A Novel Block Sparse Reconstruction Method for DOA Estimation With Unknown Mutual Coupling

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***Abstract*—In this letter, we consider the direction-of-arrival (DOA) estimation in the presence of unknown mutual coupling in application to uniform linear arrays (ULAs). A novel method is proposed that treats the DOA estimation as a block sparse signal reconstruction problem with a modified array manifold matrix which utilizes the information of entire array output. A block smoothed ℓ_0 norm approximation technique is introduced to solve the problem. Finally, an iterative proximal algorithm is proposed. Simulation results are presented that show the superiority of the proposed method against other known techniques.**

***Index Terms*—Direction-of-arrival (DOA), mutual coupling, block sparse reconstruction, block smoothed ℓ_0 (BSL0).**

I. Introduction

DIRECTION-OF-ARRIVAL (DOA) estimation is a problem important in a variety of fields, including radar, sonar, mobile communications [1], [2]. Conventional high-resolution DOA estimators, such as MUSIC and ESPRIT, are well suited for ideal scenarios when there is no mutual coupling (MC) between antennas. In practice, the received signals obtained by arrays are usually disturbed by MC, which severely degrades the DOA estimation performance. For example, in multiple-input multiple-output (MIMO) systems, which are widely investigated for future generations of communications, antennas are closely spaced which causes a high risk of MC among antennas [3].

The MC effect is generally characterized by MC matrix (MCM) which depends on both the array geometry and antenna type. The MCM of a uniform linear antenna (ULA) array has a banded symmetric Toeplitz structure [4]. It is pointed out in [5] that MC is independent of DOAs for omni-directional array elements.

Several methods have been devised for the DOA estimation under MC [5]–[7]. In [5], the direction independent MC is treated as direction dependent complex array gains, and the DOAs and MCM are jointly estimated. A rank-reduction method is proposed in [6] for uniform rectangular arrays. In [7], a subspace-based iterative method is proposed for uniform circular array.

In the past few years, sparse signal representation (SSR) and compressed sensing techniques have made remarkable achievements in fields of statistical signal analysis and parameter estimation. Due to the superiority in resolution and robustness to noise, SSR exploiting the spatial sparsity has been introduced into DOA estimation. In [8], a sparse reconstruction model for DOA estimation is presented and an algorithm named ℓ_1-SVD is proposed to analyze this model. An ℓ_1-SVD-like method under unknown MC for ULA has been proposed in [9], which takes advantage of the banded symmetric Toeplitz structure of MCM, but this method sacrifices the array aperture so that some array output data is not being used. The authors in [10] studied a joint sparse recovery of DOAs and array perturbation, in which the perturbation matrix is estimated by solving a sparse matrix completion problem. Inspired by the method in [5], an effective block sparse representation model is considered for DOA estimation in the presence of unknown MC for ULA by [11]. In order to further exploit the block sparsity of the signal, a reweighted ℓ_1-norm method [12] is introduced to solve the block sparse DOA estimation with MC, where the weighted matrix is determined by a MUSIC-like spectrum. In [13], a sparsity-inducing method over covariance matrix is proposed, which provides larger degrees of freedom and array aperture. A unified self-calibration framework for DOA estimation in the presence of non-ideal array is proposed in [14] using the sparse Bayesian learning perspective.

In this letter, a novel block sparse reconstruction method is devised and analyzed for DOA estimation with unknown MC, which first formulates a modified array manifold matrix without MC compensation, and establishes block sparse reconstruction model which differs from the models in [11], [12]. Then a family of block smoothed ℓ_0 approximation functions are utilized to exploit the block sparsity. According to proximal splitting approach, the block sparse problem is solved with the help of a iterative proximal algorithm. The proposed method utilizes entire array output data and does not require the hypothesis that the scalar parameters related to DOAs and MC are not zero [5], [11], [12].

