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**Dimensions of Design Space:  
A Decision-Theoretic Approach to  
Optimal Research Design**

**CHE Research Paper 38**



# **Dimensions of Design Space: A Decision-Theoretic Approach to Optimal Research Design**

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## Abstract

Bayesian decision theory can be used not only to establish the optimal sample size and its allocation in a single clinical study, but also to identify an optimal *portfolio* of research combining different types of study design. Within a single study, the highest societal pay-off to proposed research is achieved when its sample sizes, and allocation between available treatment options, are chosen to maximise the Expected Net Benefit of Sampling (ENBS). Where a number of different types of study informing different parameters in the decision problem could be conducted, the simultaneous estimation of ENBS across all dimensions of the design space is required to identify the optimal sample sizes and allocations within such a research portfolio. This is illustrated through a simple example of a decision model of zanamivir for the treatment of influenza. The possible study designs include: i) a single trial of all the parameters; ii) a clinical trial providing evidence only on clinical endpoints; iii) an epidemiological study of natural history of disease and iv) a survey of quality of life. The possible combinations, samples sizes and allocation between trial arms are evaluated over a range of cost-effectiveness thresholds. The computational challenges are addressed by implementing optimisation algorithms to search the ENBS surface more efficiently over such large dimensions.

**Keywords:** Bayesian decision theory; expected value of information; research design; cost-effectiveness analysis

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## 1. Introduction

Health-care systems typically face two conceptually separate, albeit concurrent, questions: firstly, should a health-care technology be adopted (reimbursed) in the light of current evidence about resource use and health outcomes? Secondly, is additional research required to support the use of the technology, and if so how much and what type of research would be most useful? Both questions are at the heart of much of the international debate about the appropriate regulation and reimbursement of health-care technologies. In general, decisions to adopt (reimburse) or reject a technology can be made on the basis of existing information or conditional on providing additional evidence to inform this decision in the future. In the United States the FDA requires claims for a technology to be “sufficiently substantiated”, based on some assessment of the benefits and costs of acquiring more information to support the claim (see Neumann et al., 2000). In England and Wales the National Institute for Health and Clinical Excellence (NICE), which issues guidance to the NHS on the use of technologies, also issues guidance on the need for further research (NICE, 2004). It has made adoption conditional on the production of further evidence and in some circumstances has made recommendations that a technology can only be used in research (Chalkidou, et al. 2007).

Addressing the question of whether and what type of research is needed becomes more challenging when the full range of different types of study and different designs (e.g., sample sizes) which could be conducted are considered. In most situations a variety of different types of study, informing different (groups of) parameters or endpoints, is usually possible. For instance, while randomised controlled trials (RCTs) are useful to investigate those clinical endpoints vulnerable to selection bias, other (not necessarily experimental) studies may be called upon to collect information about the natural history of a disease of interest or some measure of quality of life associated with particular clinical events. To start to address these research decisions, a measure of the social value of the information generated by research is needed.

### 1.1 Methodological background

The Bayesian approach to value of information has a firm foundation in statistical decision theory (see e.g. Schlaifer, 1959; Howard, 1966; Berger, 1985; Pratt et al., 1995; Raiffa and Schlaifer, 2000; Parmigiani, 2002). It provides a coherent framework to establish the social value of additional information, which has been successfully applied in other areas of research (e.g. Hammitt and Cave, 1995; Thompson and Graham, 1996; Thompson and Evans, 1997) and more recently proposed in the evaluation of health-care technologies (see e.g. Claxton and Posnett, 1996; Felli and Hazen, 1998; Claxton, 1999; Yokota and Thompson 2004; Briggs et al., 2006). In particular, an increasing number of applications in this area have incorporated an analysis of the value of perfect information (see e.g., Ades et al., 2004; Fenwick et al. 2006; Claxton and Sculpher, 2006; Philips et al., 2006; and Colbourn et al., 2007; Bojke et al., 2008). However, the demonstration and application of value of sample information analysis are more limited (see e.g. Ades et al., 2004; Willan and Pinto, 2005; Brennan and Kharroubi, 2007ab; Eckermann and Willan, 2008) and to date no attempt has been made to explore all the dimensions of design space available using these methods.

