The uncanny accuracy of God's mathematical beliefs

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Abstract: I show how mathematical platonism combined with belief in the God of classical theism can avoid Field's epistemological objection. I defend an account of divine mathematical knowledge, by showing that it falls out of my well-motivated general account of divine knowledge. I use this to provide an explanation of the accuracy of God's mathematical beliefs, which in turn explains the accuracy of our own. My arguments provide good news for theistic platonists, while also shedding new light on Field's influential objection, which should be of interest to atheistic philosophy of mathematics.

In this paper, I show how *theistic platonism*, mathematical platonism combined with belief in the God of classical theism, can respond to Hartry Field's (1989, 25-30) epistemological objection to mathematical platonism. I develop and defend a theory of divine knowledge, and show that it explains the accuracy of God's mathematical beliefs, which in turn explains the accuracy of our own. I assume that divine knowledge is not fundamentally different from our own. In particular, God's mathematical beliefs are justified by being entailed by God's systematisation of the mathematical claims presupposed by God's understanding of the physical world. This parallels mainstream contemporary platonist thought about human mathematical knowledge, and so provides an explanation that will be attractive to platonists willing to countenance the existence of God. Further, by showing what conditions knowledge like ours would have to satisfy for Field's objection to be met, I shed light on this influential objection that should also interest atheistic philosophers of mathematics. I begin by presenting platonism and Field's objection in more detail.

Platonism and its epistemological dues

Platonism is the dominant philosophy of mathematics.[1] It is typically presented as the conjunction of three metaphysical theses:

The ontological thesis: Mathematical objects (e.g. numbers, sets, and functions) exist.

The independence thesis: Mathematical objects exist mind-independently.

> **The abstractness thesis:** Mathematical objects are abstract (i.e. not spatially located or causally active).

However, I will also take platonism to entail the following epistemological theses:

> **The accuracy thesis:** Aside from a few mistakes, if mathematicians accept the mathematical proposition < *p* >, then < *p* > is true.
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> **The explanatory thesis:** Mathematical facts are explanatorily relevant to the physical world. To give a few *prima facie* compelling examples: I cannot share my 23 strawberries evenly between my three friends because there is no natural number *n* such that $\frac{23}{3} = n$; I keep failing to draw a square with the same area as a given circle, using only a compass and straight edge, because π is transcendental.[2] Opinions differ as to the nature of the explanatory connection.[3] The common thread is that a proper understanding of the physical world requires grasping certain mathematical truths.

I include these because they form part of a prominent thread of contemporary platonist thought,[4] and because it is difficult to see how platonism can do good philosophical work without them. Without the accuracy thesis, platonism would combine scepticism about mathematical practice and speculation about mathematical reality, where mathematical practice plays no role in justifying claims about mathematical reality, and mathematical reality plays no role in accounting for mathematical practice. Without the explanatory thesis, the platonist would not easily be able to account for the value of mathematics. Mathematics may deal in facts, but if those facts don't help us understand the world we inhabit, what makes them worthy of our attention?[5]

Against platonism, Field (1989, 25-30) raises an influential epistemological objection.[6] In presenting it, it will help to begin with an analogy. On a realist view of physical science, facts about stars, planets, and nebulae are mind-independent. Assuming we have accurate beliefs about such things, the accuracy of these beliefs requires explanation, which can be provided by appeal to the causal relations between stars, planets, and nebulae and the instruments we use to detect them. On platonism, mathematical objects are also mind-independent (by the independence thesis), and we have accurate beliefs about them (by the accuracy thesis). This accuracy is no less in need of explanation. Yet, it seems (by the abstractness thesis) mathematical objects do not bear any naturalistic relation to the physical world that might support such an explanation. Thus, platonism appears to imply something in need of explanation that seems inexplicable, and this is a reason to reject platonism.

Three points of clarification are in order. First, Field's objection threatens a philosophical theory, not our mathematical beliefs. It does not say that our mathematical beliefs are unjustified. Some present the objection in terms of explaining the *reliability* of our mathematical beliefs. But 'reliability' is a loaded term in epistemology, related to justification, and its use here courts misunderstanding. For example, Justin Clarke-Doane (2016) takes Field's objection to be that our mathematical beliefs are unjustified because they are unreliable in the modal sense that they could easily have been different. On my reading of Field, 'reliability' is not to be read modally, and the challenge does not concern justification. Field's objection asks for an explanation of what is according to platonism an actual fact: that our beliefs about mathematical abstracta are mostly accurate. This correlation stands in need of explanation, even if our mathematical beliefs enjoy the justification we claim for them, and even if they could not easily have been different.[7]

Second, Field's objection is best thought of as a challenge, rather than a knockdown argument. The abstractness thesis only explicitly precludes a *causal* explanation of the accuracy of our mathematical beliefs. According to platonism, there is some (non-causal) explanatory connection between mathematical objects and the physical world, which the platonist might appeal to in support of a non-causal explanation of our mathematical accuracy. All that Field has to say on this matter is that 'it is very hard to see what this supposed non-causal explanation could be' (1989, 231).

Nevertheless, even if a non-causal explanation is not impossible, Field's objection serves to highlight an explanatory debt that the platonist must pay. Merely pointing to the explanatory thesis won't meet this challenge. Our mathematical belief-forming mechanisms are the result of causal processes, and even if mathematical objects are explanatorily relevant to these processes, it is not obvious how they could have determined the content of our mathematical beliefs such that they came out accurate.

Finally, one may worry that taking platonism to entail the explanatory thesis begs the question against Field. A significant portion of the contemporary debate targets the explanatory thesis. Platonists argue that there are compelling examples of scientific explanations in which mathematics plays an explanatory role, while anti-platonists deny this. However, it is important to recognise that this debate primarily concerns the existence of a certain kind of evidence. The examples I provided above are controversial; but I only include them for illustrative purposes. I do not assume that they *are* compelling examples, nor that there are such examples to be found. I do assume that there are *prima facie* compelling examples, and that they provide *prima facie* defeasible evidence for the explanatory thesis and thus for platonism. But nominalists can agree on this. We have seen that the explanatory thesis forms part of a prominent thread of contemporary platonist thought, and that some of the philosophical work platonism aspires to do is done by it. Furthermore, Field's objection still poses a significant challenge to platonism, even when we fold in the explanatory thesis.

