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Nonlinear PI controller for systems with state constraint requirements

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***Abstract*— PI control is widely applied in industrial applications to guarantee a desired regulation for both linear and nonlinear systems. However, when a state constraint requirement is needed for the plant, conventional PI controllers fail to guarantee a desired upper bound for the system states of a generic nonlinear plant at all times. In this paper, a novel nonlinear PI controller is proposed to achieve regulation with state constraint satisfaction for a class of nonlinear systems with constant disturbances. An upper bound for the desired closed-loop system states is guaranteed based on nonlinear ultimate boundedness theory and closed-loop stability is analytically proven for the desired equilibrium point. In addition, a detailed analysis is presented for the appropriate design of the proportional and integral gains of the proposed controller. A practical example of a dc/dc power converter is investigated and simulation results demonstrate the effectiveness of the proposed nonlinear PI control compared to the conventional approach.**

I. INTRODUCTION

Integral control has been widely used in industry for the past 40 years because of its properties of asymptotic regulation and disturbance rejection. The problem of controlling linear dynamics with a PI controller is now well understood, but its application to nonlinear systems in order to guarantee the desired regulation and closed-loop system stability still remains a challenge [1], [2], [3], [4]. The existing methods leverage on the properties of the system, i.e. minimum phase, to guarantee either local or global stability [5], [6], [7]. In [8], the authors have proposed a robust nonlinear integral controller, based on high gain observers, capable of stabilizing non-minimum phase dynamics with a desired output regulation. Whereas in [9], the output feedback controller tracks references generated from an external source without using an internal model. The authors bring attention to the closed-loop performance deterioration when including an internal model in the controller structure. However, this problem can be circumvented by using a high-gain feedback controller and observer, as presented in [10] and [11].

While the aforementioned approaches offer global or semi-global stability guarantees, they do not tackle the problem of constraint satisfaction (input or state constraints) which arises from safety requirements and actuator limitations in a real engineering system. Safety of operation and stability guarantees are essential in modern processes such as power networks and chemical processes. In the former, [12] and [13] list some of the potential pitfalls of not considering the power converter limits into the control strategy, i.e. loss of stability, performance degradation, and operation outside the desired ranges because of system malfunctions. For example consider the one dimensional system $\dot{x} = -wx^3 + u$, where w is a constant unknown disturbance defined in the range $w \in [w_{min}, w_{max}] > 0$. The objective is to regulate the state x towards a desired constant reference r while satisfying the state constraint $|x| \leq x_{max}$, $\forall t \geq 0$. In general, the conventional PI controller of the form $u = k_P(r - x) + \sigma$, $\dot{\sigma} = k_I(r - x)$, can achieve the desired regulation but does not offer any guarantees when it comes to constraint satisfaction. It can easily be shown that the solution may exhibit overshoots violating constraints. Therefore, an effective management of constraints may increase the operation ranges in both the transient and the steady-state regimes.

Although the output regulation problem with input constraint satisfaction has been well studied and addressed via anti-windup methods [14], [15] or bounded integral control [16], [17], a state constraint requirement often requires more advanced control methods that change the traditional PI control architecture. For example, techniques such as Model Predictive Control (MPC), see [18] for an excellent survey on its different properties, include the constraints into their formulation and aim to exploit the behavior of the system around the constraints. These methods, however, present limitations in their implementation because they require the solution of an optimization problem online. The authors in [19] propose an MPC controller for linear systems that operates at megahertz; however, this approach is aimed at linear system and quadratic performance objectives which is at odds with nonlinear formulations of the problem. Hence, the output regulation problem with state constraint satisfaction for nonlinear systems using PI control, which is a well known and applied controller in the industry, is still an open problem.

To address this issue, in this paper, a nonlinear PI controller is proposed for a class of nonlinear systems with constant uncertainties or disturbances to achieve output regulation with a desired state constraint satisfaction. The desired state constraint is guaranteed using nonlinear ultimate boundedness theory. Furthermore, asymptotic stability is analytically proven for the desired equilibrium point without the required system states violating an upper limit during transients, which is a significant advantage compared to the conventional PI control. Furthermore, a detailed design procedure for both the proportional and the integral gains is presented to guarantee the closed-loop system stability.

