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Two-dimensional elastoplastic analysis of cylindrical cavity problems in Tresca materials

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ABSTRACT:

This paper presents analytical elastic-plastic solutions for static stress loading analysis and quasi-static expansion analysis of a cylindrical cavity in Tresca materials, considering biaxial far-field stresses and shear stresses along the inner cavity wall. The two-dimensional static stress solution is obtained by assuming that the plastic zone is statically determinate and using the complex variable theory in the elastic analysis. A rigorous conformal mapping function is constructed, which predicts that the elastic-plastic boundary is in an elliptic shape under biaxial in-situ stresses and the range of the plastic zone extends with increasing internal shear stresses. The major axis of the elliptical elastic-plastic boundary coincides with the direction of the maximum far-field compression stress. Furthermore, considering the internal shear stresses, an analytical large-strain displacement solution is derived for continuous cavity expansion analysis in a hydrostatic initial stress field. Based on the derived analytical stress and displacement solutions, the influence of the internal shear stresses on the quasi-static cavity expansion process is studied. It is shown that additional shear stresses could reduce the required normal expansion pressure to a certain degree, which partly explains the great reduction of the axial soil resistance due to rotations in rotating cone penetration tests. In addition, through additionally considering the potential influences of biaxial in-situ stresses and shear stresses generated around the borehole during drillings, an improved cavity expansion approach for estimating the maximum allowable mud pressure of horizontal directional drillings (HDDs) in undrained clays is proposed and validated.

Keywords: cavity expansion, conformal mapping function, shear stress, biaxial in-situ stresses, rotating cone penetration test, horizontal directional drilling

1 | INTRODUCTION

2 Cavity expansion theory has been extensively applied in various fields of geotechnical engineering such
3 as interpretation of cone penetration tests (CPTs), prediction of soil resistance experienced by plate
4 anchors and piles, stability analysis of tunnels and boreholes (e.g. blow out failure associated with
5 loading and local collapse associated with unloading) ¹⁻⁶. These successful applications greatly
6 stimulated the development of analytical cavity expansion solutions over the past decades. It is noted
7 that analytical elastic-plastic solutions, in general, have been derived by assuming that the cavity is
8 uniformly loaded by increasing internal normal pressures in a hydrostatic initial stress field. In reality,
9 however, the in-situ soil stresses are not uniform in most cases, and the horizontal earth pressure at rest
10 is often expressed as K_0 times of that in the vertical direction instead (i.e. $\sigma_{h0} = K_0\sigma_{v0}$) ^{7, 8}. Meanwhile,
11 a great amount of shear stress might be applied or generated around the inner cavity wall in the process
12 of rotary drilling/excavation, accommodation of screw piles or rotating penetration tests ⁹⁻¹³. It has been
13 reported that the biaxial in-situ stresses (K_0 effect) and internal shear stresses need to be additionally
14 taken into account for further improving the accuracy of the cavity expansion theory in many
15 applications ¹³⁻¹⁶, for example, in estimating the tip resistance of rotating CPTs and predicting the
16 maximum mud pressure of HDDs, in particular, at relatively shallow depths ¹⁷⁻¹⁹. To account for the
17 additional influences of these two factors during loading/expansion of a cylindrical cavity in undrained
18 clays, both static stress analysis and quasi-static expansion analysis adopting the linear elastic-perfectly-
19 plastic Tresca model are carried out in this paper.

20 The stress field developed around a cavity primarily depends on the material strength and imposed
21 boundary conditions. For a cavity with axisymmetric geometry and stress conditions, the stress analysis
22 usually can be simplified to a one-dimensional equilibrium problem, which facilitated the development
23 of a number of analytical elastic and elastic-plastic solutions ¹. When non-axisymmetric geometry
24 and/or stress boundary conditions are considered, more advanced mathematical techniques such as the
25 complex variable theory ^{20, 21}, perturbation methods ^{22, 23}, variational approaches ²⁴ and numerical
26 techniques ^{25, 26} tend to be required. Among these methods, the complex variable theory in conjunction
27 with the conformal mapping technique provides a very powerful analytical tool for the elastic analysis

28 around a cavity with various boundary conditions^{20, 27}. This method has also been extended to the
29 analysis of a cylindrical cavity surrounded by strain-hardening materials obeying the power-law^{28, 29}.
30 However, these methods cannot be directly applied to the analysis in materials characterised by elastic-
31 perfectly plastic models since the difference between elastic and plastic constitutive equations. To
32 address this problem, Galin³⁰ creatively proposed an analytical approach for the analysis of a circular
33 cavity embedded in an infinite plate under biaxial remote stretching loadings. By assuming that the
34 plastic zone is statically determinate and using the complex variable theory in the elastic analysis, both
35 constant and polynomial types of far-field stresses were studied by Galin³⁰. Although minor mistakes
36 existed in these solutions as found and improved by Ochensberger, et al.³¹ and Tokar³² respectively, it
37 is generally believed that this methodology greatly facilitated the development of analytical/semi-
38 analytical solutions for the two-dimensional elastic-plastic analysis of a series of problems with similar
39 boundary conditions³³⁻³⁸.

40 Comparing with the problem of Galin³⁰, uniform shear stresses at the inner cavity wall are additionally
41 considered in this study. As a consequence, Galin's³⁰ approach to establishing the conformal mapping
42 function cannot be applied to deal with this problem since the plastic state is no longer biharmonic³⁷.
43 Although Parasyuk³⁷ has given the form of the conformal mapping function for this problem (see
44 Appendix B) based on Harnack's theorem, the mapping function has not been completely determined.
45 Consequently, the range of the plastic zone is not known, and the elastic field cannot be obtained. In
46 order to give a complete solution to this problem, the conformal mapping function is further explored
47 by introducing Laurent decomposition theorem in the analysis of stress continuity conditions across the
48 elastic-plastic boundary, and closed-form Kolosov-Muskhelishvili complex potentials²⁰ for the elastic
49 analysis are then derived by using a Fourier series and the Cauchy integral method in this paper. This
50 analytical stress solution is validated by comparing with the finite element method and Galin's³⁰
51 solution.

52 To further demonstrate the necessity and importance of considering the internal shear stresses and/or
53 biaxial in-situ stresses in practical geotechnical problems, the newly derived static stress solution and

54 quasi-static expansion solution are applied to estimate the normal soil resistance experienced by a
55 vertically advancing cone with rotations and predict the maximum mud pressure during HDDs:

56 (1) The axial penetration resistance could be significantly reduced by rotations^{9, 18, 19}. Therefore, rotating
57 a pushing cone penetrometer is regarded as one of the most feasible physical methods to eliminate the
58 interface friction in modelling the root tip-soil interaction (lubricated interface due to the sloughing off
59 of border cells and mucilage from roots)^{9, 11, 18}. In rotating CPTs, shear stresses would be generated
60 around the sliding interface, which not only decrease the frictional resistance both on the cone tip and
61 the shaft and may also lead to additional plastic failure in the surrounding soil^{18, 19}. The reduction of the
62 axial tip resistance is usually attributed to the reorientation of the frictional resistance at the cone and
63 the additional plastic deformation caused by the generated shear stresses^{18, 19}. The former effect has
64 been explained by Bengough, et al.¹⁸ based on quasi-static equilibrium analysis. Unfortunately,
65 analytical methods accounting for the latter effect are still rarely available. According to the analogy of
66 cone penetration and continuous expansion of a cylindrical cavity^{39, 40}, an analytical quasi-static
67 expansion solution is developed to capture the additional influences of cone rotations on the normal soil
68 resistance in this paper.

69 (2) HDD becomes a popular alternative to traditional open-cut excavation for pipeline and underground
70 utility conduit installation due to its higher flexibility in operation and potential economic and
71 environmental benefits⁴¹⁻⁴³. During HDDs, it requires continuous circulation of pressurized fluid in the
72 bore to remove the cutting out, stabilize the bore formation, cool and clean the drill bit, and lubricate
73 the pipe during installation. To avoid inadvertent drilling fluid returns and also effectively return the
74 soil cutting from the boring path to the ground surface, the determination or control of the applied
75 drilling fluid pressure remains a serious concern in the design and implementation of HDDs, especially
76 the maximum allowable mud pressure⁴⁴⁻⁴⁶. Cylindrical cavity expansion solutions provide one of the
77 most commonly used theoretical methods for estimating the maximum mud pressure during HDDs^{5, 17,}
78^{42, 47}, for example, the Delft cylindrical cavity expansion model (Delft solution) proposed by Luger and
79 Hergarden⁴⁸ by extending Vesic's solution⁴⁹. In the Delft solution, the in-situ stress field is simplified
80 to be hydrostatic^{5, 47}. However, for HDD practices implemented at relatively shallow depths, it has been

81 reported that the K_0 effect may exert significant influences on the maximum allowable mud pressure¹⁷.
 82 ⁵⁰. Considering the K_0 effect, Xia and Moore¹⁷ proposed an approximate method by replacing the elastic
 83 stresses in the Delft solution with Kirsch equations (i.e. $m=0$ in Appendix A) while estimating the
 84 maximum mud pressure in undrained clays. However, the elastic and plastic stresses were derived
 85 independently in their solution, and the elastic-plastic boundary was assumed to be circular regardless
 86 of the biaxial in-situ stresses. By using the more rigorous stress solution derived in this paper, the
 87 method of accounting for the K_0 effect is further improved. Meanwhile, the shear stresses potentially
 88 generated during pilot hole boring and reaming of HDDs are also considered in the new method.

89 **2 | PROBLEM DEFINITION**

90 A sufficiently large and thick soil medium (in comparison with the cavity size) with an inside cylindrical
 91 cavity is considered. As depicted in Figure 1, biaxial initial stresses apply at infinity, and uniform shear
 92 stresses act on the inner cavity wall. The surrounding medium is loaded monotonically by normal
 93 compression pressures on the inner cavity wall to the value of p_{in} with a sufficiently slow speed. The
 94 stress and deformation analysis are conducted under the assumption of plane strain. For convenience,
 95 both Cartesian coordinates (x, y, z) and cylindrical polar coordinates (r, θ, z) with the same origin at the
 96 centre of the cavity are utilised. It is worth noting that the defined remote stress conditions are
 97 sufficiently general because we always can set a coordinate system with axes parallel to the directions
 98 of the principal stresses at infinity. Within the cylindrical polar coordinates, the stress equilibrium
 99 equations in the radial and circumferential directions, respectively, are

$$100 \quad \frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1)$$

$$101 \quad \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} = 0 \quad (2)$$

102 where σ_r , σ_θ and $\tau_{r\theta}$ are the radial, circumferential and shear stress components, respectively.

103 Taking tension as positive for normal stresses and rotation in the anticlockwise direction to the object
 104 as positive for shear stresses, the stress boundary conditions can be expressed as

$$105 \quad \sigma_r|_{r=R} = -p_{in}, \quad \tau_{r\theta}|_{r=R} = mk \quad (-1 \leq m \leq 1) \quad (3)$$

$$106 \quad P_\infty = (\sigma_y|_{y \rightarrow \infty} + \sigma_x|_{x \rightarrow \infty})/2 = -(\sigma_{v0} + \sigma_{h0})/2, \quad \tau_\infty = (\sigma_y|_{y \rightarrow \infty} - \sigma_x|_{x \rightarrow \infty})/2 = (\sigma_{h0} - \sigma_{v0})/2 \quad (4)$$

107 where k is the yield stress under pure shear loading. m gives the relationship between k and the
108 internal shear stress.

