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Joint 4-D DOA and Polarization Estimation Based on Linear Tripole Arrays

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***Abstract*—Electromagnetic (EM) vector sensor arrays can track both the polarisation and direction of arrival (DOA) parameters of the impinging signals. For crossed-dipole linear arrays, due to inherent limitation of the structure, it can only track one DOA parameter and two polarisation parameters. This problem could be solved by extending the geometry to a two-dimensional (2-D) rectangular array so that both the azimuth and elevation angles of the signal can be estimated. In this paper, instead of extending the array to a higher dimension, we replace the crossed-dipoles by tripoles and construct a linear tripole array. It will be shown that such a structure can estimate the 2-D DOA and 2-D polarisation information effectively and a dimension-reduction based MUSIC algorithm is developed so that the 4-D estimation problem can be simplified to two separate 2-D estimation problems, significantly reducing the computational complexity of the solution.**

Keywords— linear tripole array, DOA estimation, polarisation estimation, vector sensor.

I. Introduction

The joint estimation of direction of arrival (DOA) and polarization for signals based on electromagnetic (EM) vector sensor arrays has been widely studied in recent years [1]–[5]. In [1], the EM vector sensor was first used to collect both electric and magnetic information of the impinging signals, where all six electromagnetic components are measured to identify the signals. So far most of the studies are focused on the linear array structure employing crossed-dipoles, where the general two-dimensional (2-D) DOA model is simplified into a one-dimensional (1-D) one by assuming that all the signals arrive from the same known azimuth angle ϕ. In [6], a quaternion MUSIC algorithm was proposed to deal with the joint DOA (θ) and polarization (ρ, ϕ) estimation problem by considering the two complex-valued signals received by each crossed-dipole sensor as the four elements of a quaternion, where a three-dimensional (3-D) peak search is required with a very high computational complexity. In [7], a quaternion ESPRIT algorithm was developed for direction finding with a reduced complexity. Furthermore, a dimension-reduction MUSIC algorithm based on uniform linear arrays (ULAs) with crossed-dipole sensors was introduced in [8], where the 3-D joint peak search is replaced by a 1-D DOA search and a 2-D polarization search.

In practice, the azimuth angle θ and the elevation angle ϕ of the signals are unknown and they are usually different for different signals and need to be estimated together. However, the linear array with crossed-dipole sensors cannot achieve this and one solution is to extend the geometry to a two-dimensional (2-D) rectangular array, such as the uniform rectangular array (URA), at a considerable space and sensor cost. In [9], based on such a URA, a pencil-MUSIC algorithm was presented to solve the full 4-D DOA and polarization estimation problem, where with N sensors, the maximum number of resolvable signals is $2N-1$.

In this paper, motivated by simultaneously simplifying the array structure and reducing the computation complexity, instead of extending the linear array to a higher dimension, we replace the crossed-dipoles by tripoles and construct a linear tripole array [10], [11]. It will be shown that such a structure can estimate the 2-D DOA and 2-D polarisation information effectively and a dimension-reduction based MUSIC algorithm is developed so that the 4-D estimation problem can be simplified to two separate 2-D estimation problems, significantly reducing the computational complexity of the solution. The maximum number for resolvable signals is $3N-1$ for an N-sensor linear tripole array.

This paper is structured as follows. The tripole array model is introduced in Section II and the modified dimension-reduction algorithm is developed in Section III. Simulation results are presented in Section IV, and conclusions are drawn in Section V.

II. Model for array processing

Suppose there are M uncorrelated narrowband signals impinging upon a uniform linear array with N tripoles, where each tripole consists of three co-located mutually perpendicular dipoles, as shown in Fig. 1. Assume that all signals are ellipse-polarized. The parameters, including DOA and polarization of the m-th signal are denoted by $(\theta_m, \phi_m, \gamma_m, \eta_m), m = 1, 2, ..., M$. The inter-element spacing