Overall, the novelties and contributions of the paper are summarized as follows:

1) The design of a novel nonlinear PI controller for a class

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of nonlinear systems capable of guaranteeing a desired state constraint on a number of the plant states.

2) Detailed stability analysis and design considerations for the proposed controller, and comparison with the conventional PI control scheme.

In Section II, we present the problem statement and an analysis of the conventional PI controller for nonlinear systems. Section III presents the proposed controller, its corresponding stability analysis and synthesis considerations. The controller is applied to a dc/dc converter application in Section IV to highlight its importance in a practical implementation. Finally, concluding remarks are provided in Section V.

II. Problem description

A. Nonlinear plant dynamics and properties

Consider the nonlinear plant of the form:

$$\dot{x}_i = f_i(x_i, w) + g_i(w)u_i,\ \forall i = 1, 2, ..., n \quad (1)$$
$$\dot{z} = q(x, z, u, w), \quad (2)$$
$$y_i = x_i,\ \forall i = 1, 2, ..., n \quad (3)$$

with $x = [x_1 \dots x_n]^T \in R^n$, where $\bar{x} = \begin{bmatrix} x^T & z^T \end{bmatrix}^T \in R^{n+m}$, $u = [u_1 \dots u_n]^T \in R^n$ and $y = [y_1 \dots y_n]^T \in R^n$ are the state, input and output vectors, respectively, and $w \in R^l$ is a vector that contains unknown constant parameters or disturbances. The functions f, g,and q are continuous and differentiable in $(\bar{x}, u)$ and continuous in w for $\bar{x} \in D \subset R^{n+m}$, $u \in R^n$ and $w \in D_w \subset R^l$, where D and D_w are open connected sets.

The main goal is to design a controller that regulates y to a constant reference $r \in D_r \subset R^n$, where $r = [r_1 \dots r_n]^T$ and D_r is an open connected set, and guarantees a desired state constraint for the states x_i, $\forall i = 1, 2, ..., n$, of the form

$$|x_i| \le x_i^{max},\ \forall t \ge 0, \quad (4)$$

for some $x_i^{max} > 0$. It is underlined that the desired state constraint requirement is applied only to the state vector x. It is assumed that there exists a vector r such that for each pair (r, w) there is a unique pair $(\bar{x}_e, u_e)$ that depends continuously on (r, w) such that

$$0 = f_i(x_{ie}, w) + g_i(w)u_{ie},\ \forall i = 1, 2, ..., n \quad (5)$$
$$0 = q(x_e, z_e, u_e, w) \quad (6)$$
$$r_i = x_{ie},\ \forall i = 1, 2, ..., n, \quad (7)$$

where x_{ie} satisfies $|x_{ie}| < x_i^{max}$, representing the desired equilibrium point.

For system (1)-(3), consider the following assumptions:

Assumption 1: $w \in [w_{min}, w_{max}]$, with w_{min} and w_{max} being known

Assumption 2: $g_i(w) > 0$, $\forall i = 1, 2, ..., n$

Assumption 3: For each $i = 1, 2, ..., n$, there exists a continuously differentiable function $V_i(x_i, w) : D_i \times D_w \to R$, four positive constants b_{i1}, b_{i2}, c_i, δ_i with $c_i > \delta_i > 0$ and two class $\mathcal{K}$ functions a_{i1} and a_{i2} such that

$$a_{i1}(|x_i|) \le V_i(x_i, w) \le a_{i2}(|x_i|) \quad (8)$$
$$\frac{\partial V}{\partial x_i} f_i(x_i, w) \le -c_i x_i^2 \quad (9)$$
$$b_{i1}x_i \le \frac{\partial V}{\partial x_i} g_i(w) \le b_{i2}x_i. \quad (10)$$

Although the plant dynamics and above assumptions can seem restrictive, it should be mentioned that: i) several engineering applications are described by (1)-(3), e.g. power electronic converters [14] and ii) a wide class of nonlinear systems can be brought in the form of (1)-(3) using partial feedback linearization. In such case, one can obtain $f_i(x_i, w) = A_i(w)x_i$, with $A_i(w) \le 0$, and by knowing the upper/lower bounds of the uncertain parameter or disturbance w (equivalently $g(w)$), then the function $V_i = \frac{1}{2}p_i x_i^2$, with $p_i > 0$, is proven to guarantee all conditions (8)-(10).

