mathrm{LG}}) + pU(w_{\mathrm{LB}})\} \\ &\quad \geq \min_{p \in [\underline{p}_{\mathrm{L}}, \overline{p}_{\mathrm{L}}]} \{(1 - p)U(w_{\mathrm{HG}}) + pU(w_{\mathrm{HB}})\} \end{aligned} \tag{2}$$

$$\begin{aligned} &\min_{p \in [\underline{p}_{\mathrm{H}}, \overline{p}_{\mathrm{H}}]} \{(1 - p)U(w_{\mathrm{HG}}) + pU(w_{\mathrm{HB}})\} \\ &\quad \geq \min_{p \in [\underline{p}_{\mathrm{H}}, \overline{p}_{\mathrm{H}}]} \{(1 - p)U(w_{\mathrm{LG}}) + pU(w_{\mathrm{LB}})\} \end{aligned} \tag{3}$$

and the feasibility constraint

$$\lambda[(1 - p_{\mathrm{L}})w_{\mathrm{LG}} + p_{\mathrm{L}}w_{\mathrm{LB}}] + (1 - \lambda)[(1 - p_{\mathrm{H}})w_{\mathrm{HG}} + p_{\mathrm{H}}w_{\mathrm{HB}}] \leq R_\lambda, \tag{4}$$

where $R_\lambda = W - p_\lambda d$ and $p_\lambda = \lambda p_{\mathrm{L}} + (1 - \lambda) p_{\mathrm{H}}$.

5 Equilibrium

Game structure Insurance companies and insurees play the following three-stage screening game[9]:

Stage 1 At least, two insurance companies simultaneously make offers of menus of contracts.

[7] This assumption is made for simplicity and does not imply any loss of generality. Because there are only two types of insurees, all the results go through even if we allow menus to contain any finite number of contracts.

[8] All the results go through if we allow insurers to commit only to one of the two contracts in a two-contract menu.

[9] This game has also been used in Koufopoulos (2010) and Diasakos and Koufopoulos (2013) in the standard expected utility framework.

Stage 2 Insurees apply for (at most) one of the menus offered from one insurance company. If an insuree's most preferred menu is offered by more than one insurance company, he takes each insurer's menu with equal probability.

Stage 3 After observing the menus offered by their rivals and those chosen by the insurees, the insurers decide whether to withdraw or not the menus which they did not commit to at Stage 1. If a menu is withdrawn, the insurees who have chosen it go to their endowment.

We should stress here that endogeneity of commitment refers to commitment within the game and until an equilibrium has been achieved. Once an equilibrium has been reached and the two parties have signed the contract, they are fully committed to it. That is, there is no enforcement problem.

Definition of equilibrium We only consider pure-strategy Bayes–Nash equilibria. A set of menus is an equilibrium if the following conditions are satisfied:

(i) Insurees maximize their maxmin expected utility given the menus offered.
(ii) No menu in this set makes negative expected profits.
(iii) No other set of menus introduced alongside those already in the market would increase an insurer's expected profits.

We begin by examining how different assumptions about the degree of ambiguity of the two types of insurees affect the relative slopes and shapes of their indifference curves. This is important because the relative slopes and shapes of the indifference curves determine the nature of the equilibrium (pooling or separating) and whether the separating equilibria involve under- or overinsurance. There are four cases to consider which are:

Case (1) $\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}} < \overline{p}_{\mathrm{H}} < \overline{p}_{\mathrm{L}}$,
Case (2) $\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$ and $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$,
Case (3) $\overline{p}_{\mathrm{L}} > \overline{p}_{\mathrm{H}} > \underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$,
Case (4) $\overline{p}_{\mathrm{H}} > \overline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$.

In Cases (1) and (4), the indifference curves of the two types intersect twice (the single-crossing condition fails), whereas in Cases (2) and (3) the indifference curves cross only once (the single-crossing condition is satisfied). Figure 2 below illustrates these cases.

We now derive some general results which will be useful for establishing and characterizing the equilibria of our game. We first show that any equilibrium of our game must be interim incentive efficient. We then show that in any equilibrium allocation of our game, at least one of the two types will choose full insurance.

In all the figures below, $\mathrm{ZP_H}$ and $\mathrm{ZP_L}$ denote zero-profit lines of the Hs and Ls, respectively, and PZP denotes the pooling zero-profit line.

Lemma 1 *Any equilibrium allocation of our game must be interim incentive efficient.*

Proof Suppose that at Stage 1 of the game a firm offers a (pooling or separating) allocation (with or without commitment) which is not interim incentive efficient. This allocation is below the (second-best) Pareto frontier. Thus, there exist incentive compatible and feasible allocations which make both types better-off and imply strictly

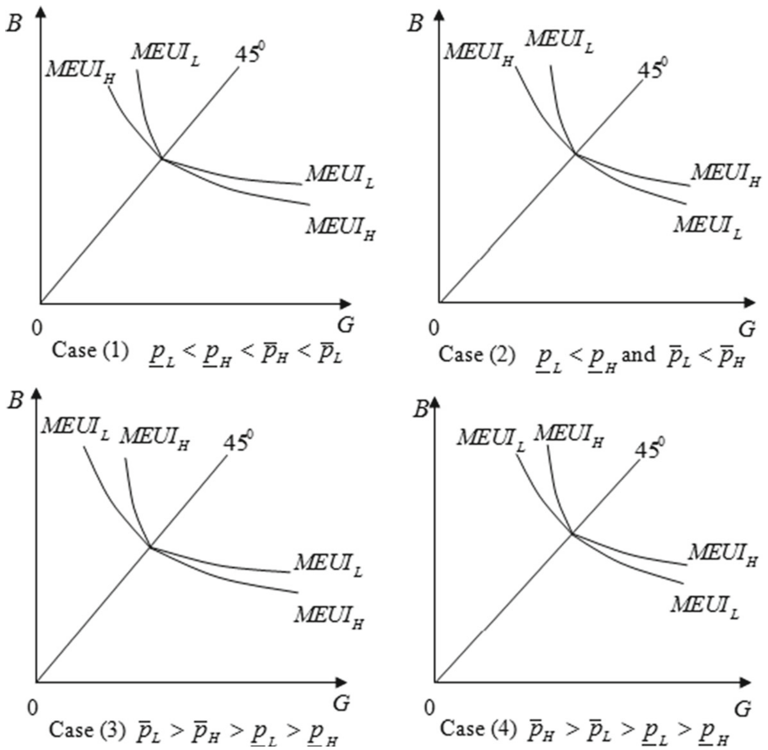


Fig. 2 Four cases of relation between relative slopes of indifference curves

positive profits for the firm(s) which offer them. As a result, at Stage 1, a new entrant can offer an allocation with commitment which is incentive compatible, profitable, and is preferred by both types of insurees to the incumbent's offer. Since the new entrant is committed to his offer, both types will take it regardless of their beliefs about the choice of the other type. Hence, the new entrant's offer will attract both types, and the incumbent's (inefficient) offer cannot be an equilibrium. □

Lemma 2 *In any equilibrium allocation of our game, at least, one of the two types of insurees buys full insurance.*

Proof Suppose that an insurer offers the menu $\{(A_\mathrm{H}, A_\mathrm{L}), c\}$, $c \in \{0, 1\}$, involving an incentive compatible and zero-profit allocation $(A_\mathrm{H}, A_\mathrm{L})$ where, for example, both types of insurees are underinsured (see Fig. 3).[10] Then, a new entrant can offer the menu $\{(A_\mathrm{H}^\mathrm{D}, A_\mathrm{L}^\mathrm{D}), 1\}$ involving another incentive compatible allocation $(A_\mathrm{H}^\mathrm{D}, A_\mathrm{L}^\mathrm{D})$ (see Fig. 3) which makes both types strictly better-off and strictly positive profits for the deviant insurer. Since the new entrant is committed to the deviant menu, both types will take it regardless of their beliefs about the choice of the other type. As a result,

[10] A similar argument applies if one type chooses underinsurance and the other overinsurance or both types choose overinsurance.

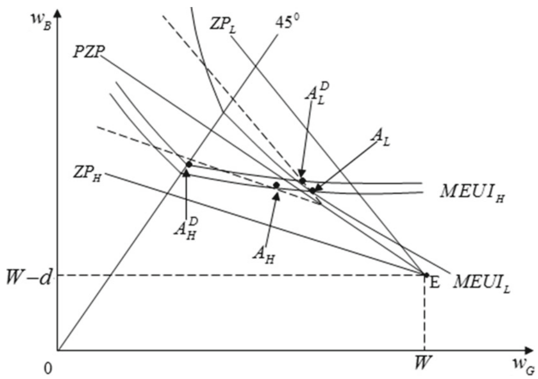


Fig. 3 At least one type buys full insurance

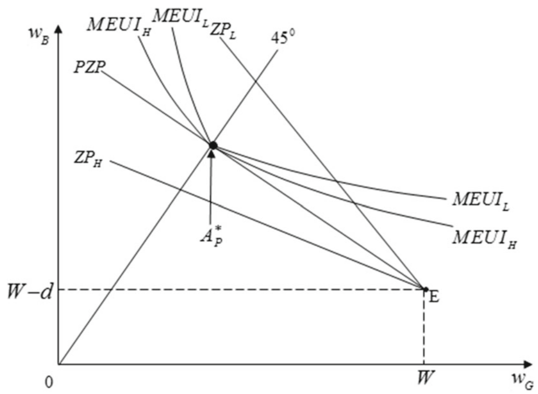


Fig. 4 Efficient pooling equilibrium

the new entrant will make a strictly positive profit and the initial menu $\{(A_\mathrm{H}, A_\mathrm{L}), c\}$ cannot be an equilibrium. Hence, in any equilibrium of our game, at least, one of the two types of insurees buys full insurance. □

Based on Lemmas 1 and 2, we show that if the condition in Case (1) is satisfied, there always exists a unique pooling equilibrium where both types of insurees buy full insurance.

Case 1 The Ls' degree of ambiguity is sufficiently higher than Hs' so that $\underline{p}_\mathrm{L} < \underline{p}_\mathrm{H} < \overline{p}_\mathrm{H} < \overline{p}_\mathrm{L}$.

Proposition 1 *Suppose that the Ls' degree of ambiguity is sufficiently higher than that of the Hs so that* $\underline{p}_\mathrm{L} < \underline{p}_\mathrm{H} < \overline{p}_\mathrm{H} < \overline{p}_\mathrm{L}$. *Then, the menus* $\{(A_P^*), 0\}$, $\{(A_P^*), 1\}$ *and any combination of them, involving the unique pooling allocation* A_P^* *where both types buy full insurance, are Bayes–Nash equilibria* (*see Fig.* 4).

Proof The Ls' indifference curves are flatter than the Hs' to the right of 45° line because $(1-\overline{p}_\mathrm{L})/\overline{p}_\mathrm{L} < (1-\overline{p}_\mathrm{H})/\overline{p}_\mathrm{H}$ and to the left of 45° line steeper, because $(1-\underline{p}_\mathrm{L})/\underline{p}_\mathrm{L} > (1-\underline{p}_\mathrm{H})/\underline{p}_\mathrm{H}$. Since the Ls' indifference curve lies inside that of the Hs, there exists no allocation which is preferred to A_P^* by the Ls and not by the Hs. Also, any allocation which is preferred to A_P^* by the Ls lies above the pooling zero-profit line. Hence, no insurer can profitably attract the Ls (or both types) and so $\{(A_P^*), c\}$ is an equilibrium.

Because the Ls' indifference curve lies inside that of the Hs, a separating equilibrium cannot exist. Also, by Lemma 2, in any equilibrium allocation, at least one type buys full insurance and so in any pooling equilibrium both types buy full insurance. Hence, $\{(A_P^*), c\}$ is the unique pooling allocation (since any other full-insurance pooling allocation implies strictly negative or positive profit for the insurers). Hence, the pooling full-insurance allocation $\{(A_P^*), c\}$ is the unique equilibrium allocation. However, the menus involving this unique pooling allocation may be offered with or without commitment which leads to multiple equilibria. □

Intuitively, ambiguity aversion increases the utility cost of under- or overinsurance. In particular, if the Ls are sufficiently more ambiguity averse than high-risk insurees, the utility cost of under- or overinsurance strictly dominates the monetary benefit of the lower per-unit premium. As a result, the Ls prefer to purchase full insurance at a high (pooling) per-unit than under- or overinsurance at a lower per-unit premium. That is, the high degree of ambiguity makes the cost of separation prohibitively high for the Ls.