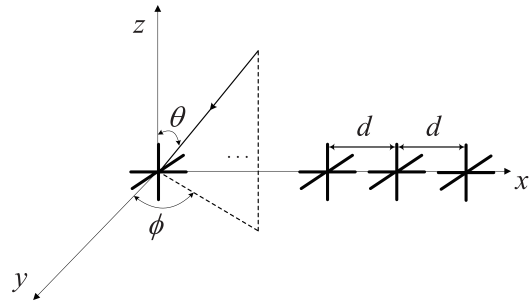

Fig. 1. Geometry of a uniform linear tripole array, where a signal arrives from elevation angle θ and azimuth angle ϕ.

d of the array is $\lambda/2$, where λ is the wavelength of the incoming signals. For each tripole, the three components are parallel to x, y and z axes, respectively. The background noise is Gaussian with zero mean and variance σ^2, which is uncorrelated with the impinging signals. The received signal at the l-th time instant at the tripole sensor array can be denoted as

$$\begin{aligned} \boldsymbol{x}[l] &= \sum_{m=1}^{M}[\boldsymbol{a}_m \otimes \boldsymbol{p}_m]\boldsymbol{s}_m[l] + \boldsymbol{n}[l] \\ &= \sum_{m=1}^{M}\boldsymbol{q}_m\boldsymbol{s}[l] + \boldsymbol{n}[l] \end{aligned} \tag{1}$$

where $\otimes$ stands for the Kronecker product, $\boldsymbol{q}_m$ is the Kronecker product of $\boldsymbol{a}_m$ and $\boldsymbol{p}_m$, and $n[l]$ is a $3N \times 1$ Gaussian white noise vector. The steering vector for the m-th signal is:

$$\boldsymbol{a}_m = [1, e^{-j\pi \sin(\theta_m)\sin(\phi_m)}, ..., e^{-j(N-1)\pi \sin(\theta_m)\sin(\phi_m)}] \tag{2}$$

and the polarization vector $\boldsymbol{p}_m$ is given by:

$$\begin{aligned} \boldsymbol{p}_m &= \begin{bmatrix} \cos\theta_m\cos\phi_m & -\sin\phi_m \\ \cos\theta_m\sin\phi_m & \cos\phi_m \\ -\sin\theta_m & 0 \end{bmatrix} \begin{bmatrix} \cos\gamma_m \\ \sin\gamma_m e^{j\eta_m} \end{bmatrix} \\ &= \boldsymbol{\Omega}_m\boldsymbol{g}_m \end{aligned} \tag{3}$$

where $\boldsymbol{\Omega}_m$ denotes the matrix containing θ_m and ϕ_m, and $\boldsymbol{g}_m$ denotes the vector containing γ_m and η_m.

The covariance matrix of the output vector $\boldsymbol{x}[l]$ is given by

$$\begin{aligned} \boldsymbol{R} &= E[\boldsymbol{x}[l]\boldsymbol{x}[l]^H] \\ &= \sum_{m=1}^{M}\boldsymbol{q}_m\boldsymbol{s}[l]\boldsymbol{s}[l]^H\boldsymbol{q}_m^H + \sigma^2\boldsymbol{I}_{3N} \\ &= \boldsymbol{R}_s + \boldsymbol{R}_n \end{aligned} \tag{4}$$

where $\boldsymbol{R}_s$ and $\boldsymbol{R}_n$ are the covariance matrices for signals and noise, respectively.

In practice, $\boldsymbol{R}$ is not available and can be estimated by averaging a finite number of snapshots, which is given by

$$\hat{\boldsymbol{R}} \approx \frac{1}{L}\sum_{l=1}^{L}\boldsymbol{x}[l]\boldsymbol{x}[l]^H = \hat{\boldsymbol{R}}_s + \hat{\boldsymbol{R}}_n \tag{5}$$

where L is the number of snapshots, and $\hat{\boldsymbol{R}}_s$ and $\hat{\boldsymbol{R}}_n$ are the corresponding estimated covariance matrices for signals and noise, respectively.

III. The Proposed Algorithm

In the following, the proposed low-complexity joint 4-D DOA and polarization estimation algorithm will be introduced based on a MUSIC-like algorithm.

