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Game theoretic handover optimisation for dense small cells heterogeneous networks

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Q2 Abstract: Ultra-dense deployment of small cells is capable of enhancing the cellular network performance in terms of capacity improvement and coverage expansion. However, this deployment results in high interference and frequent handovers, and hence, high-energy consumption is expected. In this study, the authors formulate a non-cooperative game approach in which all base stations compete in a selfish manner to transmit at higher power. Each base station in the network is considered as a player in the game. The solution of the game is obtained by finding the optimal point, namely the Nash equilibrium. The proposed method, named efficient handover game theoretic, targets to manage the handover in dense small cell heterogeneous networks. Each player in the game optimises its payoff by adjusting the transmission power so as to enhance the overall performance in terms of throughput, handover, energy consumption, and load balancing. In order to choose the preferred transmission power for each player, the payoff function takes into account the gain of increasing the transmission power, energy consumption, base station load, and unnecessary handover. The cell selection is performed using the technique for order preference by similarity to an ideal solution (TOPSIS). A game theoretical approach is implemented and evaluated for dense small cell heterogeneous networks to validate the enhancement achieved in the proposed method. Results show that the proposed game theoretical approach provides a throughput enhancement while reducing the power consumption in addition to minimise the unnecessary handover and balance the load between base stations.

Nomenclature

N_{sc}	total number of small cells
V _{ue}	user velocity
θ_k	user direction
10	Gaussian distribution with mean x and standard
	deviation (SD) y
SINR _{bsk}	signal-to-interference-plus-noise ratio received from cell
~	k
$P_{bs_k \rightarrow ue}^r$	downlink received power from cell k
$T_{bs_k \rightarrow ue}^r$	user equipment data rate from cell k
$L_{\mathrm{bs},k}$	load of cell k
$P_{\rm bs}^{t^{\rm max}}$	maximum base station transmission power
$D_{\rm bs}$	density metric
$N^{ m ho}$	unnecessary handover metric
S	number of players
A_k	set of possible strategies for player S_k
ϕ_k	payoff function for player S_k
U_k	utility function
E_k	energy cost function
L_k	load cost function
N^{ho_k}	unnecessary handover cost function
H	Hessian matrix
G	Jacobian matrix
\mathscr{P}_k	Lagrangian function
u_k, v_k	Lagrangian multipliers
w_j^{sd}	SD weight for attribute <i>j</i>

1 Introduction

The significant increase in the number of smart user equipment (UE) associated with the cellular network has led to a huge demand for network coverage capacity enhancement. The massive

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deployment of small cells (SCs) is considered as an efficient solution to cope with such demand [1]. Generally, the ultra-dense deployment of SCs is foreseen as one of the key technologies of the fifth generation (5G) networks. This kind of SC deployment can also help in offloading the traffic from the already deployed macrocell (MC) base stations, however, new challenges are introduced including the interference, handover (HO) issues, and hence, higher signalling overhead, which results in an increase in power consumption. Many types of research dealing with the problem of HO have been accomplished in the literature. Singoria et al. [2] propose a call admission control to reduce the unnecessary HO in the heterogeneous network (HetNet). The velocity of the user received signal strength (RSS) and the time required to sustain the minimum RSS for ensuring service continuity are used as HO metrics. Only low-speed UEs are allowed to perform HO to SC, while medium-speed UEs are only permitted to perform HO to SC when their traffic type is real-time traffic such as conversational traffic. In [3], we proposed a method to minimise the number of target SCs and reduce the unnecessary HOs in HetNet. A SC target list is obtained by using the distance between the UE and the SC in addition to the UE's angle of movement. Fast UEs are not permitted to perform HO to SCs. The results show improved performance in terms of SC list reduction, unnecessary HO minimisation, and network throughput enhancement. Alhabo et al. [4] proposed a method to reduce both the unnecessary HO and HO failures. An estimated time of stay (ToS) is used to remove a SC, which could lead to an unnecessary HO or HO failure, from the target HO SC list. The UE can perform HO to the SC, which gives the sufficient signal-to-interferenceplus-noise ratio (SINR) and has enough resources. The time threshold and SINR are also used to find a compromise between unnecessary HO and HO failure. Results reveal that both the unnecessary HO and HO failures have been minimised. An inbound HO method for throughput improvement and load balancing is proposed in [5]. The influence of interference and predicted ToS is combined to perform off-loading from MC tier to

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Fig. 1 HetNet system model

SC tier. An inbound HO margin based on the current serving cell load and interference level is derived so as to obtain the traffic offloading. Results show that this method has reduced the unnecessary HO and failure probability in addition to enhancing the throughput. In [6], the authors proposed a HO method for load balancing HetNets. The users are off-loaded from the congested cells by using the influence of interference. The proposed method utilises a modified A3 HO triggering condition taking into account the cell load and the interference. The results reveal enhancement in performance in terms of throughput and load balancing. Alhabo and Zhang [7] proposed two modified weighted technique for order preference by similarity to an ideal solution (TOPSIS) methods for the purpose of HO management in HetNets. The first method takes into account the entropy weighting strategy for HO metrics weighting, while the second method uses a standard deviation (SD) weighting strategy. Results reveal that the proposed methods have minimised the number of unnecessary HOs and radio link failures probability, in addition to enhancing the mean user throughput.