The following points should be made here: First, the pooling equilibrium in Proposition 1 is exclusively due to ambiguity aversion and cannot obtain in the standard expected utility framework or under over-optimism/pessimism.[11] If the insurees know accurately their accident probability, the utility cost of underinsurance will always be lower for the Ls. Hence, the Ls will always prefer underinsurance at a lower per-unit premium to full insurance at a high (pooling) per-unit premium. A similar argument applies to the case of over-optimism/pessimism because, at full insurance, the indifference curves of the two types will cross (unless they coincide, which is a zero-probability event). As a result, pooling cannot be an equilibrium and separation will always prevail. Second, this result does not depend on the maxmin formulation of ambiguity aversion we have adopted in this paper. It also obtains under smoother representations of ambiguity aversion (for example, the representations suggested by Klibanoff et al. 2005) or more general variational preferences (see Maccheroni et al. 2006). Third, the existence of the full-insurance pooling equilibrium we establish here is not driven by the indemnity (no-overinsurance) principle (as in Jeleva and Villeneuve 2004). Finally, its existence does not require the three-stage game we employ in this paper (this equilibrium exists even if we use the standard two-stage screening game).

If the condition of Proposition 1 is violated (Cases (2)–(4)), then both pooling and separating equilibria can exist. Before proceeding to characterize these equilibria, we derive some useful general results.

Lemma 3 *In any separating equilibrium of our game, the Hs take full insurance.*

[11] Unless the degree of risk aversion is infinite (Leontief preferences) in the standard expected utility framework or the perceived probabilities coincide in the case of over-optimism/pessimism (which is a zero-probability event).

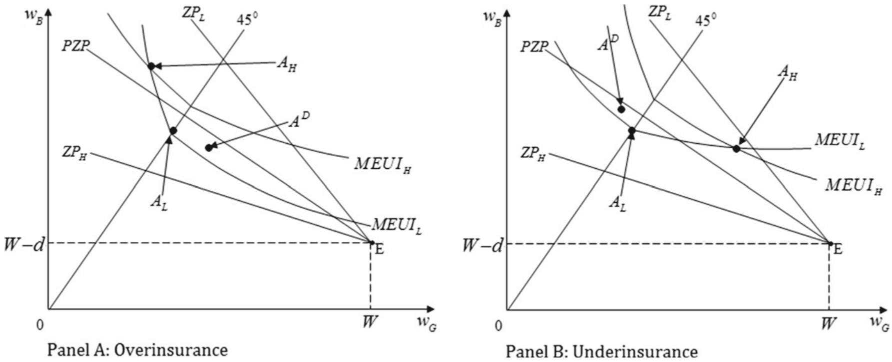


Fig. 5 The H-type always takes full insurance

Proof By Lemma 2, in any equilibrium allocation, at least one of the two types takes full insurance. Suppose that an insurer offers a menu $\{(A_{\rm H}, A_{\rm L}), c\}$, $c \in \{0, 1\}$, involving an efficient separating allocation $(A_{\rm H}, A_{\rm L})$ where the Ls choose full insurance (see Fig. 5). Then, depending on the relative degree of ambiguity, efficiency requires that the Hs choose either overinsurance or underinsurance. Also, zero profit on the menu $\{(A_{\rm H}, A_{\rm L}), c\}$ implies that $A_{\rm L}$ lies below the pooling zero-profit line (PZP) and $A_{\rm H}$ lies above it. Consider now a new entrant offering the deviant menu $\{(A^{\rm D}), 1\}$. Given the separating allocation $(A_{\rm H}, A_{\rm L})$, the deviant menu attracts either only the Ls (if $c = 1$) or both types (if $c = 0$). In either case, the deviation is profitable because $A^{\rm D}$ lies below the pooling zero-profit line. A similar argument applies if the Hs' indifference curve is steeper and the Hs choose underinsurance (see Panel B of Fig. 5). Hence, a separating equilibrium where the Ls choose full insurance cannot exist. Therefore, by Lemma 2, in any separating equilibrium the Hs choose full insurance. □

Lemma 4 *In any equilibrium of our game involving cross-subsidization across types, the Ls subsidize the Hs.*

Proof Let us start with separating equilibria. By Lemma 3, in any separating equilibrium, the Hs choose full insurance. Also, from the Ls' perspective, their accident probability is lower than that of the Hs either in the underinsurance region (Cases (2) and (4)) or in the overinsurance region (Cases (3) and (4)). As a result, the Ls will accept either under- (Cases (2) and (4)) or overinsurance (Cases (3) and (4)) in order to reveal their type and achieve a lower per-unit premium. Since the Ls are willing to bear the utility cost of under or overinsurance, they strictly prefer their contract to that chosen by the Hs. Thus, the incentive compatibility constraint of the Ls is not binding (see Fig. 5). Consider now an insurer offering a menu involving an efficient separating allocation with the Hs' contract implying strictly positive profits for the insurer. Since the Ls' incentive compatibility constraint is not binding, a new entrant can attract the Hs by offering them a menu involving a welfare improving (but still profitable) allocation. Hence, there cannot exist a separating equilibrium where the Hs subsidize the Ls. Hence, in any separating equilibrium with cross-subsidization across types, the Ls subsidize the Hs. Consider now pooling equilibria. Lemma 2 implies that any pooling equilibrium will involve both types taking full insurance. Because of Bertrand competition, in any equilibrium, insurers make zero profit. Hence, the fact that the Ls' expected income is higher than the Hs' implies that in any pooling equilibrium, the Ls subsidize the Hs. □

Lemma 5 *In any equilibrium allocation of our game involving cross-subsidization across types, the Ls' expected utility is maximized given the constraints* (2)*,* (3)*,* (4) *and the constraint implied by Lemma* 4

$$\min_{p\in[\underline{p}_{\mathrm{H}},\overline{p}_{\mathrm{H}}]}\{(1-p)U(w_{\mathrm{HG}})+pU(w_{\mathrm{HB}})\}\geq U(W-p_{\mathrm{H}}d). \tag{5}$$

Proof By Lemma 4, there cannot exist an equilibrium allocation where the Hs subsidize the Ls. Let us consider a firm which, at Stage 1, offers an interim incentive efficient allocation where the Ls subsidize the Hs' and Ls' expected utility is not maximized. This implies that there exist incentive compatible allocations which make the Ls strictly better-off and the insurer offering a menu involving one of them can make strictly positive profit on the contract chosen by the Ls. Hence, at Stage 1, a new entrant can offer a menu with commitment involving an incentive compatible allocation making strictly better-off the Ls and strictly worse-off the Hs. Because the new entrant is committed to his offer and this offer makes the Ls better-off, at Stage 2, the Ls choose the new entrant's offer regardless of the Hs' choice. If the incumbents' offer is with commitment, the Hs stay there, the incumbent is making losses and the deviant menu is clearly profit-making. If the incumbents' offer is without commitment, it will be withdrawn at Stage 3. Anticipating the withdrawal of the incumbent's offer, at Stage 2, the Hs choose the new entrant's offer. Thus, being constrained only by incentive compatibility, the new entrant can make an offer implying strictly positive profits for him. Hence, the incumbent's offer cannot be equilibrium. □

Lemmas 4 and 5 imply the following corollary:

Corollary 2 *The equilibrium allocation of our game is the solution to the following problem* (P1)*:*

$$\mathrm{P1}:\quad \max_{(w_{\mathrm{HG}},w_{\mathrm{HB}},w_{\mathrm{LG}},w_{\mathrm{LB}})\in\mathbb{R}^4_+}\quad \min_{p\in[\underline{p}_{\mathrm{L}},\overline{p}_{\mathrm{L}}]}\{(1-p)U(w_{\mathrm{LG}})+pU(w_{\mathrm{LB}})\} \tag{6}$$

satisfying the constraints (2)*–*(5)*.*

Notice that the objective function (6) *in* P1 *coincides with the objective function in the planner's problem* (*as defined in Sect.* 4) *for* $\xi=1$*. Also, constraint* (5)*, which is implied by Lemma* 4*, is due to competition and that is why it does not appear in the planner's problem.*

Lemma 6 *The constrained optimization problem* P1 *has a unique solution* $V^*=(w^*_{\mathrm{HG}},w^*_{\mathrm{HB}},w^*_{\mathrm{LG}},w^*_{\mathrm{LB}})$ *with* $w^*_{\mathrm{HG}}=w^*_{\mathrm{HB}}$*. The constraints* (3) *and* (4) *are binding.*

Proof To prove the lemma, we extend the proof of Lemma 1 in Netzer and Scheuer (2014). We first prove the statement about the constraints and then show the uniqueness of the solution.

Before we proceed, we have to first make claims regarding the regions where the solutions of the problem P1 belong to depending on the relation among the accident probabilities. Specifically, we claim that for Case (2), $A_{\mathrm{L}} \in \overline{C}^{\mathrm{U}}$ and for Case (3), $A_{\mathrm{L}} \in \overline{C}^{\mathrm{O}}$. Indeed, consider Case (2) where $\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$ and $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$. Assume $V = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}}, w_{\mathrm{LB}})$ satisfies all the constraints and $w_{\mathrm{LG}} < w_{\mathrm{LB}}$. For $\varepsilon_i > 0$ consider $\tilde{V}(\varepsilon) = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}} + \delta_1, w_{\mathrm{LB}} - \delta_2)$ for $\delta_1, \delta_2 > 0$ such that $U(w_{\mathrm{LG}} + \delta_1) = U(w_{\mathrm{LG}}) + \varepsilon$ and $U(w_{\mathrm{LB}} - \delta_2) = U(w_{\mathrm{LB}}) - \varepsilon(1 - \underline{p}_{\mathrm{L}})/\underline{p}_{\mathrm{L}}$. By construction and because $\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$, the allocation $\tilde{V}(\varepsilon)$ satisfies (2), (3) and (5), and the value of (6) is the same for both allocations $\tilde{V}(\varepsilon)$ and V for any $\varepsilon > 0$. Let

$$\Pi(\varepsilon) = \lambda\left[(1 - p_{\mathrm{L}})\Phi(U(w_{\mathrm{LG}}) + \varepsilon) + p_{\mathrm{L}}\Phi\left(U(w_{\mathrm{LB}}) - \varepsilon(1 - \underline{p}_{\mathrm{L}})/\underline{p}_{\mathrm{L}}\right)\right] \\ + (1 - \lambda)[(1 - p_{\mathrm{H}})w_{\mathrm{HG}} + p_{\mathrm{H}}w_{\mathrm{HB}}]$$

be the per capita expenditure in $\tilde{V}(\varepsilon)$. The derivative $\mathrm{d}\Pi(\varepsilon)/\mathrm{d}\varepsilon < 0$ for $0 < \varepsilon < \underline{p}_{\mathrm{L}}(U(w_{\mathrm{LG}}) - U(w_{\mathrm{LB}}))$, so that for $\varepsilon > 0$ small enough $\tilde{V}(\varepsilon)$ satisfies (4) with slack. Thus, $\tilde{V}_1(\varepsilon) = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}} + \delta_1 + \delta_3, w_{\mathrm{LB}} - \delta_2)$ for small enough $\delta_3 > 0$ satisfies all the constraints and strictly increases the value of (6). This contradicts the assumption that V is the solution of P1 and hence proves the claim.

In order to show that our claim is true for Case (3) with $\overline{p}_{\mathrm{L}} > \overline{p}_{\mathrm{H}} > \underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$, we have to consider $\tilde{V}(\varepsilon) = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}} - \delta_1, w_{\mathrm{LB}} + \delta_2)$ for $\delta_1, \delta_2 > 0$ such that $U(w_{\mathrm{LG}} - \delta_1) = U(w_{\mathrm{LG}}) - \varepsilon$ and $U(w_{\mathrm{LB}} + \delta_2) = U(w_{\mathrm{LB}}) + \varepsilon(1 - \underline{p}_{\mathrm{L}})/\underline{p}_{\mathrm{L}}$. The arguments are applied in the same way.

Constraint (4). Let us show that the constraint (4) is binding. Assume that $V = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}}, w_{\mathrm{LB}})$ is a solution of P1 that satisfies the constraint (4) with slack. To reach a contradiction, let us find $\delta_i > 0, i = 1, \ldots, 4$ such that $\tilde{V} = (w_{\mathrm{HG}} + \delta_1, w_{\mathrm{HB}} + \delta_2, w_{\mathrm{LG}} + \delta_3, w_{\mathrm{LB}} + \delta_4)$ satisfies (2), (3), (4), (5) and strictly increases the value of (6). Note that the continuity of U implies that for any $\varepsilon_i > 0, i = 1, \ldots, 4$, there exists $\delta_i > 0$ such that $U(w_{\mathrm{HG}} + \delta_1) = U(w_{\mathrm{HG}}) + \varepsilon_1$, $U(w_{\mathrm{HB}} + \delta_2) = U(w_{\mathrm{HB}}) + \varepsilon_2$, $U(w_{\mathrm{LG}} + \delta_3) = U(w_{\mathrm{LG}}) + \varepsilon_3$, $U(w_{\mathrm{LB}} + \delta_4) = U(w_{\mathrm{LB}}) + \varepsilon_4$. The choice of ε_i, in turn, depends on the particular relation among higher and lower bounds of the accident probabilities and the position of allocations $A_{\mathrm{H}} = (w_{\mathrm{HG}}, w_{\mathrm{HB}})$ and $A_{\mathrm{L}} = (w_{\mathrm{LG}}, w_{\mathrm{LB}})$ relative to the 45° line. Let us consider these choices for Cases (2)–(4) separately.