II. Data Model

Consider K narrowband signals located at far-field impinging on a ULA of M omnidirectional sensors with half-wavelength spacing. The steering vector for the direction θ is expressed as

$$\boldsymbol{a}(\theta) = [1, e^{-j\pi\sin(\theta)}, \ldots, e^{-j\pi(M-1)\sin(\theta)}]^T. \tag{1}$$

Hence, the array output vector at snapshot t is given by

$$\boldsymbol{y}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{2}$$

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where $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ denotes the ideal array manifold matrix, and $\boldsymbol{s}(t) = [s_1(t), \ldots, s_K(t)]^T \in \mathbb{C}^{K\times 1}$ are signal sources. Suppose $\boldsymbol{n}(t)$ is the complex white Gaussian noise vector with zero mean and covariance $\sigma^2\boldsymbol{I}$ and $\boldsymbol{I}$ denotes the identity matrix.

In practice there are negative effects from MC, which are charactered by an MCM. For ULA, the MCM has a banded symmetric Toeplitz structure, which is modeled as

$$\boldsymbol{C} = \text{Toeplitz}([\boldsymbol{c}^T, \boldsymbol{0}_{1\times(M-m)}]), \tag{3}$$

where $\boldsymbol{c} = [c_0, c_1, \ldots, c_{m-1}]^T \in \mathbb{C}^{m\times 1}$ with $0 < |c_{m-1}| < |c_{m-2}|, \ldots, |c_1| < |c_0| = 1$ is a complex mutual coupling coefficients vector, and Toeplitz$(\cdot)$ is symmetric Toeplitz matrix operator. Then the array output is reformulated as

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t). \tag{4}$$

III. Proposed Method

Herein, a novel block sparse reconstruction model for DOA estimation in the presence of unknown MC is derived using a modified array manifold matrix and an iterative proximal method.

A. Modified Array Manifold Matrix With Mutual Coupling

In order to estimate the DOAs and MC coefficients, the data model in (4) should be reformulated. According to (4), it can be seen that the ideal manifold matrix $\boldsymbol{A}$ is distorted by MCM, and hence, the new array steering vector $\tilde{\boldsymbol{a}}(\theta)$ can be defined as

$$\tilde{\boldsymbol{a}}(\theta) = \boldsymbol{C}\boldsymbol{a}(\theta). \tag{5}$$

The MCM (3) can be represented as

$$\boldsymbol{C} = \sum_{l=0}^{m-1} \boldsymbol{E}_l c_l, \tag{6}$$

where $\boldsymbol{E}_l$ is a symmetric matrix of the same size as $\boldsymbol{C}$, and whose (i, j)th entry is given by

$$\boldsymbol{E}_l(i,j) = \begin{cases} 1, & C(i,j) = c_l \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Then the new steering vector $\tilde{\boldsymbol{a}}(\theta)$ can be represented as

$$\begin{aligned} \tilde{\boldsymbol{a}}(\theta) &= \sum_{l=0}^{m-1} \boldsymbol{E}_l c_l \boldsymbol{a}(\theta) \\ &= c_0\boldsymbol{E}_0\boldsymbol{a}(\theta) + c_1\boldsymbol{E}_1\boldsymbol{a}(\theta) + \cdots + c_{m-1}\boldsymbol{E}_{m-1}\boldsymbol{a}(\theta) \\ &= [\boldsymbol{E}_0\boldsymbol{a}(\theta), \boldsymbol{E}_1\boldsymbol{a}(\theta), \ldots, \boldsymbol{E}_{m-1}\boldsymbol{a}(\theta)] \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{m-1} \end{bmatrix} \\ &= \boldsymbol{H}(\theta)\boldsymbol{c}, \end{aligned} \tag{8}$$

where $\boldsymbol{H}(\theta) = [\boldsymbol{E}_0\boldsymbol{a}(\theta), \boldsymbol{E}_1\boldsymbol{a}(\theta), \ldots, \boldsymbol{E}_{m-1}\boldsymbol{a}(\theta)]$. When the array geometry and MCM are fixed, the matrix $\boldsymbol{H}(\theta)$ is a function of the DOA θ.