Several theorists have suggested that platonists can meet Field's objection by adopting theism (Adams (1983, 751); Evans (2013, 121, 179-181); Rogers (2008); Thurow (2013, 1601); Field (2001, 325)). Indeed, if we were created by God, then God would have ensured in creating us that our mathematical beliefs turned out accurate. However, Dan Baras (2017) has recently demurred. According to Baras, theistic platonism is committed to the no less massive and no more explicable correlation between God's mathematical beliefs and the mathematical facts. The reason is familiar. The independence thesis takes mathematical objects to be mind-independent, and so independent of God's mind, so the accuracy of God's beliefs about them requires explanation. But the abstractness thesis seems to preclude us from providing any such explanation. God cannot causally interact with mathematical abstracta, and it is not immediately clear how the mathematical facts might non-causally guarantee the accuracy of God's mathematical beliefs. Thus, Baras argues that theistic platonism merely pushes the problem back.

This issue has been debated so far in the absence of a worked out theory of divine knowledge. To fill this gap, I will develop and defend what I call the *understanding via acquaintance* (UVA) conception of divine knowledge. Drawing on UVA, I will then show that, contra Baras, we can explain the accuracy of God's mathematical beliefs, and thereby explain our own mathematical accuracy.

Understanding via acquaintance

In this section, I present UVA and the key notions it involves. UVA is shaped by the assumption that divine knowledge has the following two properties. It is *scrutable,* in that it is not fundamentally different from human knowledge. To explain an aspect of divine knowledge, we must be able to have some understanding of what divine knowledge involves. Divine knowledge is *ideal,* in that it is the best possible way of making cognitive contact with reality, compatible with its scrutability. I now explicate the key notions of *acquaintance* and *understanding*.

***Acquaintance*:** Acquaintance is a relation of direct awareness between agents and parts of the world (e.g. physical objects, properties of physical objects, mental states, and facts or states of affairs). Acquaintance is direct in the sense that the immediate object of awareness is the object of acquaintance. In being acquainted with something, that thing, and not some mental representation of it, is the immediate object of our awareness. Acquaintance is not intentional: one cannot be acquainted with the non-existent.[8]

An intuitive example of acquaintance is awareness of pain. We apply the concept of pain to certain experiences, rather than what those experiences purport to represent. Thus, my having a pain in my left foot isn't a matter of my mentally representing that I have a pain in my foot; it is a matter of my being directly aware of a certain kind of experience. By being in pain, I perhaps mentally represent trauma in my foot; but we wouldn't normally identify this trauma with my pain. As a result, I cannot be wrong about having a pain in my foot.[9] Whether we are also acquainted with other things, such as physical objects, is moot. However, it will be useful in what follows to assume that human veridical perception of physical objects is a form of acquaintance, so as to provide vivid illustrations of my arguments. In doing so, I am not suggesting that God's manner of being acquainted with the world is anything like visual perception. Presumably, one needs certain biological faculties to visually perceive. An informative account of God's means of acquaintance would go well beyond the scope of this paper.

God is acquainted with everything with which it is possible to be acquainted. But why should divine knowledge be characterised in terms of acquaintance? Because knowledge that *p* that involves being directly aware of the fact that *p* is a better epistemic position with respect to $< p >$ than knowledge that only involves being indirectly aware of the fact that *p*. There are two reasons for this. First, the object of acquaintance is guaranteed to exist, so if one is acquainted with the fact that *p*, the fact that *p* must obtain and < *p* > must be true. So, acquaintance ensures infallibility. Second, to be acquainted with a fact is to 'see it for oneself', and thus not to depend on anything or anyone else for one's cognitive contact with reality. In this sense, acquaintance is the ultimate manifestation of the epistemic virtue of intellectual autonomy (cf. Pritchard (2016)).

Understanding: Understanding involves grasping the explanatory dependencies between things. Understanding has received most attention in the flourishing literature on scientific explanation, where it is taken to be what explanations provide (e.g. Achinstein (1983, 23); Salmon (1989, 134–135); Kitcher (2002); Lipton (2004, 30); Woodward (2003, 179); Ylikoski and Kuorikoski (2010); de Regt (2017)). Understanding is also beginning to receive attention in general epistemology (e.g. Zagzebski (2001); Kvanvig (2004); Grimm (2006); Carter and Gordon (2014); Pritchard (2016)). The growing consensus is that having a deep understanding of something is to be in a particularly strong epistemic position with respect to it. I will illustrate with an example (inspired by Carter and Gordon (2014, 6)).

Suppose a house has burned down. Upon inspecting the site, a novice firefighter notices some faulty wiring and correctly concludes that it caused the fire. The firefighter knows that the faulty wiring caused the fire, and has a shallow understanding of why the house burned down, by grasping that the faulty wiring was *somehow* responsible. Now suppose that a leading expert on exothermic chemical reactions leads an investigation, getting a team to study the wiring, examine photographs of the frequency spectrum of the flames, and so on. The scientist also knows and understands why the house burned down. But the amount of information in and explanatory coherence of the scientist's understanding is far greater (cf. Carter and Gordon (2014, 6)). The scientist fully grasps the intricacies regarding what the development of the fire depended on. She grasps, for example, what would have to be different (in oxygen levels, wiring, insulation, etc.) for the fire to have unfolded differently, or for it not to have occurred at all, granting her the ability to make sophisticated counterfactual inferences about the fire. In comparison, the firefighter's understanding only facilitates rudimentary reasoning. So we see that having a rich understanding of why something occurred is better epistemically speaking than merely knowing what is responsible for it. The ideal epistemic position should therefore involve having a maximally rich understanding of the facts.

Is understanding itself a species of propositional knowledge? I think not. Stephen Grimm (2006) shows that human understanding of physical phenomena resembles propositional knowledge in two key respects. Trying to understand physical phenomena involves mentally representing how it depends on other things, and understanding is achieved only to the extent that the representation is accurate. Thus, human understanding of empirical phenomena is *indirect*—mediated by mental representation—and *non-transparent*—we can easily be wrong about whether we have it.

Nevertheless, human understanding of physical phenomena need not be propositional. The object of our understanding is an objective explanatory dependence between things, and we grasp this by forming a mental representation that models it. But this model need not contain a propositional element that states that the physical phenomena depends on each of its determinants. An accurate causal model for physical phenomenon *P* that shows how it changes for different values of the variable *X* does not state that *X* causes *P*; yet we can grasp the explanatory dependence between *X* and *P* by grasping how changes in *X* change *P*. This need not involve grasping a further proposition.