Consider Case (2) with $\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$ and $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$. Because $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$, for any $\Delta_{24} > 0$ there exists $\Delta_{31} > 0$ such that $\frac{\overline{p}_{\mathrm{L}}}{1-\overline{p}_{\mathrm{L}}}\Delta_{24} \leq \Delta_{31} \leq \Delta_{24}\frac{\overline{p}_{\mathrm{H}}}{1-\overline{p}_{\mathrm{H}}}$. Now, fix any $\varepsilon_4 > 0$ and define $\varepsilon_1 = \varepsilon_2 = \Delta_{24} + \varepsilon_4$ and $\varepsilon_3 = \Delta_{24} + \Delta_{31} + \varepsilon_4$. Given that $A_{\mathrm{H}}, A_{\mathrm{L}} \in \overline{C}^{\mathrm{U}}$, the choice of ε_i implies that $\tilde{V}$ satisfies (2), (3) and (5) and strictly increases the value of (6). Each ε_i can be chosen sufficiently small so that $\tilde{V}$ also satisfies (4) and ensures that $\tilde{A}_{\mathrm{H}}, \tilde{A}_{\mathrm{L}} \in \underline{C}^{\mathrm{U}}$.

Consider Case (3) with $\overline{p}_{\mathrm{L}} > \overline{p}_{\mathrm{H}} > \underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$. Because $\underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$, for any $\Delta_{42} > 0$ there exists $\Delta_{13} > 0$ such that $\frac{\underline{p}_{\mathrm{H}}}{1-\underline{p}_{\mathrm{H}}}\Delta_{42} \leq \Delta_{13} \leq \Delta_{42}\frac{\underline{p}_{\mathrm{L}}}{1-\underline{p}_{\mathrm{L}}}$. Now, fix any $\varepsilon_2 > 0$ and define $\varepsilon_1 = \varepsilon_2 = \Delta_{13} + \varepsilon_3$ and $\varepsilon_4 = \Delta_{13} + \Delta_{42} + \varepsilon_3$. In this case $A_{\mathrm{H}}, A_{\mathrm{L}} \in \overline{C}^{\mathrm{O}}$, and again, by construction $\tilde{V}$ satisfies (2), (3) and (5) and strictly increases the value of (6). All ε_i can be chosen sufficiently small so $\tilde{V}$ satisfies (4) and $\tilde{A}_{\mathrm{H}}, \tilde{A}_{\mathrm{L}} \in \overline{C}^{\mathrm{O}}$.

For Case (4), ($\overline{p}_{\mathrm{H}} > \overline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$), if $A_{\mathrm{L}} \in \overline{C}^{\mathrm{U}}$, then the choice of ε_i is identical to Case (2), while if $A_{\mathrm{L}} \in \overline{C}^{\mathrm{O}}$ then the choice of ε_i is the same as in Case (3).

Constraint (3). Let us show that the constraint (3) is binding. Let $V = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}}, w_{\mathrm{LB}})$ be a solution of P1 satisfying the constraint (4) with equality and let us assume that (3) is slack. Again, as before, to reach a contradiction, let us find an allocation $\tilde{V}$ satisfying (2)–(5) and in which the value of (6) is strictly greater than in V.

Consider Case (2) ($\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$ and $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$). For $\varepsilon > 0$, consider $\tilde{V}(\varepsilon) = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}} - \delta_1, w_{\mathrm{LB}} + \delta_2)$ with $\delta_1, \delta_2 > 0$ such that $U(w_{\mathrm{LG}} - \delta_1) = U(w_{\mathrm{LG}}) - \varepsilon$ and $U(w_{\mathrm{LB}} + \delta_2) = U(w_{\mathrm{LB}}) + \varepsilon(1 - \overline{p}_{\mathrm{L}})/\overline{p}_{\mathrm{L}}$. By construction, $\tilde{V}(\varepsilon)$ satisfies (2) and (5), and the value of (6) is the same under $\tilde{V}(\varepsilon)$ and V for any $\varepsilon > 0$. Given $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$, $\tilde{V}(\varepsilon)$ also satisfies (3) for $\varepsilon > 0$ small enough. Let

$$\Pi_{\mathrm{L}}(\varepsilon) = \lambda[(1 - p_{\mathrm{L}})\Phi(U(w_{\mathrm{LG}}) - \varepsilon) + p_{\mathrm{L}}\Phi(U(w_{\mathrm{LB}}) + \varepsilon(1 - \overline{p}_{\mathrm{L}})/\overline{p}_{\mathrm{L}})]$$

be the Ls' per capita expenditure in $\tilde{V}(\varepsilon)$. Since $A_{\mathrm{L}} \in C^{\mathrm{U}}$, we have that $\mathrm{d}\Pi_{\mathrm{L}}(\varepsilon)/\mathrm{d}\varepsilon < 0$ for $0 < \varepsilon < \overline{p}_{\mathrm{L}}(U(w_{\mathrm{LG}}) - U(w_{\mathrm{LB}}))$ for $\varepsilon > 0$ small enough and $\tilde{V}(\varepsilon)$ satisfies (4) with slack. In this case, according to the previous statement, one can find an allocation satisfying all of the constraints and in which the value of (6) is strictly greater than in V, so V cannot be a solution to P1.

Consider Case (3), ($\overline{p}_{\mathrm{L}} > \overline{p}_{\mathrm{H}} > \underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$). For $\varepsilon > 0$, consider $\tilde{V}(\varepsilon) = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}} + \delta_1, w_{\mathrm{LB}} - \delta_2)$ for $\delta_1, \delta_2 > 0$ such that $U(w_{\mathrm{LG}} + \delta_1) = U(w_{\mathrm{LG}}) + \varepsilon$ and $U(w_{\mathrm{LB}} - \delta_2) = U(w_{\mathrm{LB}}) - \varepsilon(1 - \underline{p}_{\mathrm{L}})/\underline{p}_{\mathrm{L}}$. By construction and because $\underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$, $\tilde{V}(\varepsilon)$ satisfies (2), (3) and (5), and the value of (6) is the same under $\tilde{V}(\varepsilon)$ and V for small enough $\varepsilon > 0$. Let

$$\Pi_{\mathrm{L}}(\varepsilon) = \lambda[(1 - p_{\mathrm{L}})\Phi(U(w_{\mathrm{LG}}) + \varepsilon) + p_{\mathrm{L}}\Phi(U(w_{\mathrm{LB}}) - \varepsilon(1 - \underline{p}_{\mathrm{L}})/\underline{p}_{\mathrm{L}})]$$

be the Ls' per capita expenditure in $\tilde{V}(\varepsilon)$. Because $A_{\mathrm{L}} \in C^{\mathrm{O}}$, it follows that $\mathrm{d}\Pi_{\mathrm{L}}(\varepsilon)/\mathrm{d}\varepsilon < 0$ for $0 < \varepsilon < \underline{p}_{\mathrm{L}}(U(w_{\mathrm{LB}}) - U(w_{\mathrm{LG}}))$ for $\varepsilon > 0$ small enough and $\tilde{V}(\varepsilon)$ satisfies (4) with slack.

Consider Case (4), ($\overline{p}_{\mathrm{H}} > \overline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$). Assume that $A_{\mathrm{L}} \in C^{\mathrm{U}}$. For $\varepsilon > 0$, consider $\tilde{V}(\varepsilon) = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}} - \delta_1, w_{\mathrm{LB}} + \delta_2)$ with $\delta_1, \delta_2 > 0$ such that $U(w_{\mathrm{LG}} - \delta_1) = U(w_{\mathrm{LG}}) - \varepsilon$ and $U(w_{\mathrm{LB}} + \delta_2) = U(w_{\mathrm{LB}}) + \varepsilon(1 - \overline{p}_{\mathrm{L}})/\overline{p}_{\mathrm{L}}$. By construction and because $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$, $\tilde{V}(\varepsilon)$ satisfies (2), (3) and (5) for small enough $\varepsilon > 0$, and the value of (6) remains the same under $\tilde{V}(\varepsilon)$ and V. Let

$$\Pi_{\mathrm{L}}(\varepsilon) = \lambda[(1 - p_{\mathrm{L}})\Phi(U(w_{\mathrm{LG}}) - \varepsilon) + p_{\mathrm{L}}\Phi(U(w_{\mathrm{LB}}) + \varepsilon(1 - \overline{p}_{\mathrm{L}})/\overline{p}_{\mathrm{L}})]$$

be the Ls' per capita expenditure in $\tilde{V}(\varepsilon)$. Since $\mathrm{d}\Pi_{\mathrm{L}}(\varepsilon)/\mathrm{d}\varepsilon < 0$ for $0 < \varepsilon < \overline{p}_{\mathrm{L}}(U(w_{\mathrm{LG}}) - U(w_{\mathrm{LB}}))$ for $\varepsilon > 0$ small enough, $\tilde{V}(\varepsilon)$ satisfies (4) with slack.

Assume that $A_{\mathrm{L}} \in C^{\mathrm{O}}$. For $\varepsilon > 0$, consider $\tilde{V}(\varepsilon) = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}} + \delta_1, w_{\mathrm{LB}} - \delta_2)$ for $\delta_1, \delta_2 > 0$ such that $U(w_{\mathrm{LG}} + \delta_1) = U(w_{\mathrm{LG}}) + \varepsilon$ and $U(w_{\mathrm{LB}} - \delta_2) = U(w_{\mathrm{LB}}) - \varepsilon(1 - \underline{p}_{\mathrm{L}})/\underline{p}_{\mathrm{L}}$. By construction and because $\underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$, $\tilde{V}(\varepsilon)$ satisfies (2), (3) and (5) for small enough $\varepsilon > 0$, and the value of (6) is the same under $\tilde{V}(\varepsilon)$ and V. Let

$$\Pi_{\mathrm{L}}(\varepsilon) = \lambda[(1 - p_{\mathrm{L}})\Phi(U(w_{\mathrm{LG}}) + \varepsilon) + p_{\mathrm{L}}\Phi(U(w_{\mathrm{LB}}) - \varepsilon(1 - \underline{p}_{\mathrm{L}})/\underline{p}_{\mathrm{L}})]$$

be the Ls' per capita expenditure in $\tilde{V}(\varepsilon)$. Since $\mathrm{d}\Pi_{\mathrm{L}}(\varepsilon)/\mathrm{d}\varepsilon < 0$ for $0 < \varepsilon < \underline{p}_{\mathrm{L}}(U(w_{\mathrm{LB}}) - U(w_{\mathrm{LG}}))$ for $\varepsilon > 0$ small enough, $\tilde{V}(\varepsilon)$ satisfies (4) with slack and hence V cannot be a solution to P1.

Output-independent utilities for high risks Let us assume that allocation $V = (w_{\mathrm{HG}}, w_{\mathrm{HB}}, w_{\mathrm{LG}}, w_{\mathrm{LB}})$ with $w_{\mathrm{HG}} \neq w_{\mathrm{HB}}$ is a solution of P1 and satisfies the constraints (3) and (4) with equality. In order to reach a contradiction, we construct an allocation satisfying all the constraints (2)–(5) and strictly increasing the value of (6) over V. Consider again three cases.

Start with Case (2) ($\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$ and $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$). Let $w_{\mathrm{HG}} > w_{\mathrm{HB}}$. Define $\tilde{w} = \Phi((1-\overline{p}_{\mathrm{H}})U(w_{\mathrm{HG}}) + \overline{p}_{\mathrm{H}}U(w_{\mathrm{HB}}))$ and consider $\tilde{V} = (\tilde{w}, \tilde{w}, w_{LG, w_{\mathrm{LB}}})$. By construction, $\tilde{V}$ satisfies (3) and (5), and the value of (6) is the same under V and $\tilde{V}$. Since $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$ and $w_{\mathrm{HG}} > w_{\mathrm{HB}}$, it follows that $(1 - \overline{p}_{\mathrm{L}})U(w_{\mathrm{HG}}) + \overline{p}_{\mathrm{L}}U(w_{\mathrm{HB}}) > U(\tilde{w}) = (1 - \overline{p}_{\mathrm{L}})U(\tilde{w}) + \overline{p}_{\mathrm{L}}U(\tilde{w})$, so that $\tilde{V}$ satisfies (2) as well. Strict convexity of Φ implies that $\tilde{V}$ satisfies (4) with slack: $(1 - p_{\mathrm{H}})w_{\mathrm{HG}} + p_{\mathrm{H}}w_{\mathrm{HB}} > (1 - \overline{p}_{\mathrm{H}})w_{\mathrm{HG}} + \overline{p}_{\mathrm{H}}w_{\mathrm{HB}} > \tilde{w} = (1 - \overline{p}_{\mathrm{H}})\tilde{w} + \overline{p}_{\mathrm{H}}\tilde{w}$. From the above argument, the value of (6) can be increased above its value for V so it cannot be a solution to P1.