The array output model (4) can now be rewritten in the following form:

$$\begin{aligned} \boldsymbol{y}(t) &= [\tilde{\boldsymbol{a}}(\theta_1), \tilde{\boldsymbol{a}}(\theta_2), \ldots, \tilde{\boldsymbol{a}}(\theta_K)]\boldsymbol{s}(t) + \boldsymbol{n}(t) \\ &= [\boldsymbol{H}(\theta_1)\boldsymbol{c}, \boldsymbol{H}(\theta_2)\boldsymbol{c}, \ldots, \boldsymbol{H}(\theta_K)\boldsymbol{c}]\boldsymbol{s}(t) + \boldsymbol{n}(t) \\ &= [\boldsymbol{H}(\theta_1), \boldsymbol{H}(\theta_2), \ldots, \boldsymbol{H}(\theta_K)]\tilde{\boldsymbol{s}}(t) + \boldsymbol{n}(t) \\ &= \tilde{\boldsymbol{A}}\tilde{\boldsymbol{s}}(t) + \boldsymbol{n}(t), \end{aligned} \tag{9}$$

where $\tilde{\boldsymbol{A}} = [\boldsymbol{H}(\theta_1), \boldsymbol{H}(\theta_2), \ldots, \boldsymbol{H}(\theta_K)] \in \mathbb{C}^{M\times Km}$ is the modified array manifold matrix, and $\tilde{\boldsymbol{s}}(t) \in \mathbb{C}^{Km\times 1}$ is a vector defined as

$$\begin{aligned} \tilde{\boldsymbol{s}}(t) = \boldsymbol{s}(t) \otimes \boldsymbol{c} &= \begin{bmatrix} s_1(t)\boldsymbol{c} \\ s_2(t)\boldsymbol{c} \\ \vdots \\ s_K(t)\boldsymbol{c} \end{bmatrix} \\ &= [\tilde{\boldsymbol{s}}_1^T(t), \tilde{\boldsymbol{s}}_2^T(t), \ldots, \tilde{\boldsymbol{s}}_K^T(t)]^T, \end{aligned} \tag{10}$$

where $\otimes$ denotes the Kronecker product, and $\tilde{\boldsymbol{s}}_i(t) = s_i(t)\boldsymbol{c} \in \mathbb{C}^{m\times 1}, i = 1, 2, \ldots, K$, denotes a new block signal.

B. Block Sparse Reconstruction Model for DOA Estimation With Mutual Coupling

To utilize the spatial sparsity, let $\bar{\boldsymbol{\theta}} = \{\bar{\theta}_1, \cdots, \bar{\theta}_N\}$ with $N \gg M$ denotes the uniform discrete sampling grid points of spatial continuous DOA, and assume that the true DOAs $\boldsymbol{\theta}$ exactly lie on the sampling grid $\bar{\boldsymbol{\theta}}$ (i.e $\boldsymbol{\theta} \subset \bar{\boldsymbol{\theta}}$). Then an overcomplete array manifold dictionary in terms of $\bar{\boldsymbol{\theta}}$ is written as

$$\bar{\boldsymbol{A}} = [\boldsymbol{H}(\bar{\theta}_1), \boldsymbol{H}(\bar{\theta}_2), \ldots, \boldsymbol{H}(\bar{\theta}_N)], \tag{11}$$

where $\bar{\boldsymbol{A}} \in \mathbb{C}^{M\times Nm}$ has a block structure and each $M\times m$ block array steering matrix $\boldsymbol{H}(\bar{\theta}_k), k = 1, 2, \cdots, N$, corresponds to a potential DOA. As a result, the sparse-based model for the array output is given by