109 The surrounding material is regarded as homogenous and isotropic, and it is characterised with an
110 elastic-perfectly plastic model. Specifically, the elastic response is described by Hooke's law until the
111 onset of yielding which obeys the Tresca yield criterion (i.e. Equation 5).

$$112 \quad (\sigma_r - \sigma_\theta)^2 + 4\tau_{r\theta}^2 = 4k^2 \quad (5)$$

113 **3 | STATIC STRESS ANALYSIS**

114 **3.1 | Plastic region**

115 As an extension of Galin's³⁰ solution, this solution aims to provide an analytical method for the static
116 stress analysis of the soil around a cavity under conditions of: (a) the inner cavity is fully enclosed by a
117 connected plastic region, and the plastic stress field is statically determinate, (b) the plastic zone is
118 developed under monotonic loading, and no elastic unloading occurs in any case, and (c) the out-of-
119 plane stress component σ_{zz} always remains as the intermediate principal stress regardless of other
120 stresses^{33, 35}.

121 The static determinacy of the plastic field determines that the plastic stress field entirely depends on the
122 inner boundary conditions^{35, 51}. Therefore, according to the axisymmetric fact of the geometry and stress
123 boundaries at the inner cavity wall, it is reasonable to assume

$$124 \quad \frac{\partial \sigma_r^p}{\partial \theta} = 0, \quad \frac{\partial \sigma_\theta^p}{\partial \theta} = 0, \quad \frac{\partial \tau_{r\theta}^p}{\partial \theta} = 0 \quad (6)$$

125 where the superscripts e and p indicate the stress components in the elastic and plastic region respectively.

126 The plastic stresses under loading can be obtained by solving the stress equilibrium equation (i.e.
127 Equation 3) and the yield criterion (i.e. Equation 5) with the given stress boundaries (i.e. Equation 3),
128 which are known as the Mikhlin's solution^{13, 37} (i.e. Equations 7-9).

$$129 \quad \sigma_r^p = k \left\langle \ln \left[\frac{(r/R)^2 + \sqrt{(r/R)^4 - m^2}}{1 + \sqrt{1 - m^2}} \right] - \sqrt{1 - m^2} \left(\frac{R}{r} \right)^4 + \sqrt{1 - m^2} \right\rangle - p_{in} \quad (7)$$

$$130 \quad \sigma_\theta^p = k \left\langle \ln \left[\frac{(r/R)^2 + \sqrt{(r/R)^4 - m^2}}{1 + \sqrt{1 - m^2}} \right] + \sqrt{1 - m^2} \left(\frac{R}{r} \right)^4 + \sqrt{1 - m^2} \right\rangle - p_{in} \quad (8)$$

$$131 \quad \tau_{r\theta}^p = mk \frac{R^2}{r^2} \quad (9)$$

132 **3.2 | Determination of conformal mapping function**

133 The elastic-plastic interface gives the outer boundary of the plastic zone and simultaneously provides
 134 the inner boundary for computing the elastic stress field, which is, therefore, of great importance for
 135 determining the range of the developed plastic zone and deriving the elastic stresses. The location of
 136 the elastic-plastic boundary is generally determined by analysing the continuity conditions of the elastic
 137 and plastic stresses across the interface. However, under non-equal far-field stresses, the elastic stress
 138 field cannot be obtained prior to knowing its inner boundary conditions, namely the location of the
 139 elastic-plastic boundary and stresses acting on it^{30, 33, 35, 37}. Therefore, the elastic-plastic boundary is
 140 described by a general form of the conformal mapping function first (i.e. Equation B1), and the elastic
 141 stresses are represented in terms of general forms of Kolosov-Muskhelishvili complex potentials, $\Phi(\zeta)$
 142 and $\Psi(\zeta)$ ²⁰. Based on the stress continuity conditions across the elastic-plastic boundary and the far-
 143 field stress boundary conditions, the boundary values of $\Phi(\zeta)$ and $\Psi(\zeta)$ can be expressed as

$$144 \quad \begin{aligned} \Phi(\zeta) + \overline{\Phi(\zeta)} &= \frac{(\sigma_r^e + \sigma_\theta^e)}{2} \\ &= \begin{cases} k \left\langle \ln \left[\frac{\omega(\sigma)\overline{\omega(\sigma)}/R^2 + \sqrt{[\omega(\sigma)\overline{\omega(\sigma)}]^2/R^4 - m^2}}{1 + \sqrt{1 - m^2}} \right] + \sqrt{1 - m^2} \right\rangle - p_{in} & , \text{at } \gamma \quad (a) \\ P_\infty & , \zeta \rightarrow \infty \quad (b) \end{cases} \end{aligned} \quad (10)$$

$$145 \quad \begin{aligned} \frac{\overline{\omega(\zeta)}}{\omega'(\zeta)} \Phi'(\zeta) + \Psi(\zeta) &= \frac{(\sigma_\theta^e - \sigma_r^e + 2i\tau_{r\theta}^e)}{2} e^{-2i\theta} \\ &= \begin{cases} k \left\langle \frac{\sqrt{[\omega(\sigma)\overline{\omega(\sigma)}]^2 - m^2 R^4} + mR^2 i}{\omega(\sigma)\overline{\omega(\sigma)}} \right\rangle \frac{\overline{\omega(\sigma)}}{\omega(\sigma)} & , \text{at } \gamma \quad (a) \\ \tau_\infty & , \zeta \rightarrow \infty \quad (b) \end{cases} \end{aligned} \quad (11)$$

146 where $i = \sqrt{-1}$. γ represents the unit contour in the phase plane, corresponding to the elastic-plastic
147 boundary. σ is the unit complex variable, which describes points on γ . $\sigma = \cos \phi + i \sin \phi = e^{i\phi}$. and
148 $\bar{\sigma} = 1/\sigma$. ϕ is the argument of σ . The function of $\omega(\zeta)$ conformally maps the exterior of the elastic-
149 plastic boundary in the physical plane onto the exterior region of the unit circle in the phase plane;
150 $\overline{\omega(\zeta)}$ is its conjugate function. $\zeta = \xi + i\eta = \rho e^{i\phi}$. ξ and η are the real and imaginary part of the
151 complex variable ζ , respectively. ρ is the modulus of ζ . Boundary conditions of Equation 10b and
152 Equation 11b specified the behaviour of the complex potentials at infinity as

$$153 \quad \Phi(\infty) = \frac{P_\infty}{2} + O(\zeta^{-2}), \quad \Psi(\infty) = \tau_\infty + O(\zeta^{-2}) \quad (12)$$

154 Owing to the additional consideration of the internal shear stresses, the plastic state is no longer
155 biharmonic. Thus the mapping function cannot be determined by constructing a unified biharmonic
156 stress function crossing the elastic-plastic interface as proposed by Galin³⁰. Instead, Equations 10 and
157 11 are analysed based on Harnack's theorem. With this method, the form of the mapping function (i.e.
158 Equation 13) was derived by Parasyuk³⁷, and a more general derivation is presented in Appendix B.

$$159 \quad \omega(\zeta) = \alpha \left(\zeta + \frac{\beta}{\zeta} \right) \quad (13)$$

160 where $\beta = \tau_\infty / k$.

161 Equation 13 gives that the elastic-plastic boundary is in an elliptic shape while $\beta \neq 0$, whose major axis
162 is along the direction of the maximum far-field compression pressure under loading. However, the
163 parameter α in Equation 13 has not been determined so far. Consequently, the range of the plastic zone
164 and elastic stresses cannot be analytically obtained. To derive a complete solution to this problem,
165 Equations 10 and 11 are further analysed as follows.

166 As the far-field stress conditions bound the behaviour of the right-hand side of Equation 10a at infinity
167 ^{27, 52}, the continuity condition of the mean stress can be re-expressed as

$$168 \quad \Phi(\sigma) + \overline{\Phi(\sigma)} = k \ln[F(\sigma, \bar{\sigma})] + k \sqrt{1 - m^2} - p_{in} \quad (14)$$

169 where $F(\sigma, \bar{\sigma}) = \left\{ \frac{\omega(\sigma)\overline{\omega(\sigma)}/R^2 + \sqrt{[\omega(\sigma)\overline{\omega(\sigma)}]^2/R^4 - m^2}}{(1 + \sqrt{1 - m^2})\sigma\bar{\sigma}} \right\} = \frac{f(\sigma)}{\sigma} \frac{\bar{f}(\bar{\sigma})}{\bar{\sigma}^{-1}}$.

170 An elliptic elastic-plastic boundary is predicted by Equation 13, which is a closed smooth contour in
 171 the physical plane. $\ln[F(\sigma, \bar{\sigma})]$ in Equation 14 continues analytically on both sides of γ in an annulus
 172 of $0 \leq |\zeta - \sigma| \leq \infty$. Thus, based on Laurent decomposition theorem⁵³, $\ln[F(\sigma, \bar{\sigma})]$ can be decomposed
 173 as a sum of two mutually conjugate functions, $d(\zeta)$ and $\bar{d}(\zeta^{-1})$, which are analytic in Ω^+ ($|\zeta| < 1$)
 174 and Ω^- ($|\zeta| > 1$) respectively. Then by multiplying $\frac{1}{2\pi i} \frac{d\sigma}{\sigma - \zeta}$ on both sides of Equation 14 and
 175 integrating it along γ , we obtain

176 $F^+(\zeta) = \bar{d}(\zeta^{-1}) - \overline{\Phi(\zeta)}$ (15 a)

177 $F^-(\zeta) = \Phi(\zeta) - d(\zeta) - (k\sqrt{1 - m^2} - p_{in})$ (15 b)

178 where $d(\zeta) = k \ln[f(\zeta)/\zeta]$ and $\bar{d}(\zeta^{-1}) = k \ln[\bar{f}(\bar{\zeta})/\bar{\zeta}^{-1}]$. $F^+(\zeta)$ and $F^-(\zeta)$ are analytic
 179 everywhere within the region of Ω^+ and Ω^- , respectively. Now the continuity condition of the mean
 180 stress across the unit circle γ can be expressed as $F^+(\sigma) = F^-(\sigma)$. Based on Liouville's theorem,
 181 which states $F^+(\zeta)$ and $F^-(\zeta)$ are identically equal to one and same constant due to the complex
 182 potentials are bounded at infinity. The boundary values of $d(\zeta)$ and $\bar{d}(\zeta^{-1})$ are studied in the
 183 extended complex plane with the form defined in Equation 14. By analysing Equation 14, it is obtained
 184 that

185 $-\frac{P_\infty}{2} + \frac{1}{2} k \ln \left[\frac{2\alpha^2 / R^2}{1 + \sqrt{1 - m^2}} \right] = \frac{P_\infty}{2} - \frac{1}{2} k \ln \left(\frac{2\alpha^2 / R^2}{1 + \sqrt{1 - m^2}} \right) - k\sqrt{1 - m^2} + p_{in}$ (16)

186 As a result, the undetermined parameter α in Equation 13 is obtained as

187 $\alpha = \delta \text{Re}^{\frac{1}{2k} [p_{in} + P_\infty - k\sqrt{1 - m^2}]}$ (with $\delta = \left[(1 + \sqrt{1 - m^2}) / 2 \right]^{1/2}$) (17)

188 By now the conformal mapping function is completely determined. A new parameter \mathcal{D} is involved
 189 due to the additional consideration of the internal shear stresses comparing with Galin's³⁰ mapping

190 function. Equation 17 indicates that the size of the plastic region monotonically expands with an
 191 increasing expansion pressure. The lengths of the semi-major axis and the semi-minor axis of the elliptic
 192 elastic-plastic boundary can be expressed as $a_{ep} = \alpha(1+|\beta|)$ and $b_{ep} = \alpha(1-|\beta|)$ respectively. It is
 193 shown that the axis ratio (a_{ep} / b_{ep}) and axes directions of the elastic-plastic boundary entirely depend
 194 on the non-uniformity of far-field stresses (represented by the non-dimensional factor of β).