In this study, we propose a game theoretical solution, named efficient HO game theoretic (EHO-GT), using a dynamic transmission power for the base stations to enhance the performance in terms of throughput and energy efficiency. This is done by deploying a mathematical game where each base station competes to transmit power. The payoff function is defined to consider the gain from increasing the base station transmission power (the utility function) against the cost resulted from energy consumption, base station load, and unnecessary HOs performed to this base station. In order to solve the game, we proved the existence of at least one Nash equilibrium (NE). We then propose a novel EHO-GT game approach and evaluate the network performance in terms of power consumption, average SC load, unnecessary HO, and throughput. The cell selection for HO takes place by deploying multiple attribute TOPSIS technique. The remainder of this paper is organised as follows. In Section 2, an overview of the literature works is given. Section 3 presents the system model used in this work. Section 4 illustrates the proposed game theoretic approach, game solution, and TOPSIS cell selection. While Section 5 presents the results and their analysis. Finally, Section 6 draws the conclusions of this paper.

2 Related works

The energy efficiency is considered as one of the most challenging problems in dense SC HetNets. Therefore, a proper solution is needed to address it. In [8], the authors presented a power consumption mechanism, which deals with the trade-off between data traffic load and energy consumption. This mechanism improved energy efficiency by using a greedy method to switch the cell between active and idle modes. In [9, 10], centralised switching techniques are proposed to adjust the base station powers into on/off modes and transfer the UEs to the neighbouring base stations targeting to minimise the energy consumption. In [11], a method that permits the base station to modify its transmission power based on the data traffic load is presented. The base stations can minimise their transmission power rather than going into passive mode. In [12], we proposed a multi-attribute HO decisionmaking method, which jointly considers the HO problem and UE energy efficiency in HetNets. The analytical hierarchy process is used to obtain the weights of each HO metric while a grey rational

analysis is used to select the best target for HO. The results show a reduction in the number of unnecessary HOs and link failures and improvement in the UE mean energy efficiency. In [13], the authors proposed a method to enhance the energy efficiency in HetNets via power and sub-channel allocation. A resource optimisation problem is formed using convex optimisation. Results show that this method has reduced energy consumption compared to the conventional method. In [14], the authors proposed an energy efficiency method for HetNets. The base stations are distributed according to the Poisson point process distribution. The base station goes to sleep mode when its traffic load goes down aiming to maximise the energy efficiency in the network. Huang et al. [15] presented a method considering the UE association and power control in HetNets. The joint optimisation problem is formulated using a log-utility model. Results show improvement in utility energy efficiency compared to the conventional method. In [16], the authors proposed an adjustable utility function and a bargaining cooperative game for power coordination for HetNets. Results reveal that this method has enhanced energy efficiency. However, the authors of [13-16] neglected to consider the unnecessary HO and density of SCs as cost function, which may result in a high number of unnecessary HOs and an uneven load distribution in the network. Tao et al. [17] presented a sleeping mechanism for SCs to reduce the energy consumption in the network. In the MC edge, the SCs go to sleep mode and the resulted coverage gap will be compensated by the nearby range expanded SCs. The UEs connected to the sleeping SCs will be handed over to the MC. The results show improvement in energy efficiency compared to the conventional method. However, the unplanned sleeping for SCs at the MC edge may cause a link failure and result in HO failure. Additionally, handing over the UEs from the sleeping SCs to the MC may increase the unnecessary HOs and underutilise the SCs resulting in an unbalanced load.

Basically, if the base station is not activated at the right time, a connection failure will happen to cause UE's dissatisfaction. Moreover, most literature works did not consider the UE's mobility in dense SC environment. When switching the base stations between on and off modes there will be an additional increase in the signal overhead due to handing over the UEs, which were associated with an idle mode cell, to a new cell. Therefore, in this study, we consider a game theoretical approach to dynamically allow each base station to adjust its transmission power considering the state of the network in terms of the cost incurred due to the power adjusting. The proposed work considers the gain and cost in the formulation of the game. After reaching the optimum power for all base stations in the game, the base station selection is performed using the multiple attribute decision-making TOPSIS technique.

3 System model

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For the sake of clarity, we define a list of symbols as depicted in Nomenclature section.

The system model in this study consists of a two-tier HetNet, which is formed by a single MC and dense SC base stations deployed under the umbrella MC coverage area as depicted in Fig. 1.

The set of all base stations in the network $S = \{0, 1, 2, ..., N_{sc}\}$, where 0 represents the MC, which covers a radius of 500 m and N_{sc} is the total number of SCs, where each is deployed randomly according to a uniform distribution and covers a radius of 100 m. The minimum distance constraint is also taken into account to make sure that the overlapping between SCs exists. The minimum distance between MC site and SC site is set to 75 m and the SC to SC site distance is set to 40 m [1]. Users are uniformly distributed and their mobility can be defined using two parameters: UE velocity, V_{ue} , and UE direction, θ_k . These two parameters can be defined as Gaussian distribution and are updated accordingly using the following equations [18]:

$$V_{\rm ue} = \mathcal{N}(v_{\rm m}, v_{\rm std}),\tag{1}$$

$$\theta_k = \mathcal{N}\left(\theta_{\rm m}, 2\pi - \theta_{\rm m} \tan\left(\frac{\sqrt{V_{\rm ue}}}{2}\right)\Delta t\right),\tag{2}$$

where $v_{\rm m}$ represents the mean velocity of the UE, $v_{\rm std}$ denotes the SD of the UE velocity, $\theta_{\rm m}$ is the previous direction of the UE, Δt is the period between two updates of the mobility model, and $\mathcal{N}(x, y)$ is a Gaussian distribution with mean *x* and SD *y*. The propagation model between the MC and the UE is defined as

$$PL_{\rm m \to ue} = 128.1 + 37.6 \log_{10}(d_{\rm m \to ue}) + \xi^*, \qquad (3)$$

where $d_{m \to ue}$ is the distance between the UE and the MC in kilometres and ξ^* is a Gaussian distribution random variable with zero mean and 12 dB SD [19].