$$\boldsymbol{y} = \bar{\boldsymbol{A}}\bar{\boldsymbol{s}}_{blk} + \boldsymbol{n}, \tag{12}$$

where

$$\begin{aligned} \bar{\boldsymbol{s}}_{blk} &= [\bar{s}_1\boldsymbol{c}^T, \bar{s}_2\boldsymbol{c}^T, \cdots, \bar{s}_N\boldsymbol{c}^T]^T \\ &= [\bar{\boldsymbol{s}}_1^T, \bar{\boldsymbol{s}}_2^T, \cdots, \bar{\boldsymbol{s}}_N^T]^T \end{aligned}$$

is an $Nm \times 1$ block sparse vector with block size m. Since there are no zero entries in the coefficients vector $\boldsymbol{c}$, the block sparsity of $\bar{\boldsymbol{s}}_{blk}$ is the same as the sparsity of $\bar{\boldsymbol{s}} = [\bar{s}_1, \bar{s}_2, \cdots, \bar{s}_N]^T$.

Now, the DOA estimation problem with unknown MC can be reformulated into the following block sparse signal reconstruction problem:

$$\min_{\bar{\boldsymbol{s}}_{blk}} \|\bar{\boldsymbol{s}}_{blk}\|_{2,0} \quad s.t. \quad \|\boldsymbol{y} - \bar{\boldsymbol{A}}\bar{\boldsymbol{s}}_{blk}\|_2 \le \varepsilon, \tag{13}$$

where $\|\cdot\|_{2,0}$ denotes ℓ_2/ℓ_0 mixed norm operator defined as $\|\bar{\boldsymbol{s}}_{blk}\|_{2,0} = \sum_{k=1}^{N} \mathcal{I}(\|\bar{\boldsymbol{s}}_k\|_2)$, and $\mathcal{I}(\cdot)$ is an indicator function

$$\mathcal{I}(x) = \begin{cases} 0, & x = 0 \\ 1, & \text{otherwise.} \end{cases} \tag{14}$$

The block reconstruction problem in (13) is mathematically intractable because it is NP-hard. To simplify the solution, an alternative block smoothed ℓ_0 approximation is considered. Specifically, complex zero-mean Gaussian family of functions are introduced to replace the ℓ_0 norm function. The smoothed ℓ_0 approximation can better exploit the sparsity than ℓ_1 norm which is a convex norm closest to ℓ_0 norm.

Consider a complex zero-mean Gaussian family of functions:

$$f_\sigma(\boldsymbol{x}) = 1 - \exp\left(-\frac{\sum_{j=1}^{m}|x_j|^2}{\sigma^2}\right), \tag{15}$$

where $\boldsymbol{x} = [x_1, x_2, \cdots, x_m]^T$. Then, we have

$$\|\bar{\boldsymbol{s}}_{blk}\|_{2,0} = \lim_{\sigma\to 0}\sum_{k=1}^{N} f_\sigma(\bar{\boldsymbol{s}}_k) = \lim_{\sigma\to 0}\boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk}), \tag{16}$$

where $\boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk}) = \sum_{k=1}^{N} f_\sigma(\bar{\boldsymbol{s}}_k)$, and σ is a scalar parameter which provides a trade-off between the approximation accuracy and smoothness. A decreasing sequence for σ can be used to circumvent jumping into the trap of local minimum [15]. As a result, the block sparse reconstruction problem with the block smoothed ℓ_0 norm penalty is expressed as

$$\min_{\bar{\boldsymbol{s}}_{blk}} \boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk}) \quad s.t. \quad \|\boldsymbol{y} - \bar{\boldsymbol{A}}\bar{\boldsymbol{s}}_{blk}\|_2 \le \varepsilon. \tag{17}$$