A. Solution Based on Direct 4-D Search

Applying eigenvalue decomposition (EVD), the signal covariance matrix $\boldsymbol{R}_s$ can be decomposed into

$$\boldsymbol{R}_s = \sum_{k=1}^{3N}\alpha_k\boldsymbol{u}_k\boldsymbol{u}_k^H \tag{6}$$

where $\boldsymbol{u}_k$ is the k-th eigenvector and α_k is the corresponding eigenvalues (in descending order). When the number of source signals M is less than $3N$, $\boldsymbol{R}_s \in \mathbb{H}^{3N\times 3N}$ will not be a full rank matrix with $rank(\boldsymbol{R}_s) = M$, and it only has M non-zero eigenvalues, which means $\alpha_1 \geq \alpha_2 \geq \cdots \alpha_M \geq \alpha_{M+1} = \cdots = \alpha_{3N} = 0$.

Since $\boldsymbol{R}_n = \sigma^2\boldsymbol{I}_{3N}$, where $\boldsymbol{I}_{3N}$ is the $3N \times 3N$ identity matrix, we can write $\boldsymbol{R}_n$ in the following form

$$\boldsymbol{R}_n = \sum_{k=1}^{3N}\sigma^2\boldsymbol{u}_k\boldsymbol{u}_k^H \tag{7}$$

Then

$$\boldsymbol{R} = \boldsymbol{R}_s + \boldsymbol{R}_n = \sum_{k=1}^{3N}\lambda_k\boldsymbol{u}_k\boldsymbol{u}_k^H \tag{8}$$

with $\lambda_k = \alpha_k + \sigma^2$. Furthermore, we can rewrite (8) into

$$\boldsymbol{R} = \boldsymbol{U}_s\boldsymbol{\Lambda}_{\mathbf{s}}\boldsymbol{U}_s^H + \boldsymbol{U}_n\boldsymbol{\Lambda}_n\boldsymbol{U}_n^H \tag{9}$$

where $\boldsymbol{U}_s = [\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_M]$ and $\boldsymbol{U}_n = [\boldsymbol{u}_{M+1}, \boldsymbol{u}_{M+2}, \cdots, \boldsymbol{U}_{3N}]$ are the eigenvectors of the signal subspace and noise subspace, respectively, and $\boldsymbol{\Lambda}_{\mathbf{s}}$ and $\boldsymbol{\Lambda}_n$ are diagonal matrices holding the corresponding eigenvalues λ_k. As the rank of the noise subspace cannot be less than 1, the DOF (degree of freedom) of the algorithm is $3N-1$.

Clearly, the composite steering vector $\boldsymbol{q}_m$ is orthogonal to the noise subspace $\boldsymbol{U}_n$, i.e.

$$\boldsymbol{U}_n^H\boldsymbol{q}_m = \boldsymbol{0} \tag{10}$$

Multiplied by its Hermitian form on the right, it becomes

$$\boldsymbol{q}_m^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{q}_m = 0 \tag{11}$$

The parameters $(\theta_m, \phi_m, \gamma_m, \eta_m)$ represent the DOA and polarization information of the m-th signal. To find the parameters via direct 4-D search, the inverse of (11) is used as the joint estimator:

$$F(\theta_m, \phi_m, \gamma_m, \eta_m) = \frac{1}{\boldsymbol{q}_m^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{q}_m} \tag{12}$$

The peaks in (12) indicate the DOA and polarization information $(\theta, \phi, \gamma, \eta)$ for impinging signals. By searching $\theta, \phi, \gamma, \eta$ within a given domain, a 4-D hypersurface can be computed. Theoretically, we can find M peaks (local maxima) in the hypersurface corresponding to the M signals.

B. Dimension Reduction

The above algorithm is based on a 4-D peak search with an extremely large computational complexity. In this part, we employ a dimension reduction method to split the 4-D search process into two 2-D searches, significantly reducing the complexity of the solution.