A Bayesian decision-theoretic approach can establish the population Expected Value of Perfect Information (EVPI) surrounding a decision to adopt or reject a technology. Population EVPI places an upper bound on the social value of additional evidence and a necessary condition for deciding to conduct further research, i.e., if the population EVPI is greater than the expected costs of further studies. The EVPI associated with specific (groups of) parameters relevant to the decision problem (EVPPI) can be established in a similar fashion. This can be used to focus further research on those parameters where additional evidence will be most valuable and can indicate the types of studies which may be most useful. However, fully informing the research decision requires a sufficient condition to be established. The Expected Net Benefit of Sampling (ENBS) for a particular type of study and sample size is the difference between the expected value of sample information (EVSI) and the costs of sampling. Therefore, the ENBS is a measure of the social value of a particular proposed research design. It provides a sufficient condition for deciding to conduct more research, i.e., if ENBS is greater than zero. It also provides the means to choose between different types of study and different designs (e.g., sample size and its allocation). The research decision can then be characterised as choosing the type of study(ies) and design(s) which maximises the ENBS.



## 1.2 Design space

In its simplest form, research design can be formulated as a sample size determination problem (see reviews from e.g. Pezeshk, 2003; Chaloner and Verdinelli, 1995): under this perspective, the design problem reduces to identifying the optimal size for each of the studies which could be conducted. The optimal sample size for a single study will be the sample size that generates the greatest ENBS. For studies which include a number of treatment arms (e.g., a RCT comparing a new technology to one or more alternatives) not only the optimal total number of patients to enrol needs to be sought, but also their optimal *allocation* to each treatment arm. Equal between-arm allocation is often adopted based on a simplifying assumption of equal variance for each arm. However, this assumption is seldom justified and, even if it were, the resource and opportunity cost of allocation to different arms is very unlikely to be the same. Therefore, when only considering a single RCT, both optimal sample size and its allocation need to be established (Claxton and Thompson, 2001).

It is seldom the case that a single RCT is the only research possibility. If more than one type of study can be conducted, then optimal sample size and its allocation can be established for each study independently. If these are mutually exclusive alternatives then the research decision is simply to choose the optimally designed single study which offers the highest ENBS. However, in many common circumstances, different types of study are not mutually exclusive and can be conducted concurrently. The social value of a combination of studies (a portfolio) is not the sum of the ENBS of each but must be estimated simultaneously. The portfolio optimisation problem becomes one of finding the allocation of patients between and within studies which provides the maximum ENBS. The optimal portfolio size and allocation will generally be different to that implied by simple combination of independent designs. The research decision problem becomes one of a choice between a number of alternative single studies or an optimally designed portfolio. This choice can be based on the ENBS offered by each.

The purpose of this paper is to demonstrate the principles and methods for the thorough assessment of all these dimensions of the design space. Its aim is to show that Bayesian decision theory can provide a coherent and useful analytical framework to estimate the social value of research, establish optimal research designs and inform the research decision. The paper proceeds by introducing an illustrative example of a simple decision model of zanamivir for the treatment of influenza, reporting estimates of cost and effect based on, the then, current information. The necessary and sufficient conditions for conducting further research reported before optimal research designs are considered in more detail. A single RCT with equal allocation is considered first, and then equal allocation is introduced. Subsequently other types of study are introduced, first optimised independently and then simultaneously as a portfolio. The research decision is then reconsidered once the analyses of all these dimensions of design space have been explored. In doing so, we adopt an optimisation algorithm which eases the computational challenges arising from using Bayesian decision theory to optimise over such large dimensions of design space.

## 2. An illustrative example

The principles and benefits of adopting a Bayesian decision theoretic approach to research decisions is illustrated using a simple example of the use of zanamivir in the treatment of influenza. NICE conducted an appraisal of zanamivir in 2000 and issued guidance (NICE, 2000b) which imposed a number of restriction and conditions on its use. In particular, treatment should only be offered when influenza is circulating in the community and for at-risk adults who present within 36 hours of the onset of influenza like illness. At-risk adults were defined as those over the age of 65 or those with chronic respiratory disease, significant cardiovascular disease, immunocompromised and those with diabetes mellitus.

The NICE appraisal of zanamivir was based on an independent assessment report (Burls et al., 2000, 2002), to which the reader is referred for full details of the evidence available at that time, and the cost-effectiveness analysis. In summary, the assessment report identified nine RCTs but the evidence for at-risk adults is based on sub-group analysis of eight of the all-adult trials and one trial which only recruited adults with chronic respiratory disease. The trials reported time to alleviation of symptoms for the influenza positive population as well as the number of complications requiring antibiotics.

The assessment report included fixed effect meta-analysis for both these endpoints. None of the trials included effect on hospitalisation or economic and quality of life data.