By defining a set

$$\mathcal{A}_\varepsilon \triangleq \{\bar{\boldsymbol{s}}_{blk} : \|\boldsymbol{y} - \bar{\boldsymbol{A}}\bar{\boldsymbol{s}}_{blk}\|_2 \le \varepsilon\}, \tag{18}$$

the constrained optimization problem (17) can be reformulated as the following unconstrained optimization problem

$$\min_{\bar{\boldsymbol{s}}_{blk}} \boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk}) + \mathcal{I}_\varepsilon(\bar{\boldsymbol{s}}_{blk}) \tag{19}$$

where $\mathcal{I}_\varepsilon(\cdot)$ is an indicator function, given as

$$\mathcal{I}_\varepsilon(\bar{\boldsymbol{s}}_{blk}) = \begin{cases} 0, & \bar{\boldsymbol{s}}_{blk} \in \mathcal{A}_\varepsilon \\ +\infty, & \bar{\boldsymbol{s}}_{blk} \notin \mathcal{A}_\varepsilon. \end{cases} \tag{20}$$

Note that the function $\boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk})$ is non-convex but gradient Lipschitz continuous [16]. The indicator function $\mathcal{I}_\varepsilon$ is convex because the set $\mathcal{A}_\varepsilon$ is convex. As a result, (19) is a non-convex optimization problem which can easily fall into the trap of local minimum.

For jumping out the trap of local minimum, the majorization-minimization (MM) [16] approach is introduced to solve the optimization (19). Since $\boldsymbol{H}_\sigma$ is gradient Lipschitz continuous, for all $\boldsymbol{x}, \boldsymbol{y}$, we have [17]

$$\boldsymbol{H}_\sigma(\boldsymbol{x}) \le \boldsymbol{Q}_\sigma(\boldsymbol{x}, \boldsymbol{y}), \tag{21}$$

where

$$\boldsymbol{Q}_\sigma(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{H}_\sigma(\boldsymbol{y}) + \langle \boldsymbol{x} - \boldsymbol{y}, \nabla_{\boldsymbol{x}}\boldsymbol{H}_\sigma(\boldsymbol{y})\rangle + \frac{L}{2}\|\boldsymbol{x} - \boldsymbol{y}\|_2^2$$

is a quadratic upper-bound of $\boldsymbol{H}_\sigma$ at $\boldsymbol{y}$, and L is the Lipschitz constant of $\nabla\boldsymbol{H}_\sigma$. The non-convex function $\boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk})$ is replaced by the surrogate function $\boldsymbol{Q}_\sigma(\bar{\boldsymbol{s}}_{blk}, \bar{\boldsymbol{s}}_{blk}^t)$, and then the non-convex problem (19) can be solved by the following iterative proximal algorithm

$$\begin{aligned}
\bar{\boldsymbol{s}}_{blk}^{t+1} &= \arg\min_{\bar{\boldsymbol{s}}_{blk}} \boldsymbol{Q}_\sigma(\bar{\boldsymbol{s}}_{blk}, \bar{\boldsymbol{s}}_{blk}^t) + \mathcal{I}_\varepsilon(\bar{\boldsymbol{s}}_{blk}) \\
&= \arg\min_{\bar{\boldsymbol{s}}_{blk}} \boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk}^t) + \langle \bar{\boldsymbol{s}}_{blk} - \bar{\boldsymbol{s}}_{blk}^t, \nabla\boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk}^t)\rangle \\
&\quad + \frac{L}{2}\|\bar{\boldsymbol{s}}_{blk} - \bar{\boldsymbol{s}}_{blk}^t\|_2^2 + \mathcal{I}_\varepsilon(\bar{\boldsymbol{s}}_{blk}) \\
&= \arg\min_{\bar{\boldsymbol{s}}_{blk}} \frac{1}{2}\|\bar{\boldsymbol{s}}_{blk} - (\bar{\boldsymbol{s}}_{blk}^t - \frac{1}{L}\nabla\boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk}^t))\|_2^2 + \mathcal{I}_\varepsilon(\bar{\boldsymbol{s}}_{blk}) \\
&= \mathrm{prox}_{\mathcal{I}_\varepsilon}(\bar{\boldsymbol{s}}_{blk}^t - \frac{1}{L}\nabla\boldsymbol{H}_\sigma(\bar{\boldsymbol{s}}_{blk}^t))
\end{aligned} \tag{22}$$

