



This is a repository copy of *Cost-efficient performance-vesting equity*.

White Rose Research Online URL for this paper:  
<http://eprints.whiterose.ac.uk/148217/>

Version: Accepted Version

---

**Article:**

Skovoroda, R., Bruce, A., Buck, T. et al. (1 more author) (2019) Cost-efficient performance-vesting equity. *Economics Letters*, 182. pp. 37-39. ISSN 0165-1765

<https://doi.org/10.1016/j.econlet.2019.06.002>

---

Article available under the terms of the CC-BY-NC-ND licence  
(<https://creativecommons.org/licenses/by-nc-nd/4.0/>).

**Reuse**

This article is distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs (CC BY-NC-ND) licence. This licence only allows you to download this work and share it with others as long as you credit the authors, but you can't change the article in any way or use it commercially. More information and the full terms of the licence here: <https://creativecommons.org/licenses/>

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.



[eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk)  
<https://eprints.whiterose.ac.uk/>

## Cost-efficient Performance-Vesting Equity

*Rodion Skovoroda\**, *Alistair Bruce*<sup>2</sup>, *Trevor Buck*<sup>3</sup>, *Ian Gregory-Smith*<sup>4</sup>

2 June 2019

[*Economics Letters*, forthcoming]

### Abstract

We analyse the incentive effects of the Performance-Vesting Equity (PVE) component of executive pay that is characterised by zero exercise price and performance-contingent vesting. We demonstrate how PVE with upward-sloping convex/concave vesting curves can be a more efficient risk-sharing and incentive alignment device than strictly convex stock options.

**Keywords:** Executive Compensation, Incentive Pay, Performance-Vesting Equity

JEL classification: G3; J3

\* Corresponding author, [Rodion.Skovoroda@nottingham.ac.uk](mailto:Rodion.Skovoroda@nottingham.ac.uk)

(1) Nottingham University Business School, The University of Nottingham, Jubilee Campus, Nottingham, NG8 1BB, UK;

(2) Nottingham Business School, Nottingham Trent University, 50 Shakespeare St, Nottingham, NG1 4FQ, UK

(3) Adam Smith Business School, Glasgow University, Glasgow, G12 8QQ, UK;

(4) Department of Economics, The University of Sheffield, Sheffield, 9 Mappin Street, S1 4DT, UK.

## 1. Introduction

Recent research suggests that using ‘plain vanilla’ options for the purpose of providing risk-averse CEOs with target levels of utility and effort incentives can be costly and less efficient than using restricted stock with zero exercise price (Dittmann and Maug (2007), hereafter DM, and Dittmann, Yu, and Zhang (2017), hereafter DYZ), i.e., the strictly convex options are too risky to be an effective risk-sharing mechanism.

While recent studies document a decline in the use of executive stock options in the USA, Bettis *et al.* (2018) show empirically that pay convexity has not declined over time. Rather, PVE, an increasingly popular element of CEO pay, contributes significantly to measures of pay convexity in recent years. PVE typically vests after three years, based on a blend of stock-price (e.g. Total Shareholder Return) and accounting (e.g. EPS) metrics. The growing use of PVE has also been reported in the UK (Buck *et al.* 2003; Carter *et al.* 2009; Skovoroda and Bruce 2017). *Can PVE and its associated pay convexity be cost-efficient?*

We demonstrate that under the standard assumptions of risk-neutral firms and risk averse CEOs, PVE can be efficient. Importantly, cost-efficient PVE does not yield strictly convex payouts – PVE payouts are convex for medium outcomes and concave for good (high) outcomes. This result is consistent with both the shape of vesting rules observed in practice (Carter *et al.* 2009) and with the shape of the optimal incentive contract (DYZ).

## 2. Model

We analyse PVE by extending models in DM, where firm risk is exogenous, and in DYZ, where it is endogenous, and show that PVE can be cost-efficient under either assumption. While DM and DYZ investigate incentive contracts of stocks and options, they do not analyse the effects of performance-contingent vesting.

### 2.1 Exogenous firm risk

The risk-averse CEO has CRRA utility  $U = U(V) - C(a) = V^{1-\gamma}/(1-\gamma) - C(a)$ , where  $V$  is the CEO’s wealth,  $\gamma \neq 1$  is the parameter of relative risk aversion,  $a$  is the CEO’s effort, and  $C(a)$

is the convex cost of effort. The CEO has initial (not firm-related) wealth  $W_0$ , reservation utility  $E(U) = U_0$ , and is paid with a mix of risk-free wages  $W$  and performance stock  $N$ . We omit stock options, since options are shown to be inefficient in DM<sup>1</sup>.

