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Peñafiel Mera, A and Balijepalli, N orcid.org/0000-0002-8159-1513 (2020) Towards improving resilience of cities: an optimisation approach to minimising vulnerability to disruption due to natural disasters under budgetary constraints. Transportation, 47 (4). pp. 1809-1842. ISSN 0049-4488

https://doi.org/10.1007/s11116-019-09984-8

Towards improving resilience of cities: an optimisation approach to minimising vulnerability to disruption due to natural disasters under budgetary constraints

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Abstract

In recent years, climate change emerged as a dominant concern to many parts of the world bringing in huge economic losses disturbing normal business/life. In particular cities are suffering from floods affecting land based transportation systems in a significant manner more frequently than ever. Many local authorities facing funding cuts are suffering from limited budgets and they are put under even higher pressure when looking for resources to recover the damaged networks. The agencies involved with post-disaster reconstruction too struggle to prioritise the network links to recover. This paper addresses the problem of road maintenance/development with the aim of improving resilience of the network by formulating the problem as a mathematical model that minimises the vulnerability to disruption due to natural incidents under budgetary constraints. This paper extends the critical link analysis from a single link being disrupted to multiple links, and for the first time proposes an objective function involving a measure of vulnerability to minimise. Metaheuristic Simulated Annealing method is used to reach near global optimal solution for a real-life network with large demand. A segment of the City of York in England has been used to illustrate the principles involved. Numerical experiments indicate that Simulated Annealing based optimisation method outperforms the 'volume-priority' heuristic approach, returning higher value for money spent. The proposed approach spreads the benefits across wider population by including more number of links in the priority list while reducing the vulnerability to disruption.

Key words: resilience; vulnerability; bi-level optimisation; Simulated Annealing; traffic assignment.

1. Introduction

Climate change has evoked a major concern for the society in terms of the capacity of cities to resist and handle natural disasters (Godschalk, 2003). These events cause colossal losses to the man-made and natural environment. It impacts national economy and affects the social fabric itself. For instance, a study published in 2015 stated that between 2003 and 2013 natural disasters caused more than $1.5 trillion damage and affected around 2 billion people (FAO, 2015). The same study shows that hydrological events had the largest percentage of occurrences during the research period. Natural disasters are, in many cases, unpredictable. However, sometimes their consequences can be mitigated, especially when the danger is known, e.g. communities settled over floodplains, seismic fault zones or hurricane prone shorelines. In these examples, appropriate hazard mitigation and prevention programmes together with well-informed policies would help to reduce the impact of catastrophes.

Despite the term 'resilient city' being an open concept, its essence describes cities with a high capacity to absorb catastrophic situations. Tulsa and Berkeley are examples of what can be called a ‘resilient city’ (Godschalk, 2003). Tulsa, after suffering a sequence of earthquakes, developed effective tax policies and funding strategies to prevent and surpass the impact of possible catastrophes. Berkeley, on the other hand, after facing consecutive tornadoes and severe thunderstorms raised a public debate that led to effective drainage plans and the relocation of about 875 buildings settled in floodplains. The concept of resilience in ecology emerges from two paradigms: the first one refers to the capacity of the ecosystem to return to its initial status after a disturbance (equilibrium) and the second paradigm approaches resilience from the side of adaptation to the new conditions, rather than returning to an initial situation (non-equilibrium) (Pickett, et al., 2004). Nonetheless, regarding urban planning, the non-equilibrium paradigm seems to be more appropriate since it is dynamic and evolutionary (Pickett, et al., 2004).

Unlike air and water-borne transportation systems, land transportation is heavily reliant on extensive land-based infrastructure such as roadways and railways. Infrastructure requires to be properly planned and maintained to deliver high-quality services. This service function can be affected by undesired events such as flooding, landslides or earthquakes deteriorating the roadway capacity and eventually causing a complete loss of connectivity. This then raises the question – while it is difficult to predict the occurrence of an event, could the city authorities have prevented the resulting disruption by preparing a maintenance plan taking account of vulnerable road links?

The problem introduced so far reflects the context of resilience to climate change, but the underlying problem also relates to budgetary allocation. Local authorities in the face of funding cuts struggle with limited budgets allocated to transport sector but the climate change impact is ever increasing thus making the job of prioritising the efforts even more challenging. More generally local authorities allocate their resources to projects on a heuristic basis. For example, Metropolitan Washington Council of Governments’ (MWCOG) web-page CLRP (2016) on project selection process says ‘*Project development can be an unpredictable process. Projects sometimes get put on a fast track when elected officials or a group of citizens take a special interest in them. Some projects move forward when they are selected as preferred alternatives in studies…'* indicating the heuristic nature of the process involved. There is also a possibility that the authorities concerned prioritise the resource allocation to the roads in the order of their usage i.e. typically allocating money to links carrying higher volumes of traffic. While this method sounds logical, it will be worthwhile investigating the underlying efficiencies and the implications in the context of vulnerability to disruption.

Bringing together the objective of improving resilience of a road network and the tentative nature of budgetary allocation involved for its maintenance/development, it is worth asking the question - can we develop a plan to ensure an optimal combination of road links and specify the extent to which they need to be developed to deliver the best possible resilience outcome? Indeed, this is the question frequently faced by agencies involved in post-disaster reconstruction planning (FEMA 2009 - Planning for disaster recovery and reconstruction - see page 60) and needs an answer in enabling them to prepare plans to survive future natural disasters in the medium to long term horizon period. Typically, post-disaster operations may also involve restoring the network for rescuing in the immediate horizon but this paper is rather focused on the medium to long term horizon which requires identifying/prioritising critical network links and preparing a plan for their maintenance/development to survive the future disasters with minimum loss to life/ property.

Thus, this research addresses the problem of road maintenance and development from the perspective of vulnerability to disruption. When undesired or disruptive events occur, their impact may reach out different sections of the network at different levels. Initially, several researchers e.g. Balijepalli & Oppong (2014), El-Rashidy & Grant-Muller (2014), Jenelius et al (2006), Scott et al (2006) among others developed methods to identify critical network links prone to incidents on the basis of a single link being disrupted at time. More recently efforts for identifying multiple critical links are on e.g. Bagloee et al (2017), Wang et al (2016).

This paper extends the analysis of critical links from a single link being disrupted to many links getting affected simultaneously and develops a mathematical model to identify a combination of the road links to improve the resilience. Relative to the literature, this paper for the first time attempts to directly minimise a measure of vulnerability rather than minimising the total travel time. The paper specifies the vulnerability minimisation problem as a Bi-Level Programming Problem (BLPP) and solves a real-life network problem with a large demand using a metaheuristic optimisation approach to reach near global optimal solution. The measure of vulnerability is a derivative of the Network Vulnerability Index (NVI), a method used to identify link's criticality (Balijepalli & Oppong 2014). NVI differs from other vulnerability measures in the literature by including serviceability variable which accounts for relative loss of capacity. The measure redefined as Worse-Case-Vulnerability (WCV) would help policymakers to assess a range of scenarios of disruption to make more informed decisions. It will also help the agencies involved in post-disaster reconstruction to prioritise the recovery of network links to benefit wider communities.

Subsequent to defining the term WCV, a mathematical model will be proposed to identify the set of links to be enhanced in order to minimise the value of WCV for improving the resilience of a city. The upper level of the BLPP in the paper aims to minimise the WCV for link capacity changes discretised into a finite number of discrete levels as in a discrete Network Design Problem (NDP) while the lower level accomplishes the Wardrop's User Equilibrium (also known as driver´s selfish behaviour, Wardrop 1952) as in a traffic assignment problem. Later in the paper, metaheuristic Simulated Annealing for optimisation combined with the Method of Successive Averages for traffic assignment will be adapted to evaluate the performance of the WCV-Problem. Then in order to evaluate the computational cost of this solution, the model will be assessed over a real-life network with different parameters.

This paper is divided into five sections including this one. The second section introduces the context of vulnerability and builds a mathematical background to the problem by reviewing relevant literature. The third section specifies the methodology in detail and sets out a metaheuristic procedure to solve the WCV problem. The fourth section illustrates the principles involved with the help of a real-life network with suitable numerical examples. Section five concludes the research.

2. Review of literature

This section will review basic concepts of network vulnerability, robustness and criticality; which will be employed to develop the WCV measure. As a provision for the model, in this section we briefly introduce the bi-level optimisation method too.

2.1. Vulnerability and criticality of road networks

In the studies of transport networks, the concepts of vulnerability, robustness, serviceability, and criticality are interrelated. The term vulnerability has various definitions (See Table 2.1), however in this document the definition suggested by Berdica (2002) will be used, who also defined the notions of robustness and serviceability. Then, it can be stated that vulnerability is the counterpart of robustness, i.e. a network highly vulnerable has a low level of robustness and vice versa (Balijepalli & Oppong, 2014).

Serviceability is the possibility of using a link during a given period and is defined as the fraction of available capacity to the standard capacity, which indicates not only whether a link can be used, but also the extent to which it can be used (Balijepalli & Oppong, 2014). For example, during flooding, some streets may become totally impassable (zero serviceability) while other roads allow partial usage (some serviceability, but not the maximum). The connection between the concepts discussed above is the following: during disruptive events, a less vulnerable (more robust) network will provide links with greater serviceability and vice versa. Notice that vulnerability and robustness refer to network´s performance and serviceability to link´s performance.

Table 2.1 Definitions of vulnerability, robustness, serviceability and criticality

Characteristic	Definition	Reference
Vulnerability	"Susceptibility for rare, big risks".	Laurentius (1994)
	"Sensitivity to threats and hazards".	Holmgren (2004)
	"The susceptibility to incidents that can result in considerable reductions in road network serviceability".	Berdica (2002)
Robustness	"Ability to cope with disturbing incidents".	Berdica (2002)
Serviceability	Possibility to use a link during a given period.	Berdica (2002)
Criticality	An affected link with great consequences.	Balijepalli & Oppong (2014)

A critical link is termed as a street or road whose damage affects the performance of the network greatly (Balijepalli & Oppong, 2014). Hence, the *criticality* of a link is defined by the level of impact provoked to the network when the link reduces its serviceability. For example, Balijepalli & Oppong assessed criticality of ten flood prone links of York´s urban road network by means of five indicators (See **Table** *2.2*). They calculated each indicator with all links being available, then, computed each of the indicators eliminating the disruption-prone links *one* at a time. As the result, ranking of criticality was created for every indicator, depending on the proportional variation provoked by removing one of the ten flood prone links.