A simple decision tree model was constructed to compare zanamivir with standard care. The proportion of influenza positive patients presenting with influenza like illness (*pip*) was based on evidence from a sponsored submission (Glaxo-Wellcome, 2000). The probability of complications requiring antibiotics and the probability of hospitalisation with standard care (*pcs* and *phs*) was based on observational data (Meier et al., 2000). The reduction in symptom days (*rsd*) was based on the results of the fixed effect meta analysis. The reduction in complications for those receiving zanamivir was based on the log-odds ratio from the same meta-analysis (*LORc*) and a similar effect on hospitalisation (*LORh*) was assumed. The improvement in quality of life due to reduced symptom time (*utl*) was based on an assumed impact on all dimensions of EQ5D. Resource use was based on published unit costs, hospital length of stay and number of GP visits (Netton and Dennett, 1998; Burls et al., 2000).

**Table 1: Parameters of the decision model for zanamivir vs. standard care**

| Parameter   | Prior                             | Likelihood   | Source   |
|---|-----------------------------------|--|--|
| Probability of influenza-positive ( <i>pip</i> )                  | $B(34, 66)$                       | $Bin(n_{Std}, pip)$                                    | Glaxo-Wellcome submission (2000)                   |
| Probability of complications under standard care ( <i>pcs</i> )   | $B(452, 548)$                     | $Bin(n_{Std}, pcs)$                                    | Study based on GPRD (Meier et al., 2000)           |
| Probability of hospitalisation under standard care ( <i>phs</i> ) | $B(25, 975)$                      | $Bin(n_{Std}, pcs)$                                    | Study based on GPRD (Meier et al., 2000)           |
| Complications LOR ( <i>LORc</i> )                                 | $N(-0.353, 3.168 \times 10^{-2})$ | Normal proxy to Binomial (Ades et al., 2004, sec. 4.1) | Fixed effects meta-analysis in Burls et al. (2002) |
| Hospitalisation LOR ( <i>LORh</i> )                               | $N(-0.353, 3.168 \times 10^{-2})$ | Normal proxy to Binomial (Ades et al., 2004, sec. 4.1) | Fixed effects meta-analysis in Burls et al. (2002) |
| Reduction in symptom days ( <i>rsd</i> )                          | $N(1.67, 0.748)$                  | $N(rsd, 363 \times (n_{Std}^{-1} + n_{Zan}^{-1}))$     | Fixed effects meta-analysis in Burls et al. (2002) |
| Annual utility gain ( <i>utl</i> )                                | $N(-0.242, 4.613 \times 10^{-3})$ | $N(utl, 0.185 \times (n_{Std} + n_{Zan})^{-1})$        | Obtained from Burls et al. (2002)                  |

A probabilistic reanalysis of this model was conducted, based only on the information which was publicly available in the assessment report at the time the guidance was issued. The uncertainty surrounding the decision to adopt zanamivir for this patient group was characterised by assigning prior distributions to each of the key parameters and are detailed in Table 1. Unit prices and resource use were assumed to be known. Monte Carlo (MC) simulation was conducted to obtain the joint distributions of costs and effects and net benefit (Stinnett and Mullahy, 1998) for a range of cost-effectiveness thresholds ( $\lambda$ ). Zanamivir is more costly but more effective than standard care with an expected incremental cost-effectiveness ratio of £51,700. The cost-effectiveness acceptability curve is illustrated in Figure 1a (Van Hout et al. 1994; Fenwick et al., 2001) and indicates that when  $\lambda = £60,000$  per QALY gained zanamivir is expected to be cost-effective but this decision is uncertain with an error probability of 0.534. It should also be noted that the probability that zanamivir is cost effective at  $\lambda = ICER$  is less than 0.5, indicating that the prior distribution of the expected incremental net benefit offered by zanamivir is positively skewed (Fenwick et al., 2001).

### 3. Is additional research required?

In absence of sunk costs or other irreversibilities (Eckermann and Willan, 2006; Griffin et al., 2007; Palmer and Smith, 2000) the technology which is expected to be cost-effective should be selected based on current information. However, a second decision also needs to be made: is additional research required to support the use of the technology, and if so how much and what type of research would be most useful?

#### 3.1 Necessary condition for further research

The Expected Value of Perfect Information (EVPI) surrounding the decision to adopt or reject a technology places an upper bound on the social value of additional evidence and a necessary condition for deciding to conduct further research, i.e., further research is potentially worthwhile only if the population EVPI is greater than the expected costs of further studies.