Based on the joint estimator and the orthogonality between $\boldsymbol{U}_n$ and $\boldsymbol{q}_m$, (10) and (12) provide a way to locate the joint parameters, where $\boldsymbol{q}_m$ includes the DOA and polarization information $(\theta, \phi, \gamma, \eta)$. In order to avoid the 4-D peak search, we separate $\boldsymbol{q}_m$ into two components: one with DOA information (θ, ϕ) only, while the other only contains the polarization information (γ, η). In this way, (10) can be changed to

$$\begin{aligned} \mathbf{0} &= \boldsymbol{U}_n^H[\boldsymbol{a}_m \otimes (\boldsymbol{\Omega}_m \boldsymbol{g}_m)] \\ &= \boldsymbol{U}_n^H[(\boldsymbol{a}_m \otimes \boldsymbol{\Omega}_m)\boldsymbol{g}_m] \\ &= [\boldsymbol{U}_n^H \boldsymbol{b}_m]\boldsymbol{g}_m \end{aligned} \tag{13}$$

where $\boldsymbol{b}_m$ is the Kronecker product of $\boldsymbol{a}_m$ and $\boldsymbol{\Omega}_m$.

Notice that $\boldsymbol{U}_n^H \boldsymbol{b}_m$ is an $(3N - M) \times 2$ vector and $\boldsymbol{g}_m$ is a 2×1 vector, (13) shows a linear relationship between the columns in $\boldsymbol{U}_n^H \boldsymbol{b}_m$, which means the vector has a column rank less than 2. Multiplied by its Hermitian form on the right, the new 2×2 product matrix cannot have a full rank, with its determinant equal to zero. Here we use $\mathbf{det}\{\}$ to denote the determinant of a matrix. Then, we have

$$\mathbf{det}\{\boldsymbol{b}_m^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{b}_m\} = 0 \tag{14}$$

We can see that $\boldsymbol{b}_m$ is dependent on the parameters (θ, ϕ) only. As a result, a new estimator can be established corresponding to θ and ϕ as [12]

$$f(\theta_m, \phi_m) = \frac{1}{\mathbf{det}\{\boldsymbol{b}_m^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{b}_m\}} \tag{15}$$

The new estimator first performs a 2-D peak search over θ and ϕ. After locating DOA parameters θ and ϕ, polarization parameters γ and η can be achieved by another 2-D search in [12]

$$f(\gamma_m, \eta_m) = \frac{1}{\boldsymbol{g}_m^H \boldsymbol{b}_m^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{b}_m \boldsymbol{g}_m} \tag{16}$$

The following is a summary of the proposed algorithm:

- Calculate the estimated covariance matrix $\hat{\boldsymbol{R}}$ from the received signals.
- Calculate the noise space $\boldsymbol{U}_n$ by applying the eigenvalue decomposition on $\hat{\boldsymbol{R}}$. The last $3N - M$ eigenvalues and the corresponding eigenvectors form the noise space.
- Use the 2-D estimator (15) to locate the DOA parameters θ and ϕ.
- Use the 2-D estimator (16) to locate polarization parameters γ and η.

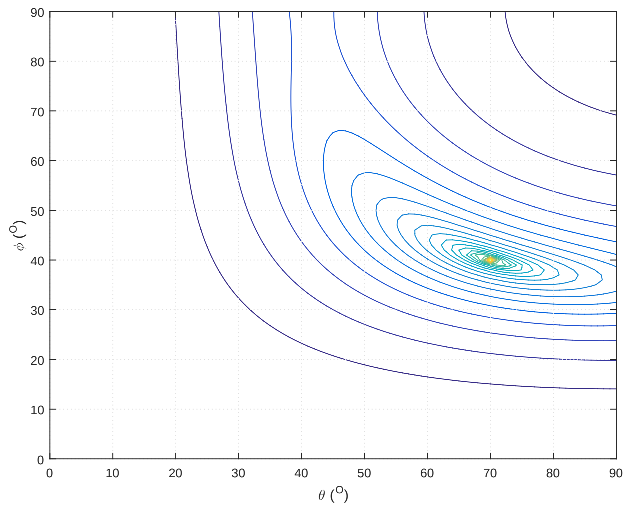

Fig. 2. DOA estimation result for a single impinging signal.