195 3.3 | Elastic stress analysis

196 The derived elastic-plastic boundary and plastic stresses provide the inner geometry and stress
 197 boundaries for the analysis of the outside elastic field. In other words, now the elastic field is equivalent
 198 to the problem that an elliptic cavity embedded in an infinite plane subjecting to non-uniform stresses
 199 at the inner cavity wall and biaxial stresses at infinity. At first, the elastic stresses are represented by
 200 the general forms of the elastic complex potentials (i.e. Equations 18 and 19) according to the complex
 201 potential method proposed by Muskhelishvili²⁰.

$$202 \quad \Phi(\zeta) = \Gamma - \frac{X + iY}{2\pi(1 + \chi)} \frac{1}{\zeta} + \Phi_0(\zeta) \quad (18)$$

$$203 \quad \Psi(\zeta) = \Gamma' + \frac{\chi(X - iY)}{2\pi(1 + \chi)} \frac{1}{\zeta} + \Psi_0(\zeta) \quad (19)$$

204 where $\Phi_0(\zeta) = \sum_{n=1}^{\infty} \frac{a_n}{\zeta^{n+1}}$, $\Psi_0(\zeta) = \sum_{n=1}^{\infty} \frac{b_n}{\zeta^{n+1}}$, which are holomorphic in the whole elastic region.

205 $\Gamma = P_{\infty} / 2$ and $\Gamma' = \tau_{\infty}$, which describe the stress conditions at infinity. X and Y are components of the
 206 resultant vector of forces acting on the elastic-plastic boundary. $\chi = 3 - 4\nu$ for the plane strain problem.
 207 ν is Poisson's ratio.

208 The complex potentials are first sought with the assumption that both the stress and displacement
 209 components remain bounded at infinity, which implies that the resultant stresses vanish at infinity²⁰.
 210 Mathematically, it requires $\Gamma = \Gamma' = 0$ and $X = Y = 0$. In this case, the complex potentials remain
 211 holomorphic in the outside region of contour γ . $\Phi_0(\zeta)$ and $\Psi_0(\zeta)$ fully satisfy the above
 212 requirements. According to the stress boundary conditions at the elastic-plastic boundary given in

213 Equations 10 and 11, it is not easy to derive the elastic complex potentials directly with simple algebraic
 214 transformations, if possible. Alternatively, the inner stress boundaries of the mean stress (i.e. Equation
 215 10a) are transformed into a Fourier series form as

$$216 \quad \Phi_0(\sigma) + \overline{\Phi_0(\sigma)} = k \ln \left[\frac{(x_{ep}^2 + y_{ep}^2)/R^2 + \sqrt{(x_{ep}^2 + y_{ep}^2)^2/R^4 - m^2}}{1 + \sqrt{1 - m^2}} \right] + k\sqrt{1 - m^2} - p_{in} - P_\infty \quad (20)$$

$$= G(\phi) = \sum_{-\infty}^{+\infty} A_n e^{in\phi} = \sum_{-\infty}^{+\infty} A_n \sigma^n$$

217 where $A_n = \frac{1}{2\pi} \int_0^{2\pi} G(\phi) e^{-in\phi} d\phi$. $x_{ep} = \alpha(1 + \beta) \cos \phi$ and $y_{ep} = \alpha(1 - \beta) \sin \phi$.

218 In Equation 20, an even function of $G(\phi)$ is formed, which is a continuous real function in terms of the
 219 argument ϕ within the interval of $0 \leq \phi \leq 2\pi$. It means that $A_n = A_{-n}$ (real numbers) and coefficients
 220 of the odd terms in $\Phi_0(\sigma)$ are infinitesimal. As $A_{-n} = \bar{A}_n$ and $\bar{\sigma} = \sigma^{-1}$, naturally both sides of Equation
 221 20 can be split into two mutually conjugate parts. As a result,

$$222 \quad \Phi_0(\sigma) = A_0 / 2 + \sum_{n=1}^{+\infty} A_{-2n} / \sigma^{2n} \quad (21)$$

223 As A_0 is vanishingly small, the requirement of $\Phi_0(\sigma)$ at infinity, namely $\Phi_0(\infty) = O(\zeta^{-2})$, is fulfilled.
 224 More strictly in the mathematical formulation, the term with coefficients of A_0 in Equation 21 is
 225 equivalently modified at the unit circle as

$$226 \quad \Phi_0(\sigma) + \overline{\Phi_0(\sigma)} = \sum_{n=1}^{+\infty} A_{-2n} \frac{1}{\sigma^{2n}} + \frac{A_0}{2|\sigma|^2} + \frac{A_0}{2} |\sigma|^2 + \sum_{n=1}^{+\infty} A_{2n} \sigma^{2n} \quad (22)$$

227 By multiplying both sides with $\frac{1}{2\pi i} \frac{d\sigma}{\sigma - \zeta}$ (here ζ is a point within Ω^-) and then integrating it along
 228 the circumference of γ , Equation 22 gives

$$229 \quad \Phi_0(\zeta) = \frac{A_0}{2|\zeta|^2} + \sum_{n=1}^{+\infty} \frac{A_{-2n}}{\zeta^{2n}} \quad (23)$$

230 Then the general form of $\Phi(\zeta)$ can be obtained by releasing the previous assumption in the process of
 231 deriving $\Phi_0(\sigma)$. The resultant vectors still equal to zero ($X = Y = 0$) because of the continuous

232 distribution of stresses along the elastic-plastic boundary. With the use of Equation 12 and 23, the first
 233 complex potential goes to

$$234 \quad \Phi(\zeta) = \frac{P_\infty}{2} + \Phi_0(\zeta) \quad (24)$$

235 Then the complex potential $\Psi(\zeta)$ is derived by integrating the continuity condition of the deviatoric
 236 stress (i.e. Equation 11a) along γ from the side of Ω^- with the Cauchy integral method. As discussed
 237 in Appendix B, all terms in Equation 11a are holomorphic in Ω^- , therefore $\Psi(\zeta)$ goes to

$$238 \quad \Psi(\zeta) = k \frac{(\beta\zeta^2 + 1)}{(\zeta^2 + \beta)} \left\langle \frac{\sqrt{\tilde{r}^4 - m^2 R^4} + mR^2 i}{\tilde{r}^2} \right\rangle - \frac{\zeta(\beta\zeta^2 + 1)}{\zeta^2 - \beta} \Phi'(\zeta) \quad (25)$$

239 where $\tilde{r}^2 = \alpha^2(1 + \beta^2 + \beta\zeta^2 + \beta\zeta^{-2})$. $\Phi'(\zeta)$ is the derivative of the complex potential $\Phi(\zeta)$ with
 240 respect to ζ , which can be easily calculated with $|\zeta|^2 = \xi^2 + \eta^2$ and $\frac{\partial}{\partial \zeta} = \frac{1}{2}(\frac{\partial}{\partial \xi} - i\frac{\partial}{\partial \eta})$. Hence,

$$241 \quad \Phi'(\zeta) = -\frac{A_0}{2\zeta|\zeta|^2} - \sum_{n=1}^{+\infty} 2n \frac{A_{-2n}}{\zeta^{2n+1}} \quad (26)$$

242 Now this stress boundary value problem is analytically solved through the combination use of the
 243 Cauchy integral method and Fourier series as shown in Equations 23-26.

244 **4 | DISCUSSION ON RESTRICTIONS FROM THE ASSUMPTIONS**

245 As stated in Section 3.1, the above static stress solution was derived based on several prior assumptions.
 246 Restrictions due to these assumptions are discussed as follows.

247 (1) The cavity is fully enclosed by a plastic region

248 This assumption requires that the inner cavity is fully enclosed by a continued plastic region. Limit
 249 conditions of this requirement will be approached while the vertices of the predicted elastic-plastic
 250 boundary in the minor axis direction just reach the cavity wall³³. It gives

$$251 \quad b_{ep} = \alpha(1 - |\beta|) \geq R \quad (27)$$

252 (2) Intermediate principal stress

253 It was assumed that the out-of-plane stress σ_{zz} always remains as the intermediate principal stress.
 254 Although this assumption is always justified for incompressible conditions, restrictions will be
 255 produced in compressible materials. The plastic stress solutions show that the principal stresses in the
 256 plastic zone vary from the inner cavity wall to the elastic-plastic boundary monotonically. Therefore, it
 257 just needs to ensure that values of σ_{zz} at the inner cavity wall and at the vertices on the major axis of
 258 the elastic-plastic boundary always remain as the intermediate principal stress³³. Relationships between
 259 the principal stresses and other stress components are given in Appendix A. With the aid of Equations
 260 7, 8, and 9, this restriction for compressible Tresca materials can be expressed as

$$261 \quad \sqrt{1-m^2} - \frac{1}{1-2\nu} + \ln(\Theta) \leq \frac{p_{in}}{k} \leq \sqrt{1-m^2} + \frac{1}{1-2\nu} \quad (28)$$

$$262 \quad \text{where } \Theta = \frac{[\alpha(1+|\beta|)/R]^2 + \sqrt{[\alpha(1+|\beta|)/R]^4 - m^2}}{1 + \sqrt{1-m^2}} > 1.$$

263 (3) Static determinacy of the plastic zone

264 The plastic stresses were derived by assuming that the plastic zone is statically determinate.
 265 Theoretically, it implies that every point in the plastic region can be connected to the cavity rim by two
 266 characteristic lines (slip-lines) of different families, and every slip-line cuts the elastic-plastic boundary
 267 only once⁵¹. Therefore, the limit condition will be reached while one, and only one, characteristic line
 268 is tangent to the elastic-plastic interface within one quadrant.