While both DM and DYZ assume that the size of the stock-based pay element  $N$  does not depend on firm performance, PVE is characterized by partial vesting  $N(s)$ ,  $N_0 \leq N(s) \leq N_0 + N_1$ , based on the firm's stock price  $s$  at maturity. The novelty of our study is in its focus on efficient vesting rules  $N(s)$ . For calibration purposes, we assume performance-contingent vesting in the form:

$$(1) \quad N(s) = N_0 + \frac{s^h N_1}{s^h + A^h},$$

where  $N_0 \geq 0$  is the number of stocks that vest across all levels of performance (i.e. non-PVE);  $N_1 \geq 0$  is the size of the PVE element where vesting is subject to performance conditions;  $A > 0$  is the level of performance that pays 50 percent of PVE. Importantly, the hill factor  $h$  affects the shape of the vesting curve – for values  $0 < h \leq 1$  vesting rule (1) is strictly concave, while for values  $h > 1$  it is convex for relatively lower values of  $s$  and concave for larger values of  $s$ .

The stock price at maturity  $T$  is log-normally distributed  $\ln(s) \sim N(\mu, \sigma^2 T)$  with density  $f(s) = f(s, \mu, \sigma^2 T)$ . Mean log-price  $\mu = \mu(a)$  is increasing and concave with CEO effort. Consistent with DM, the level of firm-specific risk  $\sigma$  is exogenous in this section, i.e. the CEO does not affect  $\sigma$ .

The CEO maximization problem, given incentive contract  $(W, N_0, N_1, A, h)$ , is:

$$\begin{aligned} \max_{a, \sigma} \{ & E(U(W_0 + W + sN(s))) - C(a) \}, \\ \text{s.t: } & \sigma = \text{const.} \end{aligned}$$

The first order condition and the incentive compatibility constraint is  $\frac{d}{d\mu} E[U(W_0 + W + sN(s))] = \frac{C'}{\mu'}$ .

Assuming the cost-minimizing risk-neutral firm wishes to implement  $a$ , the firm's problem is:

$$(2) \quad \begin{aligned} \min_{W, N_0, N_1, A, h} \{ & E(W + sN(s)) \}, \\ \text{s.t: } & E[U(W_0 + W + sN(s))] = U_0, \end{aligned}$$

---

<sup>1</sup> Dittmann, Maug and Spalt (2013) model stock options where the strike price is indexed to a benchmark and argue that indexing options would further increase compensation costs.

$$\frac{dE[U]}{d\mu} = \frac{c'(a)}{\mu'(a)}.$$

To illustrate the effect of PVE on the strengths of effort incentives  $\frac{dE[U]}{d\mu}$ , we write

$$\begin{aligned} (3) \quad \frac{dE[U]}{d\mu} &= \frac{d}{d\mu} \left[ \int_0^\infty U(W_0 + W + sN(s)) f(s, \mu, \sigma^2 T) ds \right] \\ &= \int_0^\infty U(W_0 + W + sN(s)) f'_\mu(s, \mu, \sigma^2 T) ds \\ &= \int_0^\infty \frac{dU}{d \ln(s)} f(s, \mu, \sigma^2 T) ds = \int_0^\infty s \left( N(s) + \frac{dN(s)}{d \ln(s)} \right) U' f(s, \mu, \sigma^2 T) ds \end{aligned}$$

We integrate by parts in (3) using property  $f'_\mu = -(sf)'_s$ . Equations (1) and (3) show that PVE structures ( $N_1 > 0$ ) add a positive term  $\frac{dN(s)}{d \ln(s)} > 0$  to the strengths of effort incentives. As we further show in Section 3, PVE-based incentives are powerful, i.e. cost-efficient per unit of shareholder funds: cost-efficient contracts (2) include performance stocks ( $N_1 > 0$ ), do not include restricted stocks ( $N_0 = 0$ ), and have positively-sloping vesting curves with convex/concave shapes  $h > 1$ .

## 2.2 Endogenous firm risk

While Program (2) assumes that firm risk is exogenous, DYZ study the case of endogenous risk where CEOs can also choose firm risk in addition to effort. An important feature of PVE is its ability to provide incentives to undertake risky projects with higher returns. To capture this feature, we allow CEO effort  $a$  to yield a menu of potential projects  $(\mu, \sigma)$  such that  $\mu \leq \varphi(\sigma, a)$ , where  $\varphi$  is increasing and concave in both arguments. This means, for any level of effort, more valuable projects are also riskier and the CEO faces a trade-off between risk  $\sigma$  and potential return  $\mu$ . This is also consistent with scenarios where CEOs have opportunities to reduce firm-specific risk by pursuing unrelated diversification and costly hedging strategies. These activities, if they reduce firm value, are inefficient from the shareholders' point of view.

