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Flexible transit routing model considering passengers' willingness to pay

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Abstract

This paper proposes an alternative flexible transit model with two levels of bus stops, A level and B level. A-level bus stops are fixed, while B-level bus stops are flexible and provide service only when passengers indicate a strong willingness to pay (WTP). This fare structure encourages passengers to choose bus stops with their mobile phones or computers. An optimization model of 0-1 integer-programming is formulated based on whether certain B-level stops can be serviced. With a numerical example, we compare the performance of the proposed traversing method and a tabu search algorithm, both of which are adapted to solve the model. Finally, a real case is provided to evaluate the proposed transit system against comparable systems (e.g., a fixed-route transit system and a taxi service), and the result shows that the flexible transit routing model will help both passengers and bus companies, thus creating a win-win situation.

Keywords: Public transit; flexible transit system; willingness to pay; optimal model; routing problem

1. Introduction

Urban public transport has the advantages of being efficient, reducing energy consumption and being environmentally friendly. However, conventional public transit structures are designed to contain fixed bus routes and predetermined bus stops. Moreover, the spacing between bus stops tends to be relatively large to reduce the total system cost[1], which causes long walk distances for passengers to and from the bus stops.

With the increased availability of the smartphone and electronic payment, a flexible transit system based on Internet service is gaining popularity in the public traffic (PT) field. It is possible for different user groups (e.g., the elderly, disabled, pregnant, and high-income groups) to choose their ways of participating in bus travel. These different users have different requests and degrees of willingness to pay (WTP) for bus travel. Their various WTPs reflect their specific demands, such as shorter walking distances, reduced waiting times or better service.

According to reports from different cities, many passengers were willing to consider flexible transit services as a travel option[2]–[4]. A number of bus companies have focused on flexible transit operating modes, such as DIDI minibus[5] Ruyue bus[6], and DaDa bus[7] in China. The DIDI minibus is a seven-seat car without a fixed route that is similar to a succession of passenger carpools (DIDI express), while Ruyue and DaDa bus are flexible transit systems by which bus companies collect passenger demands and design bus lines in advance. Flexible transit is also becoming popular in the US; both Uber and Lyft are trying to make some headway in it. Uber put up a service called UberHop[8], which is designed around fixed pick-up and drop-off points, just like a bus (similar to the minibus service provided by DIDI in China). Lyft[9] works with public transit agencies across the country to eliminate transportation barriers and has launched a new model for suburban mobility. Unfortunately, these market modes lack a method of drawing a compromise between the existing fixed bus lines and flexible bus lines.

Koffman[9] and Potts et al.[10] presented some common underlying features of a flexible transit system. Typically, in a flexible transit system, only some parts of the service are flexible, while some aspects of the existing fixed transit system are maintained. Proper flexible bus services that consider WTP have the advantages of being customizable, useful and necessary complements to conventional bus services. However, due to the lack of mature mobile Internet service and 'pay-by-mobile' technology, the flexible transit services of the past have lacked a flexible fare structure associated with the heterogeneity of passengers' WTP.

1.1 *Literature review*

Many researchers have focused on flexible transit systems. Flusberg[11] described the concept of the demand-responsive transportation system implemented in Merrill, Wisconsin. Daganzo [12], [13] estimated the system performance in many-to-one and many-to-many demand-responsive services and presented a preliminary study of the feasibility of checkpoint dial-a-ride systems.

In recent years, flexible transit services have been recognized as one potential solution under medium-demand or low-demand conditions in newly developed urban/suburban areas [10], [14], [15]. Quadrifoglio and Li[16] analyzed the optimal number of zones in designing feeder bus services using fixed-route and demand-responsive services. They provided a closed-form solution for regular services and an approximation formula for flexible services. The results were verified with simulations. As an extension, they explored the demand threshold between fixed-route and demand-responsive services with one vehicle. A limitation of their models was their assumption that the entire region was covered by one vehicle for both fixed-route and demand-responsive services[17], [18]. Kim and Schonfeld [19] proposed an analytical model framework for comparing operations among fixed-route only, flexible route only, and an integrated service that temporally switches between the two during peak and off-peak periods. Nourbakhsh and Ouyang[1] developed a new structured flexible-route transit system that works well at low-to-moderate demand levels. The theory works in flexible transit systems with varying degrees of success. The review by Errico et al.[20] summarizes the literature published before 2013 on semiflexible transit systems and offers a unifying model framework for representation and planning. Hickman and Blume[21], Fu[22], and Tang et al.[23] also studied transit service system optimization. However, seldom have these studies developed a compromise between the existing fixed bus line systems and the flexible travel demands in a specific service area. Achieving a win-win situation for bus companies and passengers remains a challenge for transit operators.

With the spread of smartphones and electronic payments to all aspects of life, operating systems for instant information exchange and more flexible fare structures have become available via mobile phone apps and electronic payments. Subsequently, several studies of flexible public transit structures and strategies have been conducted under this new background [24]–[27]. The transit ridership is usually sensitive to fares, travel times, wait times, and access times, among other factors[28]. Some studies have revealed that flexible transit services are promising operating policies for shaping new travel patterns in low-demand areas[28]–[31] and that passengers are generally willing to use these innovative transit systems[24], [25]. Many of these systems are well-structured flexible transit systems, but these systems do not precisely combine the actual characteristics of a flexible transit system with the heterogeneity characteristic of passengers' WTP; if we combine these two aspects, the bus company could earn more money, and passengers could get better personally designed service.

WTP is a consumer's willingness to pay for certain consumer goods or services based on the consumer's subjective appraisal of their worth. WTP is widely used when analyzing the public good of transportation systems[32]–[35]. Many studies on WTP in different cities reached a consensus that a good portion of the respondents were willing to consider flexible transit services as one of their travel options, including 40% in Merrill, Wisconsin[2], 60% in the San Francisco Bay area, California[30], and 60% in Jinan, Shandong province of China[4]. More specifically, Jie et al.[4] calculated the WTP of 2403 passenger samples collected in the Jinan Qilu Software Park, and reached a result that respondents are willing to pay an average fare of 4 Chinese Yuan (0.60 U.S. dollars equivalent) per trip to use a flexible transit service. Moreover, to our delight, respondents expressed a higher preference (65.5%) for a service that has the fixed-route attribute but with easy-to-access locations for pick-up/drop-off points, for which the flexible system of this paper is exactly suitable.