where the superscript denotes the iteration number and $\mathrm{prox}_{\mathcal{I}_\varepsilon}(\cdot)$ is the prox-operator of $\mathcal{I}_\varepsilon$. Since $\mathcal{I}_\varepsilon$ is an indicator function, the prox-operator $\mathrm{prox}_{\mathcal{I}_\varepsilon}$ is the projection onto the set $\mathcal{A}_\varepsilon$:

$$\mathrm{prox}_{\mathcal{I}_\varepsilon}(\boldsymbol{x}) = \Pi_{\mathcal{A}_\varepsilon}(\boldsymbol{x}) = \arg\min_{\boldsymbol{v}\in\mathcal{A}_\varepsilon}\|\boldsymbol{v} - \boldsymbol{x}\|_2^2. \tag{23}$$

Finally, the proposed iterative algorithm involves two levels of iterations as follows.

In the internal loop, for a fixed σ, the source signals are updated by

$$\tilde{\boldsymbol{s}}_{blk}^{t+1} = \bar{\boldsymbol{s}}_{blk}^t - \frac{1}{L}\nabla\boldsymbol{H}_\sigma(\bar{\boldsymbol{s}})_{blk}^t \tag{24}$$

and $\tilde{\boldsymbol{s}}_{blk}^{t+1}$ is projected onto $\mathcal{A}_\varepsilon$:

$$\bar{\boldsymbol{s}}_{blk}^{t+1} = \Pi_{\mathcal{A}_\varepsilon}(\tilde{\boldsymbol{s}}_{blk}^{t+1}). \tag{25}$$

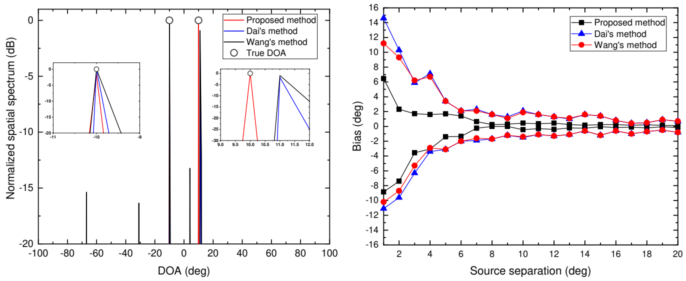
In the outer loop, the σ is updated by $\sigma = \rho\cdot\sigma$, where $0 < \rho < 1$ is a decreasing factor for σ.

Once optimization (19) is solved, the DOAs $\hat{\boldsymbol{\theta}}$ can be obtained from the spatial spectrum of $\boldsymbol{s}_{blk}$, which is block sparse, and a source signal is given as $\hat{s}_i = \hat{\boldsymbol{s}}_{blk}[(m-1)i+1], i = 1, 2, \ldots, K$, where $\hat{\boldsymbol{s}}_{blk}$ are estimates containing only non-zero block entries. The MC coefficients vector $\boldsymbol{c}$ is estimated using $\hat{\boldsymbol{c}} = \sum_{i=1}^{K}\hat{\boldsymbol{s}}_i/\hat{s}_i$, where $\hat{\boldsymbol{s}}_i$ denotes a non-zero block in $\boldsymbol{s}_{blk}$.