A decision model with unknown parameters  $\mathcal{G}$  yields (typically by MC simulation) a distribution for the net benefits  $NB_t(\mathcal{G})$  of each treatment option  $t = 1, \dots, T$  for a homogeneous patient population.

After characterising the uncertainty around model parameters  $\mathcal{G}$  by assigning prior probability distributions (see Table 1), the EVPI for an individual patient is defined as

$$EVPI = E_{\mathcal{G}} \left[ \max_{t=1, \dots, T} NB_t(\mathcal{G}) \right] - \max_{t=1, \dots, T} E_{\mathcal{G}} [NB_t(\mathcal{G})]. \quad (1)$$

Given current information, the adoption decision can only be made before the uncertainty about model parameters  $\mathcal{G}$  resolves for all evaluated treatment options, i.e., choose the alternative which maximises the *expected* net benefit  $E_{\mathcal{G}}[NB_t(\mathcal{G})]$ . In principle, perfect information about  $\mathcal{G}$  would allow maximisation of the *actual* net benefit  $NB_t(\mathcal{G})$  for any particular value of  $\mathcal{G}$ , hence yielding  $\max_t NB_t(\mathcal{G})$ ; however, since the true values are unknown, such maximum values need to be averaged over the joint distribution of  $\mathcal{G}$ , which in turn leads to  $E_{\mathcal{G}}[\max_t NB_t(\mathcal{G})]$ . Thus the EVPI is the difference in social pay-off when making decisions with perfect rather than current information (see Ades et al., 2004).

While EVPI indicates the maximum social value of information about the decision problem, it does not indicate which type of evidence (which model parameters) might be most useful. The EVPI calculation can be extended to calculate the EVPPI associated with selected (groups of) parameters (Ades et al., 2004). Having divided model parameters  $\mathcal{G} = (\varphi, \psi)$  into nuisance ( $\psi$ ) and relevant ( $\varphi$ ) parameters of interest,

$$EVPPI_{\varphi} = E_{\varphi} \left\{ \max_{t=1, \dots, T} E_{\psi|\varphi} [NB_t(\varphi, \psi)] \right\} - \max_{t=1, \dots, T} E_{\mathcal{G}} [NB_t(\mathcal{G})] \quad (2)$$

quantifies the maximum social value of acquiring information about  $\varphi$  alone. With perfect knowledge about parameters of interest  $\varphi$ , the decision-maker is able to identify the technology yielding highest expected net benefit (with respect to the residual uncertainty about  $\psi$ )  $\max_t E_{\psi|\varphi} [NB_t(\varphi, \psi)]$

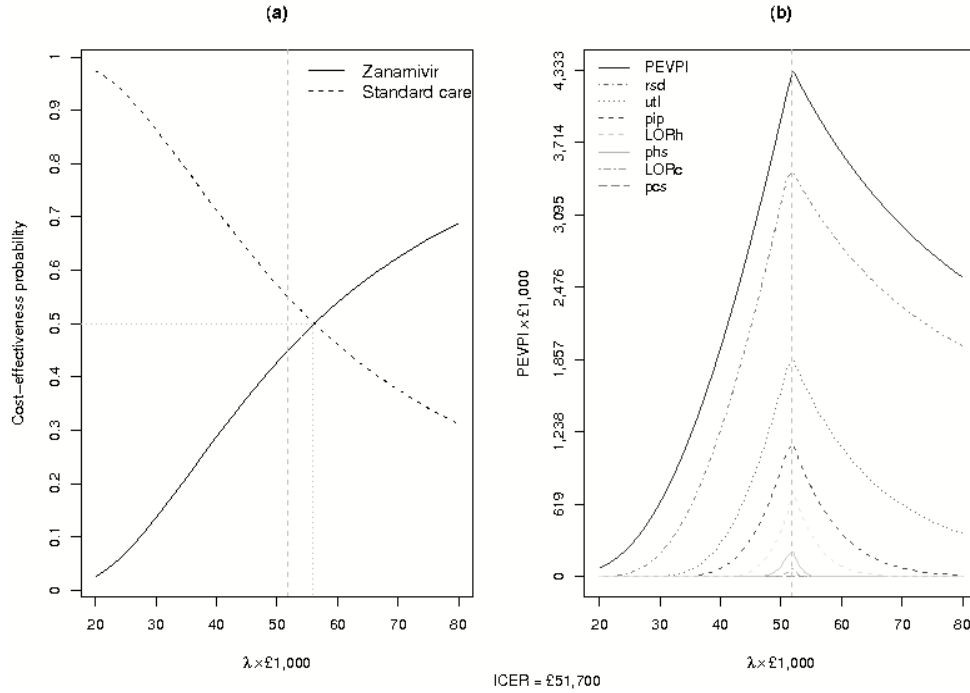
However, as in the case of EVPI, the true value of  $\varphi$  is unknown so that an additional expectation  $E_{\varphi} \left\{ \max_t E_{\psi|\varphi} [NB_t(\varphi, \psi)] \right\}$  over the prior distribution of  $\varphi$  needs to be evaluated. It should be noted that EVPI does not additively decompose into its individual EVPPI components, due to the presence of interactions occurring within the model structure.