For SC, the path loss is defined as

$$PL_{sc_i \to ue} = 38 + 30 \log_{10}(d_{sc_i \to ue}) + \xi^*, \tag{4}$$

where $d_{sc_i \rightarrow ue}$ is the distance between the UE and SC *i* in metres. The downlink SINR received from cell *k* at the UE is

$$\operatorname{SINR}_{\operatorname{bs}_{k}} = \frac{P_{\operatorname{bs}_{k} \to \operatorname{ue}}^{r}}{\sum_{\operatorname{bs} \in S, \operatorname{bs} \neq \operatorname{bs}_{k}} P_{\operatorname{bs} \to \operatorname{ue}}^{r} + \sigma^{2}},$$
(5)

where σ^2 is the noise power and $(\sum_{bs \in S, bs \neq bs_k} P_{bs \rightarrow ue}^r)$ represents the summation of the downlink power from the neighbouring cells except cell *k*, i.e. the interfering cells.

The data rate at the UE received from cell k is given by the Shannon capacity formula as

$$T_{bs_k \to ue}^r = BWlog_2(1 + SINR_{bs_k}).$$
(6)

Assuming that all the UEs in cell k have the same quality of service requirement in terms of packet arrival size. Thus, the load on cell k can be written as

$$L_{\text{bs},k} = \sum_{\forall \text{UEs}} \frac{\text{packet arrival rate} \cdot \text{mean packet size}}{T_{\text{bs}_k \to \text{ue}}^r}.$$
 (7)

4 EHO-GT approach

4.1 HO game formulation

The proposed EHO-GT method is formulated mathematically using game theory. Players in the game compete to increase their transmission power. Basically, the action played by one player in the game has an influence on the payoff of other players. The proposed game is governed by the following:

• All base stations in the game can transmit power at a range of $[0, P_{bs}^{t_{max}}]$.

• All base stations in the game share a density-specific metric $D_{\rm bs}$.

• Each base station in the game has a load metric, $L_{bs,k}$, which defines the current load on the base station.

• Each base station in the game has an unnecessary HO metric, N^{ho} , which defines the fraction of unnecessary HO compared to the total HOs in the cell.

Q6 The game is defined as $\Gamma = \{S, (A_k)_{k \in S}, (\phi_k)_{k \in S}\}\)$, where *S* is the number of players, A_k is the set of possible strategies for player S_k and ϕ_k is the payoff function for player S_k . Thus, the game components are listed below

leftmirgin=* Players: represent the base stations in the network, $(S_1, ..., S_k, ..., S_n), \forall k \in S$.

leftmiirgiin=* Strategies: each base station has a set of actions $A = (A_1, ..., A_k, ..., A_n), \forall k \in S$, where $A_k = [0, P_{bs,k}^{max}]$ is the strategy set for player S_k , and hence, $A = \prod_{k=1}^{n} A_k$.

leftmiiirgiiin=* Payoff function: it defines the cost for player S_k to transmit power at $P_{bs,k}^t$. In this study, we define the payoff function

using the gain (utility function) and cost function, which includes the energy cost, load cost, and unnecessary HO cost, all of which are defined below:

• Utility function U_k : represents the gain of player S_k for playing strategy a_k . The utility function here means the profits acquired by each base station by increasing its transmission power $P_{bs,k}^t$ aiming to maximise its gain. There are different types of utility functions, such as linear, logarithmic and exponential [20]. The utility function used in this study is the exponential utility where it has a strict concave property and its second derivative is negative, i.e.

$$U_k(a_k) = \alpha (1 - e^{-P_{\text{bs},k}^t}),$$
 (8)

where α is a predefined weighting factor and $P_{bs,k}^{t}$ is the transmission power of player S_k . Each player aims to increase its transmission power so as to maximise its utility function.

• Energy cost function $E_k(a_k, a_{-k})$: energy consumption is one of the most critical issues in dense SC HetNets. When a player increases its transmission power to maximise its utility, this will cause a negative impact by increasing the energy consumption in the network. Additionally, the dense SCs' deployment also means more power needed for operating the network. Thus, we define the energy consumption cost function as

$$E_k(a_k, a_{-k}) = \beta D_{\mathrm{bs}} P_{\mathrm{bs}, k}^t, \tag{9}$$

where β is a predefined weighting factor for energy cost function and D_{bs} is the density metric of the network [21] in a given coverage area, which can be obtained by using

$$D_{\rm bs} = \frac{|S| \pi R_{\rm sc}^2}{\pi R_{\rm m}^2},$$
 (10)

where $R_{\rm sc}$ and $R_{\rm m}$ are, respectively, the SC and MC radii. The denominator represents the area of the umbrella base station, i.e. the MC coverage area. We set up the number of SCs to 50, which means that $D_{\rm bs} \simeq 2$ and hence, the dense SCs' scenario is obtained.