IV. Simulation Results

In this section, simulation results are presented to show the performance of the proposed DOA estimation method in comparison with the Dai's method [9], Wang's method [11], MUSIC [2] and the Cramer-Rao lower bound (CRB) [18]. Herein, a ULA with $M = 14$ sensors is considered to receive two narrowband signals from directions θ_1 and θ_2, respectively. Assume that the number of MC coefficients in $\boldsymbol{c}$ is $m = 3$ with $c_1 = 0.4864 - 0.4776j$ and $c_2 = 0.2325 + 0.1914j$. For accelerating the running speed, it is not need to wait for the internal loop to converge, and so the maximum number of iterations is set to $I = 5$. The initial value of σ is set to $\sigma = 5\cdot max_i\|\bar{s}_i\|_2$ and the decreasing factor is set to $\rho = 0.9$. The spatial angle set $(-90°, 90°]$ is discretized by uniform sampling grid with spacing of 1°.

In the first simulation, the normalized spatial spectrum for two uncorrelated signals from directions $\theta_1 = -10°$ and $\theta_2 = 10°$ is shown in Fig. 1a, with signal-noise ratio SNR = 10 dB. It can be seen that all the three methods can accurately estimate direction θ_1, and but only the proposed method can accurately estimate the direction θ_2.



(a) (b)

Fig. 1. (a) Normalized spatial spectrum versus DOA; (b) Bias of proposed method versus source separation for two sources. SNR = 10 dB

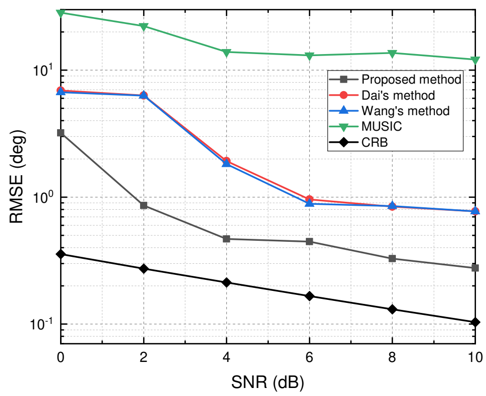


Fig. 2. RMSE of the DOA estimates versus SNR

One important aspect of the proposed method is the capability of resolving closely spaced sources. The bias between the true DOA and the estimated one is defined as $Bias = \sum_{i=1}^{T} |\theta - \hat{\theta}_i|/T$ where T denotes the number of trials used to evaluated the resolution performance of the proposed method. Suppose two uncorrelated signals with equal power are located at $\theta_1 = -10^\circ$ and $\theta_2 = -10^\circ + \Delta\theta$, respectively, where $\Delta\theta$ denotes the source separation ranging from 1° to 20° with 1° step. The estimated bias curves for our method with respect source separation $\Delta\theta$ is presented in Fig. 1b. It is seen that the bias is close to zero when source separation are higher than about 14°.

In the second simulation, we consider a scenario where two uncorrelated sources come from directions $\theta_1 = -10.1^\circ$ and $\theta_2 = 10.9^\circ$. The average root mean square error (RMSE) of DOA estimation over SNR is shown in Fig. 2, where for each SNR value we run 300 Monte-Carlo trials. It is clear that the proposed method outperforms other methods by achieving a smaller RMSE over the entire SNR range, but there is an obvious gap between the RMSE of DOA estimation obtained by the sparse-based methods and CRB. The main reason is that these sparse-based methods are based on the hypothesis that DOAs are exactly positioned at the potential points, which will lead to model error when the true DOAs are not located on the potential points. Additionally, the MUSIC algorithm also fails due to unknown MC and rank-deficiency.

V. Conclusion

In this letter, a novel sparse-based method has been proposed for DOA estimation in the presence of unknown mutual coupling for ULA. In our method, the DOAs are obtained from a block sparse reconstruction framework with a block smoothed ℓ_0 norm sparsity-promoting function, and it is solved by an iterative proximal algorithm. The proposed method can be extended to different array configurations because there is no restriction on the structure of the MCM in the derivation. Simulation results demonstrate that the proposed method achieves remarkable performance in comparison with existing sparse-based DOA estimation methods. The grid mismatch problem will be overcome in future work.

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