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On the stability analysis of a geometrically complex leaning historic structure

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Abstract. In this paper, a simple approach for the stability analysis of the southwest leaning ruined tower of the Caerphilly castle (Wales, UK) is discussed. To account for the actual complex geometry of the tower, a mesh generation procedure recently proposed by the authors is used to automatically transform the 3D point cloud surveyed from the tower into a 3D solid finite element model. A simple approach based on nonlinear static analyses is pursued to evaluate the stability condition of the tower. Geometric nonlinearity and nonlinear plastic-damaging behavior of masonry are supposed. Results show that the proposed approach could serve as a simplified method to evaluate the stability of leaning historic structures considering their actual geometry.

Keywords: leaning towers; stability; plastic-damage model; laser scanning; point cloud

1 Introduction

The structural assessment of leaning historic structures is a challenging task for engineers. A smart pioneering contribution in this field has been developed by Heyman [1], who analytically studied the safety of leaning masonry towers supposing a rigid no-tension material. This permitted to obtain a differential equation describing the fissure curve which defines the collapse mechanism and to provide useful insights on the critical inclination associated with the structural failure. Nevertheless, the hypothesis of rectangular section coupled with the absence of any irregularity along the height represent a substantial limitation of the approach, as real structures are characterized by complex and irregular geometries.

Today, with advancements in surveying techniques, such as close-range photogrammetry [2] and terrestrial laser scanning, the complex geometric features of historic structures can be captured. However, their exploitation for structural purposes is still a challenging issue. Recently, novel procedures which directly transform 3D point clouds of geometrically complex buildings into 3D finite elements models have been proposed in [3, 4, 5, 6].

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In this contribution, a simple approach for evaluating the stability analysis of geometrically complex leaning historic structures is discussed. Particularly, the southwest leaning ruined tower of the Caerphilly castle (Wales, UK) is employed as case study. To account for the actual complex and irregular geometry of the structure, a mesh generation procedure is used to automatically transform the 3D point cloud surveyed on the tower in a 3D solid finite element model. A simple approach based on nonlinear static analyses is pursued to evaluate the stability condition of the tower. Geometric nonlinearity and nonlinear continuum plastic-damaging behavior of masonry are supposed.

2 Modelling of geometrically complex historic structures

In this section, after a very brief description of the case study, the simple processing of point clouds recently discussed in [6] is used for the automatic transformation of the point cloud surveyed on the case study into its solid finite element model.

2.1 Southwest tower of the Caerphilly castle

Caerphilly castle is one of the largest medieval concentric castle in Wales, UK. The dramatic inclination of its ruined southwest tower has been noted since 1539 (Figure 1). The tower is made by rubble stone masonry and it is almost 17 meters tall and circular in plan. The diameter of the tower is approximately 9 m.
A laser scanning survey of the southwest tower has been carried out in 2014 [7]. Several scans were taken around the base of the tower, with, however, difficulties in acquiring all the surfaces of the structures due to the presence of scaffolded areas and of tourists. From the survey, the inclination of the tower has been found to be about10 degrees.

Figure 1. Southwest leaning tower of Caerphilly castle. The red circle indicates the location of the southwest leaning tower.

2.2 Automated generation of the finite element model

Detailed 3D finite element model of the southwest leaning tower of Caerphilly castle has been generated starting from a laser scanner survey, see Figure 2. Preliminary standardized operations, such as cloud cleaning and sub-sampling have been carried out on the rough point cloud (Figure 2a) in order to lower the dimension of the problem and to reduce the whole point cloud to only the tower space.

Then, the generation of the surface has been undertaken using a further simplification of the topology of the cloud points along with the idea proposed in [3] [5] [8]. Although, this algorithm produces an approximation of the surface, it was found to be adequate for structural analysis purposes since it produces watertight surface. The obtained watertight mesh, consisting of triangles, is shown in Figure 2b.

It ~~has~~ must to be pointed out that the initial point cloud presented some missing portions, due to practical complications in the surveying operations [7] thus, a projection of the bottom surface nodes has been undertaken, see Figure 2c, in order to easily apply boundary conditions as described in the following sections.

Successively, the triangles of the watertight mesh have been transformed into a boundary mesh of triangular finite elements and, finally, they have been used to generate a solid tetrahedral finite elements free mesh. This operation, which fill the volume delimitated by the watertight mesh with tetrahedrons, has been performed by means of a standard subroutine implemented in the software Abaqus. The resulting solid finite element model is shown in Figure 2c. Since the bottom face of finite element model was irregular in shape, nodes at the base of the tower were projected to a horizontal plane. It should be highlighted that every operation carried out for the generation of the solid finite element model was automatic, apart from the initial clean-up of the rough point cloud.