Let $w_{\mathrm{HG}} < w_{\mathrm{HB}}$. Define $\tilde{w} = \Phi((1-\underline{p}_{\mathrm{H}})U(w_{\mathrm{HG}})+\underline{p}_{\mathrm{H}}U(w_{\mathrm{HB}}))$. By construction, $\tilde{V}$ satisfies (3) and (5), and the value of (6) is the same under V and $\tilde{V}$. In order to show that $\tilde{V}$ satisfies (2) note that given that the constraint (3) is binding, the contract $(w_{\mathrm{LG}}, w_{\mathrm{LB}})$ should lie on the Hs' indifference curve that goes through A_{H}: $U(w_{\mathrm{LG}}) = U(\tilde{w}) - \frac{\overline{p}_{\mathrm{H}}}{1-\overline{p}_{\mathrm{H}}}U(w_{\mathrm{LB}})$. Since $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$ and $w_{\mathrm{LB}} < \tilde{w}$ (the slope of the Hs' indifference curve is negative), it follows that

$$\begin{aligned}(1 - \overline{p}_{\mathrm{L}})U(w_{\mathrm{LG}}) + \overline{p}_{\mathrm{L}}U(w_{\mathrm{LB}}) &= (1 - \overline{p}_{\mathrm{L}})\left(U\left(\tilde{w} - \frac{\overline{p}_{\mathrm{H}}}{1 - \overline{p}_{\mathrm{H}}}U(w_{\mathrm{LB}})\right)\right) + \overline{p}_{\mathrm{L}}U(w_{\mathrm{LB}}) \\ &> U(\tilde{w}) = (1 - \overline{p}_{\mathrm{L}})U(\tilde{w}) + \overline{p}_{\mathrm{L}}U(\tilde{w})\end{aligned}$$

Finally, $(1-p_{\mathrm{H}})w_{\mathrm{HG}}+p_{\mathrm{H}}w_{\mathrm{HB}} > (1-\underline{p}_{\mathrm{H}})w_{\mathrm{HG}}+\underline{p}_{\mathrm{H}}w_{\mathrm{HB}} > \tilde{w} = (1-\underline{p}_{\mathrm{H}})\tilde{w}+\underline{p}_{\mathrm{H}}\tilde{w}$ due to strict convexity of Φ which implies that $\tilde{V}$ satisfies (4) with slack. Again, the value of (6) can be increased above its value at V so V cannot be a solution to P1.

Consider now Case (3) ($\overline{p}_{\mathrm{L}} > \overline{p}_{\mathrm{H}} > \underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$). Let $w_{\mathrm{HG}} < w_{\mathrm{HB}}$. Let us define $\tilde{w} = \Phi((1 - \underline{p}_{\mathrm{H}})U(w_{\mathrm{HG}}) + \underline{p}_{\mathrm{H}}U(w_{\mathrm{HB}}))$. By construction, $\tilde{V} = (\tilde{w}, \tilde{w}, w_{\mathrm{LG}}, w_{\mathrm{LB}})$ satisfies (3) and (5), and the value of (6) is the same under V and $\tilde{V}$. Since $\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$ and $w_{\mathrm{HG}} < w_{\mathrm{HB}}$ we have $(1-\underline{p}_{\mathrm{L}})U(w_{\mathrm{HG}})+\underline{p}_{\mathrm{L}}U(w_{\mathrm{HB}}) > U(\tilde{w}) = (1-\underline{p}_{\mathrm{L}})U(\tilde{w})+\underline{p}_{\mathrm{L}}U(\tilde{w})$, so that $\tilde{V}$ satisfies (2). Strict convexity of Φ implies that $\tilde{V}$ satisfies (4) with slack: $(1 - p_{\mathrm{H}})w_{\mathrm{HG}} + p_{\mathrm{H}}w_{\mathrm{HB}} > (1 - \underline{p}_{\mathrm{H}})w_{\mathrm{HG}} + \underline{p}_{\mathrm{H}}w_{\mathrm{HB}} > \tilde{w} = (1 - \underline{p}_{\mathrm{H}})\tilde{w} + \underline{p}_{\mathrm{H}}\tilde{w}$.

As the value of the objective can be increased above its value at V so V cannot be a solution to P1.

Suppose that $w_{\mathrm{HG}} > w_{\mathrm{HB}}$. Define $\tilde{w} = \Phi((1-\overline{p}_{\mathrm{H}})U(w_{\mathrm{HG}}) + \overline{p}_{\mathrm{H}}U(w_{\mathrm{HB}}))$. $\tilde{V}$ satisfies (3) and (5), and the value of (6) is the same under V and $\tilde{V}$. In order to show that $\tilde{V}$ satisfies (2) note that given that the constraint (3) is binding, the contract $(w_{\mathrm{LG}}, w_{\mathrm{LB}})$ should lie on the Hs' indifference curve that goes through A_{H}: $U(w_{\mathrm{LG}}) = U(\tilde{w}) - \underline{p}_{\mathrm{H}}U(w_{\mathrm{LB}})/(1-\underline{p}_{\mathrm{H}})$. Since $\underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$ and $w_{\mathrm{LB}} > \tilde{w}$ (the slope of the Hs' indifference curve is negative), it follows that

$$(1-\underline{p}_{\mathrm{L}})U(w_{\mathrm{LG}}) + \underline{p}_{\mathrm{L}}U(w_{\mathrm{LB}}) = (1-\underline{p}_{\mathrm{L}})\left(U\left(\tilde{w} - \frac{\underline{p}_{\mathrm{H}}}{1-\underline{p}_{\mathrm{H}}}U(w_{\mathrm{LB}})\right)\right) + \underline{p}_{\mathrm{L}}U(w_{\mathrm{LB}})$$
$$> U(\tilde{w}) = (1-\underline{p}_{\mathrm{L}})U(\tilde{w}) + \underline{p}_{\mathrm{L}}U(\tilde{w})$$

Finally, $(1-p_{\mathrm{H}})w_{\mathrm{HG}} + p_{\mathrm{H}}w_{\mathrm{HB}} > (1-\overline{p}_{\mathrm{H}})w_{\mathrm{HG}} + \overline{p}_{\mathrm{H}}w_{\mathrm{HB}} > \tilde{w} = (1-\overline{p}_{\mathrm{H}})\tilde{w} + \overline{p}_{\mathrm{H}}\tilde{w}$ due to strict convexity of Φ which implies that $\tilde{V}$ satisfies (4) with slack.

Finally, consider Case (4) with $\overline{p}_{\mathrm{H}} > \overline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$. If $w_{\mathrm{HG}} > w_{\mathrm{HB}}$ and $w_{\mathrm{LG}} > w_{\mathrm{LB}}$ the proof is the same as for the corresponding scenario of Case (2). If $w_{\mathrm{HG}} > w_{\mathrm{HB}}$ and $w_{\mathrm{LG}} < w_{\mathrm{LB}}$, the proof is the same as for the corresponding scenario of Case (3). If $w_{\mathrm{HG}} < w_{\mathrm{HB}}$ and $w_{\mathrm{LG}} < w_{\mathrm{LB}}$, the proof is same as for the corresponding scenario of Case (3). If $w_{\mathrm{HG}} < w_{\mathrm{HB}}$ and $w_{\mathrm{LG}} > w_{\mathrm{LB}}$, the proof is identical to the corresponding scenario of Case (2).

Existence and uniqueness In order to prove existence and uniqueness of the solution of P1, let us rearrange the four-dimensional maximization problem (6) into a one-dimensional one. Let us start with Case (2). The solution to P1 must be in the form $V = (w_{\mathrm{H}}, w_{\mathrm{H}}, w_{\mathrm{LG}}, w_{\mathrm{LB}})$. The incentive compatibility constraint (3) is binding which implies $U(w_{\mathrm{LB}}) = [U(w_{\mathrm{H}}) - (1-\overline{p}_{\mathrm{H}})U(w_{\mathrm{LG}})]/\overline{p}_{\mathrm{H}}$. Moreover, constraint (2) is slack. Also, note that $w_{\mathrm{LG}} \geq w_H$, i.e., $(w_{\mathrm{LG}}, w_{\mathrm{H}}) \in \overline{C}^{\mathrm{U}}$. We can reformulate problem (6) as:

$$\max_{(w_{\mathrm{LG}},w_{\mathrm{H}})\in\overline{C}^{\mathrm{U}}} \quad \min_{p\in[\underline{p}_{\mathrm{L}},\overline{p}_{\mathrm{L}}]} \left\{ \frac{p}{\overline{p}_{\mathrm{H}}}U(w_{\mathrm{H}}) + \left(\frac{\overline{p}_{\mathrm{H}} - p}{\overline{p}_{\mathrm{H}}}\right)U(w_{\mathrm{LG}}) \right\} \tag{7}$$

subject to

$$\lambda\left[(1-p_{\mathrm{L}})w_{\mathrm{LG}} + p_{\mathrm{L}}\Phi\left(\frac{U(w_{\mathrm{H}}) - (1-\overline{p}_{\mathrm{H}})U(w_{\mathrm{LG}})}{\overline{p}_{\mathrm{H}}}\right)\right] + (1-\lambda)w_{\mathrm{H}} = R_\lambda \tag{8}$$

and constraint (5). Let

$$E(w_{\mathrm{LG}}, w_{\mathrm{H}}) = \lambda\left[(1-p_{\mathrm{L}})w_{\mathrm{LG}} + p_{\mathrm{L}}\Phi\left(\frac{U(w_{\mathrm{H}}) - (1-\overline{p}_{\mathrm{H}})U(w_{\mathrm{LG}})}{\overline{p}_{\mathrm{H}}}\right)\right] + (1-\lambda)w_{\mathrm{H}}.$$

Function E is continuously differentiable on $\mathbb{R}^2$ and is strictly increasing in w_{LG} on $\overline{C}^{\mathrm{U}}$ with $\lim_{w_{\mathrm{LG}}\to\infty} E(w_{\mathrm{LG}}, w_{\mathrm{H}}) = \infty$ due to convexity. Furthermore, E is strictly increasing in w_{H} globally with $\lim_{w_{\mathrm{H}}\to-\infty} E(w_{\mathrm{LG}}, w_{\mathrm{H}}) = -\infty$. The cross-subsidy constraint (5) can be re-arranged to $w_{\mathrm{H}} \geq W - p_{\mathrm{H}}d$. Denote the minimal choice for w_{H} as $w^- := W - p_{\mathrm{H}}d$.

We claim that $w^+(\lambda) = R_\lambda = W - p_\lambda d$ is the highest possible value for w_{H} so that $(w_{\mathrm{LG}}, w_{\mathrm{H}})$ satisfies relation (8) and simultaneously satisfies $(w_{\mathrm{LG}}, w_{\mathrm{H}}) \in \overline{C}^{\mathrm{U}}$. Indeed, consider $(w^+(\lambda), w^+(\lambda)) \in \overline{C}^{\mathrm{U}}$ satisfying (8) by construction. Any tuple $(\tilde{w}_{\mathrm{LG}}, \tilde{w}_{\mathrm{H}}) \in \overline{C}^{\mathrm{U}}$ with $\tilde{w}_{\mathrm{H}} > w^+(\lambda)$ and $\tilde{w}_{\mathrm{LG}} > w^+(\lambda)$ can be reached from $(w^+(\lambda), w^+(\lambda))$ by first increasing w_{LG} from $w^+(\lambda)$ to $\tilde{w}_{\mathrm{LG}}$ and then by increasing w_{H} from $w^+(\lambda)$ to $\tilde{w}_{\mathrm{H}}$. Both moves strictly increase E so that $(\tilde{w}_{\mathrm{LG}}, \tilde{w}_{\mathrm{H}})$ violates (8) which proves the claim.

Let $A(\lambda) = [w^-, w^+(\lambda)]$ and fix any $w_{\mathrm{H}} \in A(\lambda)$. It follows that

$$E\left(w^+(\lambda), w_{\mathrm{H}}\right) \leq E\left(w^+(\lambda), w^+(\lambda)\right) = R_\lambda$$

with strict inequality whenever $w_{\mathrm{H}} < w^+(\lambda)$. Since E is strictly increasing in w_{LG} on $\overline{C}^{\mathrm{U}}$ with $\lim_{w_{\mathrm{LG}} \to \infty} E(w_{\mathrm{LG}}, w_{\mathrm{H}}) = \infty$, there exists a unique value $H(w_{\mathrm{H}}, \lambda)$ such that $E(H(w_{\mathrm{H}}, \lambda), w_{\mathrm{H}}) = R_\lambda$ with $H(w_{\mathrm{H}}, \lambda) \geq w^+(\lambda) \geq w_{\mathrm{H}}$.