IV. Simulation Results

In this section, simulation results are provided based on a uniform linear array with N tripole sensors and an adjacent sensor spacing $d = \lambda/2$. The root mean square error (RMSE) is used as an indicator of the estimation performance, and it is calculated based on K Monte Carlo trials as follows,

$$\begin{aligned} RMSE &= \sqrt{\frac{1}{MK}\sum_{m=1}^{M}\sum_{k=1}^{K} T_{m,k}} \\ T_{m,k} &= (\theta_{m,k} - \theta_m)^2 + (\phi_{m,k} - \phi_m)^2 \\ &\quad + (\gamma_{m,k} - \gamma_m)^2 + (\eta_{m,k} - \eta_m)^2 \end{aligned} \tag{17}$$

where $\theta_{m,k}, \phi_{m,k}, \gamma_{m,k}$ and $\eta_{m,k}$ are the estimated DOA and polarisation values in the k-th trial for the m-th signal, and $\theta_m, \phi_m, \gamma_m$ and η_m are the real values of the parameters for the m-th signal.

A. Single Impinging Signal

In the first simulation, one single polarized source from $(\theta, \phi, \gamma, \eta)$=$(70°, 40°, 20°, 50°)$ impinges on a ULA with $N = 3$ tripole sensors; SNR $=$ 20dB and the snapshots number $L = 1000$. The stepsizes for searching through the four parameters $\theta, \phi, \gamma, \eta$ are all set to $1°$. Figs. 2 and 3 show the joint estimation result for all the parameters, and clearly the direction and polarisation of the source signal has been satisfactorily identified.

B. Multiple Impinging Signals

In the second simulation, we consider three polarized sources coming from $(\theta, \phi, \gamma, \eta)$=$(80°, 40°, 20°, 50°)$, $(50°, 80°, 70°, 20°)$ and $(20°, 60°, 40°, 45°)$, respectively. All other settings are the same as in the first simulation.

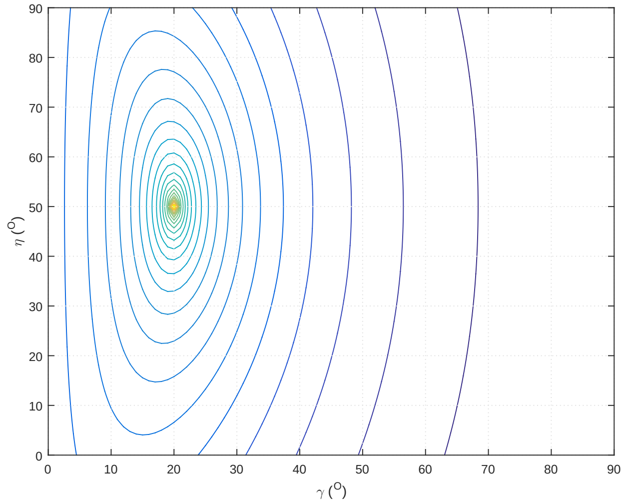

Fig. 3. Polarization estimation result for a single impinging signal.

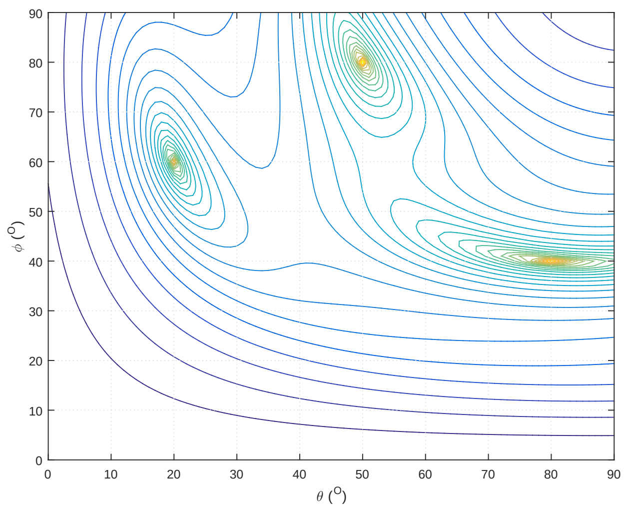

Fig. 4. DOA estimation result for multiple impinging signals.

The DOA estimation result is shown in Fig. 4, where we can see that although the number of sources are the same as number of sensors, the proposed algorithm can still effectively locate each of the signals. The reason is that the tripole sensors have expanded the covariance matrix $\boldsymbol{R}$ by three times in size. As a result, more signals can be processed by the tripole array than the traditional system.