Table 2.2 Five indicators used to estimate criticality of links

Indicator	Reference
Change in generalised cost measure	Taylor et al., (2006)
Network efficiency measure	Nagurney and Qiang (2007)
The importance measure	Jenelius et al., (2006)
Network Robustness Index	Scott et al., (2006)
Network Vulnerability Index	Balijepalli & Oppong (2014)

2.2. Network Vulnerability Index (NVI)

In the above mentioned study of link´s criticality, the Network Vulnerability Index (NVI), was introduced as a means to evaluate the impact of entire or partial degradation of the streets (links). The index is calculated as follows:

$$\mathrm{NVI} = \sum_{a \in A} \left(\frac{x_a^o}{r_a^o}\right) t_a^o - \sum_{a \in A} \left(\frac{x_a}{r_a}\right) t_a \qquad (2.1)$$

where x_a^o, t_a^o and r_a^o are traffic flow, travel time and serviceability of the link a, when the network is working with its full capacity being available. Furthermore, x_a, t_a and r_a represent traffic flows, travel time and serviceability of the link a when the available capacity of the link a has been reduced. A is the set that contains all the links in a network. Serviceability of link r_a needs explaining further to make the key notion of vulnerability even more clearer. Serviceability of a link a is defined as the fraction of total available capacity to standard capacity. Total available capacity of a link is prone to be affected by an incident leading to a drop in capacity. The standard capacity of a link is the maximum flow rate per hour and is dependent on the hierarchy of roads e.g. rural roads, motorways, urban roads etc as defined in DfT's Transport Analysis Guidance (DfT 2014). Thus the value of *r,* i.e. the serviceability of a link reduces when an incident adversely affects capacity of the link. The above definition of serviceability means that whilst multi-lane higher order road links are key to maintaining the network functionality, single-lane lower order road links are equally critical in contributing to the network connectivity which could play a significant role in reducing the disruption due to incidents. By definition, the second element in equation (2.1) is expected to be larger than the first; consequently, NVI takes negative values. If the reduction of available capacity over a link is evaluated and the resulting NVI tends to zero, it should be interpreted as the link hardly being critical. On the contrary, links' disturbances that make NVI move towards negative infinite are the most critical roads (Table 2.3). Thus, it can be noted that regardless of the heirarchy, length, width, flow or V/C ratio of a link, its degradation will impact the NVI to some extent.

Table 2.3 Interpretation of NVI to evaluate criticality of links

Link(s) **more** critical.	$-\infty \leftarrow$	NVI	$\rightarrow 0$	Link(s) **less** critical.

Hitherto it was commented about identifying isolated link failures, nonetheless, the case of simultaneous interruptions has also been studied in the recent past (see for example Bagloee et al 2017, Bell et al 2017 and Wang et al 2016). While Bell et al (2017) proposed a demand-independent capacity weighted spectral partitioning approach, both Bagloee et al (2017) and

Wang et al (2016) proposed a method involving optimising total travel time. It was revealed that when multiple links reduce their serviceability at the same time, the power value of the travel time function and travel demand play a significant role in defining criticality of links (Wang, et al., 2016). Two remarkable observations made by Wang et al. (2016) are:

- in the case of simultaneous link disruptions, the set of critical links does not equal to the combination of the most critical links identified during isolated disruptions; and
- the set of critical links not necessarily belong to the same area and thus the critical links may not be even connected.

For the simultaneous link failure case, Wang, et al. (2016) applied a piece-wise linearization of the NDP. Bagloee et al (2017) started off from identifying candidate critical links with highest traffic flow and then solved a series of discrete NDP's by using a system optimal relaxation method involving Bender's Decomposition. In this paper, we formulate a bi-level optimisation problem involving minimising the measure of vulnerability, a variant of NVI described earlier which accounts for the relative loss of capacity of roads with due consideration for all hierarchies of roads within the network.

2.3. Bi-level optimisation

Bi-level optimisation belongs to the class of hierarchical mathematical problems and is applied when there exist two goals, objectives conflicting between each other. Then, one of the goals is expressed as a constraint of the main problem. The mathematical formulation which aims to optimise this constraint is termed "lower-level" problem. Accordingly, the mathematical formulation to optimise the remaining goal is called "master" or "upper-level" problem.

The toll-setting problem is an example of BLPPs, a general formulation of it is presented above, see equations (2.2) to (2.5).

$$\min_{x \in X, y} \ F(x, y) \tag{2.2}$$

$$\text{S.t.} \quad G(x, y) \leq 0 \tag{2.3}$$

$$\min_{y} \ f(x, y) \tag{2.4}$$

$$\text{S.t.} \quad g(x, y) \leq 0 \tag{2.5}$$

Variables are classified in two groups upper-level ($x \in \mathbb{R}^n$) and lower-level ($y \in \mathbb{R}^m$). Then, $F(x, y)$ and $f(x, y)$ are the upper and lower objective functions, both with domain $\mathbb{R}^n \times \mathbb{R}^m$ and co-domain $\mathbb{R}$. While the upper and lower constraint vectors are defined by $G(x, y): \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^{l1}$ and $g(x, y): \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^{l2}$, respectively. It is important to mention that upper-level constraints include variables from both levels.

Bi-level problems and more generally multi-level problems have a close relation to the economic problem of Stackelberg, which involves agents at distinct levels: leaders and followers (Colson, et al., 2007). First applications of this category of optimisation problems were utilised for military, industrial and marketing purposes; see (Bracken & McGill, 1973;

1974; 1978). Other examples of bi-level problems are revenue management, congestion management, origin-destination matrix estimation, management of hazardous materials, NDP, Principal-agent Problem and the Stackelberg-Nash game (Colson, et al., 2007). In this paper, we formulate the WCV problem as a bi-level optimisation and describe a solution procedure using Simulated Annealing method as described in the next section.

3. Methodology

In this section, terms such as *Vulnerability Value* and Worse-Case-Vulnerability (WCV) are introduced. These ideas are derived from the Network Vulnerability Index described earlier. Later in the section, a mathematical problem to minimise the WCV is presented; it is formulated as a BLPP. Then, the Simulated Annealing is proposed as a solution approach for the upper-level problem and for the lower-level problem the Method of Successive Averages is used. The section ends with a description of the test network which is a segment of York´s urban road network.

3.1. Network context and link status

3.1.1. Definition of network context ϖ

The symbol ϖ describes the "context" of the network, this concept is used to identify the situation of every link in the network. Let´s consider an "idealistic context" defined as $\varpi = \mathrm{I}$, which supposes that every link works at its standard capacity. Then, another scenario can be the "realistic context", say, defined by $\varpi = \mathrm{R}$. In the realistic context some of the streets face partial capacity reduction, say, due to lack of periodic maintenance or minor surface damages. Other contexts could be the "pessimistic" and the "enhanced", the first one assumes that some of the streets are working with largely reduced capacities; say, due to flooding, giant holes or landslides. The final context would be "enhanced", this option considers the situation what-if some street capacities were increased. Summarizing, different contexts of the network mean different levels of available capacity for their links. Table 3.1 presents an overview of the aforementioned contexts.

Table 3.1 Network context and it's interpretation

Context	Label (ϖ)	Description
Idealistic	I	All links work at their standard capacity.
Realistic	R	Some links face partial reduction of available capacity due to lack of periodic maintenance or minor damages. However, no street suffers from total disconnection.
Enhanced	E	This context considers that some streets can increase their current standard capacity and that those links work at that higher capacity.
Pessimistic	P	This context considers that some of the links are suffering from large reduction of available capacity, e.g. flooding, landslide, giant holes, etc.

3.1.2. Link status definition and generation

The context of the network is denoted by ϖ and it is defined by the status of the links that build the network. Then, let s_a be the status of link $a \in A$, where A is the set of links. For this research, there have been defined four statuses to stand for whether a link is disrupted, degraded, intact and upgraded. The available capacity is what defines a status, viz., *disrupted links* have available capacity between 0% and 50% of the standard capacity, *degraded links* between 50% and 100% and *upgraded links* up to an arbitrary value of 120% (See Table 3.2).

Standard capacities, in the UK, are given by the DfT (1999) in their Design Manual of Road and Bridges and available capacities must be obtained by observing real-world traffic variations.

The "upgraded" status (s_a=4) is introduced to allow the model to assess the appropriateness of enhancing standard capacities of some links, where it is possible, without affecting the objective of improving the resilience of the network. Therefore, it does not make any sense for a link to take status 4 as input for the model. Because, it would signify the link as upgraded previously and its enhanced capacity is its new standard capacity.

Table 3.2 Status and available capacity

s_a	Label	Available capacity	Comments
1	Disrupted	0<AC<0.5SC	Available capacity is less than 50% of the standard capacity.
2	Degraded	0.5*SC <= AC < SC	Available capacity is larger than or equal to 50% of the standard capacity.
3	Intact	=SC	Available capacity depends on road type.
4	Upgraded	SC < SC^* <=1.2* SC	Available capacity can be increased up to 20% of the standard capacity.

SC=Standard Capacity; AC=Available Capacity; SC^*=Increased Standard Capacity.

Note: Zero available capacity is excluded as it can be modelled by removing the link from the network

3.1.3. Lower and upper link status

Assuming the capacities and thus statuses of all links within each of the contexts defined in Table 3.1 are known, then, $\mathcal{L}_a$ and $\mathcal{U}_a$ are the lower and upper status with which link a could be associated with. For e.g., if link a is known to face a large reduction of capacity in the "pessimistic context", then its lower status would be equal to 1. But, if the link a does not suffer from capacity reduction in any of the contexts, then, its lower status would be 3. Thus, the set that defines all possible statuses for any link $a \in A$, is given by $\{x|x \geq \mathcal{L}_a; x \leq \mathcal{U}_a; x \in \mathbb{N}^+\}$. Complementarily, the "enhanced context" assumes that some links can increase their standard capacity, then, the upper status of these links would be 4. The upper status for the links that cannot increase their standard capacity would be 3.

Table 3.3 shows possible combinations of lower and upper status that links can have. Note, that combination 2-3 might be the most common; which is the case of links whose lower status is associated with a minor reduction in capacity and its upper status is related to the standard capacity. Also, combinations such as 1-1, 1-2, 2-2 have not been included as it is assumed that any improvement work undertaken will bring back the status of a link to at least intact, without losing the generality of the method.

Table 3.3 Combination of upper and lower status

$\mathcal{L}_a$	$\mathcal{U}_a$	Prone to larger reduction of capacity (AC<50%)	Prone to minor reduction of available capacity (50%<=AC<100%).	Able to work at standard capacity (=100%).	Have favourable conditions to increase its standard capacity (SC^*>SC)
1	4	Yes	Yes	Yes	Yes
1	3	Yes	Yes	Yes	No
2	4	No	Yes	Yes	Yes
2	3	No	Yes	Yes	No
3	4	No	No	Yes	Yes

Note: Other combinations were supressed due to lack of realism, for example $\mathcal{L}_1$=1 and $\mathcal{U}_a$=2, which denotes a disruption-prone link that never works at its standard capacity.

SC=Standard Capacity; AC=Available Capacity; SC^*=Increased Standard Capacity.

Now that the notions of network context and link status have been introduced, their correlation can be defined. Table 3.4 maps the link status over the four contexts presented in Table 3.1.

Table 3.4 Network context defined by the status of links

Context	Label	Definition
Idealistic	I	$s_a = 3; \forall a \in A$
Realistic	R	$s_a = 2 \vee s_a = 3; \forall a \in A$
Enhanced	E	$s_a = \mathcal{U}_a; \exists a \in A$
Pessimistic	P	$s_a = \mathcal{L}_a; \exists a \in A$

3.1.4. Available capacity

Available capacity is directly related to a link´s status, in the way that for each status there exists a unique and independent available capacity. Then, the variable to denote link's available capacity is given by $y_{a,s_a} \in \mathbb{Y}$; where $\mathbb{Y}$ is a matrix of size $|A| \times 4$ and $y_{a,s_a^{\varpi}} \in \mathbb{R}^{|A| \times 4}$. Additionally, it is relevant to mention that two links with the same status not necessarily have the same available capacity. That is to say, as per the standard capacity which depends on the type of road, the available capacity on each status is specific and independent for every single link. For example, two links in status 2 might have different available capacities, it depends on the type of road and level of damage. Despite the values of $\mathbb{Y}$ must be obtained by observation of the real-world situation, for the model proposed in this paper, the elements of $\mathbb{Y}$ will be generated randomly using MS Excel® as shown in Table 3.5.