Let

$$L(w_{\mathrm{H}}, \lambda) = \min_{p \in [\underline{p}_{\mathrm{L}}, \overline{p}_{\mathrm{L}}]} \{pU(w_{\mathrm{H}}) + (\overline{p}_{\mathrm{H}} - p)U(H(w_{\mathrm{H}}, \lambda))\}.$$

Function $H(w_{\mathrm{H}}, \lambda)\colon \mathbb{R}_+ \times (0, 1) \to \mathbb{R}_+$ is continuous and continuously differentiable by the implicit function theorem, with $H(A(\lambda) \times (0, 1)) \subseteq [w^+(\lambda), \infty)$. So we can reduce (7) to one-dimensional problem

$$\begin{aligned} w^*_{\mathrm{H}}(\lambda) &= \operatorname*{argmax}_{w_{\mathrm{H}}(\lambda) \in A(\lambda)} \quad L(w_{\mathrm{H}}, \lambda) \\ &= \operatorname*{argmax}_{w_{\mathrm{H}}(\lambda) \in A(\lambda)} \quad \min_{p \in [\underline{p}_{\mathrm{L}}, \overline{p}_{\mathrm{L}}]} \{pU(w_{\mathrm{H}}) + (\overline{p}_{\mathrm{H}} - p)U(H(w_{\mathrm{H}}, \lambda))\} \end{aligned} \tag{9}$$

for which existence of a solution follows immediately by the Weierstrass theorem. To prove uniqueness, we show strict concavity of the objective function by showing that $H(w_{\mathrm{H}}, \lambda)$ is strictly concave. Let $(w^1_{\mathrm{LG}}, w^1_{\mathrm{H}})$ and $(w^2_{\mathrm{LG}}, w^2_{\mathrm{H}})$ in $\overline{C}^{\mathrm{U}}$ satisfy $E(w^1_{\mathrm{LG}}, w^1_{\mathrm{H}}) = (w^2_{\mathrm{LG}}, w^2_{\mathrm{H}}) = R_\lambda$ and $(w^1_{\mathrm{LG}}, w^1_{\mathrm{H}}) \neq (w^2_{\mathrm{LG}}, w^2_{\mathrm{H}})$. Define $w^3_{\mathrm{LG}} = \eta w^1_{\mathrm{LG}} + (1 - \eta) w^2_{\mathrm{LG}}$ and $w^3_{\mathrm{H}} = \eta w^1_{\mathrm{H}} + (1 - \eta) w^2_{\mathrm{H}}$ for $\eta \in (0, 1)$. The strict convexity of E implies that $E(w^3_{\mathrm{LG}}, w^3_{\mathrm{H}}) < R_\lambda$, which in turn implies that

$$\begin{aligned} H(w^3_{\mathrm{H}}, \lambda) &= H(\eta w^1_{\mathrm{H}} + (1 - \eta) w^2_{\mathrm{H}}, \lambda) > w^3_{\mathrm{LG}} = \eta w^1_{\mathrm{LG}} + (1 - \eta) w^2_{\mathrm{LG}} \\ &= \eta H(w^1_{\mathrm{H}}, \lambda) + (1 - \eta) H(w^2_{\mathrm{H}}, \lambda), \end{aligned}$$

which completes the proof. Cases (3) and (4) are proved analogously. □

Now that we have derived these general results, we can proceed to establish and characterize the equilibria of our game in Cases (2)–(4).

Case 2 ($\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$ and $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$)

In this case, the indifference curves of the two types of insurees intersect only once and the Ls' indifference curve is steeper as in the standard expected utility framework (see Rothschild and Stiglitz 1976). Not surprisingly, in the equilibrium which does not involve cross-subsidization across types, the results are qualitatively similar to those in Rothschild and Stiglitz. The key difference is that, because the ambiguity aversion relaxes the Hs' no-mimicking constraint, the Ls buy more insurance compared to the expected utility framework.

In order to characterize the equilibria in this case, we need to define a measure of cross-subsidization. Define the cross-subsidy as follows:

$$\chi(\lambda) = w_{\mathrm{H}}^{*}(\lambda) - (W - p_{\mathrm{H}} d).$$

Lemma 7 *Suppose ($\underline{p}_{\mathrm{L}} < \underline{p}_{\mathrm{H}}$ and $\overline{p}_{\mathrm{L}} < \overline{p}_{\mathrm{H}}$). Then:*

(i) $\chi(\lambda)$ *is continuous on* $(0, 1)$, $\lim_{\lambda\to 0} \chi(\lambda) = 0$ *and* $\lim_{\lambda\to 1} \chi(\lambda) = (p_{\mathrm{H}} - p_{\mathrm{L}})d > 0$.

(ii) *There exists* $\tilde{\lambda} \in (0, 1)$ *such that* $\chi(\lambda) = 0$ *for all* $\lambda \le \tilde{\lambda}$ *and* $\chi(\lambda)$ *is strictly increasing in* λ *for all* $\lambda > \tilde{\lambda}$.

(iii) *Moreover, there exists* $\tilde{\tilde{\lambda}} \in (\tilde{\lambda}, 1]$ *such that* $(w_{\mathrm{LG}}^{*}, w_{\mathrm{LB}}^{*}) \in C^{\mathrm{U}}$ *for all* $\lambda < \tilde{\tilde{\lambda}}$ *and* $w_{\mathrm{LG}}^{*} = w_{\mathrm{LB}}^{*} = w_{\mathrm{H}}^{*}$ *for all* $\lambda \ge \tilde{\tilde{\lambda}}$.

Proof Property (i). We first show that the solution to the maximization problem (6) is continuous in λ on $(0, 1)$. It then follows that $\chi(\lambda)$ is continuous as well.

We now transform the original constrained maximization problem (6) into an unconstrained one. Let $\mathfrak{U} = [w^{-}, (1 - p_{\mathrm{L}})W + p_{\mathrm{L}}(W - d)]$. The correspondence $A(\lambda) \equiv [w^{-}, w^{+}(\lambda)] \colon (0, 1) \rightrightarrows \mathfrak{U}$ is compact-valued and continuous. Define $Z \colon \mathfrak{U} \times (0, 1) \to \mathbb{R}$ as

$$Z(w_{\mathrm{H}}, \lambda) = \begin{cases} U(H(w_{\mathrm{H}}, \lambda)), & w_{\mathrm{H}} \le w^{+}(\lambda) \\ U(w^{+}(\lambda)), & w_{\mathrm{H}} > w^{+}(\lambda). \end{cases}$$

Since $H(w_{\mathrm{H}}, \lambda)$ is continuous in both variables, $H(w^{+}(\lambda), \lambda) = w^{+}(\lambda)$ holds and $w^{+}(\lambda)$ is continuous in λ, it follows that Z is continuous on $\mathfrak{U} \times (0, 1)$. Given that $w_{\mathrm{H}} \in A(\lambda)$, we rewrite (9) as

$$w_{\mathrm{H}}^{*}(\lambda) = \operatorname*{argmax}_{w_{\mathrm{H}} \in A(\lambda)} \{\overline{p}_{\mathrm{L}} U(w_{\mathrm{H}}) + (\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}}) Z(w_{\mathrm{H}}, \lambda)\}. \tag{10}$$

Berge's maximum principle (applied twice) implies continuity of $w_{\mathrm{H}}^{*}(\lambda)$ and $\chi(\lambda)$.

We now show that as $\lambda \to 0$, the constraint (5) must eventually become binding, so that we get $w_{\mathrm{H}}^{*}(\lambda) = w^{-} = W - p_{\mathrm{H}} d$. Consider the derivative of the objective function (9) in $w_{\mathrm{H}} = w^{-}$: $\frac{\partial L(w_{\mathrm{H}}, \lambda)}{\partial w_{\mathrm{H}}}|_{w_{\mathrm{H}} = w^{-}}$. It exists because for Case (2) the function $L(w_{\mathrm{H}}, \lambda)$ coincides with $\tilde{L}(w_{\mathrm{H}}, \lambda) = \overline{p}_{\mathrm{L}} U(w_{\mathrm{H}}) + (\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}}) U(H(w_{\mathrm{H}}, \lambda))$ in the neighborhood of the point $w_{\mathrm{H}} = w^{-}$. Indeed, since $H(w^{-}, \lambda) \ge w^{+}(\lambda) > w^{-}$, there exists a neighborhood U of $w_{\mathrm{H}} = w^{-}$ such that $(w, H(w, \lambda)) \in \overline{C}^{\mathrm{U}}$ for every $w \in U$ and hence the min operator in the definition of the function $L(w_{\mathrm{H}}, \lambda)$ is not binding in this neighborhood. The function $\tilde{L}(w_{\mathrm{H}}, \lambda)$ is continuously differentiable because the function $H(w_{\mathrm{H}}, \lambda)$ is continuously differentiable (see proof of Lemma 6) and so is $L(w_{\mathrm{H}}, \lambda)$. Using the derivative of $H(w_{\mathrm{H}}, \lambda)$ with respect to w_{H}, obtained from implicitly differentiating (8), and taking into account the fact that $H(w_{\mathrm{H}}, \lambda) \geq w_{\mathrm{H}}$, we can write the condition $\frac{\partial L(w_{\mathrm{H}},\lambda)}{\partial w_{\mathrm{H}}}|_{w_{\mathrm{H}}=w^-} \leq 0$ for $L(w_{\mathrm{H}}, \lambda)$ to be weakly decreasing in w^- as follows:

$$\begin{aligned}(1-\lambda)(\overline{p}_{\mathrm{H}}-\overline{p}_{\mathrm{L}}) &\geq \lambda\frac{\overline{p}_{\mathrm{L}}}{\overline{p}_{\mathrm{H}}}U'(w^-)\\ &\times\left[\overline{p}_{\mathrm{H}}(1-p_{\mathrm{L}})-p_{\mathrm{L}}\Phi\left(\frac{U(w^-)-(1-\overline{p}_{\mathrm{H}})U\left(H(w^-,\lambda)\right)}{\overline{p}_{\mathrm{H}}}\right)\right.\\ &\left.\times\left[(1-\overline{p}_{\mathrm{H}})U'(H(w^-,\lambda))+\frac{(\overline{p}_{\mathrm{H}}-\overline{p}_{\mathrm{L}})}{\overline{p}_{\mathrm{L}}}\right]\right] \qquad (11)\end{aligned}$$

Fixing $w_{\mathrm{H}} = w^-$, we can simplify the budget constraint (4) to

$$(1-p_{\mathrm{L}})H(w^-,\lambda)+p_{\mathrm{L}}\Phi\left(\frac{U(w^-)-(1-\overline{p}_{\mathrm{H}})U(H(w^-,\lambda))}{\overline{p}_{\mathrm{H}}}\right)=W-p_{\mathrm{L}}d,$$

implying that $H(w^-, \lambda)$ is independent of λ. The left hand side of (11) converges to a strictly positive value as $\lambda \to 0$, whereas the right hand side converges to zero. This implies that for a sufficiently small λ, the function $L(w_{\mathrm{H}}, \lambda)$ is strictly decreasing in $w_{\mathrm{H}} = w^-$. Given that $L(w_{\mathrm{H}}, \lambda)$ is strictly concave in $w_{\mathrm{H}} \in (w^-, w^+(\lambda))$ (see proof of Lemma 6), $w_{\mathrm{H}} = w^-$ is the solution of (9). Hence, for a sufficiently small λ, it holds that $\chi(\lambda) = 0$ and therefore (5) must eventually become binding as $\lambda \to 0$.

We now show that as $\lambda \to 1$, constraint (5) should become slack. This happens because inequality (11) is violated as λ becomes large enough and $w^*_{\mathrm{H}} = w^+(\lambda)$ as $\lambda \to 1$. Indeed, for any $\lambda \in (0, 1)$, the function $L(w_{\mathrm{H}}, \lambda)$ is continuously differentiable on $(w^-, w^+(\lambda))$ (see the argument above and proof of Lemma 6) and its derivative in $w_{\mathrm{H}} \in (w^-, w^+(\lambda))$ is

$$\begin{aligned}\frac{\partial L(w_{\mathrm{H}},\lambda)}{\partial w_{\mathrm{H}}} &= \overline{p}_{\mathrm{L}}U'(w_{\mathrm{H}})\\ &\quad+(\overline{p}_{\mathrm{H}}-\overline{p}_{\mathrm{L}})U'(w_{\mathrm{LG}})\frac{p_{\mathrm{L}}\Phi'(U(w_{\mathrm{LB}}))U'(w_{\mathrm{H}})+\frac{1-\lambda}{\lambda}\overline{p}_{\mathrm{H}}}{p_{\mathrm{L}}(1-\overline{p}_{\mathrm{H}})\Phi'(U(w_{\mathrm{LB}}))U'(w_{\mathrm{LG}})-\overline{p}_{\mathrm{H}}(1-p_{\mathrm{L}})}\\ &\geq U'(w_{\mathrm{H}})\left[\overline{p}_{\mathrm{L}}+(\overline{p}_{\mathrm{H}}-\overline{p}_{\mathrm{L}})\frac{p_{\mathrm{L}}+\frac{(1-\lambda)}{\lambda}\overline{p}_{\mathrm{H}}}{p_{\mathrm{L}}(1-\overline{p}_{\mathrm{H}})-\overline{p}_{\mathrm{H}}(1-p_{\mathrm{L}})}\right]\\ &= U'(w_{\mathrm{H}})\left[\frac{\overline{p}_{\mathrm{H}}(\overline{p}_{\mathrm{L}}-p_{\mathrm{L}})}{\overline{p}_{\mathrm{H}}-p_{\mathrm{L}}}+\frac{\frac{(1-\lambda)}{\lambda}\overline{p}_{\mathrm{H}}(\overline{p}_{\mathrm{H}}-\overline{p}_{\mathrm{L}})}{p_{\mathrm{L}}(1-\overline{p}_{\mathrm{H}})-\overline{p}_{\mathrm{H}}(1-p_{\mathrm{L}})}\right] \qquad (12)\end{aligned}$$