Other examples of human understanding diverge from propositional knowledge to a greater extent. For example, understanding a mathematical theorem involves being able to follow a (perhaps explanatory) proof of it. This involves grasping the propositions that comprise the proof, but also grasping how these propositions logically depend on one another, which is not a case of grasping some further proposition (cf. Zagzebski (2001, 244)). In such cases, our understanding seems to be both direct, because we grasp how the parts comprising the body of knowledge fit together by being acquainted with our mental representations of them, and transparent, because it seems at least very difficult to be wrong about whether we have this kind of understanding.

Transparent and direct understanding is better, epistemically speaking, than indirect and non-transparent understanding. Accordingly, where possible, divine understanding will be direct and transparent. Divine understanding is a non-propositional grasping of explanatory relations between things. In cases where the understanding is direct, the grasping is achieved via acquaintance with the very things that enter into the explanatory relations. In cases where the understanding is indirect, the grasping is achieved via acquaintance with some non-propositional mental representation of the explanatory connection. I can now present UVA:

> **UVA**
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> For all true propositions < *p* >, God knows that *p* iff God is acquainted with: (i) God's belief that *p*; (ii) God's understanding of the fact that *p*; and (iii) the features of (ii) that explain the truth of (i).

A note on the shape and ambition of UVA is in order. Along with mainstream epistemology, I assume that knowledge is justified true belief (JTB), plus some anti-Gettier condition (X). UVA should be taken as an account of what it takes for a case of knowledge (JTB+X) to be the best it can possibly be, epistemically speaking. Because of this, it is not a problem that divine knowledge is analysed in terms of something that provides a more demanding kind of cognitive contact with the world, namely understanding. Moreover, because I am taking understanding to be non-propositional and distinct from knowledge, there is no risk of circularity in UVA.

In the following two sections, I will motivate UVA by showing that it captures the ideal epistemic position with respect to truths of different kinds. To help in this regard, I will assume two metaphysical theses. The first is *presentism*—the view that there are no past or future facts. The second is what I will call *moderate determinism*—that there are at least some truths about the future that have a definite truth value because the facts they purport to represent are causally determined by the present. I assume these theses not because I believe them, and not because my arguments hang on them, but because they make useful case-studies of truths about the present, past, and future. Taking Travis M. Dickinson's (2019) account as a starting point, I will defend an account of divine knowledge of truths which represent existing facts with which it is possible to be acquainted—on presentism, facts about the present physical world. I will then argue that UVA is the best way of generalising this account to accommodate truths which represent facts which do not exist—on presentism and moderate determinism, truths about the past and future.

I proceed in this way for two reasons. First, I aim to motivate my account of divine mathematical knowledge by showing that it falls out of a well-motivated and general account of divine knowledge. It would certainly be more convenient to assume that the temporal and spatial extent of the physical universe is 'eternally present' to God. We could then account for all of God's knowledge of the physical world straightforwardly in terms of God's acquaintance with it. While this would simplify the account, its plausibility would then be conditional on controversial metaphysical theses. By assuming presentism and moderate determinism, I provide an account of divine knowledge that doesn't assume God's acquaintance with all the facts, and thus demonstrate that it has unconditional plausibility. Finally, assuming that God is not acquainted with all the facts brings God's epistemic predicament closer to our own, honouring my assumption that divine knowledge is scrutable, and facilitating my explanation of the accuracy of God's mathematical beliefs.

Knowledge of the present

God is acquainted with all present physical objects, properties, and facts. According to William Alston (1986), this renders God's knowledge 'infallible in a strong sense' (1986, 295), and so obviates the need to ascribe beliefs to God. However, Dickinson (2019) convincingly argues that God's knowledge would be less perfect if it did not involve beliefs. His argument is that some kind of mental representation seems necessary for knowledge:

> [N]otice that we are all immediately aware of facts right now about which we have not formed any thoughts, about which we haven't conceptualized. One should consider a patch of colour in the

> periphery of one's visual field (or the buzzing of lights or of an electrical device), which one has not (until just now) noticed, though it has been there all along as an object of awareness. Though we were (by hypothesis) directly aware of them, these non-conceptualized facts were not plausibly objects of knowledge since we didn't even notice them or form any thoughts about them. It seems that it is in the forming of thoughts that these become possible objects of knowledge. (Dickinson (2019, 6))

God's acquaintance with all the present physical facts is a perfection of God's *awareness*; but without conceptual representation of these facts, it cannot amount to *knowledge* of these facts. God's knowledge of the present physical world involves having beliefs that represent all the present physical facts. This is an *excellent* epistemic state to be in with respect to truths about the present physical world. God not only believes all the true propositions; God is also acquainted with the parts of the world that make them true. As such, God 'sees for Godself' that they are true, and thus exercises maximum intellectual autonomy. However, this is not yet an *ideal* epistemic state. Two further ingredients are needed.

The first Dickinson (2019, 10) recognises, and illustrates by example. Suppose one is confronted with with a mural filled with discrete spots that are randomly placed and of various sizes. Suppose there are exactly 1,242 spots, each of which is in clear view. Suppose that the artist assures us that there are 1,242 spots. We thus have the true belief that there are 1,242 spots, and we are acquainted with the fact that this belief truly represents. In this circumstance, perhaps we even know that there are 1,242 spots. Nevertheless, there is a better way of knowing that there are 1,242 spots.

Imagine a similar mural that instead has just three spots. If we were to see this mural, we would not need to be told by the artist that there are three dots; we would on the basis of our perception be directly aware of the very feature of the perceived state of affairs that makes true the belief that there are three spots. In this case, we would be acquainted with our belief, and what it demands of the world for its truth; and we would be acquainted with the the very thing about the fact it represents that satisfies this demand. In Dickinson's words, we would be acquainted with the *correspondence* between our belief and the fact it represents (2019, 10). This manner of speaking has the unfortunate connotation that we can be acquainted with a relation. Nevertheless, I will adopt it for convenience as shorthand for being acquainted with what enters into the correspondence, as described above.

This is a better way of knowing that there are three spots. Being acquainted with why the represented fact makes the relevant proposition true appropriately grounds one's justification for the belief in the thing responsible for its truth. It aligns one's internal justification with the ideal external justification.

In the case where there are 1,242 spots, we are not in this position. Due to our cognitive limitations, we cannot see for ourselves that there are 1,242 spots. Thus, even though we have justification for our belief, it is not aligned with the ideal external justification, and so our belief is not appropriately grounded. Hence, there is a better way of knowing that there are 1,242 spots on the mural: being acquainted with our belief that there are 1,242 spots, being acquainted with the fact that there are 1,242 spots, and being acquainted with the correspondence between the fact that there are 1,242 spots and <there are 1,242 spots>. Since God has no cognitive limitations, God knows that there are 1,242 spots on the mural by being acquainted with these three things. In fact, God knows all truths about the present physical world in this way.