269 It is known that directions of the slip-lines take an angle of $\pi/4$ with the principal stress directions in
 270 Tresca materials. While internal shear stresses apply, the radial and circumferential directions are no
 271 longer the principal stress directions in the plastic zone. As shown in Figure 2, the direction of the minor
 272 principal stress takes an anticlockwise rotation to the radial direction of the cylindrical coordinates with
 273 positive shear stresses at the inner cavity wall. Contrarily, a clockwise rotation would be caused by
 274 negative shear stresses. Here taking the anticlockwise direction of φ as positive, according to Figure
 275 3, the requirement of this assumption can be expressed as

$$276 \quad |\lambda - \theta| \leq \frac{\pi}{4} + \varphi \quad (29)$$

277 where θ is the angle between the radial direction and the x -axis. λ represents the angle between the
 278 outward normal to the elastic-plastic interface and the x -axis.

279 Angles in Equation 29 all can be expressed in terms of the unit complex variable σ as

$$280 \quad e^{2i(\lambda-\theta)} = \sigma^2 \frac{\omega'(\sigma) \overline{\omega(\sigma)}}{\overline{\omega'(\sigma)} \omega(\sigma)} = \frac{(\sigma^2 - \beta)(1 + \beta\sigma^2)}{(1 - \beta\sigma^2)(\beta + \sigma^2)} ; 2\varphi = \arcsin \left[m \frac{R^2}{\omega(\sigma)\overline{\omega(\sigma)}} \right] \quad (30)$$

281 where $\omega(\sigma) = \alpha(\sigma + \beta\sigma^{-1})$; $\overline{\omega(\sigma)} = \alpha(\sigma^{-1} + \beta\sigma)$.

282 To ensure that only one characteristic line reaches the limit condition within one quadrant, it restricts
 283 that the equality condition holds only when the function $g(\sigma) = |\lambda - \theta| - \varphi$ reaches its extremum³⁵. The
 284 extremum values of $g(\sigma)$ lie at the zero points of its first derivative. If no shear stress is applied, this
 285 restriction becomes $|\beta| \leq (\sqrt{2} - 1)$ ^{33, 35}.

286 With the defined boundary conditions, stress states developed around the inner cavity can be broadly
 287 categorized into three conditions^{26, 33}: a) purely elastic state (i.e. zone A); b) the cavity is partially
 288 surrounded by plastic regions (e.g., zone B and zone E); and c) the cavity is fully enclosed by a plastic
 289 region (e.g. zone C and zone D). The stress field in a purely elastic state can be readily calculated with
 290 the solution given in Appendix A. The limit pressure that a plastic zone starts forming from the inner
 291 cavity wall under loading is useful in estimating the plastic initiation pressure of fracturing¹⁴, and,
 292 therefore, it is also given in Equation 31.

$$293 \quad (p_{in} + P_{\infty}) \geq k\sqrt{1 - m^2} - 2|\tau_{\infty}| \quad (31)$$

294 In the other two stress states, distributions of the plastic regions are various, mainly depending on the
 295 material strength and boundary conditions^{26, 33}. By using Equations 27-31, example boundaries of
 296 different stress states are shown in Figure 4 for illustration. As previously discussed, the present solution
 297 is dedicated to problems with boundary stresses belonging to Zone C.

298 Without the internal shear stresses, Figure 4 shows that Equations 27-31 give the same results as that
 299 given by Yarushina *et al.*³³ for the problem of Galin³⁰, and the distribution patterns of the plastic regions
 300 in Zones D and E refer to Yarushina *et al.*³³. When additional shear stresses apply, the limits of different

301 stress states would change. In specific, Line 2 (representing the limit given by Equation 27) moves
302 leftwards with increasing shear stresses. Line 3 (determined by Equation 29) represents the limit of
303 static determinacy of the plastic region. It is a horizontal line with a constant value of $|\beta| \leq (\sqrt{2} - 1)$ in
304 the case without internal shear stresses, but becomes curved due to the rotation of principal stresses
305 caused by the internal shear stresses. Opposite rotations will be produced by the shear stresses with
306 different applying directions. As a result, the bounds of this requirement distribute in opposite sides of
307 the horizontal line of $|\beta| \leq (\sqrt{2} - 1)$, but they both gradually approach to $(\sqrt{2} - 1)$ with an increasing
308 value of $|P_\infty + p_{in}|/k$. Line 4 reflects the restriction specified by Equation 28. The additional shear
309 stresses exert little impact on this restriction. Example lines calculated with $p_{in} = 0$ and $\nu = 0.4$ are
310 shown in Figure 4. Line 4 moves rightwards (or leftwards) with an increasing (or decreasing) Poisson's
311 ratio, and this restriction will be released in incompressible materials.

312 **5 | SOLUTION VALIDATION AND DISCUSSION**

313 Within the permissible stress states specified by Equations 27, 28, and 29, the above solution for static
314 stress analysis under loading can be calculated with the following steps.

315 (a) determine the elastic-plastic boundary by using the mapping function given in Equation 13. Then
316 the plastic stress field can be obtained from Equations 7, 8, and 9 directly. One-to-one corresponding
317 relations between the physical plane and the phase plane can be established by using Equation 32;

$$318 \quad x = r \cos \theta = \alpha \left(\rho + \frac{\beta}{\rho} \right) \cos \phi, \quad y = r \sin \theta = \alpha \left(\rho - \frac{\beta}{\rho} \right) \sin \phi \quad (32)$$

319 (b) calculate the coefficients of the established Fourier series (e.g. Equation 23), which stay the same
320 values in the whole elastic filed (setting $n = 5$ in Equation 23 in the following calculations). The elastic
321 complex potentials given in Equations 24 and 25 can then be calculated. By substituting the complex
322 potentials into Equations 33 and 34, stresses in the elastic region can be readily obtained by separating
323 the real and imaginary parts.

$$324 \quad \sigma_x^e + \sigma_y^e = 4 \operatorname{Re}[\Phi(\zeta)] \quad (33)$$

$$325 \quad \sigma_y^e - \sigma_x^e + 2i\tau_{xy}^e = 2\left[\frac{\overline{\omega(\zeta)}}{\omega'(\zeta)}\Phi'(\zeta) + \Psi(\zeta)\right] \quad (34)$$

326 **5.1 | Comparison with Galin's solution and finite element method**

327 The analytical solution is validated by comparing with Galin's³⁰ solution (i.e. the special case of zero
 328 internal shear stress) and the finite element method (FEM) as shown in Figures 5 and 6 respectively.
 329 The FEM simulations are implemented in Abaqus/Standard 6.12 using the same linear elastic-perfectly-
 330 plastic Tresca model as the above analytical solution. Biaxial far-field stresses are applied at the outer
 331 boundaries (100 times of the cavity radius away from the centre) of the plane-strain FEM model, and
 332 the same stress boundary conditions are applied at the inner cavity wall as depicted in Figure 1. An 8-
 333 node biquadratic plane-strain quadrilateral element is utilised for meshing.

334 It was introduced that Galin's³⁰ solution is a special case that without shear stresses on the inner cavity
 335 wall of the present solution. Although different methods to decomposing the stress continuity conditions
 336 have been adopted in these two solutions, Figure 5 demonstrated that the present solution can fully
 337 recover to Galin's³⁰ solution while taking $m=0$. When internal shear stresses apply, the analytical
 338 predictions are in good agreements with the FEM simulations as shown in Figure 6. It is found that the
 339 influence of the additional shear stress mainly concentrates in the plastic region, and the shear stress
 340 rapidly attenuates from the boundary value at the cavity wall to a stable level that varies in directions
 341 and depends on non-uniformity of the far-field stresses. Overall, it is shown that the developed series-
 342 form elastic complex potentials have good convergence precision and speed in computations, and the
 343 analytical solution can accurately calculate the surrounding elastic and plastic stress fields in the defined
 344 stress state.

345 **5.2 | Distribution of elastic-plastic boundary**

346 The range of the plastic zone developed around the inner cavity can be predicted by Equation 13. It is
 347 shown in Figure 7 that additional internal shear stresses extend the range of the plastic zone, but it
 348 imposes no influence on the shape of the elastic-plastic boundary (the major to minor axis ratio remains
 349 unchanged as $a_{ep} / b_{ep} = (1 + |\beta|) / (1 - |\beta|)$). The major axis of the elliptic elastic-plastic boundary under

350 loading lies in the x -axis direction while $\beta > 0$, whereas it is along the y -axis direction while $\beta < 0$.
 351 In other words, the direction of the major axis coincides with the direction of the minor principal stress
 352 (i.e. maximum compression stress) at infinity.

353 **6 | QUASI-STATIC CAVITY EXPANSION SOLUTION IN A HYDROSTATIC** 354 **STRESS FIELD**

355 As previously discussed, rotations during cone penetrations may cause additional plastic failure in the
 356 surrounding soil comparing with non-rotated cone penetrations. It is believed that this effect contributes
 357 to the great reduction of the axial soil resistance measured by a rotating penetrometer^{18, 19}. Based on the
 358 analogy of cone penetration and continuous cavity expansion, cylindrical cavity expansion solutions
 359 have been applied to estimate the normal soil pressure acting on non-rotated cones with some successes
 360^{39, 40}. In rotating CPTs, a rotating cone displaces the ahead soil increasingly cylindrical¹⁸. Therefore, a
 361 cylindrical quasi-static expansion analysis considering internal shear stresses is developed to account
 362 for the impact of the additional plastic deformation on the normal soil resistance measured in rotating
 363 CPTs as follows.

364 The quasi-static analysis of a cylindrical cavity models the lateral soil response caused by a vertically
 365 advancing cone. The confining in-situ soil stresses in a horizontal plane are assumed to be uniform.
 366 With uniform in-situ stresses, the mapping function of the elastoplastic interface (i.e. Equation 13) will
 367 reduce to $\omega(\sigma) = \alpha\sigma$. Therefore, the inner pressure during expansion can be expressed as

$$368 \quad \frac{P_m + P_\infty^h}{k} = 2 \ln\left(\frac{r_c^h}{\delta R}\right) + \sqrt{1 - m^2} \quad (35)$$

369 where r_c^h is the elastic-plastic radius under the corresponding hydrostatic initial stress of P_∞^h .