(here we used the inequality $U'(w_{\mathrm{LG}}) \leq U'(w_{\mathrm{H}}) \leq U'(w_{\mathrm{LB}})$ for any two contracts $A_{\mathrm{H}} = (w_{\mathrm{H}}, w_{\mathrm{H}})$ and $A_{\mathrm{L}} = (w_{\mathrm{LG}}, w_{\mathrm{LB}})$ with $w_{\mathrm{H}} \in (w^-, w^+(\lambda))$ satisfying the constraints (2)–(5), see proof of Lemma 6). The first term in the parenthesis in the right hand side of (12) is positive, and the second term tends to zero as $\lambda \to 1$. Therefore, the derivative of $L(w_{\mathrm{H}}, \lambda)$ is positive for a large enough λ for all $w_{\mathrm{H}} \in$

$(w^-, w^+(\lambda))$. Due to the continuity of $L(w_{\mathrm{H}}, \lambda)$, it holds that $w^*_{\mathrm{H}} = w^+(\lambda)$ and hence, $w^*_{\mathrm{LG}} = H(w^+(\lambda), \lambda) = w^+(\lambda) = w^*_{\mathrm{H}}$ and $w^*_{\mathrm{LB}}(\lambda) = w^*_{\mathrm{LG}}(\lambda) = w^*_{\mathrm{H}}(\lambda)$. This implies

$$\lim_{\lambda\to 1} w^*_{\mathrm{LB}}(\lambda) = \lim_{\lambda\to 1} w^*_{\mathrm{LG}}(\lambda) = \lim_{\lambda\to 1} w^*_{\mathrm{H}}(\lambda) = \lim_{\lambda\to 1}(W - p_\lambda d) = (p_{\mathrm{H}} - p_{\mathrm{L}})d.$$

Property (ii). We will first show that both $1 - \overline{p}_{\mathrm{L}} U(w^*_{\mathrm{LG}}(\lambda)) + \overline{p}_{\mathrm{L}} U(w^*_{\mathrm{LB}}(\lambda))$ and $U(w^*_{\mathrm{H}}(\lambda))$ are weakly increasing in λ, and strictly so if (5) is slack. Fix $\lambda_0 \in (0, 1)$ and consider $\lambda = \lambda_0 + \delta$ for some $\delta > 0$. Note that $V^*(\lambda_0)$ satisfies all the constraints of P1 under λ. Indeed, constraints (2), (3) and (5) are satisfied trivially. $V^*(\lambda_0)$ satisfies (4) under λ if and only if

$$(p_{\mathrm{H}} - p_{\mathrm{L}})d - \left[(1 - p_{\mathrm{L}})w^*_{\mathrm{LG}}(\lambda_0) + p_{\mathrm{L}} w^*_{\mathrm{LB}}(\lambda_0) - w^*_{\mathrm{H}}(\lambda_0)\right] \geq 0, \tag{13}$$

and satisfies the budget constraint with slack if and only if the inequality is strict. On the other hand, the binding constraint (4) can be rearranged to

$$\lambda_0 \left[(1 - p_{\mathrm{L}})w^*_{\mathrm{LG}}(\lambda_0) + p_{\mathrm{L}} w^*_{\mathrm{LB}}(\lambda_0) - w^*_{\mathrm{H}}(\lambda_0) - (p_{\mathrm{H}} - p_{\mathrm{L}})d\right] + w^*_{\mathrm{H}}(\lambda_0) \\ = W - p_{\mathrm{H}} d,$$

which together with the fact that $w^*_{\mathrm{H}}(\lambda_0) \geq W - p_{\mathrm{H}} d$ from (5) implies that (13) is always satisfied (as a strict inequality whenever $w^*_{\mathrm{H}}(\lambda_0) > W - p_{\mathrm{H}} d$). Hence, if (5) is binding, the old allocation $V^*(\lambda_0)$ is still feasible under λ and the optimal value of the objective cannot decrease. If (5) is slack, the optimal value of the objective under λ must be strictly larger than under λ_0 (see the proof of Lemma 6).

Now consider the high risks' utility $U(w^*_{\mathrm{H}}(\lambda))$. If (5) is binding, it is given by $U(w^*_{\mathrm{H}}(\lambda)) = U(W - p_{\mathrm{H}} d)$ and is independent of λ. Assume now that (5) is slack, such that $w^*_{\mathrm{H}}(\lambda) \in (w^-, w^+(\lambda))$ satisfies the first-order condition $\frac{\partial L(w_{\mathrm{H}}, \lambda)}{\partial w_{\mathrm{H}}}|_{w_{\mathrm{H}} = w^-} = 0$ (the derivative exists because $w^*_{\mathrm{H}}(\lambda) \in (w^-, w^+(\lambda))$ where the function $L(w_{\mathrm{H}}, \lambda)$ is differentiable) which can be rewritten as

$$(\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}})\frac{(1-\lambda)}{\lambda} = U'(w^*_{\mathrm{H}})\left[\frac{\overline{p}_{\mathrm{L}}(1 - p_{\mathrm{L}})}{U'\left(w^*_{\mathrm{LG}}\right)} - \frac{p_{\mathrm{L}}(1 - \overline{p}_{\mathrm{L}})}{U'\left(w^*_{\mathrm{LB}}\right)}\right]. \tag{14}$$

Suppose now we increase λ and $U(w^*_{\mathrm{H}}(\lambda))$ decreases weakly. The binding incentive compatibility constraint (3) can be rearranged to

$$(1 - \overline{p}_{\mathrm{L}})U\left(w^*_{\mathrm{LG}}(\lambda)\right) + \overline{p}_{\mathrm{L}} U\left(w^*_{\mathrm{LB}}(\lambda)\right) - U\left(w^*_{\mathrm{H}}(\lambda)\right) = \left(\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}}\right)\left(U\left(w^*_{\mathrm{LG}}(\lambda)\right) - U\left(w^*_{\mathrm{LB}}(\lambda)\right)\right).$$

Because $(1 - \overline{p}_{\mathrm{L}})U(w^*_{\mathrm{LG}}(\lambda)) + \overline{p}_{\mathrm{L}} U(w^*_{\mathrm{LB}}(\lambda))$ strictly increases in λ, the term $U(w^*_{\mathrm{LG}}(\lambda)) - U(w^*_{\mathrm{LB}}(\lambda))$ must also be strictly increasing which can be possible only in the case of increasing $U(w^*_{\mathrm{LG}}(\lambda))$ and decreasing $U(w^*_{\mathrm{LB}}(\lambda))$ in λ, given that $U(w^*_{\mathrm{LG}}(\lambda))$ and $U(w^*_{\mathrm{LB}}(\lambda))$ cannot both decrease. Thus, $U'(w^*_{\mathrm{H}}(\lambda))$ is weakly increasing (by assumption), $U'(w^*_{\mathrm{LG}}(\lambda))$ is weakly decreasing and $U'(w^*_{\mathrm{LB}}(\lambda))$ is weakly increasing. This, however, contradicts the fact that both sides of Eq. (14) must decrease with λ.

Assume now that (5) is slack, i.e., $\frac{\partial L(w_{\mathrm{H}},\lambda)}{\partial w_{\mathrm{H}}} > 0$ for all $w_{\mathrm{H}} \in (w^-, w^+(\lambda))$. In this case, by the continuity of $L(w_{\mathrm{H}}, \lambda)$, we have that $U(w^*_{\mathrm{H}}(\lambda)) = U(w^+(\lambda)) = U(W - p_\lambda d)$, which is strictly increasing in λ.

Finally, if (5) is slack and $U(w^*_{\mathrm{H}}(\lambda))$ is strictly increasing at some level of λ, the same clearly holds for all $\lambda' > \lambda$. Together with the previous result that (5) must be binding in $V^*(\lambda)$ for sufficiently small and slack for sufficiently large values of λ, it follows that there exists a value $\tilde{\lambda} \in (0, 1)$ such that for all $\lambda \le \tilde{\lambda}$, constraint (5) will be binding in $V^*(\lambda)$ and neither $V^*(\lambda)$ nor $\chi(\lambda)$ change in λ, while for all $\lambda > \tilde{\lambda}$, (5) is slack and $U(w^*_{\mathrm{H}}(\lambda))$, and $\chi(\lambda)$ is strictly increasing in λ.

Property (iii). Define $\tilde{\tilde{\lambda}} \in (\tilde{\lambda}, 1)$ as

$$\tilde{\tilde{\lambda}} = \sup\left\{\lambda\colon\ \left.\frac{\partial L(w_{\mathrm{H}}, \lambda)}{\partial w_{\mathrm{H}}}\right|_{w_{\mathrm{H}}=w^*_{\mathrm{H}}(\lambda)<w^+(\lambda)} = 0\right\}.$$

The continuity of $w^*_{\mathrm{H}}(\lambda)$ and $L(w_{\mathrm{H}}, \lambda)$ together with the uniqueness of the solution of P1 implies that $w^*_{\mathrm{H}}(\tilde{\tilde{\lambda}}) = w^+(\tilde{\tilde{\lambda}}) = W - p_{\tilde{\tilde{\lambda}}} d = w_{\mathrm{LG}}(\tilde{\tilde{\lambda}}) = w^*_{\mathrm{LB}}(\tilde{\tilde{\lambda}})$.

Consider $\lambda > \tilde{\tilde{\lambda}}$. Suppose there exists a solution $V^*(\lambda)$ of the problem P1 with $(w^*_{\mathrm{LG}}, w^*_{\mathrm{LB}}) \in C^{\mathrm{U}}$. In this case, $w^*_{\mathrm{H}}(\lambda) < w^+(\lambda)$; otherwise, we get a contradiction and violation of the constraint (4). Then,

$$\begin{aligned}
\left.\frac{\partial L(w_{\mathrm{H}}, \lambda)}{\partial w_{\mathrm{H}}}\right|_{w_{\mathrm{H}}=w^*_{\mathrm{H}}(\lambda)} &= \overline{p}_{\mathrm{L}} U'\left(w^*_{\mathrm{H}}\right) \\
&\quad + (\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}}) U'(w_{\mathrm{LG}}) \frac{p_{\mathrm{L}} \Phi'\left(U\left(w^*_{\mathrm{LB}}\right)\right) U'\left(w^*_{\mathrm{H}}\right) + (1-\lambda)\overline{p}_{\mathrm{H}}/\lambda}{p_{\mathrm{L}}(1-\overline{p}_{\mathrm{H}}) \Phi'\left(U\left(w^*_{\mathrm{LB}}\right)\right) U'\left(w^*_{\mathrm{LG}}\right) - \overline{p}_{\mathrm{H}}(1-p_{\mathrm{L}})} \\
&\ge U'\left(w^*_{\mathrm{H}}\right)\left[\overline{p}_{\mathrm{L}} + (\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}}) \frac{p_{\mathrm{L}} + (1-\lambda)\overline{p}_{\mathrm{H}}/\lambda}{p_{\mathrm{L}}(1-\overline{p}_{\mathrm{H}}) - \overline{p}_{\mathrm{H}}(1-p_{\mathrm{L}}))}\right] \\
&> U'\left(w^*_{\mathrm{H}}\right)\left[\overline{p}_{\mathrm{L}} + (\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}}) \frac{p_{\mathrm{L}} + (1-\tilde{\tilde{\lambda}})\overline{p}_{\mathrm{H}}/\tilde{\tilde{\lambda}}}{p_{\mathrm{L}}(1-\overline{p}_{\mathrm{H}}) - \overline{p}_{\mathrm{H}}(1-p_{\mathrm{L}}))}\right].
\end{aligned} \tag{15}$$

Note that the left hand side derivative $\frac{\partial L^-(w_{\mathrm{H}},\lambda)}{\partial w_{\mathrm{H}}}$ exists and is continuous on $A(\lambda) \times (0, 1)$. Indeed, for any $\lambda \in (0, 1)$ and any $w_{\mathrm{H}} \in A(\lambda)$, we have that $(H(w_{\mathrm{H}}, \lambda), w_{\mathrm{H}}) \in \overline{C}^{\mathrm{U}}$ and therefore the function $L(w_{\mathrm{H}}, \lambda)$ coincides on the interval $A(\lambda)$ with the function $\tilde{L}(w_{\mathrm{H}}, \lambda) = \overline{p}_{\mathrm{L}} U(w_{\mathrm{H}}) + (\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}}) U(H(w_{\mathrm{H}}, \lambda))$ which is continuously differential on $\mathbb{R}_+ \times (0, 1)$. Furthermore, given the definition of $w^*_{\mathrm{H}}(\tilde{\tilde{\lambda}})$, it holds that

$$\begin{aligned}
&\left.\frac{\partial L^-(w_{\mathrm{H}}, \lambda)}{\partial w_{\mathrm{H}}}\right|_{\substack{w_{\mathrm{H}} = w^*_{\mathrm{H}}\left(\tilde{\tilde{\lambda}}\right) \\ \lambda = \tilde{\tilde{\lambda}}}} \\
&\quad = U'\left(w^*_{\mathrm{H}}\left(\tilde{\tilde{\lambda}}\right)\right)\left[\overline{p}_{\mathrm{L}} + (\overline{p}_{\mathrm{H}} - \overline{p}_{\mathrm{L}}) \frac{p_{\mathrm{L}} + (1-\tilde{\tilde{\lambda}})\overline{p}_{\mathrm{H}}/\tilde{\tilde{\lambda}}}{p_{\mathrm{L}}(1-\overline{p}_{\mathrm{H}}) - \overline{p}_{\mathrm{H}}(1-p_{\mathrm{L}})}\right] = 0.
\end{aligned}$$