B. Conventional PI controller

Based on the previous conditions, the desired regulation scenario can be achieved using a conventional PI controller of the form

$$u_i = k_{Pi}\left(r_i - x_i\right) + \sigma_i \quad (11)$$
$$\dot{\sigma}_i = k_{Ii}\left(r_i - x_i\right), \quad (12)$$

where k_{Pi}, $k_{Ii} > 0$ and at the desired equilibrium point, there is $\sigma_{ie} = u_{ie}$. The closed-loop system takes the form

$$\dot{x}_i = f_i(x_i, w) + g_i(w)k_{Pi}(r_i - x_i) + g_i(w)\sigma_i,\ \forall i = 1, 2, ..., n \quad (13)$$
$$\dot{\sigma}_i = k_{Ii}\left(r_i - x_i\right),\ \forall i = 1, 2, ..., n \quad (14)$$
$$\dot{z} = q(x, z, k_P\left(r - x\right) + \sigma, w), \quad (15)$$

where $k_P = diag\left\{k_{Pi}\right\}$, $\sigma = [\sigma_1 \dots \sigma_n]^T$. For system (13)-(14), stability of the equilibrium point $\begin{bmatrix} x_e^T & \sigma_e^T \end{bmatrix}^T$ can be analyzed by investigating the Jacobian matrices

$$A_i = \begin{bmatrix} \left.\frac{\partial f_i}{\partial x_i}\right|_{x_i = x_{ie}} - k_{Pi}g_i(w) & g_i(w) \\ -k_{Ii} & 0 \end{bmatrix},\ \forall i = 1,2,...,n$$

which can be proven to be Hurwitz for a suitable selection of the proportional gain k_{Pi}, i.e.

$$k_{Pi} > \max\left\{0, \left.\frac{\partial f_i}{\partial x_i}\right|_{x_i = x_{ie}} \frac{1}{\min\left\{g_i(w)\right\}}\right\}. \quad (16)$$

Now, by setting $v = v_e + \tilde{v}$, where $v = \begin{bmatrix} x^T & \sigma^T \end{bmatrix}^T$, $v_e = \begin{bmatrix} x_e^T & \sigma_e^T \end{bmatrix}^T$, $\tilde{v} = \begin{bmatrix} \tilde{x}^T & \tilde{\sigma}^T \end{bmatrix}^T$, and $z = z_e + \tilde{z}$, then (15) can be rewritten in the generic form

$$\dot{\tilde{z}} = \tilde{q}(\tilde{v}, \tilde{z}, w), \quad (17)$$

where the desired equilibrium has been shifted to the origin. When asymptotic stability at the equilibrium point v_e of (13)-(14) is guaranteed by the controller gains k_{Pi}, then if (17) is locally input-to-state stable when $\tilde{v}$ is considered as the input, according to Lemma 5.6 in [1], the equilibrium point

$\left[\begin{array}{ccc} x_e^T & \sigma_e^T & z_e^T \end{array}\right]^T$ of the closed-loop system (13)-(15) is asymptotically stable.

Although the desired equilibrium point can be proven to be asymptotically stable using a conventional PI controller, it is not guaranteed that $|x_i| \leq x_i^{max}$, $\forall t \geq 0$. This state constraint is crucial in several practical examples, such as power electronic converters and electromechanical systems [14], [12] where a current, voltage or speed is required to remain bounded below a given value at all times to avoid damaging the device. To overcome this problem a nonlinear PI controller is proposed in the sequel.