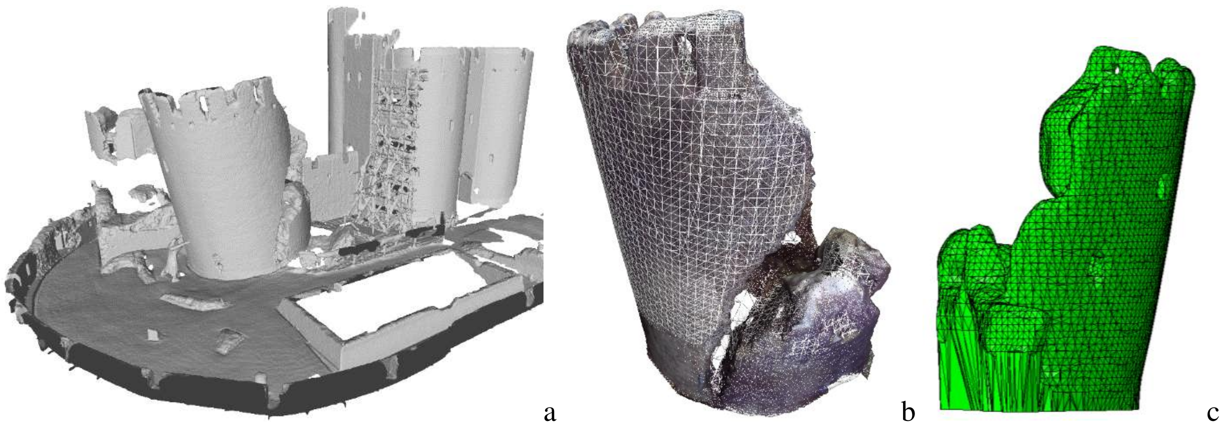

Figure 2. Automated generation of the finite element model from terrestrial laser scanning: a) rough point cloud, b) watertight mesh and c) 3D solid finite element model.

3 Stability analysis approach and results

In this section, the constitutive law used to represent the mechanical behavior of masonry, the analysis approach used for the assessment of the stability of the leaning tower, and the analyses results are presented and discussed.

3.1 Constitutive model for masonry

Although masonry is a composite material characterized by anisotropic behavior, the hypothesis of isotropic nonlinear material generally appears suitable for historic structures due to the chaotic and random texture of historic masonries [9, 10, 5]. In this study, isotropic plastic-damaging nonlinear behaviour for masonry is adopted [11]. Two independent scalar damage variables, one for the tensile damage (d_t) and the other for compressive damage (d_c) are conceived, as well as a yield function with multiple-hardening variables. The stress-strain relations under uniaxial tension and compression loading are, respectively:

$$\sigma_t = (1 - d_t)E_0(\varepsilon_t - \varepsilon_t^p) \quad \textbf{(1)}$$

$$\sigma_c = (1 - d_c)E_0(\varepsilon_c - \varepsilon_c^p) \quad \textbf{(2)}$$

where E_0 is the initial (undamaged) elastic stiffness of masonry, σ_t and σ_c are the uniaxial tensile and compression stresses, ε_t and ε_c are the uniaxial total strain in tension and in compression, ε_t^p and ε_c^p are the uniaxial plastic strain in tension and in compression.

Being the model formulated in the context of non-associated plasticity, the plastic potential is defined also by the dilatancy angle (ψ). Dilatancy angle is typically assumed for masonry equal to 10°, which is an averaged value in agreement with experimental tests [12] and previous numerical models [13]. Furthermore, the parameter ϵ that defines the rate at which the tip of the conical strength domain is smoothed to avoid numerical convergence issues is generally assumed equal to 0.1 [13]. In addition, the strength domain is characterized by the ratio f_{b0}/f_{c0} between the biaxial initial compressive strength f_{b0} and the uniaxial initial compressive strength f_{c0}. Typically, the ratio is assumed equal to $\frac{f_{bo}}{f_{co}} = 1.16$ [14]. Moreover, the constant ρ, which represents the ratio of the second stress invariant on the tensile meridian to that on the compressive meridian at initial yield, is typically assumed $\rho = 2/3$ [13].

In absence of experimental material characterization, reference to the Italian code is made to select the mechanical properties of the material. The properties adopted, referred to a cluttered stone masonry, are collected in Table 1. Finally, uniaxial inelastic strain values in compression have been chosen in agreement to [15], whereas the evolution of the scalar damage variables d_t and d_c has been kept substantially proportional to the softening of the uniaxial stresses, as adopted in several numerical campaigns [10, 13, 9].

Table 1. Mechanical properties adopted for masonry.