 Load cost function L_k(a_k, a_{-k}): represents the cost for player S_k of playing an action. Higher load means more consumption of power, thus, we define the load cost as follows:

$$L_k(a_k, a_{-k}) = \lambda L_{\mathrm{bs}, k} P_{\mathrm{bs}, k}^t, \tag{11}$$

where λ represents a predefined weighting factor for load cost function and $L_{\text{bs},k}$ is the load on base station k.

 Unnecessary HO cost function N^{ho}_k(a_k, a_{-k}): higher number of HOs means higher signalling overhead and hence higher energy consumption, in addition to uneven load distribution between cells. Therefore, we incorporate the transmission power on the cost function such that

$$N_k^{\text{ho}}(a_k, a_{-k}) = \delta N_k^{\text{unho}} P_{\text{bs},k}^t, \qquad (12)$$

where δ is a predefined weighting factor for unnecessary HO cost function and N_k^{unho} is the fraction of the number of unnecessary HO compared to the total number of HOs to base station *k*. We regard the HO as an unnecessary when an UE remains one second or less in the base station and then perform another HO.

It is worth noting that the weighting parameters α , β , λ and δ can be adjusted by the network service provider reflecting the priority of each function on the network performance. Now, the payoff function for player $S_k \forall k \in S$ can be written as

$$\phi_k(a_k, a_{-k}) = \alpha (1 - e^{-P_{\text{bs},k}^t}) - \beta D_{\text{bs}} P_{\text{bs},k}^t - \lambda L_{\text{bs},k} P_{\text{bs},k}^t$$

$$-\delta N_k^{\text{unbo}} P_{\text{bs},k}^t,$$
(13)

where $\alpha > 0$, so that the second derivative of $\phi_k(a_k, a_{-k})$ will be negative all time, i.e. concave function.

The solution of the non-cooperative game $\Gamma = \{S, (A_k)_{k \in S}, (\phi_k)_{k \in S}\}$ can be reached by finding the optimal transmission power for each player, i.e. the NE. This means that all players in the game reach optimal strategy $o_k^* = P_{bs,k}^{*^*}$, where no player can improve its payoff function by changing its current played strategy, where $o_k^* = [P_{bs,k}^{*^*}, \dots, P_{bs,k}^{*^*}]$.

Theorem 1: The game $\Gamma = \{S, (A_k)_{k \in S}, (\phi_k)_{k \in S}\}$ is a concave *n*-person game, which has at least one NE.

Proof: The strategy set $A_k = [0, ..., P_{bs,k}^{l^{max}}]$ for player S_k is closed and bounded $\forall k \in S$, which means that A_k is a compact set.

Let the two points $x, y \in A_k$ and $\zeta = [0, 1]$, where $A = \prod_{k=1}^n A_k$. The strategy vector A_k is convex $\forall k \in S$ if for any $x, y \in A_k$ and $\zeta = [0, 1], \zeta x + (1 - \zeta)y \in A_k$.

Let the Hessian matrix **H** of the differentiable payoff function $\phi_k(a_k, a_{-k}) = \alpha (1 - e^{-P_{\text{bs},k}^t}) - \beta D_{\text{bs}} P_{\text{bs},k}^t - \lambda L_{\text{bs},k} P_{\text{bs},k}^t - \delta N_k^{\text{unbo}}$ be $P_{\text{bs},k}^t$ as follows:

$$\boldsymbol{H} = \begin{bmatrix} \frac{\partial^2 \phi}{\partial P_{\text{bs},1}^{t^2}} & \frac{\partial^2 \phi}{\partial P_{\text{bs},1}^{t} P_{\text{bs},2}^{t}} & \cdots & \frac{\partial^2 \phi}{\partial P_{\text{bs},1}^{t} P_{\text{bs},n}^{t}} \\ \frac{\partial^2 \phi}{\partial P_{\text{bs},2}^{t} P_{\text{bs},1}^{t}} & \frac{\partial^2 \phi}{\partial P_{\text{bs},2}^{t^2}} & \cdots & \frac{\partial^2 \phi}{\partial P_{\text{bs},2}^{t} P_{\text{bs},n}^{t}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 \phi}{\partial P_{\text{bs},n}^{t} P_{\text{bs},1}^{t}} & \frac{\partial^2 \phi}{\partial P_{\text{bs},n}^{t} P_{\text{bs},2}^{t}} & \cdots & \frac{\partial^2 \phi}{\partial P_{\text{bs},n}^{t^2}} \end{bmatrix}.$$
(14)

By taking the second derivative of the payoff function ϕ_k , it is obvious that H is negative definite at $P'_{bs,k}$ using the leading principle minor of H, which means that it reaches a local maximum at $P'_{bs,k}$ [22] as depicted in (15). Therefore, the payoff function ϕ_k is strictly concave in A_k , $\forall k \in S$.

$$\phi_{k}^{\prime\prime} = \begin{cases} -\alpha e^{-P_{\text{bs},k}^{\prime}} & \text{for main diagonal elements} \\ 0 & \text{otherwise} \end{cases}$$
(15)

where $(\phi_k^{\check{}} < 0)$ to meet the strict concave condition. \Box

Theorem 2: The non-negative weighted sum $\omega(P_{bs,k}^t, q)$ is diagonally strictly concave if the symmetric matrix $[G(P_{bs,k}^t, q) + G'(P_{bs,k}^t, q)]$ is negative definite $\forall k \in S$, where q is the positive vector, $q = [q_1, q_2, ..., q_n]$ [23].