III. Proposed nonlinear PI controller

A. Control design to satisfy the required state constraint

By considering the same assumptions for the plant (Assumptions 1, 2 and 3) and the same regulation task at the desired constant r, which corresponds to a unique equilibrium that satisfies (5)-(7) with $|x_{ie}| < x_i^{max}$, a novel nonlinear PI controller is proposed of the form

$$
\begin{aligned}
u_i &= -k_{Pi}x_i + M_i \sin\sigma_i && (18)\\
\dot{\sigma}_i &= \frac{k_{Ii}}{M_i}(r_i - x_i)\cos\sigma_i, && (19)
\end{aligned}
$$

where M_i, k_{Pi}, $k_{Ii} > 0$ and at the desired equilibrium point, there is $\sigma_{ie} = \sin^{-1}\left(\frac{u_{ie}+k_{Pi}x_{ie}}{M_i}\right)$. Here, it is assumed that $-\frac{\pi}{2} < \sigma_{ie} < \frac{\pi}{2}$. By substituting the proposed controller (18)-(19) into the plant dynamics (1)-(3), the closed-loop system becomes

$$
\dot{x}_i = f_i(x_i, w) - g_i(w)k_{Pi}x_i + g_i(w)M_i \sin\sigma_i,\ \forall i = 1,2,...,n \qquad (20)
$$

$$
\dot{\sigma}_i = \frac{k_{Ii}}{M_i}(r_i - x_i)\cos\sigma_i,\ \forall i = 1, 2, ..., n \qquad (21)
$$

$$
\dot{z} = q(x, z, -k_P x + M\sin\sigma, w), \qquad (22)
$$

where $M = diag\{M_i\}$ and $\sin\sigma = [\sin\sigma_1 \ldots \sin\sigma_n]^T$.

Consider now a continuously differentiable function $V_i(x_i, w)$ for system (20) satisfying conditions (8)-(10) of Assumption 3. Then

$$
\begin{aligned}
\dot{V}_i &= \frac{\partial V}{\partial x_i} f_i(x_i, w) - \frac{\partial V}{\partial x_i} g(w)k_{Pi}x_i + \frac{\partial V}{\partial x_i} g(w)M_i \sin\sigma_i\\
&\leq -(c_i + k_{Pi}b_{i1})x_i^2 + b_{i2}x_iM_i\sin\sigma_i\\
&\leq -(c_i + k_{Pi}b_{i1})|x_i|^2 + b_{i2}M_i|x_i|. && (23)
\end{aligned}
$$

Considering that $c_i = \bar{c}_i + \epsilon_i \geq \delta_i > 0$, with $\bar{c}_i > 0$ and ϵ_i representing an arbitrarily small positive constant, then (23) can be rewritten as

$$
\begin{aligned}
\dot{V}_i &= -(\bar{c}_i + \epsilon_i + k_{Pi}b_{i1})|x_i|^2 + b_{i2}M_i|x_i|\\
&\leq -\epsilon_i|x_i|^2,\ \forall |x_i| \geq \frac{b_{i2}M_i}{\bar{c}_i + k_{Pi}b_{i1}}, && (24)
\end{aligned}
$$

which proves that the solution $x_i(t)$ is uniformly ultimately bounded. Therefore every solution $x_i(t)$ that starts with initial condition $x_i(0)$ satisfying

$$
|x_i(0)| \leq \frac{b_{i2}M_i}{\bar{c}_i + k_{Pi}b_{i1}}
$$

will remain in this range for all future time, i.e.

$$
|x_i(t)| \leq \frac{b_{i2}M_i}{\bar{c}_i + k_{Pi}b_{i1}},\ \forall t \geq 0.
$$

In order to guarantee the required constraint $|x_i| \leq x_i^{max}$, $\forall t \geq 0$, the controller parameters M_i and k_{Pi} can be selected to satisfy

$$
\begin{aligned}
\frac{b_{i2}M_i}{\bar{c}_i + k_{Pi}b_{i1}} &\leq x_i^{max}\\
M_i &\leq x_i^{max}\frac{\bar{c}_i + k_{Pi}b_{i1}}{b_{i2}}. && (25)
\end{aligned}
$$

This concludes the design of the controller parameter M_i to guarantee a desired upper bound for the system states x_i.