E_0 [MPa]	Poisson's coeff. [\]	Density [kg/m^3]	f_{b0}/f_{c0} [\]	ψ [\]	ϵ [\]	ρ [\]
870	0.15	1900	1.16	10°	0.1	2/3

Tensile uniaxial nonlinear behaviour			Compressive uniaxial nonlinear behaviour		
Stress [MPa]	Inelastic strain	d_t [\]	Stress [MPa]	Inelastic strain	d_c [\]
0.02	0	0	1.0	0	0
0.002	0.0001	0.9	1.1	0.001	0
			0.1	0.007	0.9

3.2 Analyses performed and results

Dead load is initially applied to the structure by means of an incremental procedure, considering clamped boundary conditions at the base (Figure 3). Particularly, geometric nonlinearity is considered to account for large-displacement effects, which in this case are expected to play a fundamental role.

The resulting vertical stress contour plots and the tensile damage contour plot of the structure are depicted in Figure 3a-b, respectively, for the last incremental step, corresponding to the full application of the dead load.

By inspecting Figure 3a, it clearly appears a significant concentration of vertical stresses at the base of the structure along the edge where the tower is leaning. In particular, the vertical stresses appear to increase approximately in a linear way as we move perpendicularly from the axis c, shown in Figure 3a trough a dotted red line, to the edge of the base of the tower. In addition, from Figure 3b appears that the tower experiences significant cracking (red elements in Figure 3b) with the application of dead load only, suggesting that possible failure could occur due to overturning of the taller part.

Then, as it can be observed, the static condition of the actual configuration of the tower induces significant stress in a quite narrow portion of terrain (we did not retrieve any information about the foundations structures, so we believe into the hypothesis that the foundations do not present any widening, as commonly occurs for medieval structures). Concerning the worst-case scenario, although no information about the terrain under the tower is available, the narrow portion of soil subjected to significant stresses could present degradation due to long-term weathering effects of the ground condition (e.g. seasonal variation of water level, extreme flooding effects etc. [16]). This phenomenon is usually connected with a soil-settlement which, in a very simplified and preliminary way, can be assumed proportional to the magnitude of the vertical stress.

Based on the above considerations, a simplified stability analysis for the leaning tower could consist in applying a distribution of imposed vertical displacement which varies linearly in a perpendicular direction with respect to the axis c, which represent in this case a sort of cylindrical hinge, to the edge of structure's base. The imposed vertical displacement is incrementally increased until the structure reaches collapse.

Figure 4 collects the results of the nonlinear static differential settlement analysis. In particular, Figure 4a shows the magnified deformed shape of the structure at collapse. Furthermore, Figure 4b depicts the horizontal top displacement – vertical base settlement curve, where the displacements are computed at points H and V in Figure 4a, respectively. Finally, Figure 4c shows the tensile damage contour plots at collapse.

As can be noted in Figure 4b, when the vertical base settlement reaches a value close to 60mm, the horizontal top displacement starts to increase while the base settlement remains constant. This condition can be considered as the collapse of the structure. Furthermore, it is worth noting that the inclination of the base which leads to collapse of the tower is about 1.4°, a value very close to the critical inclination evaluated by means of 3D homogenized limit analysis (about 1.5°) in [6] for the same case study. Finally, the collapse mechanism (Figure 4c) is governed by the overturning of the taller part of the tower, which shows further vertical fissures at collapse.

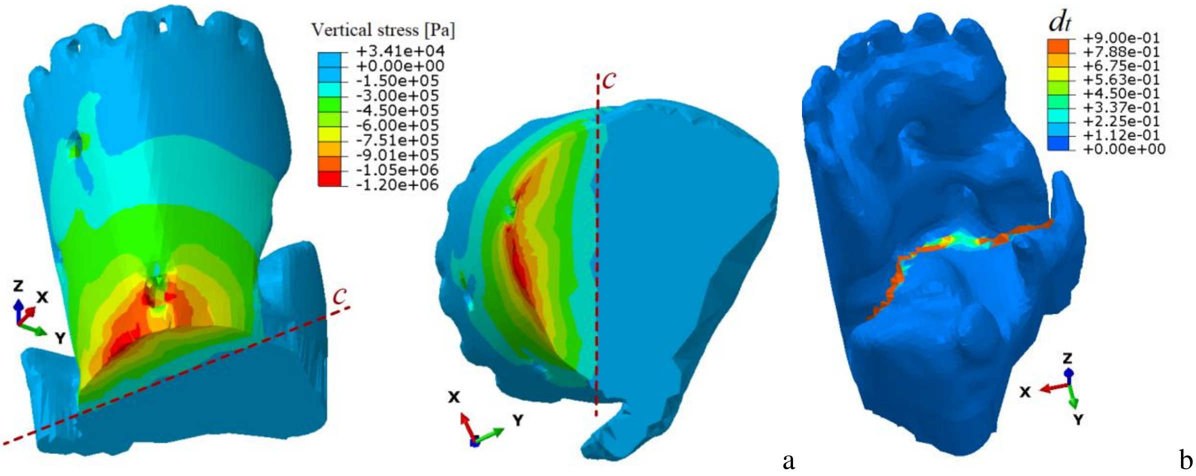

Figure 3. Nonlinear static analysis under dead load conditions: a) vertical stress and b) damage contour plots.