The CEO maximization problem now is:

$$\begin{aligned} \max_{a, \sigma} \{ & E [ U ( W_0 + W + sN(s) ) ] - C(a) \} \\ s. t: & \mu \leq \varphi(\sigma, a), \end{aligned}$$

and the first-order condition is  $\frac{dE[U]}{d\mu} = C'/\mu'$ , and  $-\left(\frac{dE[U]}{d\sigma}\right) / \left(\frac{dE[U]}{d\mu}\right) = \varphi'_\sigma(\sigma, a)$ . While  $\frac{dE[U]}{d\mu}$  measures the strengths of effort incentives, ratio  $-\left(\frac{dE[U]}{d\sigma}\right) / \left(\frac{dE[U]}{d\mu}\right)$  defines CEOs' risk-taking incentives. If the firm wishes to implement  $(a, \sigma)$ , the cost-minimisation problem now is:

$$(4) \quad \begin{aligned} & \min_{W, N_0, N_1, A, h} \{E(W + sN(s))\}, \\ & \text{s.t: } E[U(W_0 + W + sN(s))] = U_0, \\ & \frac{dE[U]}{d\mu} = C'/\mu', \\ & -\left(\frac{dE[U]}{d\sigma}\right) / \left(\frac{dE[U]}{d\mu}\right) = \varphi'_\sigma(\sigma, a). \end{aligned}$$

### 3. Empirical calibrations

In Table 1 (Panel A), we estimate the values of  $U_0$ ,  $dE[U]/d\mu$ , and  $-\left(\frac{dE[U]}{d\sigma}\right) / \left(\frac{dE[U]}{d\mu}\right) = \varphi'_\sigma$  for a representative CEO with a benchmark (not cost-efficient) contract of non-performance-contingent restricted stocks with unconditional vesting. Relevant parameters are at the median of the sample in DM. The benchmark contract costs \$16.483 million to the firm over three years. Results are shown for values 2, 3, and 4 of risk-aversion parameter  $\gamma$ .

Panel A shows that, while for  $\gamma = 2$  the benchmark non-PVE contract ensures CEO's (local) risk-neutrality ( $\varphi'_\sigma = 0.002 \approx 0$ ), for higher values of risk-aversion 3 and 4 it induces conservative selection of projects ( $\varphi'_\sigma = 0.237$  and  $\varphi'_\sigma = 0.444$ ) and might deter the CEO from risky projects with higher expected returns.

Optimal contracts when risk incentives are relevant: Panel B, Table 1, shows our main result: PVE with convex/concave shapes  $h > 1$  can efficiently induce higher levels of risk-taking (i.e. lower levels of  $\varphi'_\sigma$ ) than contracts with flat unconditional vesting while providing the CEO with the same utility and the same effort incentives as the benchmark contract. In particular, convex/concave PVE can efficiently induce risk-neutral selection of projects ( $\varphi'_\sigma = 0$ ). We show this by solving Program (4) while taking the benchmark values of utility  $U_0$  and effort incentives  $dE[U]/d\mu$  as relevant constraints and assuming a range of risk incentives  $\varphi'_\sigma$  that the firm might choose to target that includes the

benchmark levels ( $\varphi'_\sigma = 0.002$  for  $\gamma = 2$ ,  $\varphi'_\sigma = 0.237$  for  $\gamma = 3$ , and  $\varphi'_\sigma = 0.444$  for  $\gamma = 4$ , marked in bold). While inducing higher levels of risk-taking increases the compensation costs, this may be efficient for the firm.

*Optimal contracts when risk incentives are irrelevant:* While convex/concave PVE contracts induce higher levels of risk-taking, they are also more cost-efficient than flat vesting contracts. Higher efficiency savings are possible if CEOs do not influence firm risk (the assumption behind Program (3)). In Panel C, Table 1, we take the values of  $U_0$  and  $dE[U]/d\mu$  from the benchmark contract as constraints to solve Program (2). For  $\gamma = 4$ , the cost-efficient contract costs \$15.885 million and saves about \$598,000 over three years while providing the CEO with the same utility and the same effort incentives as the benchmark contract.

#### **4. Conclusions**

Overall, under the standard assumptions of risk-neutral firms and risk averse CEOs, upward-sloped convex/concave vesting curves are efficient under a wide range of parameters. Simply put, PVE is less risky than options and can be used more efficiently in incentive contracts.