The final ingredient is a maximally rich *understanding* of the present physical facts. This will consist in acquaintance with the rich network of dependencies in which the fact represented by each true proposition is embedded. Again, talk of acquaintance with dependencies is to be understood as short hand for acquaintance with what enters into them. To return to our example, God will be acquainted with the rich network of dependencies in which the fact that there are 1,242 spots on the wall is embedded. God will be acquainted with the chemical composition of the paint, the molecular structure of the wall, the exact size and shape of every spot, and so on. In being thus acquainted, God knows precisely all the possible changes relevant to whether there are 1,242 spots on the wall. For example, God knows what it would take for the spots to fade or be removed, and God knows all the possible combinations (in terms of size and shape) of spots compatible with there being 1,242 on the wall, and so on. We can now characterise divine knowledge of the present physical world as follows:

> **UVA$_{ppw}$**
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> For all true propositions < *p* > concerning the present physical world, God knows that *p* iff God is acquainted with the following: (i) God's belief that *p*; (ii) God's understanding of of the fact that *p*; and (iii) the correspondence between the fact that *p* and (i).

Knowledge beyond the present

On presentism and moderate determinism, knowledge about the past and the future is knowledge of truths that represent no existing fact. We must generalise UVA$_{ppw}$ so that it accounts for divine knowledge of such truths. Condition (i) need not be tampered with: belief is an intentional relation, so whether or not a fact exists has no effect on our (or God's) ability to believe the proposition that represents it. We can also leave condition (ii) alone, since understanding can be indirect, and thus achieved via acquaintance with a mental representation. However, (iii) requires that God be acquainted with the fact represented, so it must be suitably generalised.

Dickinson (2019, 12) attempts this by locating correspondence as an instance of a more general kind of cross-categorical entailment, holding between facts and true propositions—what he calls *entailment**. According to Dickinson, a fact entails* < *p* > iff its obtaining guarantees < *p* >'s truth. The force of 'guarantees' is left unclear, but Dickinson illustrates with the following example (2019, 12). The fact that Jones is in the room makes true the proposition that Jones is in the room, and this proposition in turn entails that someone is in the room. Dickinson takes the fact that Jones is in the room to entail* that someone is in the room. In this example, entailment* holds via a chain of truthmaking and entailment. A chain of relations is only as modally strong as its weakest link, and it is orthodoxy to take truthmaking to be metaphysically necessary (Merricks (2007, 5), Cameron (2008, 107), Shaffer (2008, 10), and Goff (2010); see Asay (2016) for arguments for the orthodoxy). So, I take it that entailment* is a metaphysically necessary relation, even if entailment requires something stronger. Thus, we can say that a fact entails* < *p* > iff < *p* > is true in all metaphysically possible worlds in which the fact obtains.

In cases where < *p* > represents an existing fact, the fact that *p* clearly necessitates < *p* >. After all, it is < *p* >'s *truthmaker*. However, in cases where there is no existing fact, Dickinson (2019, 13) appeals to facts about God instead, namely God's perfection. This raises difficulties. There are many different facts that necessitate a given truth. For example, suppose I have five million hairs on my body. Besides its truthmaker, the fact that

God believes the proposition presumably also necessitates that I have five million hairs on my body. So, if the condition is merely that God be acquainted with some fact or other that necessitates the truth of the relevant proposition, why in cases where there is a truthmaker is it acquaintance with the truthmaker, rather than God's perfection, that helps constitute God's knowledge? Worse still, it looks as though, for a given necessary truth < *p* >, God can know that *p* by virtue of being acquainted with any contingent fact, since < *p* > will be true in any metaphysically possible world in which that fact obtains.

Being acquainted with the correspondence between the fact that *p* and < *p* > is part of the ideal epistemic state with respect to knowing that *p*, not just because being so acquainted necessitates the truth of the belief that *p*, but also because the corresponding fact *explains* the truth of the belief. As a maximally virtuous knower, God will, where possible, 'see it for Godself' by be acquainted with the fact that explains the truth of God's beliefs.

What does 'seeing it for Godself' amount to when there is no existing fact represented? Recall that understanding a fact is being acquainted with the tapestry of dependencies in which it is embedded. On presentism and moderate determinism, past facts no longer exist, but can have effects in the present, and future facts do not exist yet, but some have causal antecedents in the present. Acquaintance with these present facts can furnish a sufficiently rich understanding.

To illustrate, suppose it is true that the sun will rise on 19/02/2060. By hypothesis, the corresponding fact does not yet exist, but plausibly certain present facts ensure that it will: the laws governing planetary motion in our solar system, the mass of the earth, and so on. Being acquainted with these facts, grasping how they hang together and conspire to bring about the fact that the sun will rise on the 19/02/2060, is to have a maximally rich understanding of this fact.

If God believes sun will rise on 19/02/2060 because God is acquainted with this fact's causal antecedents, then God's belief is necessarily true. There is a relation between the facts that causally determine the fact that the sun will rise on 19/02/2060, and the truth of God's belief that the sun will rise on 19/02/2060, such that the former explains the latter. Being acquainted with this relation aligns God's internal justification with the ideal external justification. God knows all causally determined future facts in this way.[10]

We turn now to truths about truths concerning the past. The sun rose on 19/02/1989. By hypothesis, the corresponding fact no longer exists. However, there are present facts which would not obtain, had the sun not risen on 19/02/1989. For example, the sun would not have risen today, and God would not remember it rising on 19/02/1989 if it hadn't happened. Thus, a sufficiently rich understanding of the fact that the sun rose on 19/02/1989 is achievable via acquaintance with the present facts that depend on its having obtained. God's perfection, and the immutability of causal laws, means the existence of these causal consequences necessitates that the relevant fact obtained. Further, the existence of these consequences explains the truth of God’s belief. Being acquainted with this explanatory relation aligns God’s justification for the belief with the ideal external justification.

We have seen that we can generalise the third condition in UVA$_{ppw}$ to accommodate truths representing non-existent facts by restating it in terms of God’s grasping what explains the truth of God’s belief. Thus, UVA is an adequate generalised characterisation of divine knowledge:

> **UVA**
>
> For all true propositions < *p* >, God knows that *p* iff God is acquainted with the following three things: (i) God’s belief that *p*; (ii) God’s understanding of the fact that *p*; and (iii) the features of (ii) that explain the truth of (i).