Table 3.5 Random generation of available capacity for each status to be used in the study

$\mathbf{s_a}$	Available Capacity	Comments	MS Excel Formula
1	$0.001^* < \mathbf{y_{a,s_a}} < 0.5 * SC$	Available capacity is less than 50% of the standard capacity	=(RAND()*0.5)* Standard capacity
2	$0.5 * \text{SC} \leq \mathbf{y_{a,s_a}} < \text{SC}$	Available capacity is greater than or equal to 50% of the standard capacity	=(1-RAND()*0.5)* Standard capacity
3	$= \text{SC}$	Available capacity depends on road type	= Standard capacity
4	$\text{SC} < \mathbf{y_{a,s_a}} \leq 1.2 * SC$	Available capacity can be increased up to 20% of the standard capacity	=(1+RAND()*0.2)* Standard capacity

SC=Standard Capacity

**For purposes of modelling travel behaviour, it was assumed that no link can have 0 available capacity, it would constitute the removal of the links from the network. However, a low enough value of available capacity will deter drivers from using the link.*

3.2. Network Vulnerability Measure (NVM)

In section 2 where the methods employed to identify road network criticality were discussed, we also described the Network Vulnerability Index (NVI), which differs from the Network Robustness Index (NRI). In particular, NVI introduces the variable serviceability which allows quantifying criticality based not only on volumes of traffic and travel time but also on the magnitude of capacity loss. This makes the vulnerability metric a highly useful one as it responds to the change in capacity of any type of road within the road network duly accounting for the share of contribution made by lower order links within the network.

NVI serves to generate a ranking of links' criticality, by fully or partially reducing the capacity of a single road (Balijepalli & Oppong, 2014). However, proofs point that when multiple failures occur simultaneously, the set of most critical links may include some of the links that were not critical through single failure evaluation (Wang, et al., 2016). In this research the concept of NVI will be used to arrive at a set of links which minimises the vulnerability due to disruption using a network-wide measure. The measure, henceforth referred as Network Vulnerability Measure (NVM), aims to offer an insight to the road network´s state. For example, in an idealistic context wherein all the streets are working at 100% of their capacity the NVM will be lower than if one or more links reduce their capacities.

When a link fails, NVI is calculated with the equation (2.1), which contains two parts, *before* and *after* disruption values. Variables x_a, r_a and t_a are obtained by dropping link a or partially reducing its available capacity.

$$NVI \quad = \sum_{a \in A} \left(\frac{x_a^o}{r_a^o} \right) t_a^o \; - \sum_{a \in A} \left(\frac{x_a}{r_a} \right) t_a$$

Note that if the first part is eliminated from the equation (as shown below), the formula can still be used to refer to a link´s criticality. This simplified version of NVI conserves its interpretation; NVI values tending to negative infinite mean higher vulnerability and the opposite for values approaching zero. However, in no case the indicator should take value of zero, it would mean that no flows have been assigned to the network ($x_a = 0; \; \forall a \in A$).

$$NVI \quad = - \sum_{a \in A} \left(\frac{x_a}{r_a} \right) * t_a$$

Using the previous simplification, it was defined the so-called *vulnerability value*, denoted with, $\widetilde{V_\varpi}$. ϖ denotes the context wherein each link has a certain available capacity. Then, the vulnerability value is formulated as follows.

$$\widetilde{V_\varpi} = \sum_{a \in A} \left(\frac{x_a^\varpi}{r_a^\varpi} \right) * t_a^\varpi \qquad (3.1)$$

Additionally, the negative sign was eliminated, thus the interpretation changes. Values of $\widetilde{V_\varpi}$ tending to positive infinite represent higher vulnerability and the opposite for values tending to zero.

Since serviceability r_a^ϖ is calculated as the proportion of the available capacity to the standard capacity (Balijepalli & Oppong, 2014), then (3.1) can be rewritten as (3.2). Y_a being the standard capacity of link a and y_a^ϖ the available capacity of link a within the context ϖ.

$$\widetilde{V_\varpi} = \sum_{a \in A} \left(\frac{x_a^\varpi}{\left(\frac{y_a^\varpi}{Y_a} \right)} \right) * t_a^\varpi \qquad (3.2)$$

Thus far, we have described the concept of $\widetilde{V_\varpi}$ whose values are related to the network's size, trip demand and travel time function. That is to say, a small network with low level of congestion would produce a smaller value of $\widetilde{V_\varpi}$ than a larger highly congested network. Complementarily, to standardize the analysis of vulnerability, the *Network Vulnerability Measure* (NVM) is introduced, which compares *vulnerability values* of two different contexts. The measure is represented with Υ^ϖ, and is given by the following:

$$\Upsilon^{\varpi} = \frac{\widetilde{V_{\varpi}}}{\widetilde{V_0}} \tag{3.3}$$

$\widetilde{V_0}$ is the vulnerability value of the base scenario and it is arguable how to calculate it. It could be the outcome of using a road network working at standard capacities. But it is also valid to use the available capacity of the network at the time of performing the study, which may differ from the standard capacity. However, in this research $\widetilde{V_0}$, will be calculated using standard capacities.

3.2.1. Worse-Case-Vulnerability (WCV)

Worse-Case-Vulnerability is the *Network Vulnerability Measure* of the worse scenario, i.e. when all links take their least possible status. Then, to understand further sections the reader is suggested to question themselves on the following:

Consider the possibility of investing in one or more links, in order to raise its lower status to a higher level. Then,

- which link or links should raise their lower status, in order to minimise WCV?
- to what status should it (or they) be raised to?
- what would be the investment needed to raise the lower status from one level to another?

3.2.2. Costs of upgrading Lower Status

Enhancement costs in continuous NDP (or CNDP, hereafter) are defined by linear functions, i.e. a monetary unit of cost for each capacity unit covered. However, it has been suggested that this linear approach is not accurate since costs depend directly on the complexity of the intervention. Karlaftis et. al. (2007) employs a "nonlinear cost" approach to optimise fund allocation for recovering the transport network after natural disasters.

As inputs for the WCV problem (proposed in further sections), three types of road works were defined: *Periodic works or Maintenance*, *Special works or Constructions* and *Development or Improvement*. The type of intervention depends on the actual lower status $\mathcal{L}_a$ and its desired lower status, denoted as $\mathcal{L}'_a$ (See Table 3.6).

Table 3.6 Work required to upgrade the lower status of a link and description of each type of intervention

From Original $\mathcal{L}_a$	To Optimised $\mathcal{L}_a^*$	Type of requirements	Description
1 1 1	2 3 4	Special works / Construction	These are activities whose need cannot be estimated with any certainty in advance. The activities include emergency works to repair landslides and washouts that result in the road being cut or made impassable. Winter maintenance works of snow removal or salting are also included under this heading. A contingency allowance is normally included within the recurrent budget to fund these works, although separate special contingency funds may also be provided.
2	3	Periodic works / Maintenance	These include activities undertaken at intervals of several years to preserve the structural integrity of the road, or to enable the road to carry increased axle loadings. The category normally excludes those works that change the geometry of a road by widening or realignment. Works can be grouped into the work types of preventive, resurfacing, overlay and pavement reconstruction. Examples are resealing and overlay works, which are carried out in response to measured deterioration in road conditions. Periodic works are expected at regular, but relatively long, intervals. As such, they can be budgeted for on a regular basis and can be included in the recurrent budget. However, many countries consider these activities as discrete projects and fund them from the capital budget.
2 3	4 4	Development / Improvement	These are construction works that are identified as part of the national development planning activity. As such, they are funded from the capital budget. Examples are the construction of by-passes, or the paving of unpaved roads in villages.

Source: The World Back Group (2016)

Before explaining how the cost is calculated, the difference between "capacity reduction" and "road damage" should be understood. Whilst, road damage refers to the physical harm expressed in distance units, capacity reduction refers to traffic flows in which units can be vehicle per hour or Passenger Car Units. Note that an equal length of road damage for two independent links may have different rates of capacity reduction, i.e. the length of the damage and the traffic capacity are independent from each other.

The parity "road damage – capacity reduction" has to be the outcome of *in situ* expert highway engineer's evaluation. Nonetheless, in this study, it is generated randomly using the criteria presented in Table 3.7, viz., when a link takes status 1, it is assumed to have between 25% and 100% of its extent damaged; when a link takes status 2, the percentage of length damaged is assumed less than 25%. Then, when a link is at status 3, i.e. it is intact, the damage is 0%. Finally, for those links with the possibility to increase their standard capacities, when they take status 4 it is assumed that intervention will be carried in less than 20% of its extent.

Table 3.7 Criteria to generate the extent of road damage, depending on lower and upper status

Status options		Percentage of link's length damaged			Percentage of link's length to be improved
$\mathcal{L}_a$	$\mathcal{U}_a$	At Status 1	At Status 2	At Status 3	At Status 4
1	4	25% - 100%	0.001% - 25%	0%	0% - 20%
1	3	25% - 100%	0.001% - 25%	0%	*
2	4	*	0.001% - 25%	0%	0% - 20%
2	3	*	0.001% - 25%	0%	*
3	4	*	*	0%	0% - 20%

**No applicable, the link cannot be in this status.*
The parity Status-Damage presented in this table is exclusively to generate parameters of this study, further researches should include real cases.

The costs of upgrading the lower status ($\mathcal{L}_a$) of a link are dependent on the extent of the link length to be intervened, the type of work required and the type of road. Table 3.8 presents some figures that can be used to generate enhancement costs. In this study it was assumed, as a relaxation, that all the streets are single two lanes (S2). Then, the variable $C_a^{\mathcal{L}_a,\mathcal{L}'_a} \in \mathbb{C}$ denotes the cost of elevating the lower status of the link a, from $\mathcal{L}_a$ to $\mathcal{L}'_a$. $\mathbb{C}$ is a matrix of size $|A| \times 4$ and the following inequality holds $\mathcal{L}_a \leq \mathcal{L}'_a \leq \mathcal{U}_a$.

Table 3.8 Range of maintenance and construction costs
(Financial figures obtained from different sources, presented in £ million/Km)

Type of Work	Type of road.	Cost range (£m/Km)
Periodic works / Maintenance[1]	All	0.004 [2 4] – 0.152 [3 4]
Special works / Construction [5]	S2	0.9 – 1.7
	WS2	1.3 – 2.3
	D2AP	1.8 – 3.4
	D3AP	2.6 – 4.8
	D2M	2.5 – 4.5
	D3M	3.2 – 6.0
	D4M	4.0 – 7.4
Development / Improvement[2]	All	0.1 – 1.4

[1]*Source: The World Bank Group (2000).*
[2]*Lower limit for resealing.*
[3]*Upper limit for structural overlays.*
[4]*Converted according to 1 USD 0.775 GBP.*
[5]*Source: DfT (1997).*

3.3. Worse-Case-Vulnerability Problem (WCV-P)

Worse-Case-Vulnerability Problem is formulated to identify the optimal set of links in which lower status should be elevated in order to reduce vulnerability in a pessimistic context. As mentioned in the previous sections NVM differs from other measures by including into analysis the magnitude of capacity losses. That is to say, importance will be given not only to higher order links carrying large volumes of traffic but also to the links with large losses of capacity notwithstanding whether they are of higher or lower in order. The model is formulated as a bi-level mathematical problem, having a budget limit and the traffic assignment problem as another constraint.