## References

- Bettis, J.C., Bizjak J, Jeffrey L. Coles, J.L., and Kalpathy, S., 2018. Performance-vesting provisions in executive compensation, *Journal of Accounting and Economics*, 66 (1), pp 194–221.
- Buck, T., Bruce, A., Main, B. G., and Udueni, H., 2003. Long term incentive plans, executive pay and UK company performance, *Journal of Management Studies*, 40(7), pp. 1709-1727.
- Carter, M., Ittner, C., and Zechman, S., 2009. Explicit relative performance evaluation in performance-vested equity grants, *Review of Accounting Studies*, 14(2-3), pp. 269-306.
- Dittmann, I. and Maug, E., 2007. Lower salaries and no options? On the optimal structure of executive pay. *The Journal of Finance*, 62(1), pp. 303-343.
- Dittmann I., Maug E., and Spalt O.G., 2013. Indexing executive compensation contracts, *Review of Financial Studies*, 26(12), pp. 3182–3224
- Dittmann, I., Yu, K.C. and Zhang, D., 2017. How important are risk-taking incentives in executive compensation? *Review of Finance*, 21(5), pp. 1805-1846.
- Skovoroda, R. and Bruce, A., 2017. Shifting the goalposts? Analysing changes to performance peer groups used to determine the remuneration of FTSE 100 CEOs. *British Journal of Management*, 28(2), pp. 265-279.



**Table 1. Cost-efficient PVE.**

This Table describes the risk incentives, cost, and the average expected vesting ratio of the benchmark incentive contract of restricted stocks and that of the optimal incentive contracts of performance stocks for a representative CEO where parameters are close to the median of the sample in Dittmann and Maug (2007), (see Panel A, Table I therein). The parameters are: initial non-firm wealth  $W_0 = \$6.86$  million, three-year wage  $W = \$3.783$  million, standard deviation of log returns  $\sigma = 0.335$  and maturity  $T = 3$  years. The value of restricted stocks in the benchmark contract at maturity is \$12.7 million. Without loss of generality, the expected stock price at maturity is \$10, which implies mean log-price  $\mu = 2.13$  and  $N_0 = 1.27$  million for the quantity of restricted stocks in the benchmark contract. Results are shown for values 2, 3, and 4 of the risk-aversion  $\gamma$ .

Risk-aversion	Risk incentives induced; lower values indicate CEO is less likely to pursue risk-reduction strategies that also reduce firm value.	Parameters of contracts: Three-year wage $W$ ; $N_0$ - quantity of restricted stocks with vesting not conditional on performance; $N_1$ - quantity of performance-contingent stocks; $A$ - median vesting location parameter; $h$ - hill coefficient.					Risk-neutral Cost to the firm; Expected percentage of vested stocks;	
$\gamma$	$\varphi'_\sigma$	$W$ (\$ m)	$N_0$ (m)	$N_1$ (m)	$A$ (\$)	$h$	Cost (\$ m)	Expected % of vested stocks
Panel A: Benchmark incentive contract of restricted stocks (not cost-efficient)								
2	<b>0.002</b>	<b>3.783</b>	<b>1.27</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>16.483</b>	<b>100</b>
3	<b>0.237</b>	<b>3.783</b>	<b>1.27</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>16.483</b>	<b>100</b>
4	<b>0.444</b>	<b>3.783</b>	<b>1.27</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>16.483</b>	<b>100</b>
Panel B: Cost-efficient PVE that solves Program (4) for alternative risk incentives $\varphi'_\sigma$ while providing the CEO with the same utility $U_0$ and effort incentives $dE[U]/d\mu$ as the benchmark contract in Panel A.								
2	<b>0.002</b>	<b>7.917</b>	<b>0</b>	<b>0.990</b>	<b>5.996</b>	<b>6.787</b>	<b>16.403</b>	<b>70.5</b>
	0	7.932	0	0.990	6.017	6.774	16.405	70.3
3	<b>0.237</b>	<b>7.748</b>	<b>0</b>	<b>0.952</b>	<b>5.530</b>	<b>10.4</b>	<b>16.270</b>	<b>75.8</b>
	0.1	8.508	0	0.986	6.752	10.21	16.617	64.4
	0	8.939	0	1.049	7.61	11.08	16.997	56.9
4	<b>0.444</b>	<b>7.517</b>	<b>0</b>	<b>0.927</b>	<b>5.049</b>	<b>15.178</b>	<b>16.094</b>	<b>80.8</b>
	0.2	8.639	0	1.045	6.928	16.442	17.188	63.2
	0	9.254	0	1.498	8.379	28.421	20.081	50.6
Panel C: Cost-efficient PVE that solves Program (2) while providing the CEO with the same utility $U_0$ and effort incentives $dE[U]/d\mu$ as the benchmark contract in Panel A. Program (3) assumes that firm risk is exogenous, CEO does not set firm risk, and risk incentives $\varphi'_\sigma$ are irrelevant.								
2	0.157	6.368	0	1.033	4.246	17.88	<b>16.271</b>	87.9
3	0.383	6.386	0	1.002	3.994	26.50	<b>16.087</b>	90.0
4	0.577	6.358	0	0.976	3.751	37.65	<b>15.885</b>	91.8