1.2 *Objectives and contributions*

Better service deserves higher fares. Passengers can receive a higher level of service if they have a higher WTP. Additionally, bus companies can make more income if they can provide more customized service and offer a higher quality of service. This paper aims to integrate these interesting ideas (i.e., flexible transit service, passenger heterogeneity in their WTP) into the design of a flexible-route transit system. In a fixed transit system, all the bus stops are of the same class and are served without considering the net operating income of these stops, thus leading to a low benefit. Therefore, in this paper, we propose a flexible-route transit system with two levels of stops (A-level bus stop and B-level bus stop), as shown in Fig. 1. A-level bus stops are fixed-service stops that are served by all the buses, while B-level bus stops are flexible and provide service only when there are active requests from passengers and a high WTP.

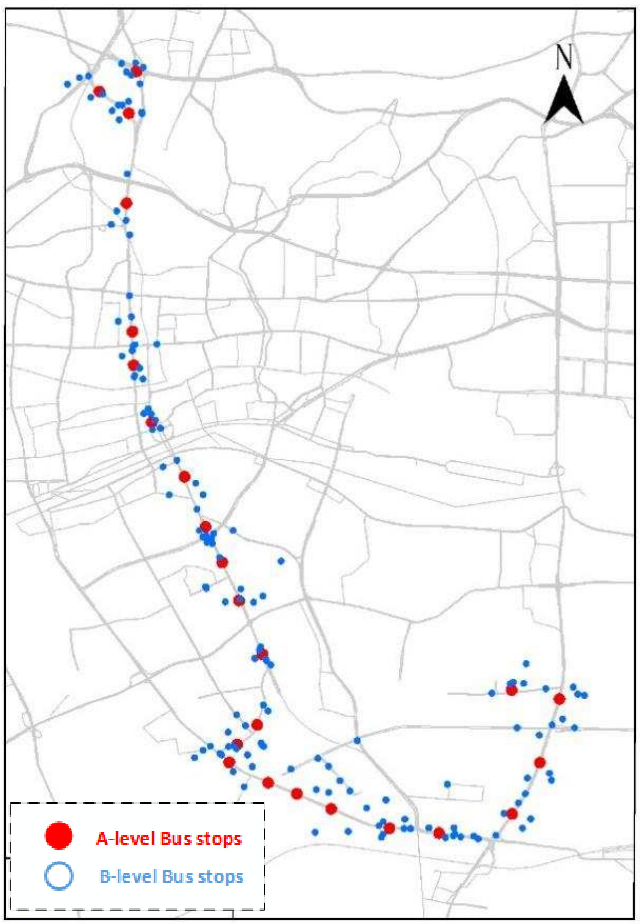


Fig. 1 Schematic diagram of two levels of stops

With the increased availability of the mobile Internet and instant messaging (IM), it has become possible to process simple requests and responses [36]. Internet-based bus service is a future star of the PT field, but it might take a long time for passengers to become accustomed to making their reservations online; therefore, it is essential to select a receptive operation mode. This mode can coordinate the existing mode of waiting at the bus stop directly with a new mode of making personal requests in advance. Such an appropriate system for passengers can lead to a win-win situation for passengers of both of these modes, especially in areas that lack para-transit systems and during off-peak hours.

The remainder of this paper is organized as follows. Section 2 outlines the methodology of the model. Section 3 addresses a numerical example, in which we compare the performances of the proposed traversing method with the tabu search algorithm and then analyze the sensitivity to the relevant parameters. Section 4 presents a real-world case of the No. 87 bus in Guangzhou and compared it with alternative transit systems. Section 5 concludes the paper.

2. Methodology

2.1 *Operation characteristics*

In this paper, two levels of bus stops (A level and B level) are considered. As shown in Table 1, A-level stops are fixed, and passengers can board at A-level bus stops whether they make reservations or not. However, they are encouraged to book in advance because the bus company would consider the time cost of passengers at the A-level bus stop only when they made reservations in advance.

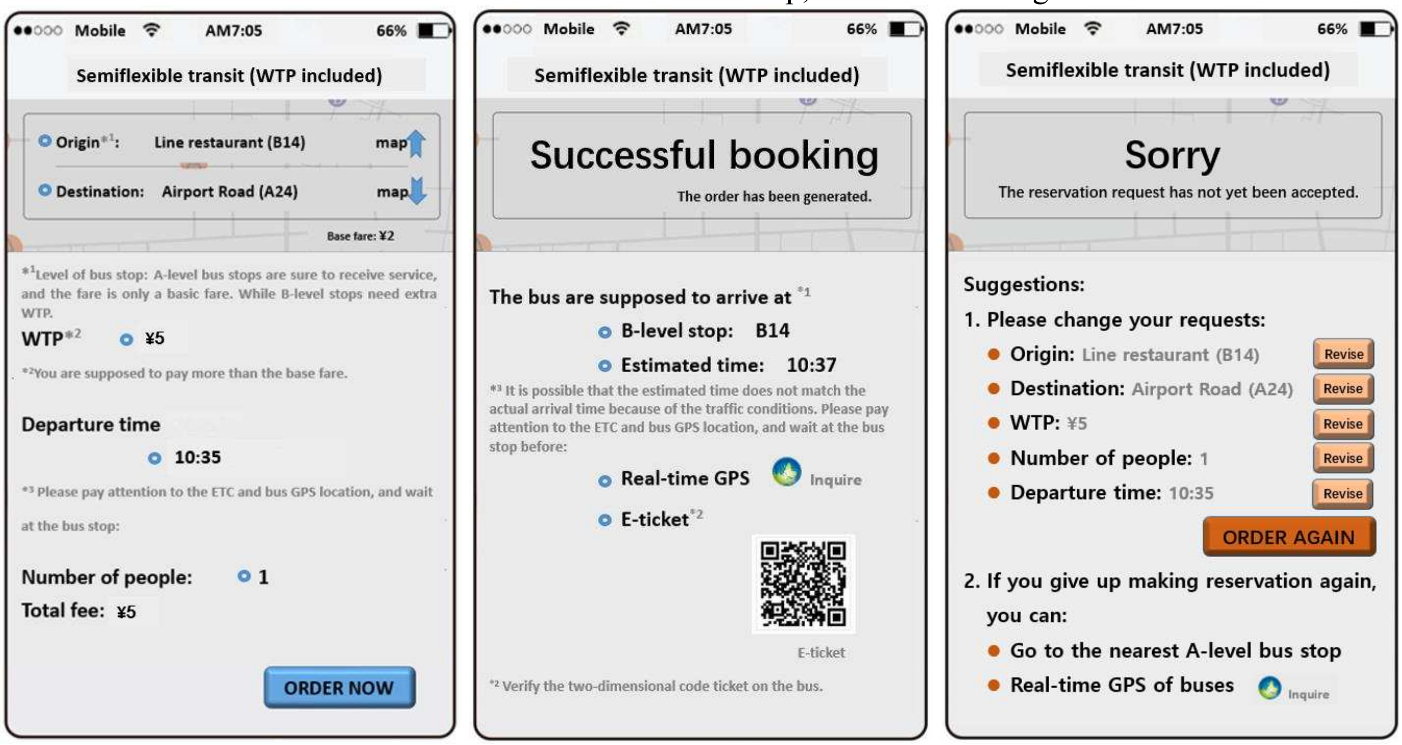
In contrast, all passengers at B-level bus stops must make requests online. B-level bus stops are flexible and provide service only in the case of reservations and sufficient WTPs. All passengers, whether they board at A or B level bus stops, can alight from the bus only at A-level stops. Regarding the fare structure, passengers who wish to board at A-level bus stops would pay for only the base fare from their point of origin to their destination, but passengers at B-level stops are supposed to make a reservation indicating their travel origin and destination (OD), departure time and their total WTP (no less than the base fare).