Knowledge of the third kind

Mathematical facts cannot be the objects of acquaintance. Mathematical objects are causally inert, so they cannot be the objects of human perception. But why assume that God cannot be acquainted with them? Because, for God to be acquainted with them, mathematical objects must enter into a relation whereby God is directly aware of them. This relation cannot be causation; but it must be causation-like, since the mathematical objects must in some sense impinge on God's cognition. So, God's acquaintance with the mathematical facts requires a *sui generis* relation that is like causation, apart from its incompatibility with platonism. We might call this 'supernatural causation'. Positing this relation raises three problems.

First, it violates the spirit of platonism; calling the relation 'supernatural causation' serves only to preserve the letter (see Baras (2017, 485-486)). I am not merely trying to provide a legal explanation; I am trying to provide one that platonists might find attractive, so long as they countenance the existence of God. For this reason, positing supernatural causation seems like a bad idea. Second, positing supernatural causation violates the assumption that divine knowledge is scrutable, and thus undermines my aim of replying to Field's objection. The notion of supernatural causation is a *sui generis* one that is beyond our ken, so appealing to it in characterising divine mathematical knowledge would destroy our understanding of this aspect of divine knowledge, and along with it any hope of explaining the accuracy of God's mathematical beliefs.

We have so far considered two kinds of knowledge: knowledge of truths that represent existing facts with which acquaintance is possible; and knowledge of truths which represent non-existing facts. Mathematical knowledge is knowledge of a third kind: knowledge of truths that represent existing facts with which acquaintance is not possible. Following UVA, the task at hand is to locate what God's understanding of mathematical facts involves, and the facts with which God can be acquainted that explain the truth of God's mathematical beliefs.

As with other domains, I will treat divine mathematical knowledge as not fundamentally different from, but an ideal version of, human mathematical knowledge. Broadly speaking, there are two sources of human mathematical knowledge, and so two sources of divine mathematical knowledge: extra- and intra-mathematical.

***Extra-mathematical*:** Our best scientific theories indispensably involve apparent reference to and quantification over mathematical objects. The empirical evidence for these theories gives us reason to think they are true, and thus gives us reason to think that mathematical objects exist. So runs the *indispensability argument* for platonism (attributed to Quine (1948) and Putnam (1971)).

Things have come a long way since Quine and Putnam. Field (1980) attempted to show that science can be done without mathematics. While his project was impressive and enlightening, most now consider it doomed (see Macbride (1999) for an excellent survey), conceding that mathematics is indispensable to science. However, the spirit of Field's objection—that mathematics is not confirmed by its role in science—lives on. Several theorists (e.g. Leng (2010), (2012); Melia (2000); Yablo (2012)) argue that mathematics merely serves to represent physical things that could not otherwise be represented.

In response, platonists appeal to cases where scientists apparently appeal to the properties of mathematical objects to explain physical phenomena (see note 4 for references). According to these theorists, such examples provide *prima facie* evidence for the explanatory thesis and thus platonism. This is where contemporary platonists look for external justification for platonism.

God has a maximally rich understanding of the physical facts. On platonism, mathematical objects are explanatorily relevant to the physical facts, so, in having a perfect understanding of the physical facts, God must grasp how the mathematical facts help determine the physical facts. This also facilitates understanding for the mathematical facts themselves, via grasping the dependencies into which they enter.

In a sense, I'm claiming that mathematics is explanatory indispensable to God's theory of the physical world, so one might worry that I am pre-judging the outcome of the ongoing debate described above. I am not. I repeat: the debate concerns the existence of a certain kind of evidence: whether *our* current best science explains physical phenomena by appeal to the properties of mathematical objects. However this debate turns out, platonism, as I have characterised it, assumes that there is an explanatory connection between mathematical objects and the physical world. Thus, an *ideal* understanding of the physical world will involve a grasping this connection. This is compatible with the possibility that *our* best understanding of the physical world merely draws on mathematics as a tool for representing purely physical dependencies.

***Intra-mathematical*:** Some mathematical truths strike us as immediately obvious of their respective domains. These intuitions form part of the support we claim for our axiomatic mathematical theories: a minimum requirement for a mathematical theory is that it saves at least a substantial range of our mathematical intuitions. Beyond that, axioms are justified to the extent that they exhibit certain theoretical virtues. As far as possible, they should strike us as obvious and distinctive of their domain. But they should facilitate proofs of interesting and useful theorems beyond those of which we are already convinced, perhaps shedding new light on other domains—they should be *fruitful*. By maximising such virtues, a system of axioms earns the status of being part of the best systematisation of our mathematical intuitions. Less obvious mathematical claims are then justified by being entailed by the axioms.

Sometimes, theoretical virtues pull in different directions, so the task is to find the best overall balance. Take the following axioms characterising the rules of addition for the real numbers:

A1: For all $x,y \in \mathbb{R}$, $x + y \in \mathbb{R}$.

A2: For all $x,y,z \in \mathbb{R}$, $(y + x) + z = x + (y + z)$.

A3: For all $x,y \in \mathbb{R}$, $x + y = y + x$.

A4: There is a number $0 \in \mathbb{R}$ such that $x + 0 = x = 0 + x$ for all $x \in \mathbb{R}$.

A5: For each $x \in \mathbb{R}$, there is a number $(-x) \in \mathbb{R}$ such that $x + (-x) = 0 = (-x) + x$.

A1 through A5 are obvious: they make explicit things implicit in our concept of number. However, they do not capture anything distinctive of the real numbers, and they are not fruitful with respect to the real numbers. To address this, we might include the completeness axiom, with the help of the following definitions. If S is a set of real numbers, then S is *bounded above* iff there is some N such that $x \leq N$ for all $x \in S$. The *supremum* of a set S is the *least upper bound* of S; that is, the smallest number N such that, for all $x \in S$, $x \leq N$.

A6: Every non-empty set of real numbers which is bounded above has a supremum.

A6 is less obvious; its status as an axiom is justified by its capturing something distinctive about its domain—that there aren't any 'gaps'—and its fruitfulness. For example, it allows us to prove the pre-theoretically compelling *Archimedean property* that, given any real number x, there is an integer n such that $n > x$.