After obtaining the DOA parameters, the polarization parameters can then be extracted successfully by another 2-D peak search and the results are presented in Figs. 5-7.

C. RMSE Results

Now we provide the RMSE result of the estimation versus a varied SNR based on $L = 10000$ snapshots and $K = 100$ Monte Carlo trials. Three cases are studied in the simulation with M = 1, 3 and 5 impinging signals, respectively. In the first case, one signal comes

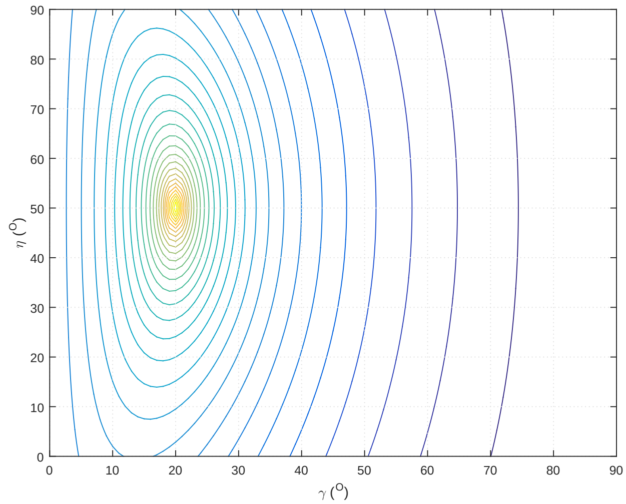

Fig. 5. Polarization estimation result with $\theta = 80^\circ$ and $\phi = 40^\circ$.

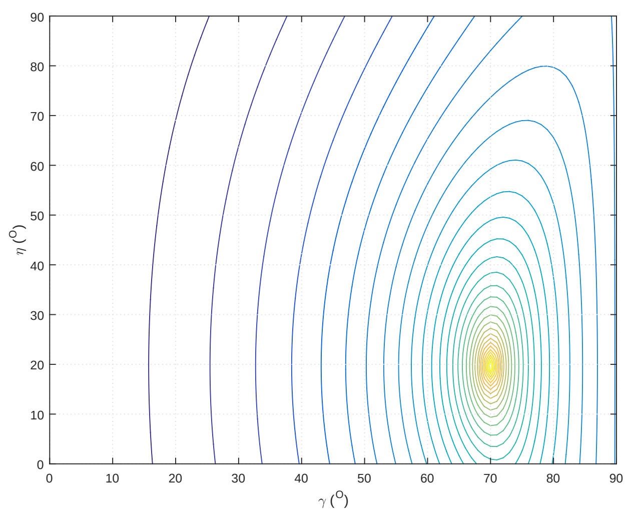

Fig. 6. Polarization estimation result with $\theta = 50^\circ$ and $\phi = 80^\circ$.

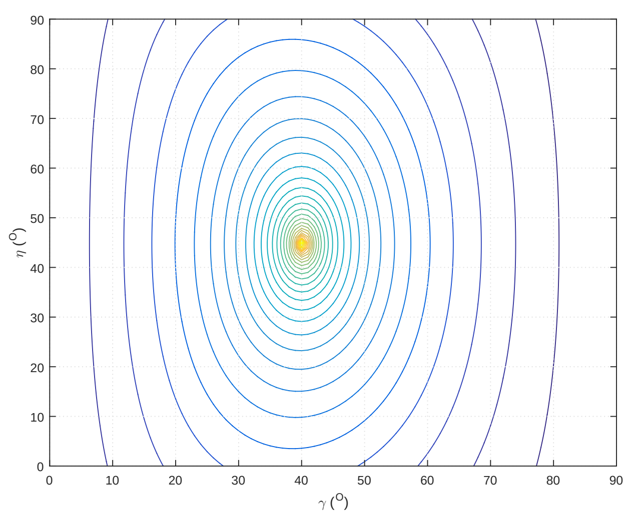

Fig. 7. Polarization estimation result with $\theta = 20^\circ$ and $\phi = 60^\circ$.