Proof: We can express the non-negative weighted sum $\omega(P_{\text{bs},k}^t, q)$ as the summation of ϕ_k , i.e.

$$\omega(P_{\mathrm{bs},k}^{t},\boldsymbol{q}) = \sum_{k=1}^{n} q_{k} \phi_{k}(P_{\mathrm{bs},k}^{t}), \quad \forall k \in S, \ q_{k} \ge 0$$
(16)

For each fixed q, a related mapping $g(P_{bs,k}^t, q)$ is defined as the gradients $\nabla_k \phi_k(P_{bs,k}^t)$, i.e.

$$\boldsymbol{g}(\boldsymbol{P}_{\mathrm{bs},k}^{t},\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{q}_{1}\nabla_{1}\boldsymbol{\phi}_{1}(\boldsymbol{P}_{\mathrm{bs},1}^{t}) \\ \boldsymbol{q}_{2}\nabla_{2}\boldsymbol{\phi}_{2}(\boldsymbol{P}_{\mathrm{bs},2}^{t}) \\ \vdots \\ \boldsymbol{q}_{n}\nabla_{n}\boldsymbol{\phi}_{n}(\boldsymbol{P}_{\mathrm{bs},n}^{t}) \end{bmatrix},$$
(17)

where $g(P_{\text{bs},k}^{t}, q)$ is the pseudo-gradient of $\omega(P_{\text{bs},k}^{t}, q)$ and $\nabla_{k}\phi_{k}(P_{\text{bs},k}^{t}) = \alpha e^{-P_{\text{bs},k}^{t}} - \beta D_{\text{bs}} - \lambda L_{\text{bs},k} - \delta N_{k}^{\text{unho}}, \forall k \in S.$

As stated earlier, when the symmetric matrix $[G(P_{bs,k}^{l}, q) + G'(P_{bs,k}^{l}, q)]$ is negative definite, then $\omega(P_{bs,k}^{l}, q)$ is diagonally strictly concave [23]. Therefore, we define the Jacobian matrix $G(P_{bs,k}^{l}, q)$ of $g(P_{bs,k}^{l}, q)$ with respect to $P_{bs,k}^{l}$ as follows:

$$G(P_{\mathrm{bs},k}^t, q) =$$

$$\begin{bmatrix} q_1 \frac{\partial^2 \phi}{\partial P_{\text{bs},1}^{t^2}} & q_1 \frac{\partial^2 \phi}{\partial P_{\text{bs},1}^{t} P_{\text{bs},2}^{t}} & \dots & q_1 \frac{\partial^2 \phi}{\partial P_{\text{bs},1}^{t} P_{\text{bs},n}^{t}} \\ q_2 \frac{\partial^2 \phi}{\partial P_{\text{bs},2}^{t} P_{\text{bs},1}^{t}} & q_2 \frac{\partial^2 \phi}{\partial P_{\text{bs},2}^{t^2}} & \dots & q_2 \frac{\partial^2 \phi}{\partial P_{\text{bs},2}^{t} P_{\text{bs},n}^{t}} \\ \vdots & \vdots & \ddots & \vdots \\ q_n \frac{\partial^2 \phi}{\partial P_{\text{bs},n}^{t} P_{\text{bs},1}^{t}} & q_n \frac{\partial^2 \phi}{\partial P_{\text{bs},n}^{t} P_{\text{bs},2}^{t}} & \dots & q_n \frac{\partial^2 \phi}{\partial P_{\text{bs},n}^{t}} \end{bmatrix}.$$
(18)

Obviously, the symmetric matrix $[\boldsymbol{G}(P_{\text{bs},k}^{t},\boldsymbol{q}) + \boldsymbol{G}'(P_{\text{bs},k}^{t},\boldsymbol{q})]$ is negative definite $\forall P_{\text{bs},k}^{t} \in S$, therefore, the non-negative weighted sum $\omega(P_{\text{bs},k}^{t},\boldsymbol{q})$ is diagonally strictly concave. This means that the game $\Gamma = \{S, (\boldsymbol{A}_{k})_{k \in S}, (\phi_{k})_{k \in S}\}$ has a unique NE (Theorem 2 [23]).