Expressions (1) or (2) provides the EVPI or EVPPI for each time this decision is made (i.e., for an individual patient or individual patient episode). These values need to be expressed for the current and future population that could benefit from further research. If  $Q_h$  patients (or episodes) enter the decision problem at year  $h = 1, \dots, H$  over the time horizon for the decision problem  $H$ , the

Population EVPI, or PEVPI at discount rate  $r$ ,

$$PEVPI = EVPI \times \sum_{h=1}^H \frac{Q_h}{(1+r)^h} \quad (3)$$

is obtained. Both  $Q_h$  and  $H$  are in principle also subject to uncertainty and proxy a complex and uncertain process of future changes (Philips et al., 2008). In this example we assume  $H = 7$  years,  $Q_h = 136,000$  and  $r = 6\%$ .



**Figure 1: Cost-Effectiveness Acceptability Curve (a) and Population Expected Values of Perfect Information (b) (vertical lines = ICER)**

Estimates of population EVPI and EVPPI over a range of cost-effectiveness thresholds are illustrated in Figure 1b. At cost-effectiveness thresholds greater than £30,000 the population EVPI is likely to exceed the costs of further investigation and further research may be worthwhile. The EVPPI associated with reduction in symptom days on zanamivir is relatively high but a more precise estimate would require experimental design to avoid selection bias. This suggests that a further randomised trial may be worthwhile. However, there are other parameters associated with significant EVPPI (most notably the quality of life associated with influenza symptoms and the probability that a patient presenting with influenza like symptoms does in fact have influenza) which may not require experimental design and could be informed by an observational study or a survey of quality of life.

### 3.2 Sufficient condition for further research

The estimates of population EVPI and EVPPI only provide an upper bound on the social value of research. However, fully informing the research decision requires a sufficient condition to be established based on the expected benefits and costs of particular proposed research designs. Sample information would allow an update of (imperfect) prior knowledge of model parameters into a *posterior* distribution for  $\mathcal{G}$  conditional on the collected data. This is formally done by combining each prior distribution with its likelihood by means of Bayes Theorem, as detailed e.g. in Gelman et al. (2004, ch. 2). Thus proposed research, generating prospective data  $x = (x_1, \dots, x_J)$  to be collected

from  $J$  samples of  $n = (n_1, \dots, n_J)$  patients, will inform parameters  $\mathcal{G}$ . The expected social value of evidence generated by the design  $n$  is then expressed by the Expected Value of Sample Information (Ades et al., 2004),

$$EVSI(n) = E_x \left\{ \max_{t=1, \dots, T} E_{\mathcal{G}|x} [NB_t(\mathcal{G})] - \max_{t=1, \dots, T} E_{\mathcal{G}} [NB_t(\mathcal{G})] \right\}, \quad (4)$$

which measures the additional value of the adoption decision when based on sample  $(n)$  rather than current information. Analogously to (2), the information about  $\mathcal{G}$  conveyed in  $x$  would allow the decision-maker to establish which treatment yields highest expected net benefit  $E_{\mathcal{G}|x} [NB_t(\mathcal{G})]$ . Due to the sampling variability surrounding  $x$ , a further expectation over its predictive distribution is however needed (first term at the RHS of (4)). At a particular cost-effectiveness threshold the EVSI converges to the EVPI as sample size increases and provides an upper bound to EVPI. As with EVPI the EVSI needs to be expressed for the population of current and future patients who will benefit from this information. The population EVSI for proposed research with sample sizes  $n$  is given by

$$PEVSI(n) = EVSI(n) \times \sum_{h=1}^H \frac{Q_h}{(1+r)^h} \quad (5)$$

The estimation of  $EVSI(n)$  poses greater challenges than EVPI because both inner and outer expectations within the first term of (4) can be computationally burdensome, especially with non-multi-linear decision models<sup>1</sup> (Ades et al., 2004). In this simple example, both log-odds ratios of complications and hospitalisation are modelled via a Normal approximation to Binomial likelihoods (see Table 1) and second-order Taylor series expansions were also invoked to ensure multi-linearity and are recommended in computations involving non-linear parameter transformations in otherwise linear net benefit functions (Ades et al., 2004,).