370 Radial displacements during continuous cavity expansions can be obtained by using the derived large-
 371 strain displacement solution in Appendix C. By letting $r = R$ and $r_0 = R_0$ in Equation C8, the radius
 372 ratio (r_c^h / R) of the elastic-plastic boundary to the current cavity at any expansion instant can be
 373 expressed as Equation 36.

$$\begin{aligned}
& \left[1 - \frac{k}{2G} \sqrt{1 - m^2} \left(\frac{R}{r_c^h} \right)^4 \right]^2 \left(\frac{r_c^h}{R} \right)^2 - \left(\frac{R_0}{R} \right)^2 \\
& = \frac{1}{\varpi^2 - 1} \left[\left(\frac{(r_c^h / R)^2 + \sqrt{(r_c^h / R)^4 - m^2}}{1 + \sqrt{1 - m^2}} \right)^\varpi (1 + \varpi \sqrt{1 - m^2}) - \left[\left(\frac{r_c^h}{R} \right)^2 + \varpi \sqrt{\left(\frac{r_c^h}{R} \right)^4 - m^2} \right] \right]
\end{aligned} \tag{36}$$

While regarding the material as incompressible (i.e. $\nu = 0.5$), the displacement solution of Zhou et al.¹³ can be recovered by Equation 36 as:

$$\frac{r_c^h}{R} = \sqrt[4]{\left[\frac{G}{k} \left[1 - \left(\frac{R_0}{R} \right)^2 \right] \right]^2 + m^2} \tag{37}$$

Equation 36 gives that a limit value of r_c^h / R exists during continuous expansions. This limit can be approached by putting $R_0 / R \rightarrow \infty$. In the special case of zero internal shear stress (i.e. $m = 0$), the limit ratio of r_c^h / R given by Yu¹ from a rigorous similarity analysis, considering the material compressibility, can be recovered as given in Equation 38, neglecting the small quantities.

$$\frac{r_c^h}{R} = \left[(\varpi - 1) \left(1 - \frac{k}{2G} \right)^2 + 1 \right]^{\frac{1}{2(\varpi - 1)}} \tag{38}$$

A rigorous quasi-static expansion solution is obtained by the combination use of Equations 35 and 36. In the simplified cases of $\nu = 0.5$ and $m = 0$, the well-known limit expansion pressure derived by Gibson and Anderson⁵⁴ (i.e. Equation 39) can be recovered by substituting the limit value of r_c^h / R into Equation 35.

$$p_{\text{limit}} = k \left[1 + \ln \left(\frac{G}{k} \right) \right] - P_\infty^h \tag{39}$$

By using Equations 35 and 36, influences of the internal shear stress on the continuous pressure-expansion response and the limit expansion pressure are depicted in Figures 8 and 9, respectively. It is shown that additional shear stresses reduce the required normal pressure during continuous expansions, and the limit expansion pressure declines with increases of the applied shear stresses and decreases of Poisson's ratio. These reductions are due to the additional plastic failure caused by the internal shear stresses. With fully mobilised shear stresses (i.e. $m=1$), a reduction of 6% of the limit expansion pressure

394 may be produced within the Tresca material, and this influence would be intensified in materials with
 395 greater interface shear stress holding capacity. In addition to the stress reorientation effect considered
 396 by Bengough, et al. ¹⁸, this solution provides an analytical method to further explain the great reduction
 397 of soil resistance measured in rotating CPTs.

398 **7 | ESTIMATION OF MAXIMUM MUD PRESSURE IN HORIZONTAL** 399 **DIRECTIONAL DRILLINGS**

400 In the cavity expansion approach, the maximum mud pressure (p_m) in HDD practices is usually
 401 expressed in terms of the maximum allowable plastic radius (R_c). Based on the derived mapping
 402 function in Equation 13, p_m in terms of R_c can be expressed as

$$403 \quad p_m = 2k \ln\left(\frac{R_c}{\delta R}\right) - P_\infty + k\sqrt{1-m^2} + u_0 \quad (40)$$

404 where u_0 is the groundwater pressure at the depth of the borehole.

405 According to Equations 13 and 35, the propagation distance of the plastic failure zone ($r_{c\theta}$) under
 406 biaxial stress conditions within the admissible stress range can be expressed as

$$407 \quad r_{c\theta} = |\omega(\sigma)| = r_c^h \left| \left(\sigma + \frac{\beta}{\sigma} \right) \right| \quad (41)$$

408 It has been demonstrated that the average cavity displacement in a biaxial stress field can be
 409 approximated by the solution derived in the corresponding hydrostatic stress condition ^{34, 55}. Therefore,
 410 the borehole radius pressurised under a uniform mud pressure is estimated by using the displacement
 411 solution given in Appendix C. With known initial borehole diameter ($2R_0 = D$) and radius of the
 412 elastic-plastic boundary, r_c^h / R can be obtained by letting $r = R$ and $r_0 = R_0$ in Equation C8 as:

$$413 \quad \begin{aligned} & (1-M)^2 - \left(\frac{R_0}{r_c^h}\right)^2 \\ & = \frac{1}{\varpi^2 - 1} \left[\left(\frac{1 + \sqrt{1-m^2}(R/r_c^h)^4}{[1 + \sqrt{1-m^2}](R/r_c^h)^2} \right)^\varpi (1 + \varpi\sqrt{1-m^2}) \left(\frac{R}{r_c^h}\right)^2 - [1 + \varpi\sqrt{1-m^2}(R/r_c^h)^4] \right] \end{aligned} \quad (42)$$

414 Neglecting the internal shear stress, r_c^h / R can be explicitly expressed as:

$$415 \quad \frac{r_c^h}{R} = \left\{ (\varpi - 1) \left[\left(1 - \frac{k}{2G}\right)^2 - \left(\frac{R_0}{r_c^h}\right)^2 \right] + 1 \right\}^{\frac{1}{2(\varpi-1)}} \quad (\text{special case of } m=0) \quad (43)$$

416 In undrained clays, the plastic volumetric change under loading is often negligible. Therefore, the
 417 displacement solution can be further simplified to the Delft solution in this case⁵⁶ as Equation 44.

$$418 \quad \frac{r_c^h}{R} = \left[\frac{k}{G} + \left(\frac{R_0}{r_c^h}\right)^2 \right]^{-\frac{1}{2}} \quad (\text{special case of } m=0 \text{ and } \nu=0.5) \quad (44)$$

419 The failure model caused by pressurized drilling fluids in clays is considered to be either tensile or shear
 420 failure⁵⁷⁻⁵⁹. The limit failure pressure varies with the failure model significantly. In general, the tensile
 421 fracture pressure is lower than that with shear failure^{17, 58}. The failure model primarily depends on the
 422 material strength and stress conditions (e.g. the K_0 effect) within homogenous materials^{50, 60}. Both
 423 elastic (e.g. Kirsch equations) and elastic-plastic (e.g. Delft solution) cavity expansion solutions are
 424 often applied to estimate the limit pressure based on different failure criteria^{5, 14, 17, 60}. Based on the
 425 cavity expansion analysis, the potential failure models under loading may refer to the discussion given
 426 in Section 4. Taking Figure 4 as an example, tensile fracture failure tends to occur within the stress
 427 ranges of Zones B and E; shear failure may dominate in the Zone C; and a mixed failure may take place
 428 in the Zone D. As discussed earlier, the present stress solution was derived under stress conditions
 429 within the Zone C. Therefore, Equations 40, 41, and 42 are preferable for estimating the maximum mud
 430 pressure of HDDs implemented under stress conditions within this stress range.

431 For horizontal boreholes excavated at relatively shallow depths, apart from the considered K_0 effect, the
 432 free ground surface and soil strength and stress gradient with depths may also influence the failure
 433 model. The cavity expansion solutions used for this application were mostly developed in an infinite
 434 plane. Therefore, different criteria have been proposed in the cavity expansion approach to estimating
 435 the maximum mud pressure during HDDs, for example, the maximum allowable plastic radius criterion
 436^{42, 61} and maximum hoop strain criterion⁵. Evaluation of the reliability of these criteria is out of the
 437 scope of this paper. Instead, the most commonly adopted maximum allowable plastic radius criterion is
 438 followed in this solution.

439 Comparing with the elastic-plastic radius calculated in the corresponding hydrostatic stress condition
440 (Equation 42), a shape factor of $|\sigma + \beta / \sigma|$ is introduced by Equation 41 due to the biaxial in-situ
441 stresses. It is shown in Figure 10 (a) that Equation 41 describes that the plastic zone develops farther in
442 the vertical direction while $K_0 < 1$ (e.g. in normally consolidated or lightly overconsolidated clays),
443 whereas the farthest yielding point is along the horizontal direction before failure while $K_0 > 1$ (e.g.
444 heavily overconsolidated clays). Similar trends have also been reported in numerical and experimental
445 studies^{56, 62}. However, it needs to point out that the propagation of the plastic zone in cases of $K_0 > 1$
446 may rapidly extend to the free ground surface at the ultimate failure stage instead of propagating farther
447 in the horizontal direction, and, consequently, caution should be exercised when the maximum
448 allowable plastic radius criterion is applied in this case. Therefore, the following discussion
449 concentrates on the performance of the present solution in cases of $K_0 \leq 1$.

450 Following the failure criterion suggested by the Delft solution, which states that the maximum allowable
451 mud pressure will be reached when the plastic region expands to a distance equalling to half of the
452 burial depth, the new method given by Equations 40-42 is applied to estimate p_m for cases while
453 $K_0 \leq 1$. While $K_0 \leq 1$, the farthest yielding point locates above the crown of the borehole with a centre
454 distance of $R_c = r_c^h (1 - \beta) = 0.5H$. Based on numerical simulation results, Xia and Moore¹⁷ reported
455 that the Delft solution may overestimate the mud pressure by up to 15% while $K_0 < 1$, and better
456 agreements with the numerical results have been achieved by their solution considering the K_0 effect⁵⁶.
457 Figure 10 (b) demonstrated that the K_0 effect is also well captured by the present solution in the given
458 stress states. Additionally, it is shown that internal shear stresses may lead to further decreases of the
459 maximum allowable mud pressure. Hence, the ignorance of the internal shear stresses may be another
460 noticeable reason that leads to that the Delft solution generally tends to overestimate p_m in practice⁴⁵.
461⁴⁷.

462 **8 | SUMMARY AND CONCLUSIONS**

463 An analytical elastoplastic stress solution was developed for a cylindrical cavity under loading in Tresca
464 materials, subjected to biaxial stresses at infinity and uniform normal and shear stresses on the cavity
465 wall. The solution was derived by three steps: a) plastic stresses were derived by assuming that the
466 plastic zone is statically determinate, b) a conformal mapping function describing the outside elastic
467 region was determined through analysing the stress continuity conditions across the elastic-plastic
468 boundary based on Laurent's decomposition theorem and Harnack's theorem, and c) Kolosov-
469 Muskhelishvili complex potentials for the elastic analysis were obtained by using the Cauchy integral
470 method and Fourier series method. Within the admissible application range, the analytical solution
471 showed excellent agreement with FEM simulations and Galin's solution in the special case without
472 internal shear stresses. It was demonstrated that both the internal shear stress and biaxial in-situ soil
473 stresses have fairly significant influences on the distribution of the plastic zone. It was shown that the
474 elastic-plastic boundary is in an elliptic shape under biaxial stress conditions, whose major axis
475 coincides with the direction of the maximum far-field compression stress under loading on the internal
476 cavity wall. The additional shear stresses extend the plastic region, but apply no influence on the shape
477 of the elastic-plastic boundary. In addition, considering the internal shear stresses, an analytical large
478 strain displacement solution was developed for a cylindrical cavity expanding in a hydrostatic in-situ
479 stress field. The derived stress and displacement solutions were applied to account for the additional
480 influence of the shear stresses generated during rotating CPTs and to estimate the maximum allowable
481 mud pressure of HDDs considering both biaxial in-situ stresses and the possibly generated internal shear
482 stresses.

483 To explain the reduction of the normal soil resistance in rotating CPTs due to the shear stresses
484 generated around the interface, an analytical quasi-static cavity expansion solution was developed by
485 combining the derived static stress solution and the continuous displacement solution. It was found that
486 the internal shear stresses would lead to additional plastic failure in the surrounding soil. As a result,
487 the required normal pressure during expansions and the limit normal expansion pressure decline,
488 depending on the shear stress level. These findings further explained the great reduction of the tip

489 resistance measured in rotating CPTs in addition to the reorientation effect of the interface frictional
 490 resistance.