Table 1 Boarding and alighting strategies

Bus stop level	Boarding		Alighting	WTP required
	Direct waiting	Book in advance		
A	√	√	√	×

B	×	√	×	√

In this system, all the passengers show their WTP individually (without considering and knowing the WTPs of other passengers) and book with an understanding of the rules of operation. Furthermore, all passengers should ensure that their booking account has sufficient funds and automatically accept the rules of bus fare deduction. Fig. 2 (a) shows a reservation request on the App. There is a response deadline to stop receiving new orders. It takes a few seconds for the system to calculate the best route (generally no more than 30 seconds with a tabu search). Next, every passenger would get a notice of a successful reservation or rebooking suggestions. If passengers book successfully, as shown in Fig. 2 (b), they will receive a notice of their bus details and E-ticket. If a passenger's service request is not accepted, they will receive an apology notice as well as some suggestions, as shown in Fig. 2 (c), which may encourage them to raise their personal WTP and place another order or wait at the nearest A-level bus stop, where the bus is guaranteed to arrive.



(a) Make reservation **(b) Booking success** **(c) Booking failure**

Fig. 2 App schematic

2.2 *Notation and definition*

To simplify the model, we assume that all passengers in the system travel rationally and that they would not give up taking a bus once they made a reservation successfully. Table 2 defines the notation used in this paper.

Table 2 Notation

Symbol	Definition
Sets and indices	
A	Set of A-level bus stops, $A_n = \{A_1, A_2, \dots, A_m\} \in A$
B	Set of B-level bus stops, $B_i \in B$
B_i	Set of B-level bus stops, $B_i \in B$
L	Set of bus line connections, $L_{ij} \in L$
R	Set of passengers at bus stop B_i, $r \in R$
K	Set of selected B-level bus stops between bus stop A_n and bus stop A_{n+1}, $K = \{\forall i, x_{B_i} = 1\}$, $K \in B$
Parameters and functions	
Y	Total system income (Yuan)
P	Total ticket income (Yuan)

P_A	Total ticket income of A-level bus stops (Yuan)
P_B	Total ticket income of B-level bus stops (Yuan)
C	Total cost of the system of operation (Yuan)
C_1	Base operation cost (Yuan/km)
C_2	Time cost of passengers on the bus (Yuan/km)
C_3	Time cost of passengers who have already made a reservation at the following A-level bus stops (Yuan/km)
P_{OD}^0	Base fare from the origin bus stop (O) to the destination bus stop (D)
C_A	Cost of all A-level bus stops (Yuan)
C_B	Cost of all deviated B-level bus stops (Yuan)
C_{B_i}	Total cost of deviation of bus stop B_i (Yuan)
n_{B_i}	Number of passengers on the bus over a road segment of B_i
$n_{A_n}^O$	Number of passengers that board at A_n
$n_{B_i}^O$	Number of passengers that board at B_i
$n_{A_n}^D$	Number of passengers that alight at A_n
ΔL_{B_i}	Relative detour distance of B_i
P_{B_i}	Minimum WTP of all passengers at stop B_i (Yuan)
$P_{B_i}^r$	WTP of passenger r at bus stop B_i (Yuan), $r \in R$
θ	Bypass distance ratio
ς	Passenger load factor, $\zeta_{min} \le \zeta \le \zeta_{max}$
Q	Vehicle capacity
N	Maximum number of B-level stops that serve as a bus tour
Δ	Absolute difference of total system income between a tabu search and the traversing method (Yuan)
Y_{Tabu}	Total system income with a tabu search (Yuan)
Y_{Tm}	Total system income with the traversing method (Yuan)
δ	Percentage of absolute difference Δ
α	Strength of WTP
variables	
X	B-level bus stops, $X = \{x_{B_1}, x_{B_2}, x_{B_3}, \ldots, x_{B_i}, \ldots, x_{B_I}\}$
x_{B_i}	Binary variable:$x_{B_i} = 1$ if B-level bus stop i is served; $x_{B_i} = 0$ otherwise. $x_{B_i} \in X$

2.3 *2.3 Formulation*

In this model, Eq. (1) shows that the system income consists of two main parts: ticket income and B stops deviation cost (we omitted some fixed cost of this operation system).

$$\max_{x_{B_i} \in X} Y = P - C_B \tag{1}$$

In Eq. (2), the system income (Y) includes ticket fares from A-level and B-level bus stops. Eqs. (3) and (4) show that the ticket income of the A-level bus stops is only the base fare, and the ticket income of the B-level bus stops is from their base fare and the extra WTP amount. The base fare (P_{OD}^0) varies with the mode of the fare structure in different cities.

$$P = P_A + P_B \tag{2}$$

$$P_A = \sum_{O \in A} \sum_{D \in A} P_{A_O} * n_{A_O A_D}, \mathrm{O} < \mathrm{D} \tag{3}$$

$$P_B = \sum_{B_i \in B} \sum_{A_n \in A} \left(P_{B_i A_n}^0 + P_{B_i}\right) * n_{B_i A_n} * x_{B_i} \tag{4}$$
$$x_{B_i} \in X, B_i \in B, A_n \in A$$