B. Stability analysis

Although the state constraint is guaranteed, in order to analyze the stability of the desired equilibrium point $\left[\begin{array}{cc} x_e^T & \sigma_e^T \end{array}\right]^T$, where $x_{ie} \in (-x_i^{max}, x_i^{max})$, $\sigma_{ie} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\forall i = 1, 2, ..., n$, the Jacobian matrix of system (20)-(21) is calculated as

$$
A_i = \begin{bmatrix} \left.\frac{\partial f_i}{\partial x_i}\right|_{x_i = x_{ie}} - k_{Pi}g_i(w) & g_i(w)M_i\cos\sigma_{ie}\\ -\frac{k_{Ii}}{M_i}\cos\sigma_{ie} & 0 \end{bmatrix}, \forall i = 1,2,...,n.
$$

Thus, the equilibrium point $\left[\begin{array}{cc} x_e^T & \sigma_e^T \end{array}\right]^T$ will be asymptotically stable when

$$
k_{Pi} > \max\left\{0, \left.\frac{\partial f_i}{\partial x_i}\right|_{x_i = x_{ie}} \frac{1}{\min\{g_i(w)\}}\right\}, \qquad (26)
$$

which matches the condition (16) of the conventional PI control. Hence, by selecting k_{Pi} according to (26), then M_i can be chosen to satisfy (25). For the remaining dynamics, similarly to the analysis of the conventional PI controller, (22) can be rewritten in the generic form

$$
\dot{\tilde{z}} = \bar{q}(\tilde{v}, \tilde{z}, w), \qquad (27)
$$

with the desired equilibrium being shifted at the origin $\tilde{z} = 0$. As explained in Subsection II-B, if (27) is locally input-to-state stable with respect to the input $\tilde{v}$, according to Lemma 5.6 in [1], the equilibrium point $\left[\begin{array}{ccc} x_e^T & \sigma_e^T & z_e^T \end{array}\right]^T$ of the closed-loop system (20)-(22) is asymptotically stable.

It is highlighted that the asymptotic stability of the desired equilibrium point $\left[\begin{array}{cc} x_e^T & \sigma_e^T \end{array}\right]^T$ has been proven in a neighborhood of the equilibrium point (as in the case of the conventional PI control). In order to prove that the asymptotic stability holds in the entire constrained range $x_i \in [-x_i^{max}, x_i^{max}]$, the nonlinear dynamics of (20)-(21) should be further investigated. In particular, for each i, (20)-(21) describe a second-order system. In order to guarantee that the solution will converge to the unique equilibrium $\left[\begin{array}{cc} x_{ie} & \sigma_{ie} \end{array}\right]$, it should be proven that no limit cycles exist in the range $x_i \in [-x_i^{max}, x_i^{max}]$. Based on the Bendixon theorem [20], in order to prove the non-existence of limit cycles the following expression

$$
\frac{\partial f_i}{\partial x_i} - k_{Pi}g_i(w) - \frac{k_{Ii}}{M_i}(r_i - x_i)\sin\sigma_i
$$

should not vanish or change sign. Since $|x_i| \leq x_i^{max}$, then $\frac{\partial f_i}{\partial x_i} - k_{Pi} g_i(w) < 0$ can be guaranteed by selecting k_{Pi} as

$$k_{Pi} > \max\left\{0, \max\left\{\frac{\partial f_i}{\partial x_i}\right\} \frac{1}{\min\{g_i(w)\}}\right\}, \quad (28)$$

which covers the condition (26) as well. Then no limit cycles will exist in the constrained range if

$$\frac{\partial f_i}{\partial x_i} - k_{Pi} g_i(w) + \frac{k_{Ii}}{M_i} \max|r_i - x_i| < 0, \forall t \geq 0. \quad (29)$$

Given that $x_i \in [-x_i^{max}, x_i^{max}]$, then the expression $\max|r_i - x_i|$ can be analytically calculated and condition (29) can be satisfied by a suitable choice of the integral gain k_{Ii} as

$$k_{Ii} < \min\left\{k_{Pi} g_i(w) - \frac{\partial f_i}{\partial x_i}\right\} \frac{M_i}{\max|r_i - x_i|}. \quad (30)$$

The importance of the proposed nonlinear PI controller and the design procedure for selecting the controller gains is better understood in a real-example application as explained in the following section.