This section introduces the mathematical formulation of the WCV-P. Parameters, variables, objective functions and constraints are also explained.

3.3.1. Parameters

A: Set of links.
N: Set of nodes.
N^{OD}: Set of Origin and Destination nodes, $N^{OD} \subset N$.
OD: Set of O-D pairs, defined by $OD: \{(i,j); \forall i,j \in N^{OD}\}$
$D_{(i,j)}$: Demand of trips from i to j, $\forall (i,j) \in OD$.
$\mathcal{L}_a$: Initial lower status of link $a \in A$; $\mathcal{L}_a \in \mathcal{L}$.
$\mathcal{U}_a$: Upper status of link $a \in A$; $\mathcal{U}_a \in \mathcal{U}$.
$C_a^{\mathcal{L}'_a}$: Cost of elevating the lower status of link a to $\mathcal{L}'_a$.
B: Available budget.
$\hat{q}_a$: Standard capacity of link a.

$Q_{a,\mathcal{L}'_a}$: Matrix of available capacities, where $a \in A$. Reads as "available capacity of link a when its lower status change to $\mathcal{L}'_a$". Note that with $\mathcal{L}'_a = 3$, then $Q_{a,3} = \hat{q}_a$.
R: Set of routes.
$R_{(i,j)}$: Set of routes that connect the O-D pair $(i,j) \in OD$. $R_{i,j} \subset R$.
$\widetilde{V_0}$: Vulnerability value of base scenario, i.e. $s_a = 3$ and $r_a^0 = 1$; $\forall a \in A$. Then, $\widetilde{V_0} = \sum_{a \in A}(x_a^0) * t_a^0$.
α_a: Free flow travel time expressed as a parameter of the travel time function.
β_a: Second parameter of the travel time function.

3.3.2. Indices

a: Index to denote and arc, element of A
(i,j): O-D pair, element of ; $\forall i,j \in N^{OD}$.
r: Route $r \in R_{(i,j)} \subset R$.
$\mathcal{L}'_a$: Lower level status of link a during the critical scenario. The following holds $\mathcal{L}_a \leq \mathcal{L}'_a \leq \mathcal{U}_a$ and $\mathcal{L}'_a \in \mathbb{N}^+$.

3.3.3. Variables

$\tau_a(v_a, q_a)$: Travel time function and its parameters.
$\tilde{V}$: Vulnerability value of critical scenario, i.e. $s_a = \mathcal{L}_a$; $\forall a \in A$.
$f_r^{(i,j)}$: Traffic flow on route $r \in R_{(i,j)}$ for each $(i,j) \in OD$.
q_a: Available capacity of link a.

3.3.4. Decision variables

Master problem:

$x_{a,\mathcal{L}'_a}$: Binary variable that takes 1 if lower status of link a changes to $\mathcal{L}'_a$; 0 otherwise.

Second level problem:

v_a: Traffic flow on link a.
$\delta_{r,a}^{(i,j)}$: Takes 1 if route r that connects O-D pair (i,j) uses link a.

3.3.5. Objective Functions

Master problem:

Υ: WCV will be used as the objective function of the problem of the master problem.

Second level problem:

T: Individual travel time of the objective function of the second level problem.

3.3.6. Worse-Case-Vulnerability Problem formulation

The following bi-level problem is formulated to minimise WCV, which is denoted with ϒ (See section 3.2). Among the constraints is the driver´s behaviour or traffic assignment problem. This last constraint constitutes the second-level problem, which objective function is to minimise individual travel time, whilst following the Wardrop's First Principle.

The model is the following:

$$Min\left[\Upsilon = \frac{\tilde{V}}{\widetilde{V_0}}\right] \tag{3.4}$$

s.t.

$$\tilde{V} = \sum_{a\in A}\left(\frac{v_a}{\left(\frac{q_a}{\hat{q}_a}\right)}\right) * \tau_a \tag{3.5}$$

$$q_a = \sum_{\mathcal{L}'_a \in (\mathcal{L}_a,\mathcal{U}_a)} x_{a,\mathcal{L}'_a} * Q_{a,\mathcal{L}'_a} \qquad \forall a \in A \tag{3.6}$$

$$\sum_{\mathcal{L}'_a \in (\mathcal{L}_a,\mathcal{U}_a)} x_{a,\mathcal{L}'_a} = 1 \tag{3.7}$$

$$\sum_{a\in A}\sum_{\mathcal{L}'_a \in (\mathcal{L}_a,\mathcal{U}_a)} C_a^{\mathcal{L}'_a} * x_{a,\mathcal{L}'_a} \leq B \tag{3.8}$$

$$(v_a) \in argmin\left[T = \sum_{a\in A}\int_0^{v'_a} \tau_a(\omega, q_a)\, d\omega\right] \tag{3.9}$$

s.t.

$$\sum_{r\in R^{(i,j)}} f_r^{(i,j)} = D_{(i,j)} \qquad \forall (i,j) \in OD \tag{3.10}$$

$$f_r^{(i,j)} \geq 0 \qquad \forall (i,j) \in OD \quad \forall r \in R \tag{3.11}$$

$$v'_a = \sum_{a\in A}\sum_{(i,j)\in OD} f_r^{(i,j)} * \delta_{r,a}^{(i,j)} \qquad \forall a \in A \tag{3.12}$$

The objective function of the master problem is presented in equation *(3.4)*. The constraints are expressed in the equations *(3.5)* until *(3.9)*. The vulnerability value is calculated in *(3.5)*, viz., the vulnerability after elevating the lower level of the links that minimise WCV. In *(3.6)* the model calculates the available capacity of link a and *(3.7)* forces every link to have only one lower status. One important constraint is *(3.8)*, which confirms that enhancement costs do not exceed the budget. Finally, *(3.9)* guarantees the accomplishment of the User Equilibrium. This constraint in itself, is the objective function of the second-level problem, for which constraints are stated from *(3.10)* to *(3.12)*. The traffic flow is conserved by *(3.10)*, and *(3.11)* ensures non-negativity of flow. *(3.12)* imposes the definitional constraints linking the path flows to the link flows. The travel time in *(3.9)* is calculated using the standard Bureau of Public Roads (1964) style function, $\tau_a(v'_a, q_a) = \alpha_a \left[1 + \beta_a \left(\frac{v'_a}{q_a}\right)^{\theta}\right]$.

3.4. Simulated annealing based solution procedure

Simulated Annealing (SA) method was developed under the idea that there exists a useful connection between statistical mechanics and combinatorial optimisation (Kirkpatrick, et al., 1983). The following notions introduced by Liu (2001) ease the comprehension of the method.

- ***Initialization***: Set the initial feasible status of the system and the initial "temperature". The initial temperature can be defined by trial and error.
- ***Markov Length*** (ML): is the number of random displacements performed during each temperature.
- ***Cooling schedule:*** is the reduction rate of temperature, e.g. it has been applied 20% reduction for the first 12 reductions and 50% afterward (Xu, et al., 2009).
- ***Step Size:*** the extent of displacements, i.e. how many and how big are the modifications during each random generation.
- ***Neighbouring solutions:*** are the set of feasible solutions. They can be generated departing from the previous feasible solution.
- ***Stopping Criteria:*** this value indicates the final temperature was achieved and the process must stop.

The system is represented through a road network, whose elements are described in the sections 3.3.1 until 3.3.4. The objective function is represented by the WCV, described explicitly in the section 3.2.1.

3.4.1. Initialization

The initialization is none other than the departing point from where SA begins to seek for better solutions. As the initial solution, it will be based on a Larger-traffic-More-importance approach, which seems to be a very logical strategy. This approach identifies the links with the larger volume of traffic during an *idealistic context*, i.e. with all the links working at standard capacity. Then a *pessimistic context* is assumed, i.e. all links work at their lower status. It may be noted that this may be too pessimistic as the chance of so many links simultaneously getting affected may not be high. However, it would serve as a good benchmark to compare alternative strategies. Subsequently, the lower status is upgraded beginning with the links with larger traffic under the *idealistic context*, up until the budget allows it. The WCV measure is calculated according to the new lower levels of the links. The algorithm of this approach is presented in Box 1.

Box 1: Pseudocode for initialising the simulation

1st Step:	Perform MSA under the base scenario (*idealistic context*; $\mathcal{L}'_a = 3$) and calculate the vulnerability value $\widetilde{V_0}$.
2nd Step:	Make a ranking of links with larger volumes of traffic.
3rd Step:	Perform MSA under critical scenario (*pessimistic context;* $\mathcal{L}'_a = \mathcal{L}_a$).
4th Step:	Begin increasing the lower status of links with larger volumes of traffic to its upper status ($\mathcal{L}'_a = \mathcal{U}_a$) and repeat the operation until the budget allows it. The new set of lower status will be denoted by $\mathcal{L}^1_a$.
5th Step:	Perform traffic assignment with the set of upgraded lower statuses ($\mathcal{L}^1_a$) and calculate the vulnerability value $\tilde{V}$.
6th Step:	Calculate the WCV, $\Upsilon_1 = \frac{\tilde{V}}{\widetilde{V_0}}$.

Then, Υ_1 which is produced under the new set of lower status will stand for the initial solution of the Simulated Annealing. This solution will be hence referred as **Strategy One (S1)**.

3.4.2. The generator of displacements / neighbourhood solutions

During each iteration of the SA one neighbourhood solution is generated as follows in Box 2.

Box 2: Pseudocode for generating displacements and neighbourhood solutions

1st Step:	Generate the subset $\tilde{A} \subset A$, which contains links whose lower status can be upgraded. $\tilde{A}$ also excludes links that cannot be upgraded because the remaining budget is not enough to upgrade them
2nd Step:	Select a random link $l = RandomSelect[\tilde{A}]$.
3rd Step:	Select a random status S_l of l, excluding its current lower status.
4th Step:	$S_l = RandomSelect[\{\mathcal{L}_l, \ldots, \mathcal{U}_l\} \backslash \{\mathcal{L}'_l\}]$.
5th Step:	Assign the new lower status to link l, $\mathcal{L}'_l = S_l$.

3.4.3. Initial, final temperatures, cooling scheme, Markov Length

There is no standard method to set *initial and final temperatures*, besides, in the literature is commonly noted that those values should be obtained by trial and error. The random walk product of the generation of neighbourhood solution is strongly connected with the values assumed as temperatures, e.g. a very high initial "temperature" allows the algorithm to accept bad solutions with a high probability. On the other hand, very low initial "temperatures" does not allow the model to explore different feasible regions. Consequently, the values of temperatures will be presented individually for each one of the instances to be evaluated in the further sections.

The *cooling scheme* refers to the velocity at which the temperature decreases. Some studies use fixed rates of refrigeration and others use variable rates that quicken the refrigeration as the temperature approaches the lower temperature. This last method will be employed in this research. For the first seven iterations, the temperature decreases by 20% and thereafter by 50%.