In Eq. (5), because all the A-level bus stops are sure to be served, the operation cost of the A-level bus stops is fixed. To simplify the model, we omitted the operation cost of the A-level bus stops (C_A) from the optimization process. The total cost of the operating system is thus the sum of all the B-level bus stops (C_B) that are served in this system and the increased cost incurred by the B stop because of the detour. The total cost of each bus stop B_i has three components in Eq. (6): the base operation cost (C_1), the time cost to all passengers on the bus (C_2) and the time cost of passengers who have already booked at the following A-level stops (C_3).

$$C_B = \sum_{B_i \in B} C_{B_i} * x_{B_i}, x_{B_i} \in X, B_i \in B \tag{5}$$

$$C_{B_i} = \Delta L_{B_i} * (C_1 + n_{B_i} * C_2 + \sum_{f=n+1}^{N} n_{A_f}^{O} * C_3) \quad (6)$$
$$B_i \in B, A_f \in A, f = n+1 \leq m$$

Eq. (7) counts the passenger number on the bus over a road segment of B_i.

$$n_{B_i} = \sum_{k=1}^{n} n_{A_k}^{o} + \sum_{s=0}^{i} n_{B_s}^{O} * x_{B_s} - \sum_{k=1}^{n} n_{A_k}^{D} \quad (7)$$
$$A_k, A_n \in A, 0 \leq k \leq n$$
$$B_s, B_i \in B, 0 \leq s \leq i$$

Eq. (8) states the value of ΔL_{B_i}, which is the detour distance of B_i. In the region between bus stops A_n and A_{n+1}, the B-level bus stops run from B_p to B_q. If there are many B-level bus stops between A-level bus stops A_n and A_{n+1}, the route between the B-level bus stops is determined according to the Dijkstra shortest path algorithm.

$$\Delta L_{B_i} = \left(L_{A_n B_p} + \sum_{p \leq i \leq j \leq q} L_{B_i B_j} + L_{B_q A_{n+1}} - L_{A_n A_{n+1}} \right) * \frac{L_{A_n B_i} + L_{B_i A_{n+1}}}{\sum_{p \leq i \leq q} (L_{A_n B_i} + L_{B_i A_{n+1}})} \quad (8)$$
$$B_i, B_j, B_p, B_q \in K, p \leq i \leq j \leq q$$
$$A_n, A_{n+1} \in A$$

2.3.1 Minimum WTP

For each B-level bus stop, there is a minimum WTP (P_{B_i}) because of the detour of the main bus line. The extra WTP amount raised by passengers at B_i should be no less than the total cost of the detour. Eq. (9) shows this restriction, and Eq. (10) is the collection of all the individual WTP amounts at bus stop B_i at the same time.

$$P_{B_i} \geq C_{B_i}, B_i \in B \quad (9)$$
$$P_{B_i} = \sum_{r \in R} P_{B_i}^{r}, B_i \in B, r \in R \quad (10)$$

2.3.2 Service constraints

A few rules are established to effectively limit the length of the bus line, such as the limitation of the detour ratio in Eq. (11) and the restriction of the maximum number of B-level stops that are served in a bus tour in Eq. (12).

$$\frac{L_{A_n B_p} + \sum_{p \leq i \leq j \leq q} L_{B_i B_j} + L_{B_q A_{n+1}}}{L_{A_n A_{n+1}}} \leq \theta \quad (11)$$
$$B_i, B_j, B_p, B_q \in K, p \leq i \leq j \leq q$$
$$A_n, A_{n+1} \in A$$
$$\sum_{B_i \in B} x_{B_i} \leq N, B_i \in B \quad (12)$$

We also constrain the passenger load factor between ζ_{min} and ζ_{max} in Eq. (13) to keep the system operating smoothly.

$$\zeta_{min} \leq \frac{n_{B_i}}{Q} \leq \zeta_{max}, B_i \in B \quad (13)$$

2.4 *Method of solution*

The model is a 0-1 integer-programming problem, $x_{B_i} = 1$ if B-level bus stop i is served; $x_{B_i} = 0$ otherwise. The solutions of this type of problem fall into two categories: precise algorithms [36] and heuristic algorithms[37]. The enumeration method, which is a precise algorithm, fails when the number of situations in the corresponding case is too large. The heuristic algorithm generates an initial solution according to the given constraints and then improves the performance of the initial solution based on an algorithm to obtain a satisfactory solution. The algorithms that optimize the initial solution are mostly intelligent algorithms, such as tabu search[38]–[41], ant colony[41]–[43], simulated annealing[44]–[47] and genetic algorithms[47]. A tabu search is a metaheuristic that guides a local heuristic search procedure in exploring the solution space beyond local optimality. The tabu search algorithm has a strong ability to solve this problem and was used by Ruisanchez et al.[47] to efficiently solve bilevel optimization public transport route models.

In this paper, when the feasible solution range is relatively small, we utilize the traversing method, and a coding scheme is proposed based on the sequence of the complete binary tree of this method. There are approximately 2^n total feasible solutions. When the feasible solution range is broad, we apply the tabu search algorithm to deal with these cases to meet the need for fast solutions in a practical application. We present the general framework of our tabu search algorithm below:

1) Find an initial feasible solution:

We solve a current solution (we start from $x_{B_i} = 0$), then search several solutions in the neighborhood of this current solution and finally choose one of the best solutions as the new current solution. In addition, in this paper, there is a maximum for the total number of B-level bus stops that are served in a single bus tour ($\sum_{B_i \in B} x_{B_i} \leq N$); therefore, we limit the number of B-level bus stops that served, which means there are no more than a certain number of $x_{B_i} = 1$. This rule improves the accuracy and speeds up the calculation.

2) Record the optimal local solution:

To avoid repeatedly searching the optimal local solution, we record the optimal local solution in a search list. When the current solution and its neighborhood are all in the tabu list, we again produce a random solution as the current solution.

3) Compute a termination rule:

Once the optimal solution does not update for a period of 100 steps, we force the termination of the tabu search and output the optimal solution.

3. Numerical example 1

We generated a small numerical example with only 5 A-level stops and 21 B-level stops. The distances between the bus stops are from the real data of the No. 25 bus line of Guangzhou, and the data of the passengers' OD demands, departure times and the WTP of each passenger were determined randomly according to a specific rule (very similar to numerical example 2).