491 Considering the K_0 effect and influences of the shear stresses generated during drillings, a new
 492 approximate method for estimating the maximum allowable mud pressure during HDDs was developed.
 493 With the same maximum allowable plastic radius as in the Delft solution, the new method predicted
 494 lower maximum mud pressures than those by the Delft solution in cases of $K_0 < 1$ and showed that the
 495 maximum mud pressure declines with a decreasing value of K_0 . These findings are consistent with that
 496 reported in numerical simulations. In addition, it was found that the internal shear stresses may also
 497 cause a considerable reduction of the maximum mud pressure in HDDs, which suggested that this effect
 498 may need to be considered while a significant amount of shear stresses between the rotating drill
 499 bit/reamer and the surrounding soil are unavoidably generated during drillings.

500 **Appendix A. Purely elastic stress solutions and expressions of principal stresses**¹

$$501 \quad \sigma_r = P_\infty \left(1 - \frac{R_0^2}{r^2}\right) - \tau_\infty \left(1 + \frac{3R_0^4}{r^4} - \frac{4R_0^2}{r^2}\right) \cos 2\theta - p_{in} \frac{R_0^2}{r^2} \quad (\text{A1})$$

$$502 \quad \sigma_\theta = P_\infty \left(1 + \frac{R_0^2}{r^2}\right) + \tau_\infty \left(1 + \frac{3R_0^4}{r^4}\right) \cos 2\theta + p_{in} \frac{R_0^2}{r^2} \quad (\text{A2})$$

$$503 \quad \tau_{r\theta} = \tau_\infty \left(1 - \frac{3R_0^4}{r^4} + \frac{2R_0^2}{r^2}\right) \sin 2\theta + mk \frac{R_0^2}{r^2} \quad (\text{A3})$$

$$504 \quad \sigma_1 = \frac{\sigma_r + \sigma_\theta}{2} + \left[\left(\frac{\sigma_r - \sigma_\theta}{2} \right)^2 + \tau_{r\theta}^2 \right]^{1/2} \quad (\text{A4})$$

$$505 \quad \sigma_3 = \frac{\sigma_r + \sigma_\theta}{2} - \left[\left(\frac{\sigma_r - \sigma_\theta}{2} \right)^2 + \tau_{r\theta}^2 \right]^{1/2} \quad (\text{A5})$$

$$506 \quad \sigma_2 = \nu(\sigma_1 + \sigma_3) \quad (\text{A6})$$

507 **Appendix B. Re-derivation of the mapping function**³⁷

508 To transform the exterior of the elastic-plastic boundary in the physical plane onto the exterior region
 509 of the unit circle in the phase plane, a general form of conformal mapping function is introduced⁶³ as

$$510 \quad \omega(\zeta) = \alpha' \zeta + \alpha'_0 + \sum_{n=1}^{\infty} \frac{\alpha'_n}{\zeta^n} \quad (\text{B1})$$

511 Due to the symmetry of the geometry and stress boundaries, the mapping function $\omega(\zeta)$ has the
 512 following features³⁵.

$$513 \quad \omega(\zeta) = -\omega(-\zeta) \quad , \quad \omega(\zeta) = \overline{\omega(\bar{\zeta})} \quad (B2)$$

514 As a result, α_0 and coefficients of the even order terms are equal to zero, and remaining coefficients
 515 are real numbers. $\omega(\zeta)$ can be rewritten as

$$516 \quad z = x + iy = \omega(\zeta) = \alpha\zeta + \sum_{j=0}^{\infty} \frac{\alpha_{2j+1}}{\zeta^{2j+1}} \quad (B3)$$

517 By multiplying both sides of Equation 11a with $\frac{1}{2\pi i} \frac{d\sigma}{\sigma - \zeta}$, a Cauchy integral is established as:

$$518 \quad \frac{1}{2\pi i} \int_{\gamma} \left[\frac{\bar{\omega}(\sigma^{-1})}{\omega(\sigma)} \Phi'(\sigma) + \Psi(\sigma) \right] \frac{d\sigma}{\sigma - \zeta} = \frac{k}{2\pi i} \int_{\gamma} \left\langle \frac{\sqrt{[\omega(\sigma)\overline{\omega(\sigma)}]^2 - m^2 R_0^4 + m R_0^2 i}}{\omega(\sigma)\overline{\omega(\sigma)}} \right\rangle \frac{\overline{\omega(\sigma)}}{\omega(\sigma)} \frac{d\sigma}{\sigma - \zeta} \quad (B4)$$

519 The right part of Equation B4 should be bounded at infinity^{27, 52}, hence terms of $j \geq 1$ in the mapping
 520 function vanish as shown in Equation B5.

$$521 \quad \frac{\overline{\omega(\sigma)}}{\omega(\sigma)} = \frac{\bar{\omega}(\sigma^{-1})}{\omega(\sigma)} = \frac{\bar{\alpha}\sigma^{-1} + \bar{\alpha}_1\sigma + \bar{\alpha}_3\sigma^3 + \dots}{\alpha\sigma + \alpha_1\sigma^{-1} + \alpha_3\sigma^{-3} + \dots} = \frac{\bar{\alpha}_1}{\alpha} + M(\sigma) \quad (B5)$$

522 where $M(\sigma)$ is analytic on the exterior of contour γ , and $M(\infty) = 0$. Similarly, we find

$$523 \quad \frac{\bar{\omega}(\sigma^{-1})}{\omega(\sigma)} \Phi'(\sigma) = N(\sigma) \quad (B6)$$

524 where $N(\sigma)$ is analytic on the exterior of contour γ , and $N(\infty) = 0$.

525 Meanwhile, $\Psi(\sigma)$ is holomorphic in Ω^- (including infinity points). Finally, according to Harnack's
 526 theorem²⁰, integrating Equation B4 along γ from Ω^+ side gives

$$527 \quad \alpha_1 = \bar{\alpha}_1 = \frac{\tau_{\infty}}{k} \alpha \quad (B7)$$

528 Therefore, the mapping function is in the form of Equation 13. Detailed knowledge about the complex
 529 variable methods in elasticity refers to the references of Muskhelishvili²⁰ and England⁶³.

530 **Appendix C. Displacement solution for continuous expansions in a hydrostatic stress field**

531 For a cavity deforms in a hydrostatic initial stress state, the radial displacement during continuous
 532 expansions can be obtained by a one-dimensional deformation analysis. Note that Zhou et al.¹³ presented
 533 an analytical expansion solution for this problem by ideally regarding the material as volumetric
 534 incompressible. More generally, a new large-strain displacement solution considering the material
 535 compressibility is presented here. The elastic displacement is derived based on the small strain theory,
 536 and finite strain definitions are adopted in the plastic deformation analysis^{64, 65}.

537 During purely elastic expansions, the radial displacement (u_r) can be expressed as

$$538 \quad u_r = r - r_0 = \frac{p_{in} + P_\infty^h}{2G} \left(\frac{R}{r}\right)^2 r \quad (C1)$$

539 According to the elastic stress solution (i.e. Equations A1, A2, and A3, ignoring the angle-dependent
 540 terms) and the yield criterion (i.e. Equation 5), it is found that a plastic zone starts forming from the
 541 inner cavity wall while $p_{in} \geq k\sqrt{1-m^2} - P_\infty^h$. While plastic zone appears, the radial displacement in the
 542 outside elastic zone can be obtained by replacing p_{in} with the compression pressure at the elastic-
 543 plastic boundary ($p_{r_c^h}$) and R with the radius of the elastic-plastic boundary (r_c^h) in the solution of
 544 Equation C1. $p_{r_c^h}$ can be calculated by substituting the elastic stress solution into the yield criterion as

$$545 \quad p_{r_c^h} = k\sqrt{1-m^2} \left(\frac{R}{r_c^h}\right)^4 - P_\infty^h \quad (C2)$$

546 Then the radial displacement at the elastic-plastic boundary is obtained based on the elastic solution as

$$547 \quad u_r|_{r=r_c} = \frac{kr_c^h}{2G} \sqrt{1-m^2} \left(\frac{R}{r_c^h}\right)^4 \quad (C3)$$

548 Due to the plastic volumetric strain rate is zero in Tresca materials, the compressibility equation in the
 549 plastic region¹ can be expressed as

$$550 \quad \dot{\varepsilon}_r + \dot{\varepsilon}_\theta = \frac{1-2\nu}{2G} [\dot{\sigma}_r + \dot{\sigma}_\theta] \quad (C4)$$

551 With the initial stress boundary conditions, integrating Equation C4 gives

552 $\varepsilon_r + \varepsilon_\theta = \frac{1-2\nu}{2G}[\sigma_r + \sigma_\theta - 2P_\infty^h]$ (C5)

553 By adopting the definition of logarithmic strain to characterise the accumulative deformation, the radial
554 strain and circumferential strain respectively are

555 $\varepsilon_r = \ln \frac{dr}{dr_0}$, $\varepsilon_\theta = \ln \frac{r}{r_0}$ (C6)

556 By substituting the plastic stresses and Equation C6 into Equation C5, it gives

557 $\ln \left[\frac{r}{r_0} \frac{dr}{dr_0} \right] = \varpi \ln \left[\frac{r^2 + \sqrt{r^4 - m^2 R^4}}{(r_c^h)^2 + \sqrt{(r_c^h)^4 - m^2 R^4}} \right]$ (with $\varpi = \frac{(1-2\nu)k}{G}$) (C7)

558 With the use of Equation C3, Equation C7 can be integrated over the interval $[r, r_c^h]$, leading to

559
$$\left[1 - \frac{k}{2G} \sqrt{1 - m^2 \left(\frac{R}{r_c^h} \right)^4} \right]^2 (r_c^h)^2 - r_0^2$$

$$= \frac{1}{\varpi^2 - 1} \left\{ \left(\frac{(r_c^h)^2 + \sqrt{(r_c^h)^4 - m^2 R^4}}{r^2 + \sqrt{r^4 - m^2 R^4}} \right)^\varpi (r^2 + \varpi \sqrt{r^4 - m^2 R^4}) - [(r_c^h)^2 + \varpi \sqrt{(r_c^h)^4 - m^2 R^4}] \right\}$$
 (C8)

560 Now the continuous radial expansion of a cylindrical cavity in a hydrostatic in-situ stress field can be
561 modelled by using Equation C8 without limitation of the deformation level.