Markov Length is the number of neighbourhood solutions generated at each temperature. Then, having Markov Length very long will increase the computation time; in fact, it can worsen the solution in first stages and therefore it needs more "temperature" stages in order to find a good solution. Regarding computational time, the size of Markov Length is even more important for nested iterative solutions as the one proposed in this study. Thus, after a few preliminary tests, it was found that 20 iterations per stage of temperature produce good results with an acceptable computational time.

Considering the cooling scheme mentioned above some tests were performed to define appropriate temperatures and it was found that initial temperature of 0.005 and the final temperature of 0.0001 throw results deemed to be good for the research. The only case where these values did not seem to improve WCV was for the network with 50% of the available budget. So in this unique case, an initial temperature of 0.001 and a final temperature of 0.0005 were used. For each pair of temperatures, the cooling scheme generates 14 and 13 stages, respectively, which when multiplied with the Markov Length result in 280 and 260 iterations.

3.4.4. Pseudocode

In addition to the notation provided in sections 3.3.1 until 3.3.4, the following list is required for the upcoming algorithm.

Υ_{min}: Minimum WCV in the last iteration of SA.
Υ_{abs}: Absolute minimum WCV.
$\mathcal{L}^{min}$: Set of lower statuses that produces Υ_{min}.
$\mathcal{L}^{abs}$: Set of lower statuses that produces Υ_{abs}.

Box 3 shows the main steps involved in minimising the WCV.

Box 3: Pseudocode for Simulated Annealing procedure

Initialization.
    Perform **Strategy One** and obtain the new set of lower statuses, $\mathcal{L}^1$.
    Perform MSA using capacities according to $\mathcal{L}^1$.
Calculate initial value of objective function, Υ_1.
    Make $\Upsilon_{min} = \Upsilon_{abs} = \Upsilon_1$ and $\mathcal{L}^{min} = \mathcal{L}^{abs} = \mathcal{L}^1$
    Set $T = T^{high}$, T_{low} and ML.
While $T \geq T_{low}$ repeat…
    For $n^{ml} = 1$ until $n^{ml} = ML$, repeat…
        Generate a neighbourhood solution, $\mathcal{L}'$.
        Perform MSA using capacities according to $\mathcal{L}'$.
        Calculate new value of objective function, Υ'.
        Make $\Delta\Upsilon = \Upsilon' - \Upsilon_{min}$
        If $\left(\Delta\Upsilon \leq 0 \text{ or RandomReal}[0,1] \leq e^{\frac{\Delta\Upsilon}{T}}\right)$
            If yes: Make Υ_{min}=Υ' and $\mathcal{L}^{min} = \mathcal{L}'$.
            If no: Do nothing.
        End If.
        If ($\Upsilon_{min} \leq \Upsilon_{abs}$)

If yes: Make $\Upsilon_{abs} = \Upsilon_{min}$ and $\mathcal{L}^{abs} = \mathcal{L}^{min}$
If no: Do nothing.
End If.
End For.
If $(T \geq T_{low} * 0.8^7)$
If yes: Make $T = T * 0.8$
If no: Make $T = T * 0.5$
End If
End While
Show Υ_{min} , Υ_{abs}, $\mathcal{L}^{min}$, $\mathcal{L}^{abs}$.

This exponent in $T \geq T_{low} * 0.8^7$ in Box 3 above defines that in the first 7 iterations the temperature will reduce by 20% and thereafter by 50%. Despite the SA not guaranteeing the optimal, for problems with a large number of decision variables, it approaches a very good solution with lesser computational load. Complementarily, the Method of Successive Averages is a technique to model driver's behaviour which has been proved to converge to user equilibrium. One of the advantages of using MSA is its simplicity, nonetheless, one drawback is its slow convergence to the equilibrium solution. However, utilizing a loose convergence in MSA aids in reducing the computational burden.

3.5. Remarks on the methodology

Worse Case Vulnerability Problem as defined in the paper has been set up as a Discrete Network Design Problem (DNDP) as the objective is to identify a set of links whose improvement from their lower status to a possible upper status would reduce the vulnerability measure. DNDP's are common in NDP literature (see for example, Xu et al 2017, Wang et al 2013 among many others) wherein a set of new links to construct, or binary capacity improvement decisions are explored. On the other hand, Continuous Network Design Problems (CNDP) are developed to determine optimal value of some variable such as toll, capacity (see Farahani et al 2013 for a comprehensive review of NDP's). In this research we chose to set up the WCV-P as a DNDP as discretising the capacity change into a finite number of discrete levels vastly reduces the computational burden especially when solving the problem for a real-life network with large demand. The following paragraphs explain further the reasons for choosing the particular objective function used and the metaheuristic solution method applied.

3.5.1. Comments on the objective function

Minimising directly an objective function involving a measure of vulnerability is new to the field of vulnerability analysis. Thus far the literature reviewed attempted to minimise total travel time in a bid to assess a degraded network within the normative system optimal state. Whilst the system optimal objective of minimising total travel time is known to be a good benchmark to compare network states, the literature recognises that minimising total travel time makes some drivers travel longer to achieve efficiency (Boyce & Xiong 2004, Jahn et al 2005). In this research our aim is to minimise the vulnerability to disruption to keep up the connectivity levels which is completely different to the objective of improving efficiency. The objective function of WCV-P is aptly designed to include all types of roads to keep up the connectivity levels, be they higher order major roads carrying large volume of traffic or a lower order single lane roads in the neighbourhood connecting homes with activities.

3.5.2. Comments on the solution procedure

Metaheuristic SA procedure has been used for solving the optimisation involved as SA is found to be the most commonly used method alongside of Genetic Algorithm (GA) as noted by Farahani et al (2013). Furthermore, SA has been found to outperform GA when the demand is large (see Xu, et al 2009 for further details). SA is also attractive in achieving near global optimal solution even without the need for having the gradient information from the flow response (Friesz et al 1993, Meng and Yang 2002). SA is a stochastic method which avoids getting stuck with a local optimum by accepting solutions not only that decrease the value of objective function but also the worse solution which satisfies the acceptance criterion with a probability. During the minimisation process, the probability of accepting worse solution gradually tends to zero with an appropriate cooling schedule. For this reason, SA method does not get stuck with the local optimum and the final accepted solution reaches near global optimum.

4. Numerical examples

4.1. Experimental network

The model proposed above will be evaluated using a section of the York´s urban road network.

The following notions will help understanding the experiments.

- *Base Scenario*: situation when all links work at their standard capacity.
- *Critical Scenario*: pessimistic context when each link is at its lower status.
- *Strategy Do-Nothing* (DN): this strategy assumes that no action is taken to improve the WCV of the *critical scenario*.
- *Strategy One* (S1): this strategy assumes that some improvements are developed prioritizing the *links with larger volumes* of traffic.
- *Strategy Simulated Annealing* (SA): this approach is the result of modelling the *WCV-P* using SA.
- *Total Funding Requirement* (TFR): it is the amount of money needed to guarantee that during the critical scenario the status of all links is no lower than their upper-status, i.e. Lower-status = Upper-status.
- *Available Budget*: it is a proportion of the TFR expressed as percentage.

All the programming required for this study was developed using *Wolfram Language* including the Method of Successive Averages and Simulated Annealing. Additionally, the *Wolfram Language* was used to generate graphics. Complementarily, all the parameters and information on the network was stored in MS Excel which was taken as input by the program.

4.1.1. Description of York´s urban road network

This example is used to evaluate performance of the WCV-P and SA over a network of realistic size. The segment of York´s road network to be used is located to the south-west from the city centre, see Figure 4.1.

The data related to this network was initially coded in SATURN traffic assignment software (Van Vliet & Hall, 2004). Amongst the data extracted from the underlying SATURN model of York include - node coordinates, link length/capacity, freeflow/capacity times and travel time function parameters. The travel time function used in this study is the one proposed by the Bureau of Public Roads (1964). Figure 4.2 below shows the network using the function P1X of SATURN on the left and the network coded using *Wolfram Language* on the right.

The network consists of:

- 251 links, each of them represent a street.
- 108 dummy links: these arcs are not considered in the process of traffic assignment, as they are used exclusively to represent the connection between the centroid nodes and the network.
- 40 O-D nodes: these nodes connect with junction nodes through dummy links.
- 110 junction nodes: These can be signalized junctions, priority junctions or round abouts.
- An O-D matrix containing 5800 trips for the segmented portion of York which has 40 zones.

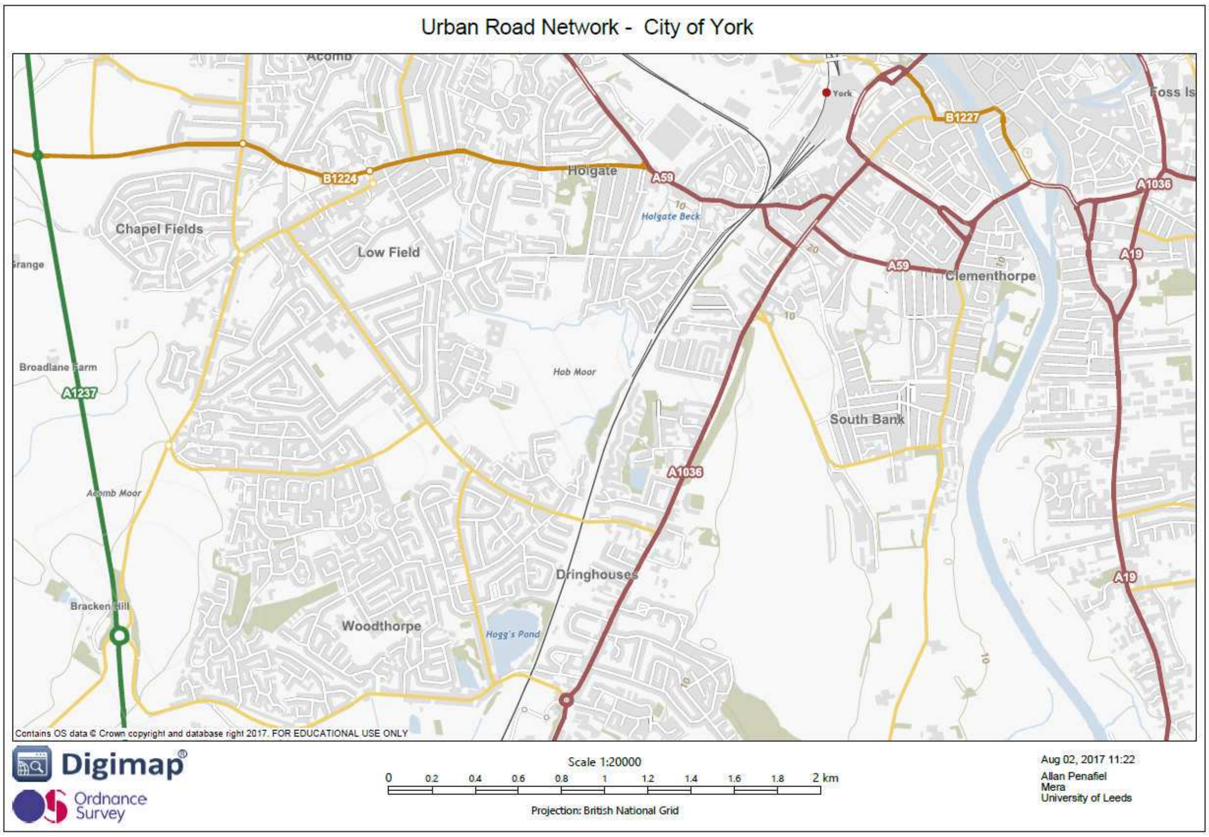

Figure 4.1 Area of interest of York´s urban road network. Source:digimap.edina.ac.uk