3.1 *Model solution*

The model is solved using both the traversing method and the tabu search algorithm to determine the accuracy of the tabu search in this model. Table 3 shows the details of the optimal solution, and Fig. 3 shows the schematic diagram of a route for case 1. The total system income (Y) using the traversing method is 95.7, whereas the result using the tabu search algorithm is 93.339.

$$\Delta = |Y_{Tabu} - Y_{Tm}| \tag{14}$$

$$\delta = \frac{\Delta}{Y_{Tabu}} * 100\% \tag{15}$$

The result using the tabu search algorithm is slightly lower (the absolute difference $\Delta = 2.361$; $\delta = 2.47\%$) than that of the traversing method, which is acceptable in practical applications. The operating time for the tabu search is only 0.21 seconds, much lower than the 2918.8325 s for the traversing method.

Table 3 Optimal solution for case 1

	Tabu search result	**Traversing method result**
Best X	[1 1 1 1 1 0 0 1 1 0 0 0 0 0 0 0 1 0 0 0 0]	[1 1 1 0 1 1 0 1 1 0 0 0 0 0 0 0 1 0 0 0 0]
Steps	108	2097151
Update steps	8	17
Time	0.21 s	2918.8325 s
Best Y (Total system income)	93.339	95.7
		$\Delta = 2.361; \delta = 2.47\%$

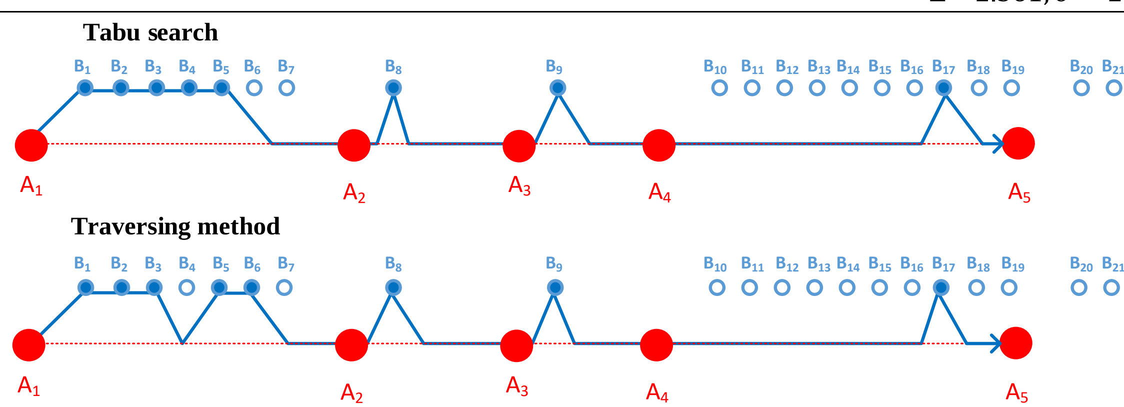


Fig. 3 Schematic diagram of a route for case 1

As we can see from Fig. 3, the routes produced by the traversing method and the tabu search in the A_1A_2 district are slightly different. The traversing method is the precise solution and selects bus stops B_1, B_2, B_3, B_5, and B_6, while in the tabu search, the optimal result is B_1, B_2, B_3, B_4, and B_5. We also find that the absolute difference is only 2.47%, which is acceptable in a real-world case when a quick response is required.

3.2 *Sensitivity analysis*

The result of the optimization is closely related to the parameters chosen; therefore, we analyzed the sensitivity of the following vital parameters:

- P_{B_i} (WTP)
- C_2 and C_3 (Time cost)
- N (Maximum B-level stops served)

3.2.1 Strength of WTP

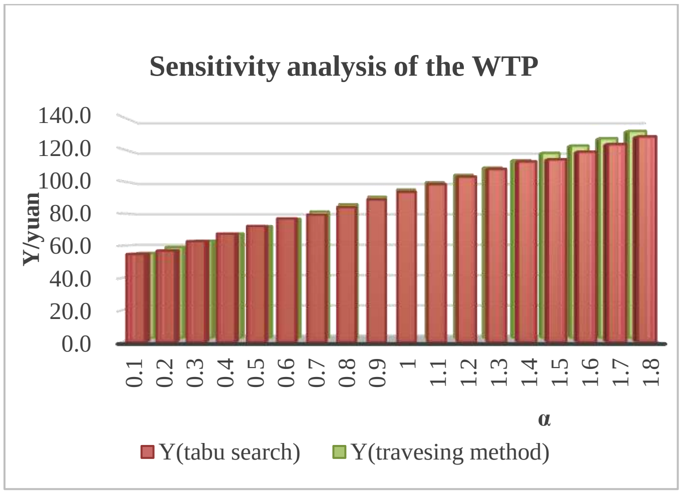


Fig. 4 Sensitivity analysis of the WTP

Table 4 Regression analysis

	Equation	R-squared
Traversing method	$Y = 46.655 * \alpha + 49.171$	0.99985
Tabu search	$Y = 43.178 * \alpha + 50.3024$	0.99837

Upon increasing the strength of the WTP ($0.1 \leq \alpha \leq 1.8$), the value of Y increases, as shown in Fig. 4. We solved the model with different strengths of WTP by means of the traversing method and a tabu search and reached similar values for the total system income (Y). The line chart indicates the following results:

(1) Y and P_{B_i} are positively correlated. We obtained the equations in Table 4 by performing a regression analysis of Y with the strength of the WTP. In this regression analysis, the sample shows good linearity; R-squared is 0.99985, which can better reflect the appropriate characteristics. The value of Y is logically sensitive to increases in the WTP.

(2) We tested twenty-one situations and found that the tabu search is effective in solving this model. We compared the results of the traversing method and the tabu search, and in 4 of the 18 (22.22%) situations, the results from the traversing method are identical to those of the tabu search. The greatest gap in the set of results occurred when $\alpha = 1.4$ ($\Delta = 5.8$, $\delta = 4.87\%$), which means that the precise algorithm and the heuristic algorithm both work well in this case.