God's mathematical beliefs will be perfectly systematised. God grasps axiomatic propositions that capture obvious and distinctive truths about the mathematical objects presupposed in God's understanding of the physical world, while ensuring maximal fruitfulness. This will, of course, rely on God's judgements about the best overall balance of theoretical virtues. God will also immediately see all the deductive consequences of these axioms. Thus, all of the mathematical truths will strike God as immediately obvious. To illustrate the sense in which God can 'see' all of the consequences of God's mathematical beliefs, consider a case where our own ability to see consequences fails:

$$\lim_{n\to\infty}(1+\frac{1}{n}) = 1$$

$$\lim_{n\to\infty}(1+\frac{1}{n})^n = e$$

The first of these says that, as n tends to infinity, $(1+\frac{1}{n})$ tends to 1. This is immediately obvious. We can see that, as n gets larger and larger, $\frac{1}{n}$ gets closer and closer to 0, and so $(1+\frac{1}{n})$ gets closer and closer to 1. The second, however, is less obvious. We might be tempted to think that, since $(1+\frac{1}{n})$ tends to 1 as n tends to infinity and $1^n = 1$ for any n, $(1+\frac{1}{n})^n$ tends to 1 as n tends to infinity. This intuition is due to a failure to see the incremental effect of increasing the exponent as the denominator of the fraction inside the parentheses is increased. It is due to a limitation in our cognitive capacities. God, however, is able to immediately see the incremental effect of increasing the exponent.

We must now identify the facts with which God can be acquainted that explain the truth of God's mathematical beliefs. God's mathematical beliefs are derived abductively: they are formed as part of God's best-systematised explanation of the physical world. The truth of abductively derived conclusions are determined by two things: the truth of the premises; and the abductive judgements on the basis of which the conclusions are derived. In this case, the premises are the explananda: the physical facts to be explained. God is acquainted with these. And God is acquainted with God's abductive judgements on the basis of which the mathematical conclusions are derived. But how do these ingredients explain the truth of God's mathematical beliefs? Because God is a perfect epistemic agent, so God's judgements cannot err.

I close this section by dealing with two possible objections. First, abductive judgements provide at best defeasible justification. So, abductive judgements cannot necessitate the truth of the beliefs derived from them. But, if God's abductive judgements *cannot* err, then God's abductive judgements *do* necessitate the truth of the beliefs derived from them. So, the explanation of the truth of God's mathematical beliefs is one we cannot make sense of, violating my commitment to the scrutability of divine knowledge.

To reply, I will attempt to flesh out the notion of a perfect epistemic agent in the hope of making sense of the idea that God's abductive judgements cannot err. Here is one way to go.[11] We start with an equation, derived from Bayes' Theorem (see Earman (2000)), which runs informally as follows. Suppose we have a group of witnesses. For each witness: the likelihood that they will judge *p* true is more likely when *p* is true than when *p* is false—the witnesses are *relatively reliable*; and the likelihood that each witness judges *p* true is not made more or less likely than any of the other witness's judgements—the witnesses are *independent*. Then, as the number of witnesses judging *p* true approaches infinity, the probability that *p* is true tends to 1. We can then take the perfect epistemic agent, God, to be one that satisfies the following. Let *n* be the number of relatively reliable and independent witnesses. Then, for any value of *n*, the probability of *p* being true given that God judges *p* true is greater than the probability of *p* being true given that each witness judges *p* true. It follows that, for any *p*, the probability that *p* is true given that God judges *p* true is arbitrarily close to 1. And this applies to abductive and non-abductive judgements alike. In this way, we can make sense of the idea that God's abductive judgements cannot err.

The second objection is that the idea of God deriving mathematical beliefs via abductive judgements about the physical world is hard to reconcile with God's creative power. God is supposed to have created the physical universe. So, if the physical universe is one way rather than another because certain mathematical facts obtain, and if God has no control over which mathematical facts obtain, then God would have bumped up against the mathematical facts in creating the universe. My response is to point at that, when I say that God's mathematical beliefs are derived via abductive judgements about the physical world, I do not mean to suggest that God surveyed the physical world, then wondered why it is the way it is, then inferred God's mathematical beliefs as the best explanation. Rather, I mean to say that God's mathematical beliefs are rationally grounded in abductive judgements regarding the physical world. I make no claims about the aetiology of God's mathematical beliefs. I am open to the possibility that God formed God's mathematical beliefs while creating the universe.

A supernaturalistic explanation

We are at last in a position to see how theistic platonism avoids Field's objection. To do so, we must first reply to Baras's objection that the accuracy of God's mathematical beliefs cannot be explained. For Baras's objection to inherit the dialectical force of Field's, it must take the form of a debunking argument. Field's objection provides a reason for rejecting platonism while granting the justification claimed for the view. Platonism yields an explanatory debt that we have principled reasons to believe cannot be settled, and this 'tends to undermine the belief in mathematical entities, despite whatever reason we might have for believing in them' (1989, 26).

Thus, Baras must allow the theistic platonist to appeal to the resources available to her, and grant the defeasible justification claimed for her view. But this means granting the legitimacy of the kinds of supernaturalistic explanations that theists indulge in, and take to support their view. For example, theists tend to think that God's creating the universe is the best explanation for its existence. This is an explanation that appeals to the properties of a supernatural entity—a supernaturalistic explanation. This reveals that it is legitimate to answer Baras's objection by providing a supernaturalistic explanation of the accuracy of God's mathematical beliefs; that is, one that appeals to the properties of God.

Such an explanation falls out of UVA. God's perfection as an epistemic agent adequately explains the accuracy of God's mathematical beliefs. We saw that the rational grounds of God's mathematical beliefs are God's abductive judgements. Since God is a perfect epistemic agent, God's abductive judgements cannot err. Thus, God's perfection as an epistemic agent adequately explains the accuracy of God's mathematical beliefs. Moreover, this explanation in terms of UVA is not mere speculation. We saw that UVA can account for divine knowledge of a variety of different kinds of truth, and is not hostage to the nature of time and causation. UVA is an independently-supported general account of divine knowledge, and this support is transferred to the explanation of divine mathematical knowledge that falls out of it. From this firm footing, we can move on to explain the accuracy of our own mathematical beliefs, and thus respond to Field's objection, by claiming that God made sure in creating us that our own abductive judgements would not lead us astray.

Herein lies the crucial difference between theistic and atheistic platonism. On atheism, our abductive judgements are, for all we know, not only sensitive to the way the world really is. This is especially true in the case of mathematics, where we have no causal feedback from the domain of investigation. Our judgements may be a guide to what best allows limited cognitive creatures like ourselves to navigate the world; but this is as much to do with our own cognitive limitations and idiosyncrasies as it is with how the world really is. By contrast, on theism, God ensures that our judgements produce accurate mathematical beliefs; and God's perfection ensures that God's judgements rationally ground accurate mathematical beliefs.