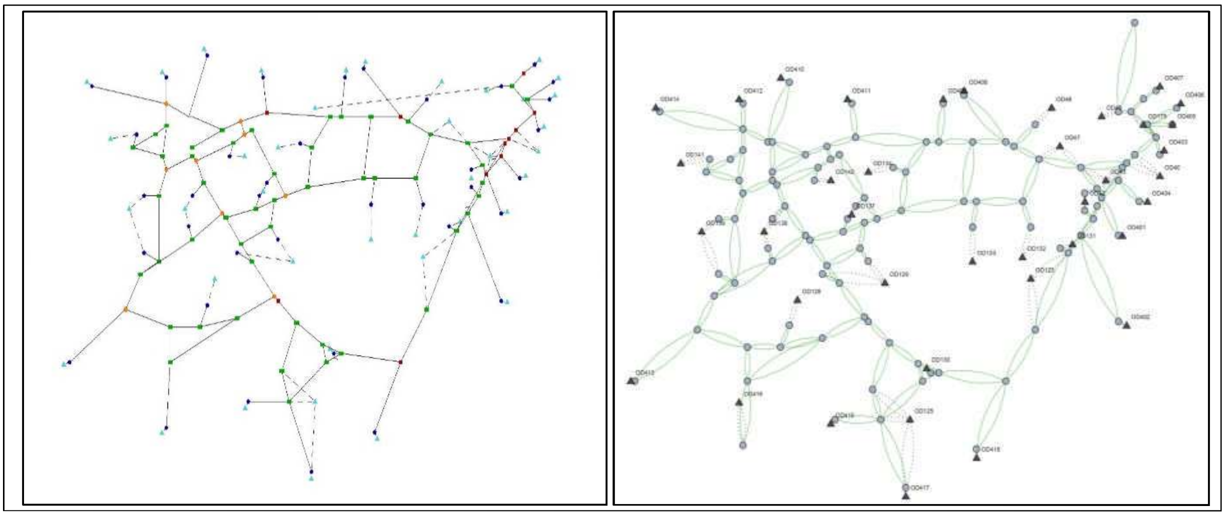

Figure 4.2 Segment of York´ s urban road network represented in SATURN (left) and Wolfram (right)

4.2. Numerical results, findings

This section firstly describes the results of traffic assignment performed for the *do-nothing scenario.* The initial values of WCV and travel time for the DN strategy are obtained by performing traffic assignment under the *critical scenario*. Subsequently this section describes the results for the two main test scenarios viz., S1 and SA with an available budget of 50% of TFR. Finally this section extends the analysis by describing the results at an aggregated level after performing a series of optimisations with 10%, 20%, 30% and 40% of the TFR. SA is performed fifteen times (random walks) with five different budgets (10%-50%), thus a total of 75 instances were simulated. The best and worst solutions for each of the fifteen instances of SA within an available budget will be defined as SA-best and SA-worst. For this network, TFR is £8.54 million if all links are improved from their lower status $\mathcal{L}_a$ to upper status $\mathcal{U}_a$ $\forall a \in A$.

4.2.1. Do-Nothing Strategy

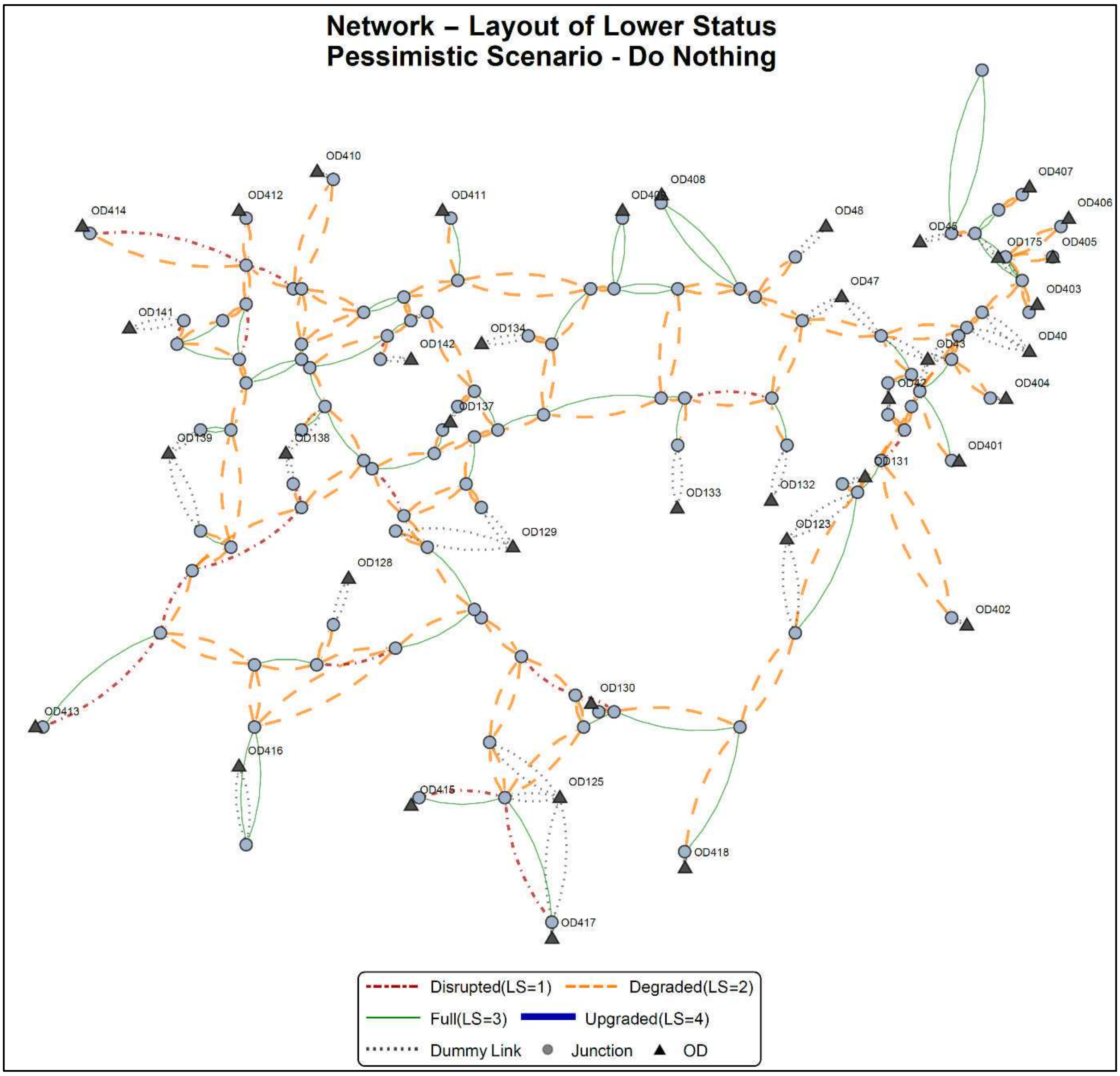

Figure 4.3 Layout of link status under pessimistic critical scenario

The total travel time, convergence, *vulnerability values* and WCV for the *base scenario* and for the DN strategy of the *critical scenario* (Figure 4.3) are given in **Table *4.1***.

Table 4.1 Summary of Base scenario and Do-Nothing strategy

	Base	Do-Nothing
Total Travel Time, Sec	1.362035784*10^6	3.693286923*10^9
Convergence	0.01563951479	0.003214267431
Vulnerability Value ($\widetilde{V_{\varpi}}$)	1.362035784*10^6	5.990034654*10^11
Worse-Case-Vulnerability (Υ^{ϖ})	-	439785.4096

4.2.2. WCV under S1 and SA with available budget limited to 50% of TFR

With 50% of the funds required, the S1 generated a WCV of 0.989, which corresponds to a travel time of 1.348×10^6 seconds. These outcomes are achieved with an investment of 99.99% of the available budget. Note that WCV value is lower than 1, it means this solution outperforms even the base scenario which is a perfectly feasible outcome to an NDP. It is not incoherent considering that, with such a constrained fund available, the solution involves a *critical scenario* wherein some links indeed transcend the initial standard capacities. Thus, the new critical scenario would have a lower travel time than the base scenario. It can be witnessed by comparing Table 4.2 and Table 4.1.

Table 4.2 Results of 15 instances of SA with available budget 50%

Instance No	Min WCV	Iteration No	Investment(£m)	% Invested	Travel Time(Sec)
S1	0.988988416	-	4.268648956	99.99	1.347889302*10^6
1	0.984022013	-	-	-	-
2	0.983690829	-	-	-	-
3	0.982225527	-	-	-	-
4	0.986068964	-	-	-	-
5	0.980692162	-	-	-	-
6	0.981942546	-	-	-	-
7	0.985258912	-	-	-	-
8	0.987065777	-	-	-	-
9	0.983870333	-	-	-	-
10	0.983401138	-	-	-	-
11	0.984526846	-	-	-	-
12	0.981402363	-	-	-	-
13	0.98685229	-	-	-	-
14	0.980287926	76(Best)	4.246602674	99.48	1.348136415*10^6
15	0.988390919	75(Worst)	4.237007342	99.26	1.348060203*10^6

Note: Total funding requirement: £8.54 million. Then, the available budget is: £ 4.27 million.

The set of instances of SA was generated using a high temperature of 0.001 and low temperature of 0.0005. The Markov Length was 20. With these parameters, the SA's best and worst solutions were found in instances 14 and 15 respectively. SA-best solution requires an investment of £4.247 million and SA-worst £4.237 million. Finally, the total travel time is not considerably different from S1, for either of the best/worst solutions of SA.

Figure 4.4 shows the random walk of the WCV along all the iterations. For the case of SA-best, the first forty solutions accepted worse WCV compared to S1. However, after iteration 20 there exist a clear decreasing tendency. Complementarily, the random walk of the investment and total travel time is presented in Figure 4.5.