IV. A DC/DC POWER CONVERTER EXAMPLE

A. System and controller design

Consider the dynamic equations of the dc/dc boost converter connected to a resistive load R, given as in [21]:

$$L\frac{di}{dt} = -ri - (1-u)v + V_{in} \quad (31)$$

$$C\frac{dv}{dt} = (1-u)i - \frac{v}{R} \quad (32)$$

where $V_{in} > 0$ is the dc input voltage, L is the converter inductance with a series resistance r, C is the converter capacitance, $\bar{x} = \begin{bmatrix} i & v \end{bmatrix}^T$ is the state vector and u is the control input describing the duty-ratio input of the converter, which has physical bounds defined as $u \in [0, 1]$. The dc/dc converter can achieve a higher dc voltage v at its output compared to the input voltage V_{in}. The main task is to regulate the converter current i to a constant reference i^{ref}, while maintaining a desired constraint $|i| \leq i_{max}$, where $i_{max} > 0$ represents the maximum allowed current of the converter to avoid damage of the device. It is assumed that L, C, r and R are not accurately known, i.e. $L \in [L_n - \Delta L, L_n + \Delta L] > 0$, $C \in [C_n - \Delta C, C_n + \Delta C] > 0$, $r \in [r_n - \Delta r, r_n + \Delta r] > 0$ and $R \in [R_n - \Delta R, R_n + \Delta R] > 0$, where L_n, C_n, r_n and R_n are the corresponding known nominal quantities and that both the current i and voltage v can be measured. By defining the control input u as

$$u = 1 - \frac{V_{in} - \bar{u}}{v} \quad (33)$$

and replacing it in (31)-(32), the converter dynamics take the form

$$\frac{di}{dt} = -\frac{r}{L}i + \frac{1}{L}\bar{u} \quad (34)$$

$$\frac{dv}{dt} = \frac{V_{in} - \bar{u}}{Cv}i - \frac{v}{CR} \quad (35)$$

$$y = i, \quad (36)$$

which is in the form of (1)-(3) considering the control input $\bar{u}$. Note that $g(w) = \frac{1}{L} > 0$ with $L \in [L_n - \Delta L, L_n + \Delta L]$, which confirms that both Assumptions 1 and 2 hold. Since (34) is a linear dynamic equation, then by considering the function $V = \frac{1}{2}Li^2$, which satisfies (8), there is

$$\frac{\partial V}{\partial i} f = -ri^2$$

$$\frac{\partial V}{\partial i} g = i,$$

hence (9) and (10) are also satisfied with $c = r > 0$ and $b_1 = b_2 = 1$, yielding that Assumption 3 holds as well. As a result, both the conventional PI controller and the proposed nonlinear PI controller can be implemented to regulate $\bar{x}$ to the desired unique equilibrium $\bar{x}_e = \begin{bmatrix} i_e & v_e \end{bmatrix}^T$, where $i_e = i^{ref} \in [-i_{max}, i_{max}]$ and $v_e = \sqrt{Ri^{ref}(V_{in} - ri^{ref})}$. It is underlined that due to its physical properties, the boost converter output voltage is always higher than the input voltage, i.e. $v \geq V_{in} > 0$. For any proportional gain $k_P > 0$, the equilibrium point $i_e = i^{ref}$ of (34) will be asymptotically stable since conditions (16) and (26) will be satisfied for both the conventional and the proposed PI controller. For the proposed controller, from (25), parameter M should satisfy

$$M \leq i_{max}(\bar{r} + k_P), \quad (37)$$

where $\bar{r} = r - \epsilon$, for an arbitrarily small positive number ϵ. To this end, M can be selected as

$$M = i_{max}(r_n - \Delta r - \epsilon + k_P), \quad (38)$$

which satisfies (37) using the known system parameters. Finally, from (30), the integral gain k_I should satisfy

$$k_I < \min\left\{\frac{k_P + r}{L}\right\} \frac{M}{\max|i^{ref} - i|}. \quad (39)$$