3.2.2 Time cost analysis

The parameters of the time cost (C_2: passengers on the bus; C_3: passengers waiting at the following A-level bus stops) are very important to the total system income; therefore, we performed a sensitivity analysis on C_2 and C_3 ($0 \leq C_2 \leq 1$, $0 \leq C_3 \leq 1$) with 10,000 combinations. The value of Y (solved via a tabu search) varies with the values of C_2 and C_3, as shown in Fig. 5. The following results were reached:

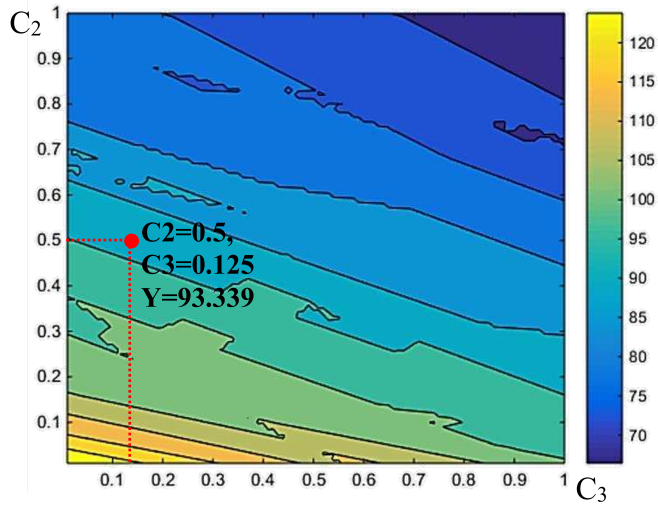


Fig. 5 Sensitivity analysis of the time cost

(1) The optimal model decreases when C_2 or C_3 increases.

$$Y = -37.461C_2 - 14.187C_3 + 115.0685 \quad (16)$$

R-squared is 0.96498 in the regression analysis. In Eq. (16), the sensitivity of the time cost to passengers on the bus (C_2) is significantly greater than that of the time cost to passengers with reservations at the subsequent A-level stops (C_3).

(2) When parameters C_2 and C_3 range from 0 to 1, the values range from 60 to 130, which is an appropriate interval. We chose $C_2 = 0.5$ and $C_3 = 0.125$ as the median parameters.

(3) At many points, the local optimal solution changes. The nearly smooth border in Fig. 5 is also strong evidence of the effectiveness of the tabu search algorithm.

3.2.3 Parameter N analysis

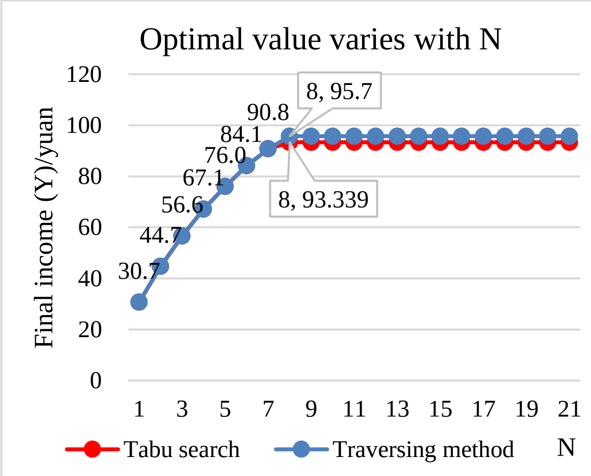


Fig. 6 Sensitivity analysis of parameter N

An excessive number of B-level stops is undesirable because the passengers will suffer if the new flexible bus system wastes too much time. Fig. 6 provides details. We found that the value of Y increases when $N \leq 8$, although Y does not always increase with an increase in N ($N \leq 8$).

We also compared the results of a tabu search with those of the traversing method: when $N \leq 7$, the results of the traversing method were identical to the results of the tabu search; when $N \geq 8$, the results were slightly different ($\Delta = 2.361$; $\delta = 2.47\%$).

Upon analyzing the vital parameters P_{B_i}, C_2, C_3 and N, we reached the same conclusion that the tabu search is an effective optimization solution method for this model. Thus, we can use a tabu search instead of the traversing method when the time is limited or when the case is complicated.

4. Numerical example 2

Case 2 is a case involving the No. 87 bus in Guangzhou. This bus line starts from Yijing bus stop and ends at Airport Road bus stop and has a history from the year 1980. Though many people work and live along the bus line, the metro stops there are inconvenient. Table 5 shows detailed information on this bus line.

Table 5 Information on the NO. 87 bus line

Name	Guangzhou NO. 87 bus
Operating time:	06:00-23:00
Ticket price:	￥2
Starts and arrives:	Yijing Stop →Airport Road Stop
Peak hours:	7:00-9:00 am, 18:00-19:00 pm
Line length:	17.4 km
Number of A stops:	24
Bus company:	Guangzhou NO. 1 bus company

4.1 *Data preparation*

4.1.1 Bus stops and distance

In this case, all the A-level bus stops are from the existing bus stops (24 bus stops in Fig. 7).

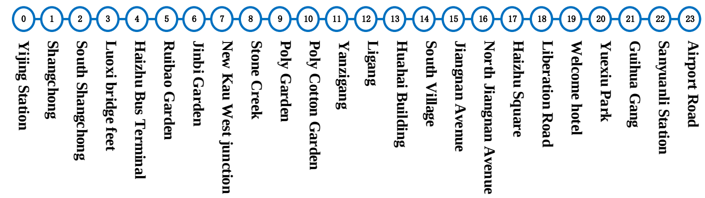


Fig. 7 A-level bus stops

We also selected 138 potential B-level stops. These potential B-level stops were determined based on real nearby demands. The B-level stops are sufficient and covered all the travel request origins of passengers. Passengers can choose wherever they want as their departure place, and it would be included in the set of B-level bus stops. In Fig. 1, they are along the existing A-level bus line, and most of them are located near the entrances of residential areas, supermarkets, drugstores, office buildings, primary schools, kindergartens, etc.

Then, we invoke the Accurate Position Indicator (API) from Gaode Map (an online mapping service in China) to obtain the real bus route distances for the 162 stops and estimated the travel time between any two points using an average bus speed of 30 km/h.

4.1.2 Demand requests

According to the IC-card data from March 2017, we found that the No. 87 bus line in Guangzhou is commute-characterized. The bus departure intervals are nine minutes, and there is an average of 13.37 bus shifts per day (9 minutes per shift), with an average of 43.67 passengers for a whole bus shift during off-peak hours and 95.22 passengers during peak hours. Then, we selected data from the peak hours and off-peak hours and obtained the average A-level demand requests separately. We select an average day to determine the A-level bus stop demand requests.