Hence, at the point $w^+(\tilde{\tilde{\lambda}})$, it holds that

$$\left.\frac{\partial L^-(w_H,\lambda)}{\partial w_H}\right|_{\substack{w_H = w^+\left(\tilde{\tilde{\lambda}}\right)\\ \lambda=\tilde{\tilde{\lambda}}}} = \left.\frac{\partial \tilde{L}^-(w_H,\lambda)}{\partial w_H}\right|_{\substack{w_H = w^+\left(\tilde{\tilde{\lambda}}\right)\\ \lambda=\tilde{\tilde{\lambda}}}} = \left.\frac{\partial \tilde{L}(w_H,\lambda)}{\partial w_H}\right|_{\substack{w_H = w^+\left(\tilde{\tilde{\lambda}}\right)\\ \lambda=\tilde{\tilde{\lambda}}}}$$

$$= U'\left(w_H^*\left(\tilde{\tilde{\lambda}}\right)\right)\left[\overline{p}_L + (\overline{p}_H - \overline{p}_L)\frac{p_L + (1-\tilde{\tilde{\lambda}})\overline{p}_H/\tilde{\tilde{\lambda}}}{p_L(1-\overline{p}_H) - \overline{p}_H(1-p_L)}\right] = 0.$$

Finally, note that $\frac{\partial L^-(w_H,\lambda)}{\partial w_H}\Big|_{\substack{w_H = w_H^*(\tilde{\tilde{\lambda}})\\ \lambda=\tilde{\tilde{\lambda}}}} = 0$ which follows from the definition of $\tilde{\tilde{\lambda}}$ and the continuity of the functions $\frac{\partial L^-(w_H,\lambda)}{\partial w_H}$ and $w_H^*(\lambda)$. Hence, this fact and (15) imply that $\frac{\partial L(w_H,\lambda)}{\partial w_H}|_{w_H=w_H^*(\lambda)} > \frac{\partial L(w_H,\lambda)}{\partial w_H}\Big|_{\substack{w_H = w_H^*(\tilde{\tilde{\lambda}})\\ \lambda=\tilde{\tilde{\lambda}}}} = 0$. This means that the function $L(w_H,\lambda)$ is increasing in w_H and therefore $w_H^*(\lambda) = w^+(\lambda)$. This contradicts our earlier assumption. □

Proposition 3 *Suppose that* $\lambda \in [0,\tilde{\lambda}]$ *and the separating allocation* (A_H^*, A_L^*) *is the solution of the problem* P1. *Then, the menus* $\{(A_H^*, A_L^*), 0\}$, $\{(A_H^*, A_L^*), 1\}$ *or any combination of them, involving the unique separating allocation* (A_H^*, A_L^*)*, are Bayes–Nash equilibria with underinsurance and no cross-subsidization across types.*

Proof Lemma 5 implies that the only candidate for an equilibrium in our game is the solution of P1. According to Lemmas 6 and 7, the unique solution of P1 is (A_H^*, A_L^*), where $A_H^* = (W - p_H d, W - p_H d)$ and the contract $A_L^* = (w_{LG}^*, w_{LB}^*)$ is defined by the intersection of the Hs' binding incentive compatibility constraint $U(W - p_H d) = \overline{p}_H U(w_{LB}^*) + (1-\overline{p}_H)U(w_{LG}^*)$ and zero-profit condition $p_L w_{LB}^* + (1-p_L)w_{LG}^* = W - p_L d$.

We now show that the menu $\{(A_H^*, A_L^*), c\}$ involving the separating allocation (A_H^*, A_L^*) is an equilibrium. By Lemma 6, there is no allocation which can profitably attract both types. Also, because, by Lemma 6, the Hs' incentive compatibility constraint is binding and A_L^* implies zero profit, there is no contract that can profitably attract only the Ls (see Fig. 6). Hence, the separating allocation (A_H^*, A_L^*) is the unique equilibrium allocation. However, the menus involving this unique separating allocation may be offered with or without commitment which leads to multiple equilibria. □

Compared to the standard expected utility framework (Rothschild and Stiglitz 1976), the Hs choose the same contract in equilibrium in both cases $A_H^* = A_H^{EU}$. In contrast, the Ls' equilibrium contract under ambiguity aversion, A_L^*, involves more coverage than the corresponding contract in the standard expected utility model, A_L^{EU} (Fig. 6). This difference is due to the fact that ambiguity aversion relaxes the Hs' incentive compatibility constraint allowing the Ls to buy more insurance and move closer to the efficient insurance level.

Proposition 4 *Suppose that* $\lambda \in (\tilde{\lambda}, \tilde{\tilde{\lambda}})$ *and the separating allocation* (A_H^{**}, A_L^{**}) *is the solution of problem* P1. *Then, the menu* $\{(A_H^{**}, A_L^{**}), 0\}$ *which involves underinsurance*

Fig. 6 Efficient separating equilibrium with no cross-subsidies and underinsurance

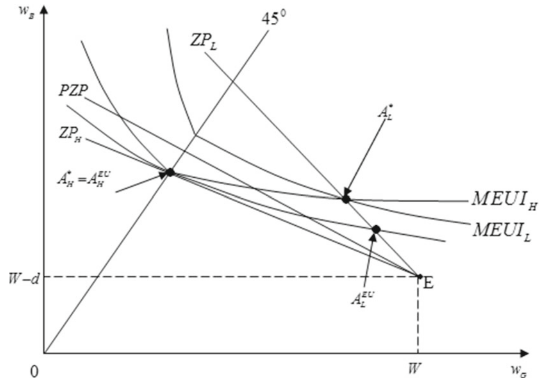
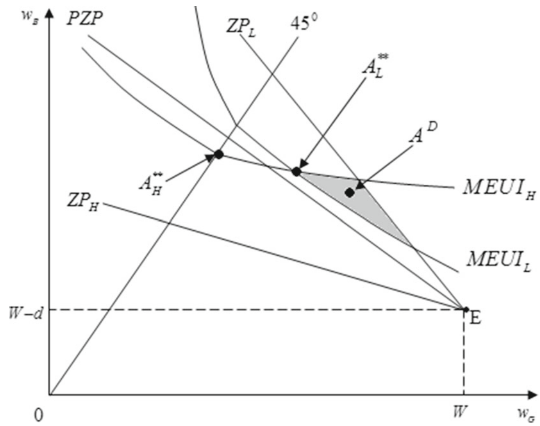


Fig. 7 Efficient separating equilibrium with cross-subsidies and underinsurance



and cross-subsidy across types is the unique Bayes–Nash equilibrium of our game (*see Fig.* 7).[12]

Proof According to Lemma 5, the allocation $(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**})$, which is the solution of P1, is the only candidate for an equilibrium. Statement (iii) of Lemma 7 implies that this allocation is separating and involves cross-subsidization across types.

We first show that $\{(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**}), 1\}$ cannot be an equilibrium. Suppose that, at Stage 1, an insurer offers $\{(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**}), 1\}$. Consider a new entrant who offers the deviant menu $\{(A^{\mathrm{D}}), 1\}$ (Fig. 7). Because the incumbent has committed to the menu $\{(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**}), 1\}$, the deviant menu $\{(A^{\mathrm{D}}), 1\}$ attracts only the Ls and so $\{(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**}), 1\}$ becomes loss-making and cannot be an equilibrium.

Suppose now that, at Stage 1, an insurer offers the separating allocation $(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**})$ through two single-contract menus: $\{(A_{\mathrm{H}}^{**}), c\}$ and $\{(A_{\mathrm{L}}^{**}), c\}$, where $c \in \{0, 1\}$. From the argument above, if the insurer commits to both menus, his strategy will

[12] Notice that if we employed the two-stage screening game widely used in applied theory papers (e.g., Rothschild and Stiglitz 1976), the nonexistence of equilibrium problem would arise in cases the Rothschild–Stiglitz separating allocation was not interim incentive efficient.

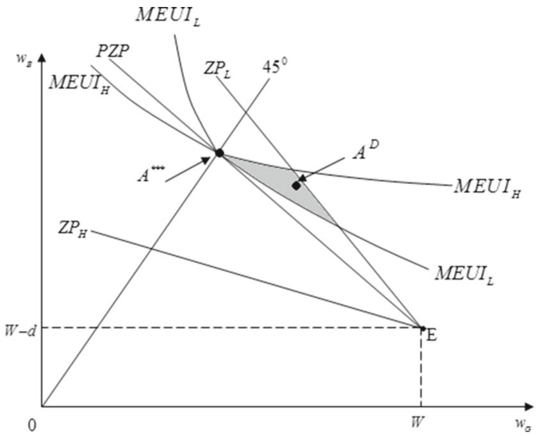
be loss-making and so it cannot be an equilibrium strategy. Clearly, the strategy of offering $\{(A_{\mathrm{H}}^{**}), 1\}$ and $\{(A_{\mathrm{L}}^{**}), 0\}$ will also be loss-making for the insurer choosing it. Hence, it cannot be an equilibrium strategy either. Finally, consider the case where the insurer offers $\{(A_{\mathrm{L}}^{**}), c\}$ and $\{(A_{\mathrm{H}}^{**}), 0\}$, where $c \in \{0, 1\}$. Then, at Stage 3, the insurer would withdraw the loss-making menu $\{(A_{\mathrm{H}}^{**}), 0\}$. Anticipating that, all insurees would choose the menu $\{(A_{\mathrm{L}}^{**}), c\}$ which would also become loss-making (it lies above the pooling zero-profit line). Thus, this strategy cannot be an equilibrium strategy. Therefore, the menu $\{(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**}), 0\}$ is the only candidate for equilibrium.

There are two different types of deviations which could potentially destroy the equilibrium: One is to offer a menu involving an allocation different from $(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**})$, and the other one is to offer a menu involving the allocation $(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**})$ but different strategies.

We start with the first type of deviations. Because the separating allocation $(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**})$ is the solution of P1, there cannot exist an incentive compatible allocation which attracts both types profitably (see Lemma 6). Thus, the potentially profitable deviations would be the ones that attract only the Ls. Since the aim is to attract only the Ls, without loss of generality, we can consider only the menus consisting of a single contract. This contract should be in the shaded area in Fig. 7. Suppose now that a new entrant offers the deviant menu $\{(A^{\mathrm{D}}), 1\}$ (Fig. 7). Given the incumbent's menu, at Stage 2, the Ls will choose the deviant menu. As a result, the incumbent's menu becomes loss-making and so it will be withdrawn at Stage 3. Anticipating that, the Hs will also choose the deviant menu at Stage 2 and so the deviant menu becomes loss-making too. Finally, suppose that a new entrant offers the deviant menu $\{(A^{\mathrm{D}}), 0\}$. This off-the-equilibrium path sub-game has two equilibria: (i) both types take the deviant menu and so it becomes loss-making; (ii) both types stay in the incumbent's menu. In either case, the deviant strategy is not profitable. Therefore, there is no profitable deviation of this type.