4.2 HO game solution

In the previous section, we mathematically proved the existence of NE, we need to compute the optimal game solution for each player S_k . This is done by choosing a strategy that maximises its payoff function $\phi_k(P_{\text{bs},k}^t)$. The optimal transmission power $P_{\text{bs},k}^{t^*} \forall k \in S$ is in the range $0 \leq P_{\text{bs},k}^t \leq P_{\text{bs},k}^{t^{\text{max}}}$. Therefore, the optimisation problem can be written as

$$\begin{array}{l} \underset{P_{bs,k}^{t} \in A_{k}}{\text{maximise}} \quad \phi_{k}(P_{bs,k}^{t}, P_{bs,-k}^{t}), \\ \underset{P_{bs,k}^{t} \in A_{k}}{\text{subject to}} \quad P_{bs,k}^{t} \geq 0, \\ P_{bs,k}^{t} \leq P_{bs,k}^{t^{\max}}, \quad \forall k \in S. \end{array}$$

$$(19)$$

To solve the above non-linear optimisation problem, we define the Lagrangian function \mathcal{P}_k and the Lagrangian multipliers u_k and v_k for player S_k , $\forall k \in S$ as follows:

$$\mathcal{P}_{k} = \phi_{k}(P_{\text{bs},k}^{t}, P_{\text{bs},-k}^{t}) + u_{k}P_{\text{bs},k}^{t} + v_{k}(P_{\text{bs},k}^{t^{\max}} - P_{\text{bs},k}^{t}),$$
(20)

The Karush–Kuhn–Tucker conditions [24] of the maximisation problem for player S_k are

$$\begin{split} u_{k}, v_{k} &\geq 0, \\ P_{\text{bs},k}^{t} &\geq 0, \\ P_{\text{bs},k}^{t^{\max}} - P_{\text{bs},k}^{t} &\geq 0, \\ \nabla_{P_{\text{bs},k}^{t}} \phi_{k}(P_{\text{bs},k}^{t}, P_{\text{bs},-k}^{t}) + u_{k} \nabla_{P_{\text{bs},k}^{t}}(P_{\text{bs},k}^{t}) \\ &+ v_{k} \nabla_{P_{\text{bs},k}^{t}}(P_{\text{bs},k}^{t^{\max}} - P_{\text{bs},k}^{t}) = 0, \\ u_{k}(P_{\text{bs},k}^{t}), v_{k}(P_{\text{bs},k}^{t^{\max}} - P_{\text{bs},k}^{t}) = 0. \end{split}$$

The problem above can be solved as follows:

• When $P_{\text{bs},k}^t = 0$ and $v_k = 0$

$$\alpha e^{0} - \beta D_{bs} - \lambda L_{bs,k} - \delta N_{k}^{unho} + u_{k} = 0$$
$$u_{k} = \beta D_{bs} + \lambda L_{bs,k} + \delta N_{k}^{unho} - \alpha$$

The solution $P_{bs,k}^t = 0$ is feasible, if the condition $(u_k > 0)$ holds and it is as follows:

$$\beta D_{bs} + \lambda L_{bs,k} + \delta N_k^{\text{unno}} \ge \alpha$$

When $P_{bs,k}^t = P_{bs,k}^{t^{\text{max}}}$ and $u_k = 0$
 $\alpha e^{-P_{bs,k}^t} - \beta D_{bs} - \lambda L_{bs,k} - \delta N_k^{\text{unho}} - v_k = 0$
 $v_k = \alpha e^{-P_{bs,k}^t} - \beta D_{bs} - \lambda L_{bs,k} - \delta N_k^{\text{unho}}$

The solution $P_{bs,k}^{t} = P_{bs,k}^{max}$ is feasible, if the condition $(v_k > 0)$ holds and it is as follows:

$$\beta D_{bs} + \lambda L_{bs,k} + \delta N_k^{\text{unho}} \leq \alpha e^{-P'_{bs,k}}$$
• When $u_k = 0$, $v_k = 0$ and $(0 < P_{bs,k}^t < P_{bs,k}^{t^{\text{max}}})$

$$\alpha e^{-P'_{bs,k}} - \beta D_{bs} - \lambda L_{bs,k} - \delta N_k^{\text{unho}} = 0$$

$$e^{-P'_{bs,k}} = \frac{\beta D_{bs} + \lambda L_{bs,k} + \delta N_k^{\text{unho}}}{\alpha}$$

$$P_{bs,k}^t = \ln\left(\frac{\alpha}{\beta D_{bs} + \lambda L_{bs,k} + \delta N_k^{\text{unho}}}\right)$$

Therefore, the game solution for player S_k , $\forall k \in S$, is the optimum transmission power $P_{bs,k}^{t^*}$, which can be expressed as follows:

$$P_{\text{bs},k}^{t^*} = \begin{cases} 0 & \text{if condition } A \\ P_{\text{bs},k}^{\text{max}} & \text{if condition } B \\ \ln\left(\frac{\alpha}{\beta D_{\text{bs}} + \lambda L_{\text{bs},k} + \delta N_k^{\text{unho}}}\right) & \text{otherwise} \end{cases}$$
(21)

where condition A and condition B, respectively, are

$$\beta D_{\rm bs} + \lambda L_{{\rm bs},k} + \delta N_k^{\rm unho} \ge \alpha, \tag{22}$$

$$\beta D_{\rm bs} + \lambda L_{{\rm bs},k} + \delta N_k^{\rm unho} \le \alpha {\rm e}^{-P_{\rm bs}^t,k} \,. \tag{23}$$

The optimum transmission power $P_{bs,k}^{t^*}$ is the NE and the solution of the game.

4.3 Cell selection

After adjusting the transmission power for each cell, we use multiple criteria HO including data rate, UE velocity, and cell load.