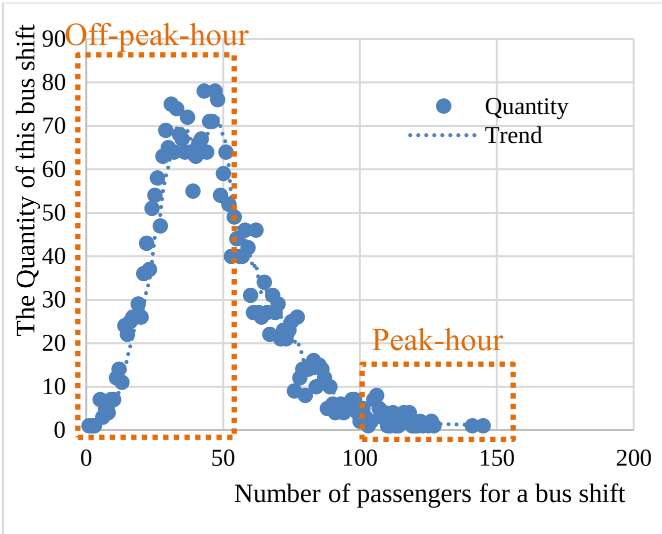


Fig. 8 Bus IC-card data of each NO. 87 bus in Guangzhou (March 2017, the whole 30 days)

The demand for the A-level OD matrix comes from the data, while the B-level demand is generated randomly according to the following rules:

We considered the land use of each B stop (as shown in Table 7, we marked them with residence, workplace, or business). The properties of the land around B-level stops can help us generate the initial range of random numbers. In the morning peak, different initial number ranges were used for residence (0-8), workplace (0-4), and business (0-4). Then, the distance (between the B-level bus stop and its nearest A-level bus stop) determines its conversion factor: for the distances from 0 to 300 meters, 300 to 500 meters, 500 to 1000 meters, and over 1000 meters, we multiply the values by different conversion factors.

Table 6 Number of B-level stops with particular land properties

Land properties	Level 1* (High)	Level 2 (Medium)	Level 3 (Low)	Total
Workplace	22	13	22	49
Business	28	11	10	49
Residence	17	19	13	40
Total	67	43	28	138

*: Housing/rent price levels (High, Medium, Low), All housing price data are obtained from https://guangzhou.anjuke.com/

4.1.1 WTP generation

In this case, it is hard to get the real B-level OD demands and WTPs because there are no B-level bus stops in the market. Thus, we developed a series of rules and generated these data randomly according to the duration of the transit time, the distance from the nearest bus stop, and nearby housing prices.

All passengers have a WTP within an initial random range that is generated according to the length of their transit time (less than four stops, five to ten stops and more than ten stops have different initial WTPs). Then, the distance conversion factors between each B-level bus stop and its nearest A-level bus stop are also multiply to the initial WTP. Finally, the housing/rent price also provides conversion factors: high, medium and low housing/rent prices each have different values. These steps result in the generation of each passenger's WTP.

Then, we examined the randomly generated WTPs with a questionnaire survey in Guangzhou and references from the past literature. The survey is about the WTPs of passengers at some B-level bus stops. Moreover, we performed a comparison of them and found that the generated WTPs are acceptable.

Table 7 WTP comparison and modification

City	WTP (survey)	WTP*4 (generated)	Percentage error
San Francisco [3] *1	$10.00	-	-
Jinan [25]	¥4	-	-
Guangzhou (B2) *2	¥4.33	¥4.35	0.46%
Guangzhou (B72) *3	¥3.85	¥3.85	0%

*1 $10.00 for a 30-min trip, based on a survey conducted in the San Francisco Bay area in 2004.
*2*3 based on a survey we did in early 2018.
*4 The generated WTP of this paper (WTP modified)

The WTP for a flexible transit service was relatively high, and over half of the respondents showed a WTP for a flexible transit service. By using the method of propensity score matching, we obtain a series of acceptable WTPs for all the passengers. Though these WTPs lack data (i.e., not all the B-level bus stops were included in the survey), it is still informative and provides an excellent reference for this numerical example. We can calculate the model using these data.

4.2 *Results and evaluation*

We chose parameters for the analysis according to the bus capacity ($Q = 75$), bus fare structure ($P_{OD}^0 = 2$) and passenger tolerance ($\theta = 3, \zeta_{min} = 0.3, \zeta_{max} = 2, N = 10$) in Guangzhou. According to the characteristics of Guangzhou, we selected various values for C_2 and C_3 (peak hours $C_2 = 1$ and $C_3 = 0.25$, off-peak hours $C_2 = 0.5$ and $C_3 = 0.125$). We will thoroughly discuss the parameter analyses later in section 4.3.

The number of feasible solutions increased to 2^{138}, which was too large for a traversing search method. The tabu search algorithm performed well (115 search steps in off-peak hours and 109 steps in peak hours) and converged quickly (23.12 seconds in off-peak hours and 22.05 seconds in peak hours).

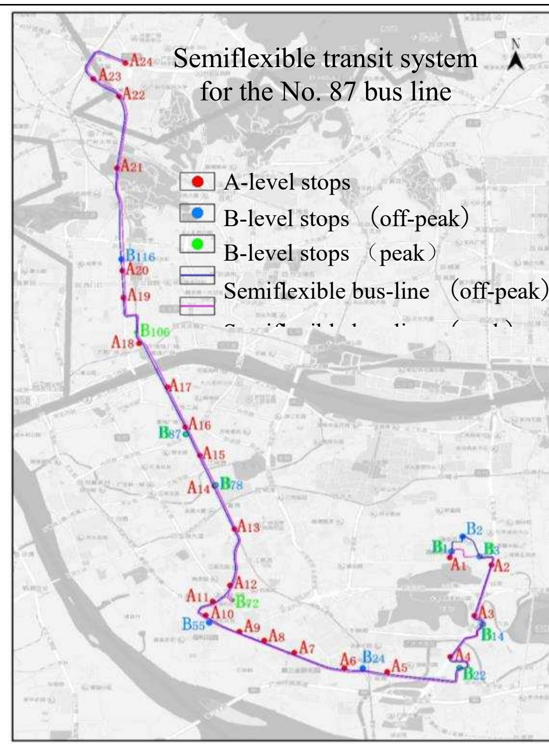
According to the results shown in Tables 5 and 6, eight B-level stops were served during peak hours, and ten B-level stops were required during off-peak hours. The specific B-level stops were different. During peak hours, bus stops B_1, B_3, B_{14}, B_{22}, B_{72}, B_{78}, B_{87} and B_{106} are serviced instead of bus stops B_1, B_2, B_3, B_{14}, B_{22}, B_{24}, B_{55}, B_{78}, B_{87}, and B_{116}. Regarding the specific bus stops, B_2 (where the passengers are residents of level 2 and might have a lower WTP) and B_{24}, B_{55} and B_{116} (in the W area, where there may be fewer passengers during morning peak hours) are serviced only during off-peak hours. Stops B_{72} and B_{106} have more passengers with a higher WTP and a lower detour distance, so they are serviced during peak hours. Moreover, the more buses that arrive at B-level stops, the higher the fare income; however, during peak hours, the time cost to passengers is relatively high, and the model limits the number of bus stops to maintain smooth system operations.