The second type of potential deviations involves the same allocation but different strategies. Suppose that a new entrant offers two menus $\{(A_{\mathrm{L}}^{**}), c\}, c \in \{0, 1\}$ and $\{(A_{\mathrm{H}}^{**}), c\}, c \in \{0, 1\}$. The menu $\{(A_{\mathrm{H}}^{**}), 0\}$ is loss-making and since the new entrant is not committed to it, it will be withdrawn at Stage 3. Anticipating that, none of the Hs will choose $\{(A_{\mathrm{H}}^{**}), 0\}$ at Stage 2. Notice that the Ls are indifferent between $\{(A_{\mathrm{H}}^{**}, A_{\mathrm{L}}^{**}), 0\}$ and $\{(A_{\mathrm{L}}^{**}), c\}$. As a result, in this out-of-equilibrium sub-game, there are two types of equilibria: (i) all Hs and Ls choose the incumbent's menu and so the equilibrium is not upset; (ii) the insurees believe that some Ls (a strictly positive measure of them) will choose $\{(A_{\mathrm{L}}^{**}), c\}$ at Stage 2. So, the incumbent's menu becomes loss-making, and it will be withdrawn at Stage 3. Given this belief, both types will choose $\{(A_{\mathrm{L}}^{**}), c\}$ at Stage 2. As a result, $\{(A_{\mathrm{L}}^{**}), c\}$ becomes loss-making and the new entrant cannot make any profit. Suppose that a new entrant offers two menus $\{(A_{\mathrm{L}}^{**}), c\}$ and $\{(A_{\mathrm{H}}^{**}), 1\}$. The resulting out-of-equilibrium sub-game has two equilibria. (i) The belief is that the proportion of the Ls who switch to the deviant menu is bigger than λ. In this case, the incumbent's menu becomes loss-making and will be withdrawn in Stage 3. Hence, both types take the corresponding deviant menu. As a result, the deviation does not make a strictly positive profit. (ii) The belief is that the proportion of the Ls who switch to the deviant menu is smaller than λ. In this case, the incumbent's menu remains profitable and will not be withdrawn in Stage 3. In contrast, the deviant

Fig. 8 Efficient pooling equilibrium



is loss-making. Hence, there is no profitable deviation, and the unique equilibrium of our game is $\{(A^{**}_{\mathrm{H}}, A^{**}_{\mathrm{L}}), 0\}$. □

The intuition behind this separating equilibrium is straightforward: The Ls accept to subsidize the Hs in order to relax the Hs' incentive compatibility constraint, buy more insurance and reduce the disutility cost of underinsurance. If the Ls' proportion is sufficiently high, the utility benefit (due to higher insurance) exceeds the cost of subsidization. The existence of this equilibrium relies on two elements: (i) in equilibrium, both types choose the same menu (but different contracts within the menu); (ii) the equilibrium menu is offered without commitment. Notice, however, that in our game, these two key elements emerge endogenously (they are optimal responses by the players).

Proposition 5 *For any* $\lambda \in (\tilde{\tilde{\lambda}}, 1]$, *the menu* $\{(A^{***}), 0\}$ *involving the pooling allocation* $A^{***} = (W - p_\lambda d, W - p_\lambda d)$, *which is the solution of* P1, *is the unique Bayes–Nash equilibrium of our game* (*see Fig.* 8).

Proof According to Lemma 5, only the solution to P1 can be a candidate for equilibrium. Statement (iii) of Lemma 7 implies that the pooling allocation A^{***} is the solution.

We first show that $\{(A^{***}), 1\}$ cannot be an equilibrium. Suppose that, at Stage 1, an insurer offers $\{(A^{***}), 1\}$. Consider a new entrant who offers the deviant menu $\{(A^{\mathrm{D}}), 1\}$ (see Fig. 8). Because the incumbent has committed to his offer, the deviant menu $\{(A^{\mathrm{D}}), 1\}$ attracts only the Ls and so $\{(A^{***}), 1\}$ becomes loss-making. Hence, it cannot be an equilibrium.

We now show that $\{(A^{***}), 0\}$ is an equilibrium. The only way to potentially destroy the equilibrium is to offer a menu involving an allocation different from A^{***}. Because A^{***} is the solution of P1, there cannot exist an incentive compatible allocation which attracts both types profitably (see Lemma 6). Thus, the potentially profitable deviations would be the ones that attract only the Ls. Again, without loss of generality, we consider only menus consisting of a single contract. This contract can only be in the shaded

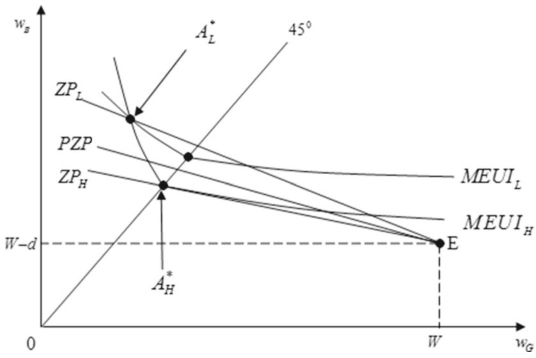


Fig. 9 Efficient separating equilibrium with no cross-subsidies and overinsurance

area on Fig. 8. Suppose now that a new entrant offers the deviant menu $\{(A^{\mathrm{D}}), 1\}$. Given the incumbent's menu, at Stage 2, the Ls will choose the deviant contract. As a result, the menu becomes loss-making and so it will be withdrawn at Stage 3. Anticipating that, the Hs will also choose the deviant menu at Stage 2 and so the deviant menu becomes loss-making. Finally, suppose that a new entrant offers the deviant menu $\{(A^{\mathrm{D}}), 0\}$. This off-the-equilibrium path sub-game has two equilibria: (i) both types take the deviant menu and so it becomes loss-making; (ii) both types stay in the incumbent's menu. In either case, the deviant strategy is not profitable. Therefore, there is no profitable deviation, and $\{(A^{***}), 0\}$ is the unique equilibrium of our game. □

The basic intuition for the existence of this pooling equilibrium is similar to that in Proposition 1. That is, the high degree of ambiguity makes the cost of separation prohibitively high for the Ls. However, there are two main differences between the two pooling equilibria: First, the pooling equilibrium of Proposition 1 exists regardless of the proportion of the Ls whereas that of Proposition 5 exists only if the proportion of Ls is sufficiently high. Second, the pooling equilibrium of Proposition 1 exists even if the standard two-stage screening game is used while that of Proposition 5 does not. Finally, it should be noticed that the pooling equilibrium of Proposition 5 does not exist under standard expected utility. In the latter case, regardless of the Ls' proportion, a separating equilibrium always arises (except for the limiting case where the Ls' proportion is 1).[13]

Case 3 $\overline{p}_{\mathrm{L}} > \overline{p}_{\mathrm{H}} > \underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$

In this case, the indifference curves of the two types intersect only once but, contrary to the standard expected utility framework, the Ls' indifference curves are flatter. Hence, if the equilibrium is separating, it involves the Ls taking overinsurance. This result, which is due to ambiguity aversion, is in sharp contrast with those of the expected utility model (Fig. 9).

Lemma 8 *Suppose* $\overline{p}_{\mathrm{L}} > \overline{p}_{\mathrm{H}} > \underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$. *Then:*

[13] See, for example, Diasakos and Koufopoulos (2013) and Netzer and Scheuer (2014).

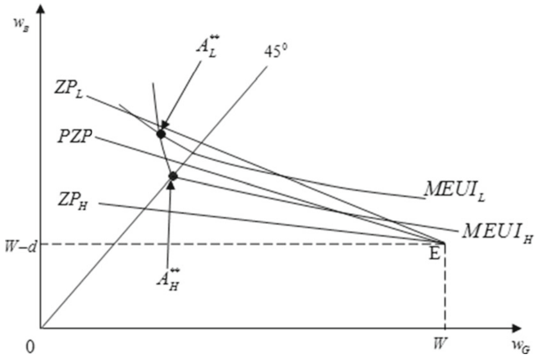


Fig. 10 Efficient separating equilibrium with overinsurance and cross-subsidies

(i) $\chi(\lambda)$ *is continuous on* $(0, 1)$, $\lim_{\lambda\to 0} \chi(\lambda) = 0$ *and* $\lim_{\lambda\to 1} \chi(\lambda) = (p_{\mathrm{H}} - p_{\mathrm{L}})d > 0$.

(ii) *There exists* $\tilde{\lambda} \in (0, 1)$ *such that* $\chi(\lambda) = 0$ *for all* $\lambda \leq \tilde{\lambda}$ *and* $\chi(\lambda)$ *is strictly increasing in* λ *for all* $\lambda > \tilde{\lambda}$.

(iii) *Moreover, there exists* $\tilde{\tilde{\lambda}} \in (\tilde{\lambda}, 1]$ *such that* $(w^*_{\mathrm{LG}}, w^*_{\mathrm{LB}}) \in C^{\mathrm{U}}$ *for all* $\lambda < \tilde{\tilde{\lambda}}$ *and* $w^*_{\mathrm{LG}} = w^*_{\mathrm{LB}} = w^*_{\mathrm{H}}$ *for all* $\lambda \geq \tilde{\tilde{\lambda}}$.

Proof The proof is similar to the one of Lemma 7. □

Proposition 6 *Suppose that* $\lambda \in [0, \tilde{\lambda}]$ *and the separating allocation* $(A^*_{\mathrm{H}}, A^*_{\mathrm{L}})$ *is the solution of the problem* P1. *Then, the menus* $\{(A^*_{\mathrm{H}}, A^*_{\mathrm{L}}), 0\}$, $\{(A^*_{\mathrm{H}}, A^*_{\mathrm{L}}), 1\}$ *or any combination of them, involving the unique separating allocation* $(A^*_{\mathrm{H}}, A^*_{\mathrm{L}})$*, are Bayes–Nash equilibria with overinsurance and no cross-subsidization across types.*

Proof Similar to Proposition 3. □

Because the lower bound of the Ls' accident probability, $\underline{p}_{\mathrm{L}}$, is higher than that of the Hs, $\underline{p}_{\mathrm{H}}$, the utility cost of overinsurance is lower for the Ls. As a result, the insurers can separate the two types of insurees by offering contracts involving overinsurance. The Ls prefer overinsurance at a lower per-unit premium to full insurance at a high (pooling) per-unit premium. In contrast, because the Ls' highest accident probability, $\overline{p}_{\mathrm{L}}$, is greater than the corresponding probability of the Hs, $\overline{p}_{\mathrm{H}}$, the utility cost of underinsurance is higher for the Ls. Hence, insurers cannot profitably attract the Ls by offering contracts involving less than full coverage (underinsurance). Therefore, there can exist separating equilibria involving overinsurance but not underinsurance. Notice that if we impose the no-overinsurance restriction (indemnity principle), the unique equilibrium would be pooling with full insurance.

Proposition 7 *Suppose that* $\lambda \in (\tilde{\lambda}, \tilde{\tilde{\lambda}})$ *and the separating allocation* $(A^{**}_{\mathrm{H}}, A^{**}_{\mathrm{L}})$ *is the solution of problem* P1. *Then, the menu* $\{(A^{**}_{\mathrm{H}}, A^{**}_{\mathrm{L}}), 0\}$ *which involves overinsurance and cross-subsidy across types is the unique Bayes–Nash equilibrium of our game* (*see Fig.* 10).

Proof Similar to Proposition 4. □

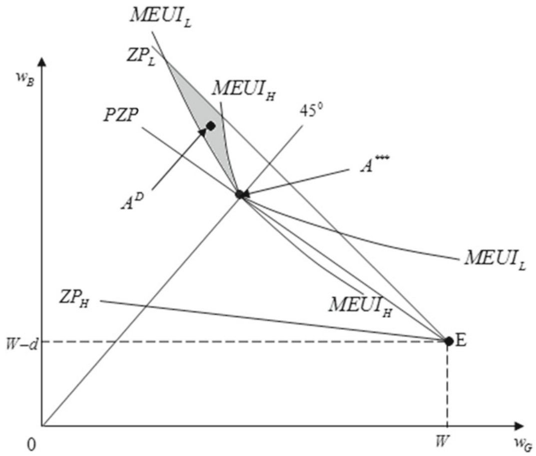


Fig. 11 Efficient pooling equilibrium

Proposition 8 *For any* $\lambda \in (\tilde{\tilde{\lambda}}, 1]$*, the menu* $\{(A^{***}), 0\}$ *involving the pooling allocation* $A^{***} = (W - p_\lambda d, W - p_\lambda d)$*, which is the solution of* P1*, is the unique Bayes–Nash equilibrium of our game* (*see Fig.* 11).

Proof Similar to Proposition 5. □

Case 4 $\overline{p}_{\mathrm{H}} > \overline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{L}} > \underline{p}_{\mathrm{H}}$

In this case, the Ls' indifference curves are steeper in the underinsurance region and flatter in the overinsurance region. Therefore, depending on the relative slopes of the indifference curves of the two types, there can exist either pooling or separating equilibria involving either under- or overinsurance which are similar to those in Cases (2) and (3), respectively.

6 Conclusions

In this paper, we examine the impact of ambiguity aversion on the equilibrium allocation in competitive insurance markets with asymmetric information. We derive a number of interesting results which are due to ambiguity aversion. First, for some parameter values, there exists a unique pooling equilibrium where both types of insurees buy full insurance. This result is driven by ambiguity aversion as it cannot obtain under standard expected utility. Second, we show that under ambiguity aversion, the equilibrium contract of the Ls is closer to their first-best one than under standard expected utility. In fact, ambiguity aversion relaxes the (binding) incentive compatibility constraint of the Hs. As a result, the Ls buy more insurance (while still revealing their type) and move closer to their first-best allocation.

Another distinguishing feature of our model is that the mechanism we employ in this paper is optimal (the equilibrium is always interim incentive efficient). This implies that the results discussed above are driven by ambiguity aversion and not by the sub-optimality of the mechanism used. To the best of our knowledge, none of the existing papers in this literature considers the optimality of the mechanism employed and so it is not clear whether their results are only driven by ambiguity aversion or by the (possible) sub-optimality of the mechanism.

Finally, although in this paper we have focused on insurance markets, the introduction of ambiguity aversion into an asymmetric information framework may have interesting implications for other issues as well. The design of managerial compensation schemes, the choice between self employment and being an employee, the design of financial contracts and other corporate finance issues are only some of them.

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