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We adopt one of the well-known multiple attribute decision marking techniques, i.e. TOPSIS [25], to select the proper HO target cell. The three criteria are all weighted based on the SD weighting technique [26] to rate the importance of each metric on HO decision. The SD weighting technique computes the weights of each metric in terms of the SD and gives a small weight for a metric if the value of this metric is identical for all available cells. In other words, metrics with small SD are given smaller weights and vice versa. The best cells are ranked according to TOPSIS and the highest ranked cell is chosen as the HO target.

The cell selection for HO using TOPSIS procedures can be expressed as follows:

Step 1: The decision matrix, D, is formed by mapping the alternatives against the attributes as shown

$$\boldsymbol{D} = \begin{bmatrix} x_{11} & x_{12} & x_{1n} \\ x_{21} & x_{22} & x_{2n} \\ x_{31} & x_{32} & x_{3n} \\ \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & x_{mn} \end{bmatrix},$$
(24)

where each row represents one alternative, and the columns represent their correspondent attributes, n = 1, ..., 3, $m = 0, 1, 2, ..., N_{sc}$, x_{ij} represents the value of the *j*th attribute (HO metric) for the *i*th alternative (base station).

Step 2: The decision matrix is then normalised as shown in (25)

$$x_{ij}^{\text{norm}} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}, \ x_{ij}^{\text{norm}} \in [0, 1],$$
(25)

where x_{ij}^{norm} is the *j*th normalised attribute of the *i*th alternative.

Step 3: The normalised matrix is weighted. Thus, the weighted normalised decision matrix can be written as

$$\boldsymbol{D}^{n,\boldsymbol{w}} = \begin{bmatrix} x_{11}^{\text{norm}} \cdot w_1 & x_{12}^{\text{norm}} \cdot w_2 & x_{13}^{\text{norm}} \cdot w_3 \\ x_{21}^{\text{norm}} \cdot w_1 & x_{22}^{\text{norm}} \cdot w_2 & x_{23}^{\text{norm}} \cdot w_3 \\ x_{31}^{\text{norm}} \cdot w_1 & x_{32}^{\text{norm}} \cdot w_2 & x_{33}^{\text{norm}} \cdot w_3 \\ \vdots & \vdots & \vdots \\ x_{m1}^{\text{norm}} \cdot w_1 & x_{m2}^{\text{norm}} \cdot w_2 & x_{m3}^{\text{norm}} \cdot w_3 \end{bmatrix}$$
(26)
$$= \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \\ \vdots & \vdots & \vdots \\ d_{m1} & d_{m2} & d_{m3} \end{bmatrix}$$
subject to $\sum_{i \in n} w_i = 1$, (27)

where d_{ij} is the *j*th weighted normalised attribute of the *i*th alternative, i.e. $d_{11} = x_{11}^{norm} \cdot w_1$, $d_{12} = x_{12}^{norm} \cdot w_2$ and so on. The SD weighting technique [26] measures the weights of each attribute in terms of the SD. It gives a small weight for an attribute if the value of this attribute is identical for all available alternatives. The weights can be measured using the SD technique as

$$w_j^{\rm sd} = \frac{\sigma_j}{\sum_{k=1}^n \sigma_k},\tag{28}$$

$$\sigma_{j} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (x_{ij}^{\text{norm}} - \mu_{j})^{2}},$$
(29)

$$\mu_j = \frac{1}{m} \sum_{i=1}^m x_{ij}^{\text{norm}},$$
(30)

Table 1 Simulation parameters	
simulation time	1200 s
MC radius	500 m
SC radius	100 m
number of SCs	50
bandwidth	20 MHz
MC maximum transmission power	46 dBm
SC maximum transmission power	30 dBm
UE velocity	{0, 10, 20, 30, 40,
	50, 60, 70, 80,
	90, 100} km/h
(packet arrival rate \cdot mean packet size)	180 kbps
$(\alpha, \beta, \lambda, \delta)$	(14, 7, 7, 7)



Fig. 2 Average SC power consumption



Fig. 3 SC average load

where σ_j and μ_j are, respectively, the SD and the mean value of the *j*th normalised attribute.

Step 4: The weighted normalised decision matrix is used to find the ideal positive solution (best alternative, which has the best attribute values, denoted as z^+) and the ideal negative solution (worst alternative, which has the worst attribute values, denoted as z^-) by

$$z^{+} = \left\{ (\max_{i \in m} D_{ij}^{n,w} \mid j \in j^{+}), (\min_{i \in m} D_{ij}^{n,w} \mid j \in j^{-}) \right\}$$

= $\left\{ d_{1}^{+}, d_{2}^{+}, d_{3}^{+} \right\},$ (31)

$$z^{-} = \left\{ (\min_{i \in m} D_{ij}^{n,w} \mid j \in j^{+}), (\max_{i \in m} D_{ij}^{n,w} \mid j \in j^{-}) \right\}$$

= $\left\{ d_{1}^{-}, d_{2}^{-}, d_{3}^{-} \right\},$ (32)

where j^+ is the set with the attributes having a positive impact (i.e. the higher value the better), and j^- is the set with the attributes having a negative impact (i.e. the lower value the better).