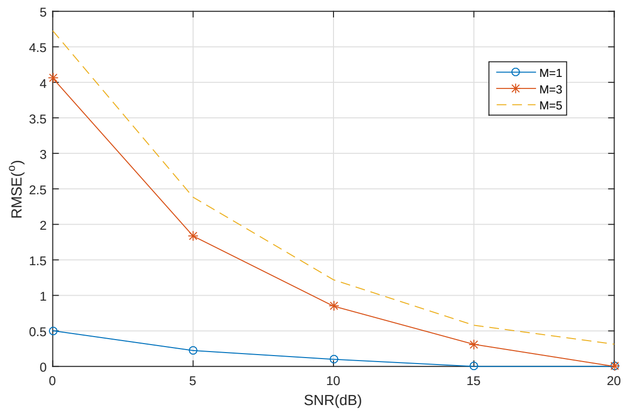

Fig. 8. RMSE versus SNR for different number of impinging signals.

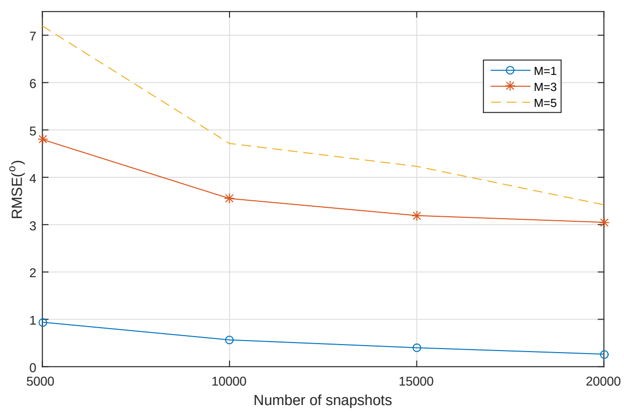

Fig. 9. RMSE versus snapshot number for different number of impinging signals.

from $(\theta,\phi,\gamma,\eta)$=$(70^\circ,40^\circ,20^\circ,50^\circ)$; in the second case, three signals come from $(\theta,\phi,\gamma,\eta)$=$(80^\circ,40^\circ,20^\circ,50^\circ)$, $(50^\circ,80^\circ,70^\circ,20^\circ)$ and $(20^\circ,60^\circ,40^\circ,45^\circ)$; in the third case, five signals come from $(\theta,\phi,\gamma,\eta)$=$(10^\circ,10^\circ,20^\circ,50^\circ)$, $(30^\circ,30^\circ,70^\circ,20^\circ)$, $(45^\circ,45^\circ,40^\circ,45^\circ)$, $(60^\circ,60^\circ,10^\circ,20^\circ)$ and $(80^\circ,80^\circ,60^\circ,70^\circ)$. Fig. 8 gives the RMSE curves from SNR = 0 dB to 20 dB. With the increase of SNR, the RMSE for each curve decreases showing a better estimation performance. Moreover, with the same SNR, the single input case outperforms the multiple input case. The algorithm suffers a performance loss with more and more input signals.

Next, we investigate the RMSE results versus a varied snapshots number. With a fixed SNR=0 dB, we keep the DOA and polarization parameters of all signals unchanged in each case. The results are presented in Fig. 9. Compared with the other curves, the single input case holds the lowest RMSE. In general, with more input signals, RMSE increases. Besides, the RMSE slowly drops for each curve with the increase of snapshot numbers. The drop rate trends to be sharper when there are more input signals.

V. Conclusion

Due to inherent limitation of the crossed-dipole linear arrays, it can only estimate one DOA parameter and two polarisation parameters. In order to simultaneously estimate both the 2-D DOA and 2-D polarisation parameters of the impinging signals, we could increase the dimension of the array and construct a planar crossed-dipole array. To avoid this, a linear tripole array has been introduced. It has been shown that such a structure can estimate the 2-D DOA and 2-D polarisation information effectively. Moreover, a dimension-reduction based MUSIC algorithm has been developed so that the 4-D estimation problem can be simplified to two separate 2-D estimation problems, significantly reducing the computational complexity of the solution.

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