Since the proposed controller guarantees the state constraint $|i| \leq i_{max}$ according to the analysis in Subsection III-A and $\left|i^{ref}\right| < i_{max}$, then $\max\left|i^{ref} - i\right| = 2i_{max}$. As a result, taking into account the choice of M from (38) and the range of the uncertain parameters $L \in [L_n - \Delta L, L_n + \Delta L] > 0$ and $r \in [r_n - \Delta r, r_n + \Delta r] > 0$, inequality (39) becomes

$$k_I < \frac{(k_P + r_n - \Delta r)(r_n - \Delta r - \epsilon + k_P)}{2(L_n + \Delta L)}. \quad (40)$$

By setting $i = i_e + \tilde{i}$, $v = v_e + \tilde{v}$ and $\sigma = \sigma_e + \tilde{\sigma}$, then (35) becomes

$$\frac{d\tilde{v}}{dt} = \frac{V_{in} + k_P(i_e + \tilde{i}) - M\sin(\sigma_e + \tilde{\sigma})}{C(v_e + \tilde{v})}(i_e + \tilde{i}) - \frac{v_e + \tilde{v}}{CR}$$

$$= \tilde{q}(\tilde{x}, \tilde{v}, w). \quad (41)$$

Considering a set $D_{\tilde{v}}$ for $\tilde{v}$, where $\tilde{v} > -v_e$ in $D_{\tilde{v}}$, then both $\frac{\partial \tilde{q}}{\partial \tilde{v}}$ and $\frac{\partial \tilde{q}}{\partial \tilde{x}}$ are bounded in $D_{\tilde{v}}$, where $\tilde{x} = \begin{bmatrix} \tilde{i} & \tilde{\sigma} \end{bmatrix}^T$. In addition, the unforced system, i.e. for $\tilde{x} = 0$, becomes

$$\frac{d\tilde{v}}{dt} = \frac{V_{in} i_e}{C(v_e + \tilde{v})} - \frac{v_e + \tilde{v}}{CR}. \quad (42)$$

TABLE I

CONVERTER PARAMETERS

Parameters	Values	Parameters	Values
L	$12mH$	L_n	$10mH$
ΔL	$5mH$	r	$8m\Omega$
r_n	$10m\Omega$	Δr	$8m\Omega$
C	$120\mu F$	C_n	$100\mu F$
ΔC	$50\mu F$	$R,\ R_n$	10Ω
V_{in}	$10V$	ΔR	5Ω

The Jacobian of system (42) results in

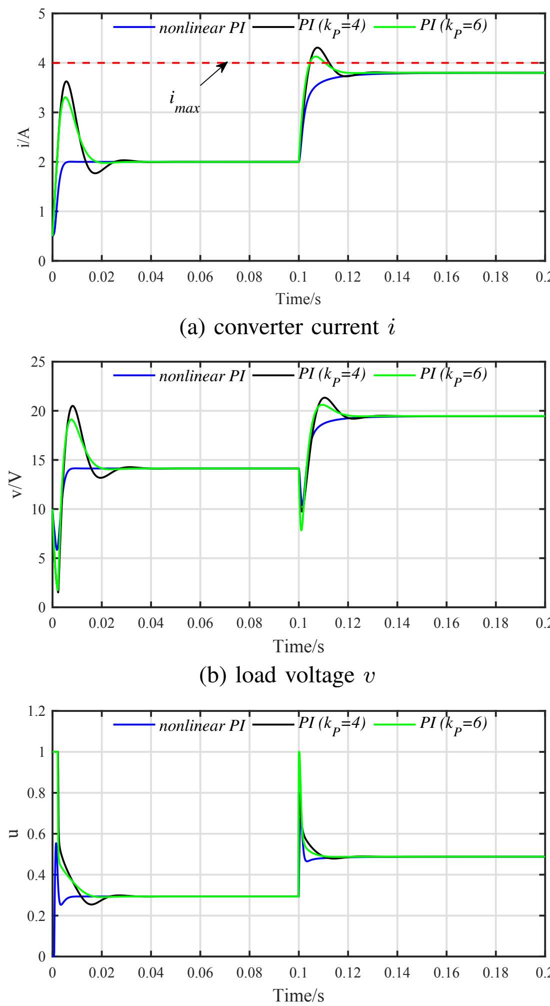
$$A_v = -\frac{V_{in} i_e}{C v_e^2} - \frac{1}{CR} = -\frac{1}{CR}\left(\frac{V_{in}}{V_{in} - r i^{ref}} + 1\right) < 0$$