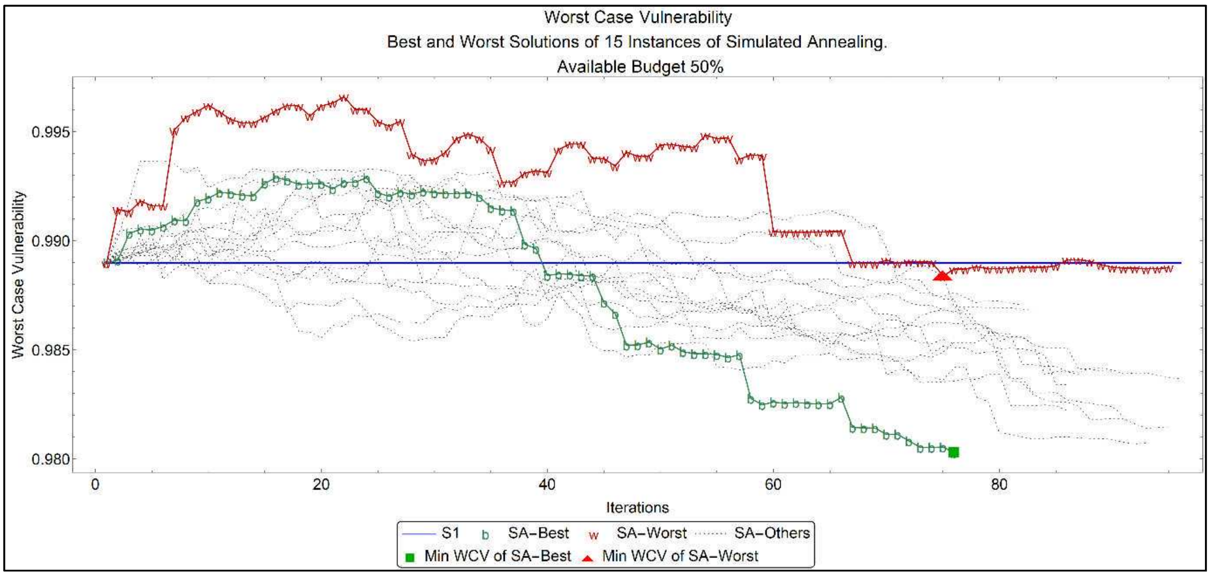

Figure 4.4 Worst Case Vulnerability with an available budget of 50% of TFR

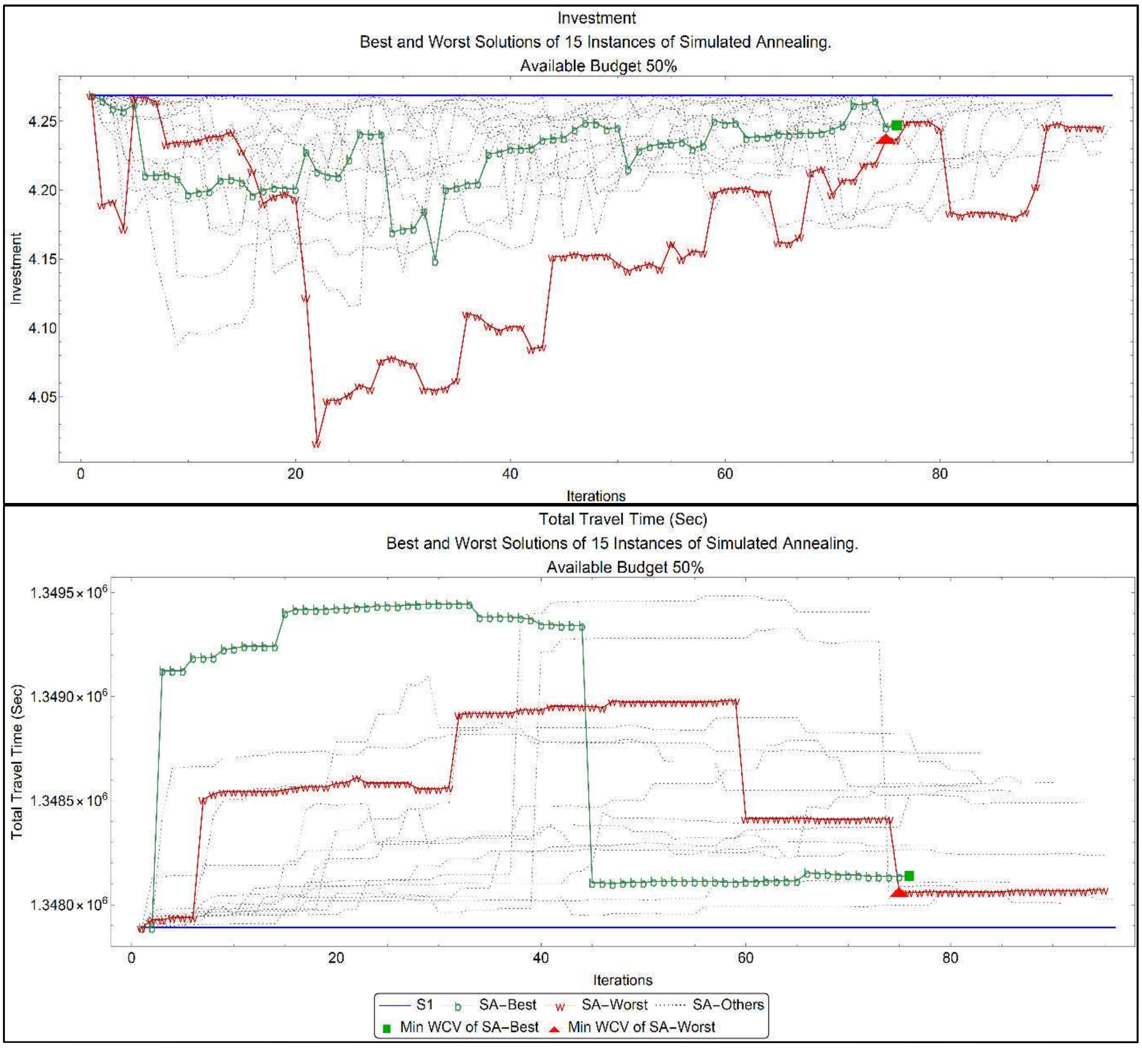

Figure 4.5 Investment (top) and total travel time (bottom) profiles with an available budget of 50% of TFR

With this level of available budget, the S1 suggests that eight links have to remain in *status 1* during the critical scenario (Figure 4.6). The number of links that would be left in *status 2* reduces from 168 with the DN strategy to 59 with S1 strategy. For *status 3* S1 suggests 110 links, 51 more than the DN approach. In the case of links with the possibility for an increase of capacity, S1 suggest 74.

On the other hand, SA suggests the following distribution of statuses for the *critical scenario*:

- 7 links to remain in *status 1*, it is one less than S1.
- 36 links remain in *status 2*.
- The number of links that takes *status 3* is 137.
- 71 links are recommended to increase their available capacity further than the actual standard capacity (*status 4*).

The number of links by their status during a critical scenario thus significantly differs from S1 to SA. This variation may also involve a totally different set of links; it is evidenced in the Figure 4.7.

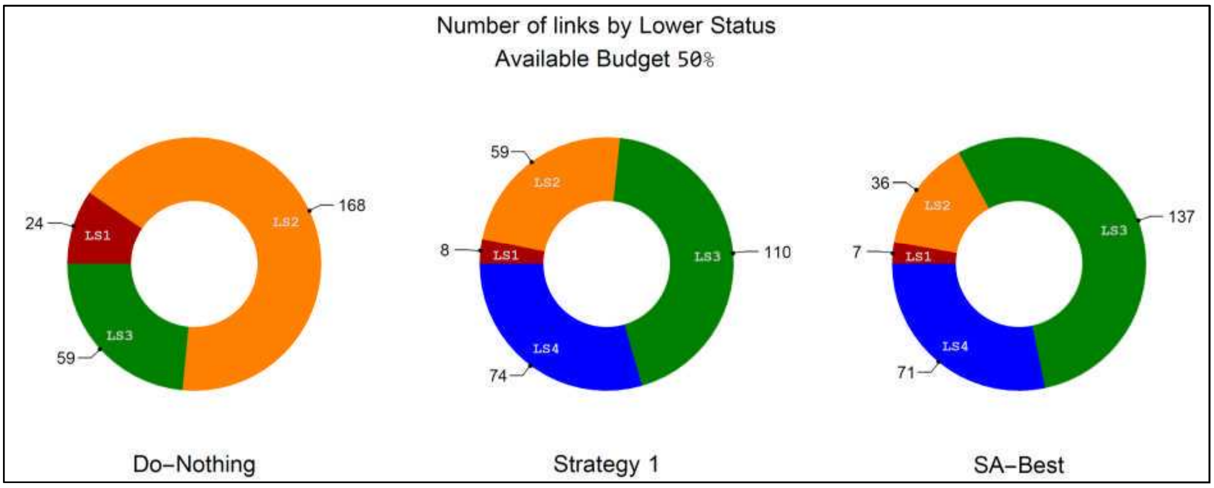

Figure 4.6 Number of links by Lower Status: Do-nothing(left), Strategy 1 (Middle) and Simulated Annealing (right)

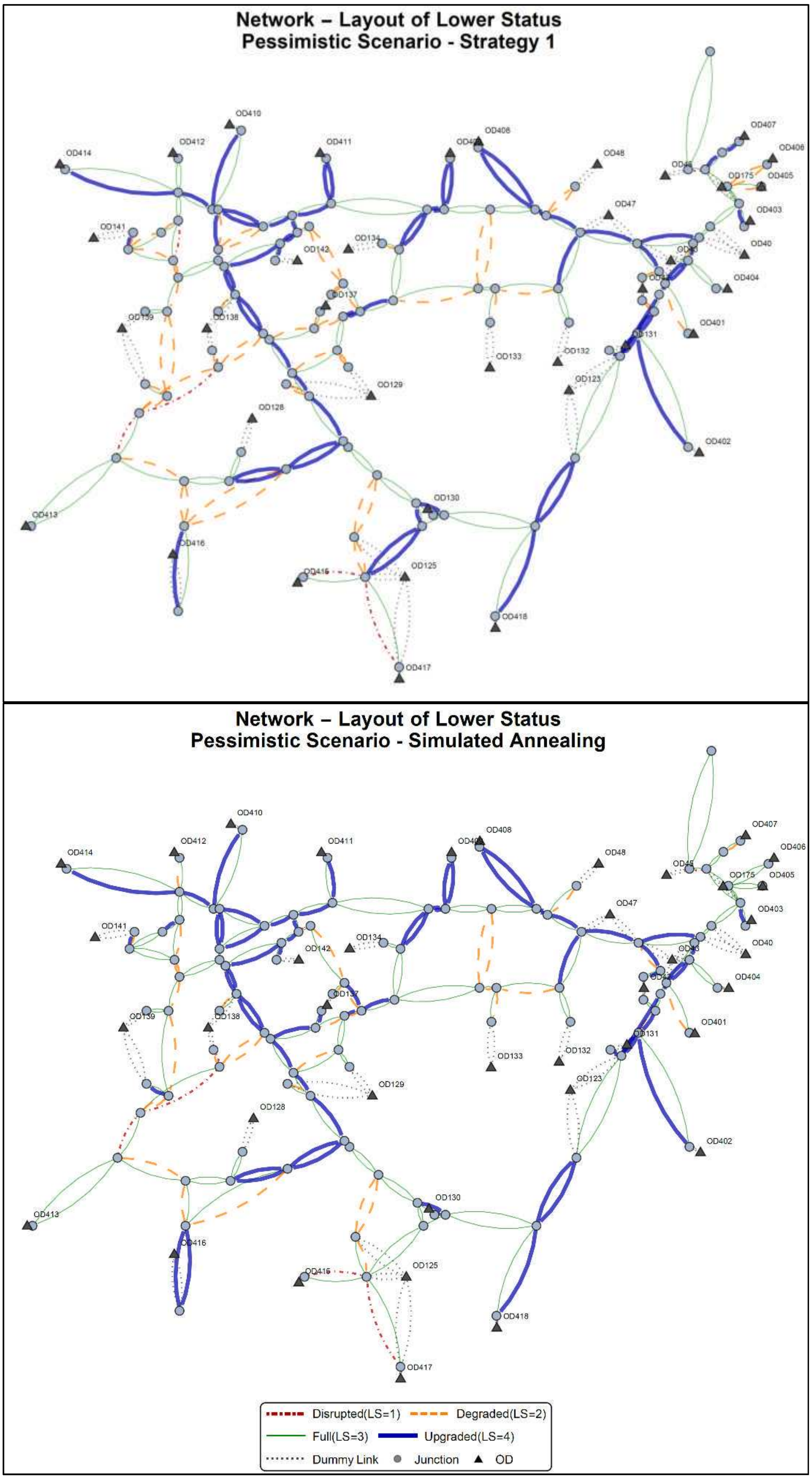

Figure 4.7 Network representation of two different sets of Lower Status: Strategy 1 (Top) and Simulated Annealing (Bottom).