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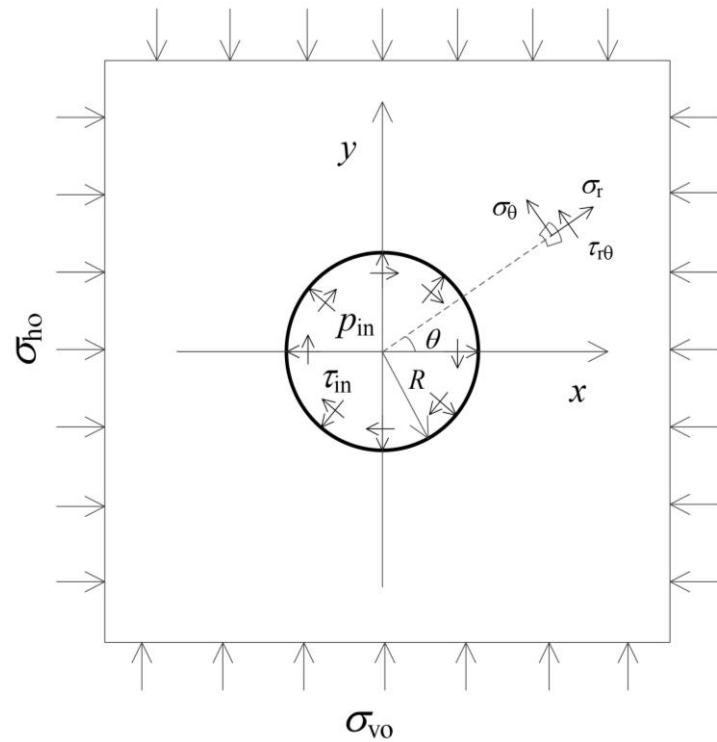
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710 **Figures:**

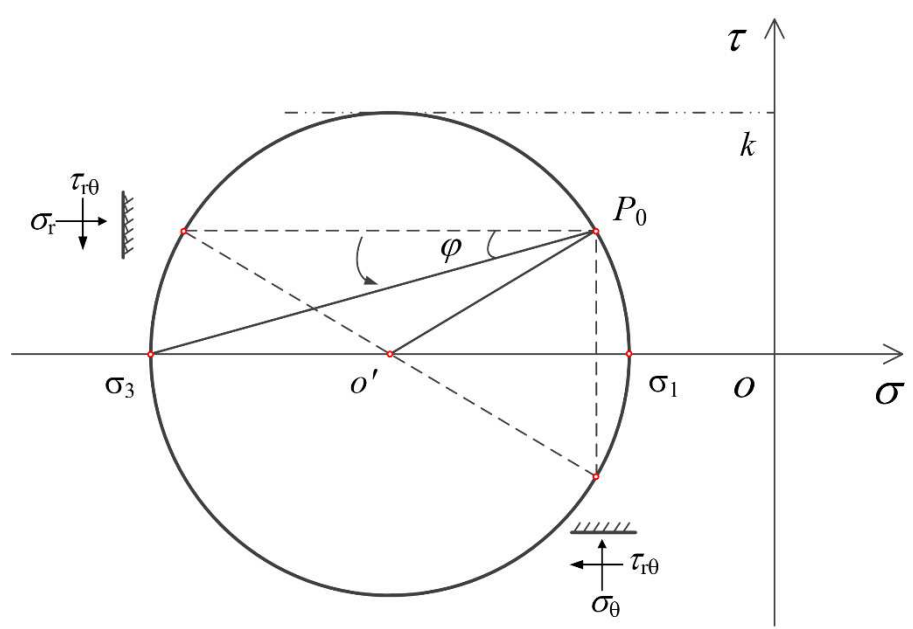


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Figure 1 Stress boundaries and coordinate systems

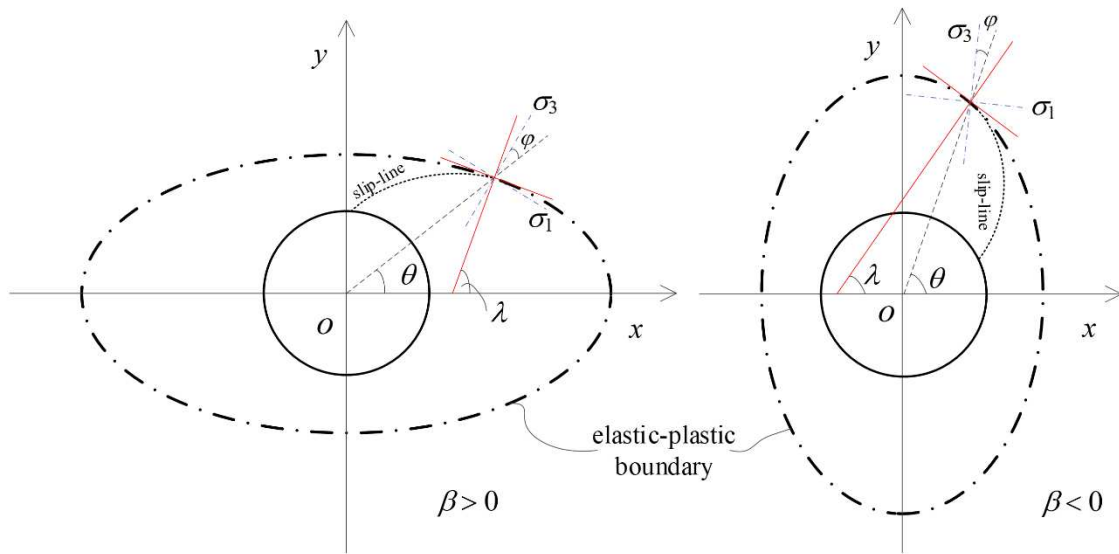
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Figure 2 Direction of principal stresses



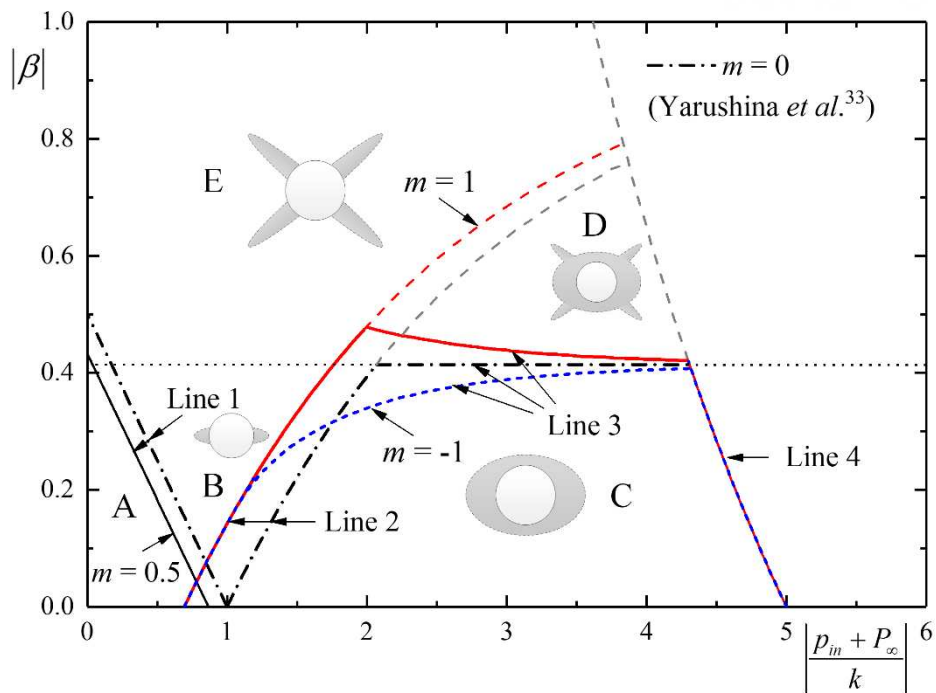
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717 Figure 3 Illustration of slip lines intersecting with the elastic-plastic boundary (taking $m > 0$ as an

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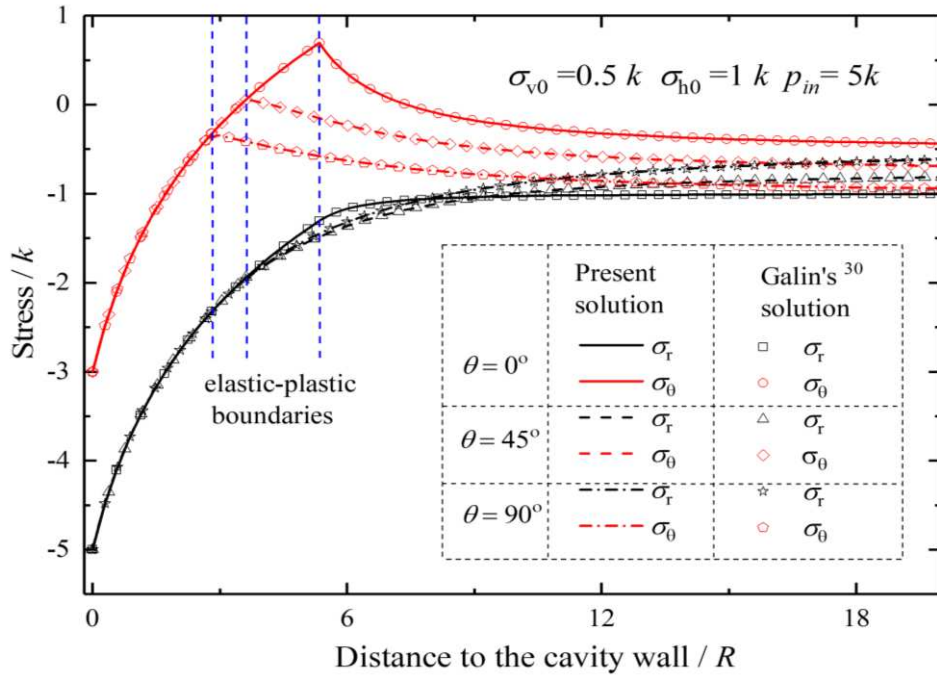
example)

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721 Figure 4 Example limits of different stress states ($\nu = 0.4$)

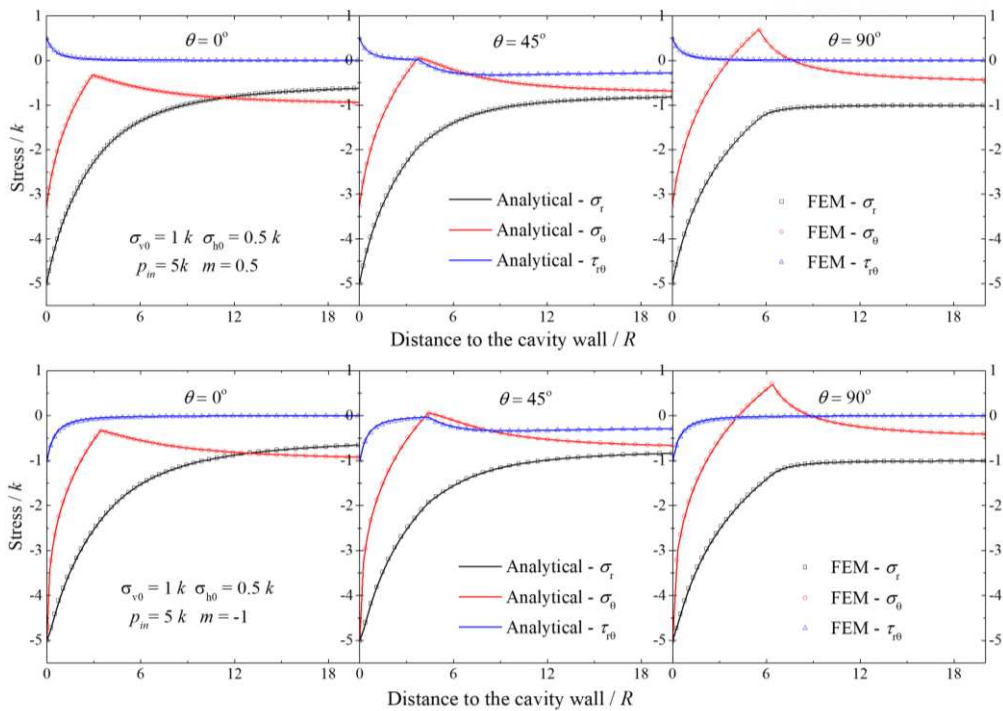


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Figure 5 Comparison between the present solution with $m=0$ and Galin's³⁰ solution