and therefore the origin of (42) is asymptotically stable. Then according to Lemma 5.4 in [1], system (41) is locally input-to-state stable. As a result, the desired equilibrium point $\begin{bmatrix} i_e & \sigma_e & v_e \end{bmatrix}^T$ of system (34)-(35) with both the conventional PI and the proposed nonlinear PI controller will be asymptotically stable.

B. Simulation results

To demonstrate the proposed nonlinear PI controller performance in comparison to the conventional PI control, the dc/dc converter system of (31)-(32) was simulated using the parameters shown in Table I. For the conventional PI controller, the integral gain is selected as $k_I = 1200$, while two different values are tested for the proportional gain $k_P = 4$ and 6. For the proposed nonlinear PI controller gains, the design procedure mentioned in the previous subsection is followed, providing the selection $k_P = 20$ and $k_I = 1.33 \times 10^4$.

The desired scenario is for the converter to regulate initially the current i to a desired value $i^{ref} = 2A$, while at the time instant $t = 0.1s$ the reference current changes to $i^{ref} = 3.8A$. It is required that the converter current i remains limited below $i_{max} = 4A$ at all times, i.e. even during transients. As it is illustrated in Fig. 1(a), both the proposed and the conventional PI controllers (with both proportional gains) manage to regulate the converter current to any desired value. The converter voltage reaches the expected steady-state value $v_e = \sqrt{Ri^{ref}(V_{in} - ri^{ref})}$, as demonstrated in Fig. 1(b). However, when the conventional PI controller is applied, the desired state constraint $|i| \leq i_{max}$ is not guaranteed at all times, since during the transient response the current i violates the desired maximum value (Fig. 1(a)). On the other hand, as expected from the theoretical analysis, the proposed nonlinear PI controller leads the converter current to the desired regulation without violating the maximum bound. It should be highlighted that for a different choice of the proportional and integral gains, it is possible that the conventional PI controller can maintain the current below the maximum value for the given regulation scenario. However, there is no analytic method for calculating the gains and guarantee that $|i| \leq i_{max}, \forall t \geq 0$, for different values of



(a) converter current i

(b) load voltage v

(c) duty-ratio input u

Fig. 1. Simulation results of a dc/dc power converter using the conventional and the proposed nonlinear PI controller

the i^{ref} or different load cases, opposed to the proposed approach which guarantees the desired state constraint at all times. Furthermore, from the duty-ratio input performance shown in Fig. 1(c), it is clear that for a larger value of k_P for the conventional PI controller, the input value will exceed the value of 1, which represents the physical limit of the converter. As a result, the proposed nonlinear PI controller offers a superior performance during transients and state constraint satisfaction, while the analysis presented in this paper offers a rigorous methodology for the selection of the proportional and integral gains.

V. CONCLUSIONS

A novel nonlinear PI controller was proposed in this paper to guarantee accurate output regulation and state constraint satisfaction. The proposed controller can be applied to a wide class of nonlinear systems with constant uncertainties or disturbances. Asymptotic stability of the desired equilibrium point and a given upper bound for the desired system states were analytically proven. A design procedure for the controller gains was also presented. The superiority of the proposed nonlinear PI controller compared to the conventional approach was demonstrated in a practical example consisting of a dc/dc power electronic converter application.

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