4.2.3. WCV under S1 and SA with a range of available budgets

The previous section described the results of S1 and SA with a budget availability of 50% of TFR. This section describes the results of WCV for a range of budget availabilities viz., 10%, 20%, 30% and 40%. Table 4.3 compares the differences in values of WCV, investment and total travel time between SA and S1 strategies for varying budget availabilities. Among all the instances, the larger reduction of WCV was 30.9%, related to the SA-worst with a budget of 30% of the TFR. Despite this solution to require less investment than S1, the difference is very small, only 0.062%. Additionally, a drawback is the increment of total travel time, 1.27% more than S1. Nonetheless, the travel time is less than the DN strategy (not shown in Table). Finally, the worst solution was found in the series SA-best with available budget of 40% of TFR. This solution was not able to improve WCV, then, it assumed S1 as its minimum solution. In summary, SA-based strategy almost always improved the WCV compared to S1. SA solutions almost always used up less amount of the available budget and produced similar travel time as that of S1.

Table 4.3 Outputs of Simulated Annealing compared against strategy S1

Target Budget	Approach	(A)	(B)	(C)	(D)
10%	SA Best	-0.000021%	88.143	-0.011%	-0.0006%
10%	SA Worst	-0.000146%	1598.884	-0.187%	-0.0025%
20%	SA Best	-0.000119%	2085.624	-0.122%	-0.0003%
20%	SA Worst	-0.000003%	1675.11	-0.098%	0.0003%
30%	SA Best	-0.422286%	12628.945	-0.493%	0.3364%
30%	SA Worst	-30.962%	1588.039	-0.062%	1.2796%
40%	SA Best	0.000000%	0	0.000%	0.0000%
40%	SA Worst	-10.939380%	2501.854	-0.073%	-0.694%
50%	SA Best	-0.879736%	22046.282	-0.516%	0.0183%
50%	SA Worst	-0.060415%	31641.614	-0.741%	0.0127%

(A) Difference of WCV SA-S1 (%).
(B) Difference of Investment SA- S1 (£).
(C) Difference of Investment SA- S1 (%).
(D)Difference of Total travel time SA- S1(%).

The computational time required to arrive at the final solution has an average of slightly over 700 seconds. For the first two sets of instances, specifically for available budgets of 10% and 20% of the TFR, the algorithm required around 400 seconds (See Table **4.4**). Then, when the available budget is 30% of TFR the algorithm employs a mean of 1109.8 seconds. Furthermore, for available budgets of 40% and 50% of TFR, the computational time seems to have a declining tendency (See Figure **4.8**).

Table 4.4 Average computational time and Simulated Annealing parameters by available budget

Number of Temperature Stages:			14 and 13*
Markov Length:			50
MSA Number of Iterations:			30 (or δ<2%)
Budget	Sample Size	Mean (Sec)	St. Dev
10%	15	400.990625	18.80512618
20%	15	397.465625	24.74593996
30%	15	1109.875	106.3788296
40%	15	979.359375	109.8143583
50%	15	620.8770833	60.53504863
All	75	701.7135417	305.1442873

CPU: Intel® Core ™ i7-6500U CPU @2.5GHz 2.60GHz
RAM: 8.00GB
**Only for 50%*

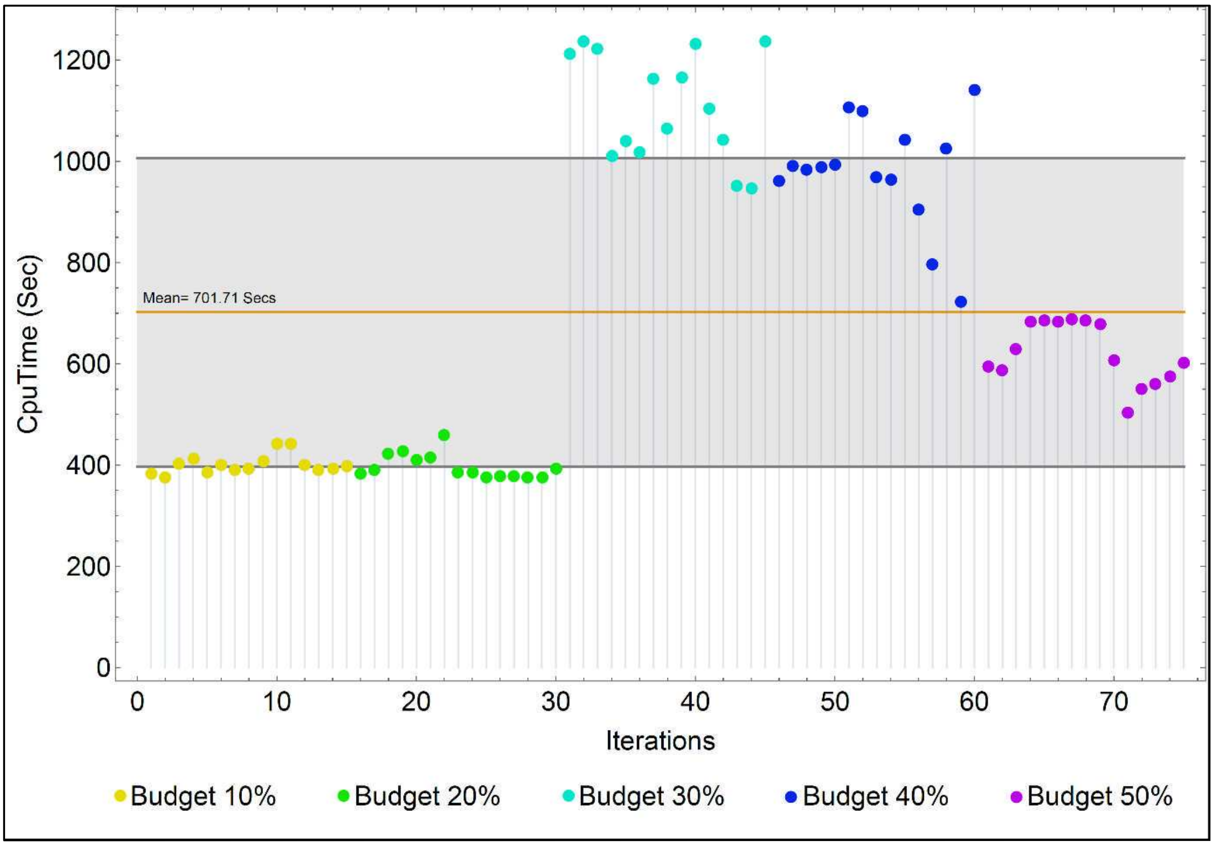

Figure 4.8 Computation requirements of 75 instances of S (Each colour in the graph denotes the percentage of available budget used).

5. Concluding remarks

The Worse-Case-Vulnerability was introduced as a measure to compare vulnerability to disruption under different scenarios that an urban road network can undergo. To compute the WCV it requires the knowledge of set of links that will be affected by incidents and an estimation of their capacity losses. Then, a decision maker can contrast the variation of WCV when the network is working at full capacity against another situation that involves loss of available capacity. The WCV is calculated as the division of the so-called *vulnerability value* of the event of interest by the *vulnerability value* of the base scenario. In this study, the base scenario assumes network operating at full capacity. Therefore, the following holds:

- $WCV = 1$ if the critical scenario is equal to the base scenario, viz., vulnerability value of the critical scenario equals the vulnerability value of the base scenario.
- $WCV > 1$ when the *critical scenario* represents a network suffering capacity reductions. By definition, WCV should be bigger than 1.
- $WCV < 1$ only in the case of testing enhancement projects that end up with a *critical scenario* more suitable than the *base scenario*. Note this situation should only be possible in theoretical situations.

Worse-Case-Vulnerability Problem (WCV-P) was formulated as a mathematical model to minimise WCV. The decision variables of the problem are the status of the links during the *critical scenario*. Additionally, the model considers that the enhancement costs are limited by budget. Thus, the model requires to have estimation of the capacity losses during the *critical scenario* as well as the costs for developing the solutions that will avoid future reductions of capacity.

A solution for the WCV-P was generated using the metaheuristic Simulated Annealing coupled with a user equilibrium based traffic assignment solved by the Method of Successive Averages. This solution was tested using a network representing a segment of the urban road network of the City of York in UK.

5.1. Policy implications

There are three main implications to the policy aiming at improving the resilience of a road network:

- Agencies involved in post disaster reconstruction planning as well as the local authorities facing potential threat of disruption due to incidents can develop a plan to minimise the disruption to normal life/business by solving the WCV-P. While developing the plan they can consider the constraint on budget availability too and develop robust solutions involving identifying a set of road links with an extent of capacity to which they need to be maintained/developed.
- In almost all cases, outcomes of Simulated Annealing reduced WCV using a smaller percentage of available funds compared to 'volume-priority' heuristic approach, indicating higher returns per £ spent.
- Simulated Annealing allowed more streets for improvement in its list of priority roads than the 'volume-priority' approach, which signifies that more neighborhoods and thus larger population to benefit from the authority's planning efforts thus distributing the benefits equitably across different parts of the city.

Simulated Annealing is an algorithm easy to implement and quick to run. It does not require specialist software though a few notions of programming are helpful. The algorithm can be implemented on a personal computer and the average time required to arrive at a solution, with a network of 251 links and 110 junctions, is 11.6 minutes. Thus, computing resources required are not too onerous to arrive at an efficient solution to improve the resilience of a network.

In this study, hypothetical capacities were generated for each status of links. It was done as a means to obtain sufficient data to assess the performance of the network. However, the model is formulated to receive data from observation of real variation in capacities during *critical scenarios*.

Finally, the demand was assumed to be fixed as the model considers peak hour during when the demand is inelastic. However, for off-peak applications with discretionary trips, it would be pertinent to consider elastic demand by introducing an objective function that combines WCV and consumer's surplus.

Acknowledgements

The authors acknowledge the financial support received from the SENECYT, Ecuador for funding the studies of the first author at ITS, University of Leeds, UK.

Authors' contribution

Specific contributions of this paper are listed as below.

- Extends the critical link analysis to a combination of critical links to improve resilience;
- Directly minimises an objective function involving a measure of vulnerability;
- Simulated Annealing based solution method has been used to arrive at near global optimal solution to the vulnerability minimising problem;
- Considers a range of budgetary constraints for reinforcing the network; and
- Compares 'volume-priority' heuristic approach with vulnerability optimisation outcomes.

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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