Did I need to provide a theory of divine mathematical knowledge to make this case? Couldn't I have just appealed to God's omniscience to explain the accuracy of God's mathematical beliefs? No. This simpler explanation raises a question of explanatory priority: is God omniscient because God knows all the truths, or does God know all the truths because God is omniscient? If the former, then one cannot appeal to God's perfection to *explain* the accuracy of God's mathematical beliefs, since the explanatory priority runs the wrong way. Having motivated an account of divine mathematical knowledge independently, I can settle this issue in a principled manner. On my account of divine mathematical knowledge, God's perfect abductive judgements provide the rational grounds for God's mathematical beliefs in a way that explains their accuracy.

Conclusion

I have shown how theistic platonism can reply to Field's epistemological objection to mathematical platonism. My reply is premised on a God whose knowledge is not fundamentally different from our own. The epistemology of divine mathematics parallels contemporary platonist thought about the epistemology of human mathematics. This, I hope, renders the overall package attractive to contemporary platonists. Moreover, it sheds some light on Field's influential objection that I hope atheistic philosophers of mathematics will find interesting. We saw that it was the fact that God's abductive judgements cannot err that ultimately explained the accuracy of God's mathematical beliefs. In contrast, on atheism, human abductive judgements may tell us more about our limitations than they tell us about reality. To reply to Field's objection without appeal to God, we must locate something about the kinds of abductive judgements from which our mathematical beliefs are derived that explains the truth of our mathematical beliefs.

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References

ACHINSTEIN, P. (1983) *The Nature of Explanation* (Oxford: Oxford University Press).

ADAMS, R. M. (1983) 'Divine necessity', *Journal of Philosophy*, **80**, 741–752.

ASAY, J. (2016) 'The facts about truthmaking: an argument for truthmaker necessitarianism', *Ergo*, **3**, 493–500.

ALSTON, W. (1986) 'Does God have beliefs?', *Religious Studies*, **22**. 287–306.

BAKER, A. (2005) 'Are there genuine mathematical explanations of physical phenomena?', *Mind*, **114**, 223–238.

BAKER, A. (2016a) 'Mathematics and explanatory generality', *Philosophia Mathematica*, **25**, 194–209.

BAKER, A. (2016b) 'Parsimony and inference to the best explanation', *Synthese*, **193**, 333–350.

BAKER, A. and COLYVAN, M. (2011) ‘Indexing and mathematical explanation’, *Philosophia Mathematica*, **19**, 323–334.

BANGU, S. (2013) ‘Indispensability and explanation’, *Journal for the Philosophy of Science*, **64**, 255–277.

BARAS, D. (2017) ‘A reliability challenge to theistic platonism’, *Analysis*, **77**, 479–487.

BARON, S. (2014) ‘Optimisation and mathematical explanation: doing the Levy´ walk’, *Synthese*, **191**, 459–479.

BARON, S., COLYVAN, M., and RIPLEY, D. (2017) ‘How mathematics can make a difference’, *Philosophers’ Imprint*, **17**, 1–29.

BENACERRAF, P. (1973) ‘Mathematical truth’, *Journal of Philosophy*, **70**, 661–679.

BONJOUR, L. (2003) ‘A version of internalist foundationalism’, in L. BonJour and E. Sosa (eds) *Epistemic Justification: Internalism vs. Externalism, Foundations vs. Virtues* (Malden, MA: Blackwell).

BROWN, J. R. (2012) *Platonism, Naturalism, and Mathematical Knowledge* (New York: Routlege).

CAMERON, R. P. (2008) ‘Truthmakers, realism, and ontology’, *Philosophy*, **83**, 107–128.

CARTER, J. A. and GORDON, E. C. (2014) ‘Objectual understanding and the value problem’, *American Philosophical Quarterly*, **51**, 1–13.

CLARKE-DOANE, J. (2016) ‘What is the Benacerraf problem?’, in F. Pataut (ed.) *Truth, Objects, Infinity: New Perspectives on the Philosophy of Paul Benacerraf* (Dordrecht: Springer).

COLYVAN, M. (2002) ‘Mathematics and aesthetic considerations in science’, *Mind*, **111**, 69–74.

COLYVAN, M. (2010) ‘There is no easy road to nominalism’, *Mind*, **119**, 285–306.

COLYVAN, M. (2012) ‘Road work ahead: heavy machinery on the easy road’, *Mind*, **121**, 1031–1046.

DE REGT, H. W. (2017) *Understanding Scientific Understanding* (Oxford: Oxford University Press).

DICKINSON, T. M. (2019) ‘God knows: acquaintance and the nature of divine knowledge’, *Religious Studies*, **55**, 1–16.

EVANS, C. S. (2013) *God and Moral Obligation* (Oxford: Oxford University Press).

FIELD, H. (1980) *Science Without Numbers* (Oxford: Blackwell).

FIELD, H. (1989) *Realism, Mathematics, and Modality* (Oxford: Basil Blackwell).

FIELD, H. (2001) *Truth and the Absence of Fact* (Oxford: Oxford University Press).

FUMERTON, R. (1995) *Metaepistemology and Skepticism* (Lanham MD: Rowman and Littlefield).

FUMERTON, R. 2001. ‘Classical foundationalism’, in M. R. De Paul (ed.) *Resurrecting Old-Fashioned Foundationalism* (Lanham MD: Rowman and Littlefield).

GÖDEL, K. (1964) ‘What is Cantor’s continuum problem?’, in P. Benacerraf and H. Putnam (eds) *Philosophy of Mathematics*, second edition (Cambridge: Cambridge University Press).

GOFF, P. (2010) 'Orthodox truthmaker theory cannot be defended by cost/benefit analysis' *Analysis*, **70**, 45–50.

GRIMM, S. (2006) 'Is understanding a species of knowledge?', *British Journal for the Philosophy of Science*, **57**, 515–535.

HASAN, A. and FUMERTON, R. (2017) 'Knowledge by acquaintance vs. description', in E. N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*.

KITCHER, P. (2002) 'Scientific knowledge', in P. Moser (ed.) *The Oxford Handbook of Epistemology* (New York: Oxford University Press).

KVANVIG, J. (2003) *The Value of Knowledge and the Pursuit of Understanding* (New York: Cambridge University Press).

LANGE, M. (2017) *Because Without Cause: Non-Causal Explanations in Science and Mathematics* (Oxford: Oxford University Press).

LENG, M. (2010) *Mathematics and Reality* (Oxford: Oxford University Press).

LENG, M. (2012) 'Taking it easy: a response to Colyvan', *Mind*, **121**, 983–995.

LIGGINS, D. (2018) 'The reality of Field's epistemological challenge to platonism', *Erkenntnis*, **83**, 1027–1031.