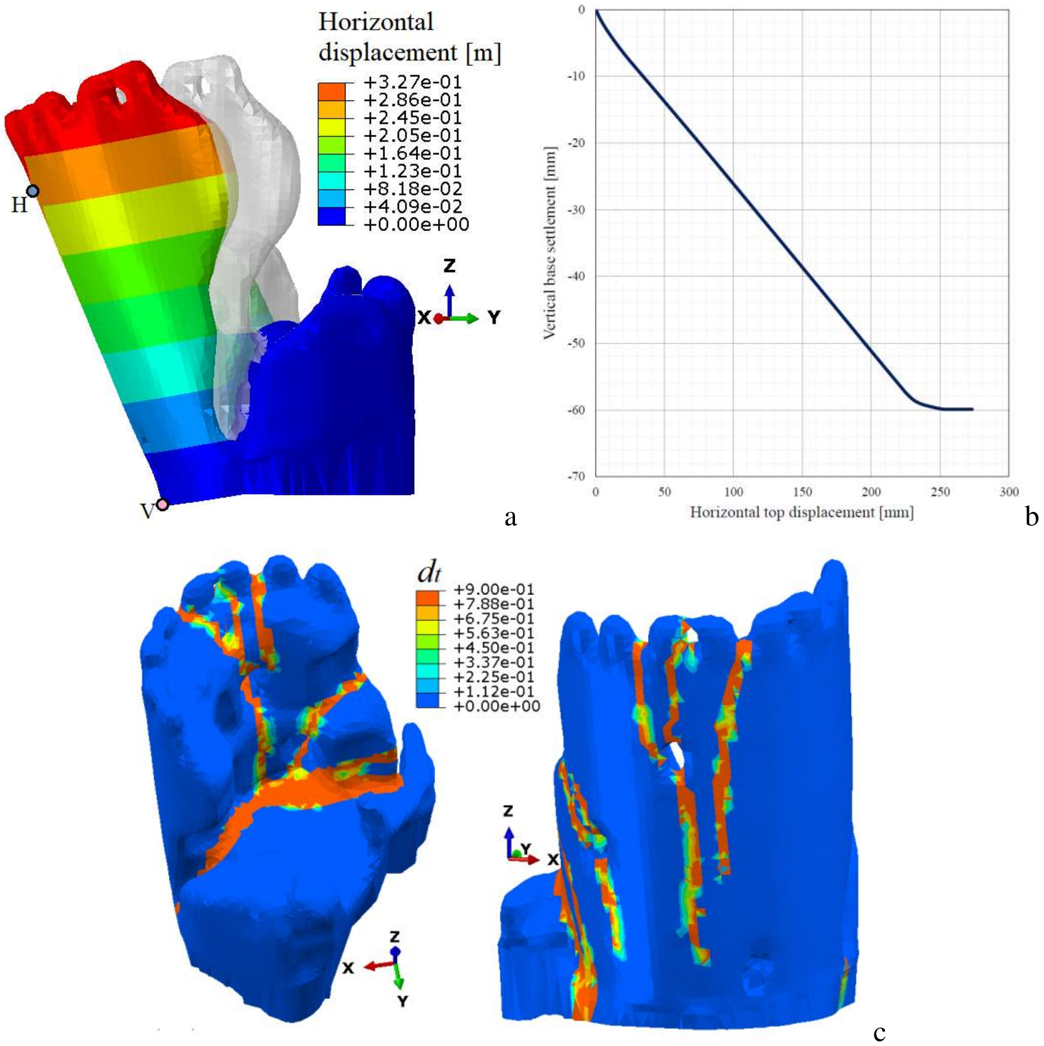

Figure 4. Nonlinear static differential settlement of the base: a) horizontal top displacement – vertical base displacement curve, b) magnified deformed shape and c) damage contour plots at collapse.

4 Conclusions

In this contribution, a simple approach for evaluating the stability analysis of geometrically complex leaning historic structures has been discussed. Particularly, the southwest leaning ruined tower of the Caerphilly castle (Wales, UK) has been employed as case study. To account for the actual complex geometry of the structure, a mesh generation procedure recently proposed by the authors has been used to automatically transform the 3D point cloud surveyed on the tower in a 3D solid finite element model. A simple approach based on nonlinear static analysis has been pursued to evaluate the stability condition of the structure, supposing geometric nonlinearity and nonlinear plastic-damaging behavior of masonry. It has been found that the structure in its actual configuration is quite close to its collapse condition. Indeed, a further inclination of about 1.4° could be deleterious for the structure.

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