Step 5: Measure the Euclidean distance between each alternative and both the positive and negative ideal solutions as

$$dist^{+} = \sqrt{\sum_{j=1}^{n} (D_{ij}^{n,w} - d_{j}^{+})^{2}}, \quad \forall i = 1, ..., m$$
(33)

dist⁻ =
$$\sqrt{\sum_{j=1}^{n} (D_{ij}^{n,w} - d_j^{-})^2}, \quad \forall i = 1, ..., m$$
 (34)

Step 6: The ranking network vector, **Rn**, is obtained so as to measure

$$\boldsymbol{Rn} = \frac{\operatorname{dist}^{-}}{\max\left(\operatorname{dist}^{-}\right)} - \frac{\operatorname{dist}^{+}}{\min\left(\operatorname{dist}^{+}\right)}, \quad \forall i = 1, \dots, m.$$
(35)

Step 7: The ranking network vector is then ranked in descending order and the best alternative from Rn vector is chosen as a target (i.e. the HO target cell)

$$HO_{target} = \arg \max_{i \in m} \mathbf{Rn}(i).$$
(36)

5 Performance and results analysis

In this section, the proposed EHO-GT method is implemented, evaluated, and compared against the conventional method, in which the cells are not able to optimise their transmission power, in terms of power consumption, average SC load, unnecessary HO probability, and throughput. The conventional method adopts TOPSIS for cell selection. Each cell in the network dynamically adjusts its transmission power according to the solution of the EHO-GT method. Then, the cell selection is performed using the TOPSIS technique. Simulation parameters are listed in Table 1.

5.1 Power consumption

For different velocities, the average SC power consumption with regard to the number of users is depicted in Fig. 2. Comparing the proposed EHO-GT method with the conventional method at 30 km/h, at all user densities the EHO-GT gives better performance. For example, when the number of users is 20, the EHO-GT has a 6.5% reduction in the average SC power consumption compared to the conventional method. It is observed for the proposed EHO-GT method that the higher the velocity the lower consumption in power. This is because more SC will increase their transmission power when low speed users approach their coverage area. On the other hand, low power consumption for higher velocities is due to the association of the users with MC and reducing the transmission power of SCs.

5.2 Average load

The average SC load versus the number of UEs with the consideration of different velocities is depicted in Fig. 3. It can be seen that for all velocities the proposed EHO-GT method has outperformed the conventional method as the latter does not optimise the transmission power prior to HO. For the proposed EHO-GT method, at high velocity (e.g. 90km/h), the SC load is the lowest because most high-speed users will be connected to the MC due to reducing/deactivating the SC transmission power. The opposite is happening with a low velocity of 30 km/h because more users will be associated with the SC and the load increase with the increase of the number of users.

Q7 Furthermore, in Fig. 4, for 40 UEs and variable densities of SCs, the average SC load is shown. When the SCs is 10–20, the SC



Fig. 4 SC average load



Fig. 5 Unnecessary HO probability

load gets reduced because the load will be distributed among the increased number of SCs. When the number of SCs is 20–40, the SC load gets increased because some of the SCs will go to sleep mode causing an increase in the load in the active SCs. Additionally, when the number of SCs is 40–50, the load gain will Q8sharply reduce due to the distribution of load among the increased number of SCs.

5.3 Unnecessary HO

Fig. 5 shows the probability of unnecessary HO with respect to the number of users and for different velocities. We defined the unnecessary HO when the UE starts a HO process to cell k and leaves the cell after one second. We can observe that our proposed EHO-GT method has outperformed the conventional method. For instance, comparing the two methods at 20 UEs and at a velocity of 30 km/h, the EHO-GT has about 51% reduction in the unnecessary HO and this percentage increases with the increase in the number of UEs. Generally, with the EHO-GT, the lower the velocity the lower the unnecessary HO since high-speed UEs are likely to cause frequent HOs. The unnecessary HO increases with the increase in the UE numbers (i.e. load increases) affecting the load and unnecessary HO terms in the payoff function in (13), and hence, the increase occurs.

5.4 Throughput

For different UE densities, the averaged SC throughout is depicted in Fig. 6. It is obvious that the EHO-GT method has outperformed the conventional method. For the EHO-GT, the average SC throughput for high-speed UEs is the lowest compared to the lower speed UEs because the former tends to select the MC while the latter tends to select the SC in TOPSIS cell selection. Generally, the average SC throughput for all UE densities goes down after a

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Fig. 6 Average SC throughput versus UE velocity

40 km/h velocity because the high-speed UEs connect to the MC and few numbers of UEs connect to the SC.

6 Conclusion

In this study, we used the game theory approach to optimise the transmission power of the SC aiming to find the optimal power for all cells in the network. The payoff function for each player (each cell) is formulated mathematically using utility (gain) and cost functions where each cell selfishly aims to increase its transmission power to improve its utility. The cost function includes the influence of SC density, cell load, and unnecessary HO. The proposed EHO-GT method is solved mathematically by finding the NE. The cell selection is then performed by deploying the multiple criteria TOPSIS technique to choose the best HO target cell. Furthermore, we have implemented, evaluated and compared the proposed EHO-GT method with the conventional method where the power optimisation policy is not present. Simulation results reveal that the proposed EHO-GT method outperformed the conventional method in terms of power consumption, SC average load, unnecessary HO, and throughput. For example, with 30 km/h velocity and 20 users, the proposed EHO-GT method has an improvement of 6.5, 43, 51 and 81% over the conventional method in terms of power consumption, average SC load, unnecessary HO, and average SC throughput, respectively. The achieved results validate the efficiency of our proposed method, which can be adopted in an ultra-dense SC environment. Q9

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