Table 8 Results of case 2

	Y/yuan	Steps	Update steps	B_i
Off-peak hours	355.79	111	11	B_1; B_2; B_3; B_{14}; B_{22}; B_{24}; B_{55}; B_{78}; B_{87}; B_{116}
Peak hours	317.9	108	8	B_1; B_3; B_{14}; B_{22}; B_{72}; B_{78}; B_{87}; B_{106}

Table 9 Analyses of B-level bus stops

		Location between	**Description**[*1]	**Housing prices**[*2]
Off-peak hours ($N = 10$)	B_1	A1-A2	R	High (L1)
	B_2	A1-A2	R	Medium (L2)
	B_3	A1-A2	R	Low (L3)
	B_{14}	A2-A3	W	-
	B_{22}	A4-A5	B	-
	B_{24}	A5-A6	W	-
	B_{55}	A9-A10	W	-
	B_{78}	A13-A14	B	-
	B_{87}	A15-A16	R	Medium (L2)
	B_{116}	A20-A21	W	-
Peak hours ($N = 8$)	B_1	A1-A2	R	High (L1)
	B_3	A1-A2	R	Low (L3)
	B_{14}	A2-A3	W	-
	B_{22}	A4-A5	B	-
	B_{72}	A11-A12	B	-
	B_{78}	A13-A14	B	-
	B_{87}	A15-A16	R	Medium (L2)
	B_{106}	A18-A19	R	High (L1)



[*1] R: Residential area; W: Working area; B: Business area
[*2] L1: high house price; L2: medium house price; L3: low house price

4.3 *Sensitivity analysis*

The sensitivity analyses also highlight that the time cost is closely related to the optimal route of the system. The value of Y varies with C_2 and C_3 in Fig. 8. The total system income (Y) ranges from ¥317.9 to ¥454.79 when the parameters are $0 \le C_2 \le 1$ and $0 \le C_3 \le 0.25$.

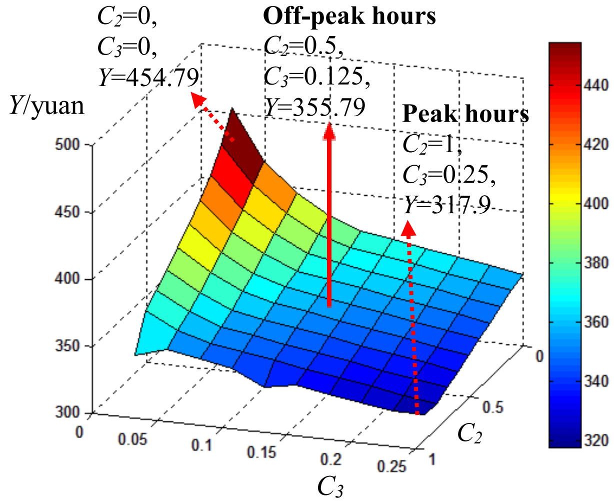


Fig. 9 The sensitivity of the time cost

$$Y = -54.182C_2 - 234.312C_3 + 417.836 \qquad (17)$$

We performed a regression analysis on these parameters. The r-squared is 0.8994, which is acceptable. In Eq. (17), C_2 is four times more effective than C_3. It is proper to choose a parameter relationship close to $C_2 = 4C_3$.

4.4 *Discussion*

In this subsection, we compare the performances of other service-providing systems that are comparable to the proposed flexible transit system.

4.3.1 Fixed-route transit system

In the fixed-route system, the total income comes from the total bus fares. According to a paper written by Nourbakhsh and Ouyang [1], the total costs include those related to the infrastructure investment, total vehicle distance, bus fleet size, passengers' walking times, passengers' wait times, and transfer discomfort. Among these, the walking time of passengers decreased considerably in the flexible system; the total vehicle distance and passenger wait times were already considered in the total system income of the flexible bus system, while the other costs (infrastructure investment and bus fleet size) are the same in these two systems. While the total system income (Y) of the flexible system is already greater than the fare ticket income of a fixed system, we found that the total income increased considerably.

Table 10 Comparison with a fixed-route bus line

Index	**Fixed bus**	**Flexible bus**	
		Off-peak hour	Peak hour
P	¥240	¥417 (+73.75%)	¥364 (+51.67%)
C	0	¥61.21	¥46.1
Y	Less than ¥240	¥355.79 (+48.24%)	¥317.9 (+32.46%)

According to the index comparison of the flexible and the existing fixed-route bus lines in case 2, the flexible transit system got a higher level of income by approximately 48.24% during off-peak hours and 32.46% during peak hours.

4.3.2 Taxi

An increasing number of people need door-to-door service. Passengers spend more money on a taxi if they want to walk a shorter distance from their origin to their transit stop. Therefore, we compared the cost of taking a taxi with that of the flexible transit in this paper. The comparison result is shown in Table 8. The savings are over four times the cost of the flexible transit system.

Table 11 Comparison with taxi

Index	**Taxi**		**Flexible bus**	
	Off-peak hours	Peak hours	Off-peak hours	Peak hours
Passengers paid	¥2501	¥1906	¥417	¥364
Passengers saved	¥2084 (+499.7%)	¥1542 (+423.6%)	-	-

5. Conclusion

In this paper, a flexible-route plan model was proposed with two bus stop levels. A-level bus stops are fixed and identical to those of the existing fixed-route bus line, whereas B-level stops provide a flexible type of bus stop and serve passengers with strong WTP. We considered the relationship between the passenger demand and passenger WTP and formulated a 0-1 integer-programming model based on whether certain B-level stops could be serviced. We compared the performance of the proposed traversing algorithm and the tabu search algorithm in a numerical example, both of which were adopted to solve the model. Finally, a real case was provided to verify the proposed method in a comparison of the proposed transit system with comparable systems (i.e., a fixed-route transit network and a taxi service). The total system income increases by 48.24% during off-peak hours and by 32.46% during peak hours compared to a fixed bus line.

This flexible transit routing model offers the advantages of energy conservation and environmental protection, helps passengers secure improved services, and benefits bus companies. Such an appropriate system can lead to a win-win situation, especially in areas that lack para-transit systems and during off-peak hours.

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