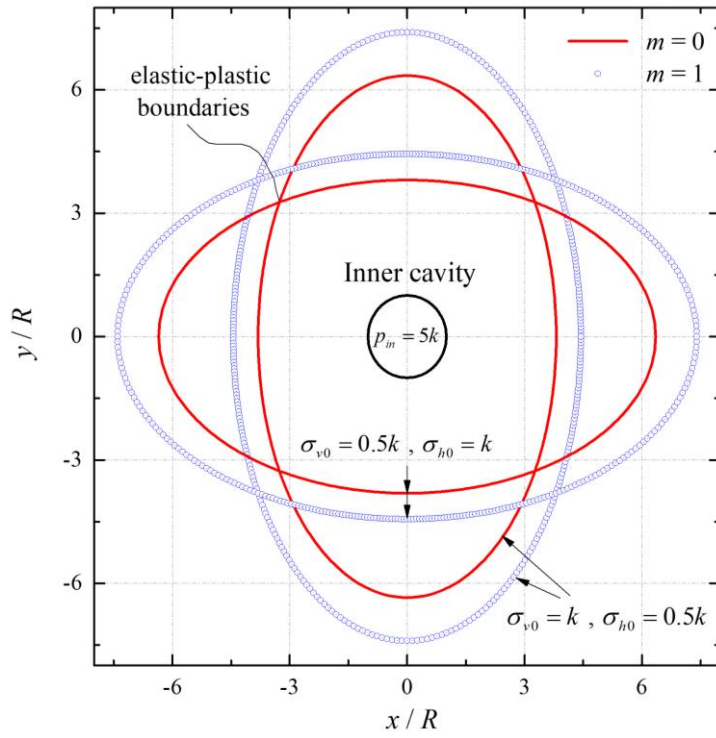
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Figure 6 Comparison of stress components between the analytical solution and FEM simulations



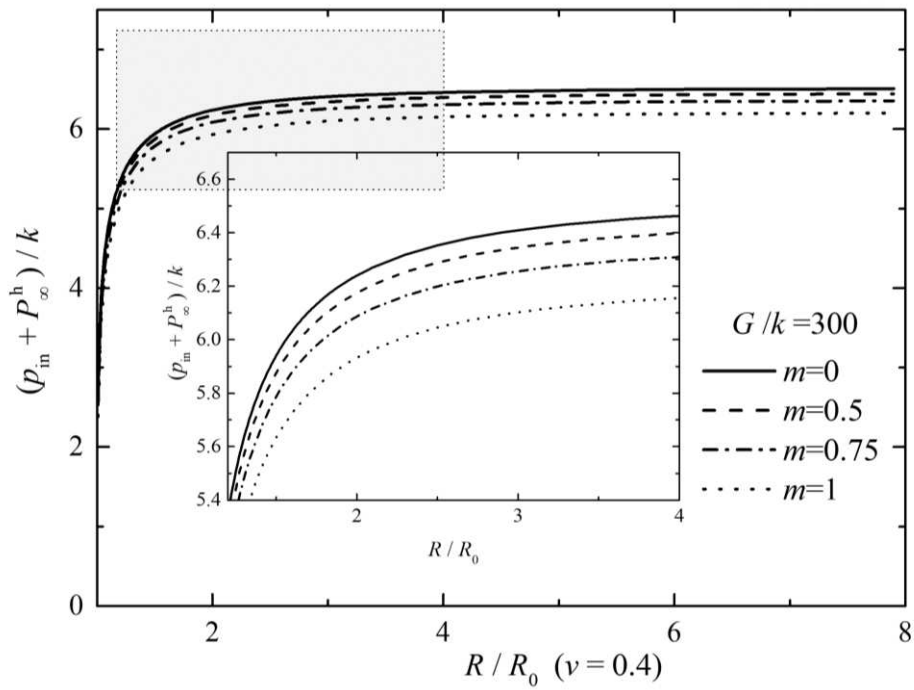
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728 Figure 7 Influences of the internal shear stress and biaxial in-situ stresses on the elastic-plastic

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boundary

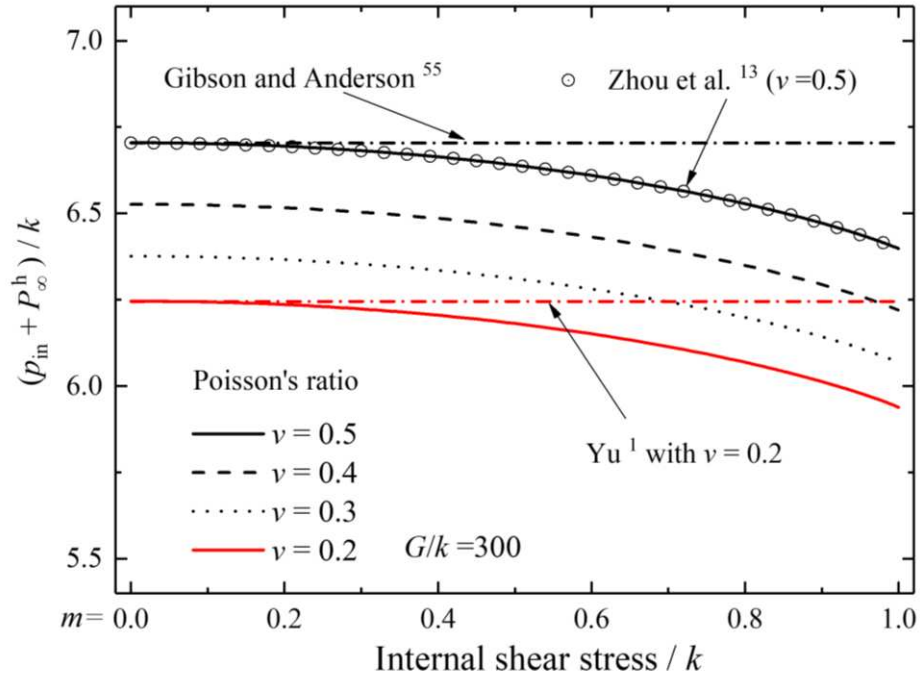
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Figure 8 Example continuous pressure-expansion curves

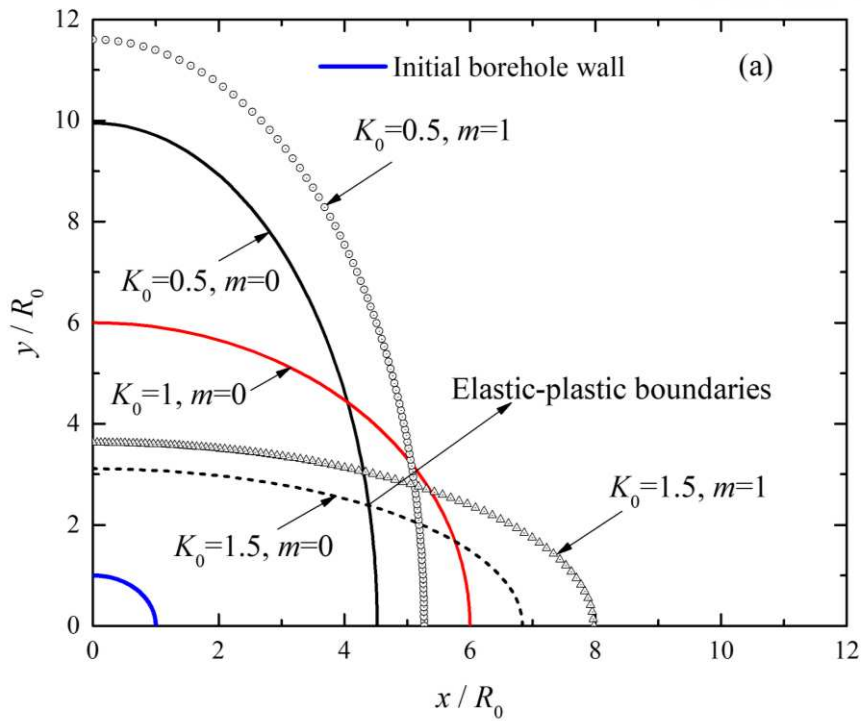


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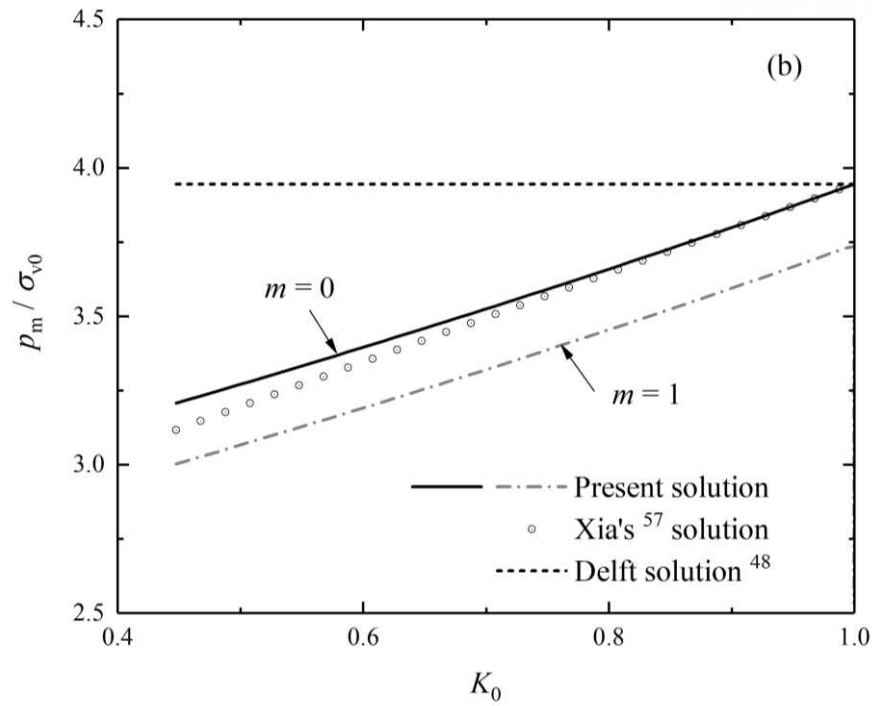
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Figure 9 Influences of the internal shear stress on the limit expansion pressure

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738 Figure 10 K_0 effect: (a) elastic-plastic boundaries with the same mud pressure; (b) limit mud pressures

739 with the same value of $R_c(=0.5H)$ ($H=3\text{m}$, $D=0.5\text{m}$, $G/k=200$, $\nu=0.5$, $k=40\text{kPa}$, $\gamma=20\text{ kN/m}^3$)