LINNEBO, Ø. (2018) 'Platonism in the philosophy of mathematics', in E. N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*.

LIPTON, P. (2004) *Inference to the Best Explanation*, second edition (New York: Routledge).

LYON, A. (2012) 'Mathematical explanations of empirical facts, and mathematical realism', *Australasian Journal of Philosophy*, **90**, 559–578.

LYON, A. and COLYVAN, M. (2008) 'The explanatory power of phase spaces', *Philosophia Mathematica*, **16**, 227–243.

MACBRIDE, F. (1999) ‘Listening to fictions: a study of Fieldian nominalism’, *British Journal for the Philosophy of Science*, **50**: 431–455.

MADDY, P. (1990) *Realism in Mathematics* (Oxford: Oxford University Press).

MELIA, J. (2000) ‘Weaseling away the indispensability argument’, *Mind*, **109**, 455–479.

MERRICKS, T. (2007) *Truth and Ontology* (Clarendon Press).

MURAT, A. (2013) ‘Pain’, in E. N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*.

PINCOCK, C. (2015) ‘Abstract explanations in science’, *British Journal for the Philosophy of Science*, **66**, 857–882.

PRITCHARD, D. (2016) ‘Seeing it for oneself: perceptual knowledge, understanding, and intellectual autonomy’, *Episteme*, **13**, 29–42.

PUTNAM, H. (1971) *Philosophy of Logic* (New York: Harper).

QUINE, W. V. (1948) ‘On what there is’, *The Review of Metaphysics*, **2**, 21–38.

ROGERS, K. A. (2008) ‘Evidence for God from certainty’, *Faith and Philosophy*, **25**, 31–46.

SALMON, W. (1989) ‘Four decades of scientific explanation’, in P. Kitcher and W. Salmon (eds) *Scientific Explanation: Minnesota Studies in the Philosophy of Science*, **13** (Minneapolis: University of Minnesota Press) 3–219.

SCHAFFER, J. (2008) ‘Truthmaker commitments’, *Philosophical Studies*, **141**, 7–19.

SHAPIRO, S. (2000) *Thinking about Mathematics* (New York: Oxford University Press).

THUROW, J. C. (2013) ‘The defeater version of Benacerraf’s problem for a priori knowledge’, *Synthese*, **190**, 1587–1603.

WARREN, J. (2017) ‘Epistemology versus non-causal realism’, *Synthese*, **194**, 1643–1662.

WOODWARD, J. (2003) *Making Things Happen: A Theory of Causal Explanation* (New York: Oxford University of Press).

YABLO, S. (2012) ‘Explanation, extrapolation, and existence’, *Mind*, **121**, 1007–1029.

YLIKOSKI, P. and KUORIKOSKI, J. (2010) ‘Dissecting explanatory power’, *Philosophical Studies*, **148**, 201–219.

ZAGZEBSKI, L. (2001) ‘Recovering understanding’, in M. Steup (ed.) *Knowledge, Truth, and Duty: Essays on Epistemic Justification, Responsibility, and Virtue* (New York: Oxford University Press).

Notes

1. See Linnebo (2018) for more on platonism, and Shapiro (2000, 201–225) for an excellent introduction to the contemporary debate.

2. The former example is from Lange (2017); the latter is critically discussed in Leng (2012).

3. See Lyon (2012), Pincock (2015), and Baron et al. (2017) for a menagerie of accounts.

4. See Baker (2005), (2016a), (2016b); Baker and Colyvan (2011); Bangu (2013); Baron (2014); Colyvan (2002), (2010), (2012); Lyon (2012); Lyon and Colyvan (2008). These authors argue for platonism by appeal to *prima facie* cases of mathematical explanation in science, so their platonism is one that includes the explanatory thesis.

5. An anonymous referee points out that there are other ways to account for the value of mathematics that do not presuppose the explanatory thesis. One is to claim that mathematics is a powerful deductive tool, enabling us to deduce non-mathematical conclusions from non-mathematical premises more efficiently than we would be able to without it (as contended by Field's (1980) programme and its followers). Another is to claim that, as well as helping with deductions, mathematics offers a rich framework of concepts for representing the physical world (as contended by 'easy-road nominalists', e.g. Leng (2010); Melia (2000)). I agree with the referee that these theses are available to the platonist. (Indeed, Brown (2012) is a platonist who ascribes to something like the easy-road view.) Nevertheless, there are two reasons why they would fail to provide an adequate platonist account of the value of mathematics. First, they are not really accounts of the value of mathematics; they are accounts of the utility of mathematics in science. What we want is an account of why mathematics itself is valuable. Second, these accounts were originally advanced in support of anti-platonism, so they assign no role whatsoever to mathematical facts or our cognitive contact with them. Thus, on a form of platonism that subscribes to one of these views, the value of mathematics would be an epiphenomenon, arising out of the fact that ascertaining mathematical facts involves considerable theoretical ingenuity; the ascertaining of the mathematical facts would not in and of itself be valuable. In contrast, commitment to the explanatory thesis renders the ascertaining of mathematical facts valuable because the facts themselves are deeply important aspects of reality.

6. Field's objection (1989, 25–30) improves on Benacerraf's (1973), by not relying on a particular theory of knowledge.

7. See Liggins (2018) for an interpretation of Field along these lines. As Liggins (2018, 1030) does, I note that the objection Clarke-Doane addresses does echo later statements of Field's (see e.g. Field 2005, 81). All I claim is that this is not the objection that Field originally presented. See Warren (2017) for a presentation and defence of the reliability objection.

8. I follow Fumerton (1995), (2001), who takes acquaintance to be a sui generis relation of direct awareness that cannot be analysed into any more familiar concepts (1995, 76). In contrast, Bonjour (2003) takes acquaintance to be an intrinsic, non-relational property of conscious states. Nothing of relevance hangs on this difference: my account is compatible with either notion of acquaintance. See Hasan and Fumerton (2017) for more on acquaintance.

9. While the nature of pain is debated, what I have said seems broadly agreed upon by perceptual theorists. Nevertheless, I only appeal to pain to help illustrate the notion of acquaintance. See Murat (2013) for more on pain.

10. What about the future that isn't causally determined? We would have to claim that there are no truths about such things for God to know. However, God is not clueless about such things. Propositions concerning the uncertain future have a certain probability of being true, given present facts, and God will have knowledge of these probabilities. This will fall short of knowledge of what will happen in these cases, but I think this is the best we can do consistent with the assumption that divine knowledge is scrutable.

11. This approach is inspired by Schindler's (2018) recent defence of scientific realism.