The  $PEVSI(n)$  needs to be compared to the costs of sampling  $C(n)$ , which includes resource and opportunity costs:

$$C(n) = \left[ c_f + \sum_{t=1}^T (c_t n_t) \right] + EVSI(n) \sum_{t=1}^T n_t + \sum_{\substack{t=1 \\ t \neq \tilde{t}}}^T \{ n_t E_{\mathcal{G}} [NB_t(\mathcal{G}) - NB_{\tilde{t}}(\mathcal{G})] \}. \quad (6)$$

The first term of (6) includes the fixed and reporting resource costs, the second term includes expected benefits from the research which will be forgone by those patients enrolled in the study (the population that can benefit from the results of the research is 'used up') and the third term includes the expected net benefit forgone by those patients allocated to the treatment(s) which are not *a priori* cost-effective.

The social value of proposed research is the difference between expected benefits ( $PEVSI(n)$ ) and costs ( $C(n)$ ) or the Expected Net Benefit of Sampling

$$ENBS(n) = PEVSI(n) - C(n), \quad (7)$$

This provides a sufficient condition for deciding to conduct more research, i.e., only if  $ENBS(n)$  is greater than zero is further research justified. It also provides the means to choose between different types of study and different designs (sample size and its allocation). The research decision can now be characterised as choosing the type of study(ies) and design(s) which maximises the ENBS.

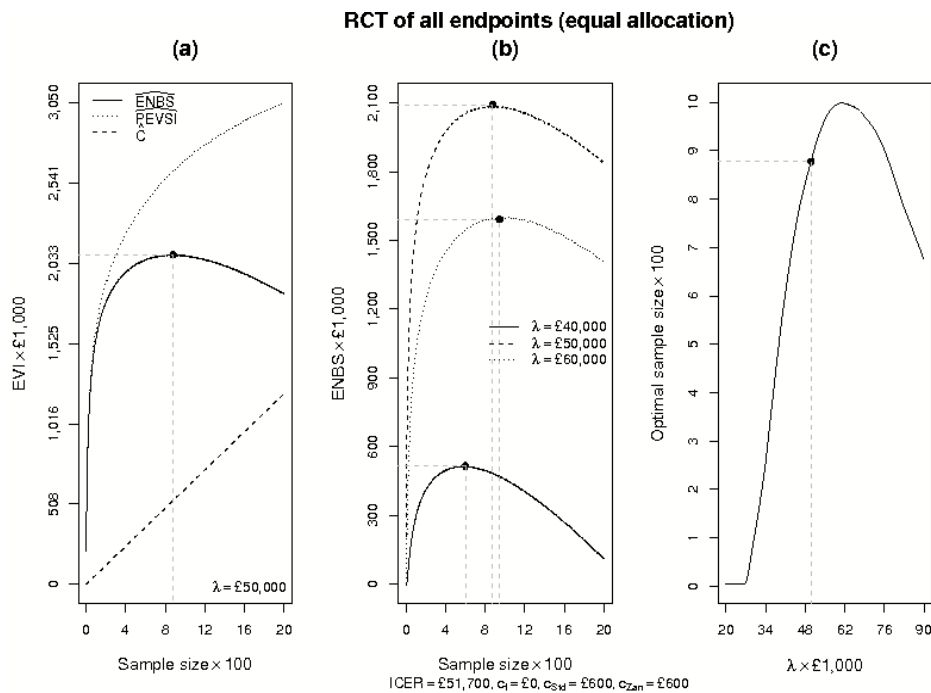
<sup>1</sup> Multi-linearity of the net benefit function allows replacement of the inner expectations in (2)-(4) with expected prior or predicted posterior values

## 4. Research design space

Ideally, the research design  $n^*$  yielding the highest ENBS over a range of sample sizes should be sought and implemented. In principle this defines a relatively straightforward integer programming problem, where the objective function  $ENBS(\cdot)$  is to be maximised over the examined design space  $D$  of study sizes  $n$ . Complications, however, arise in practice due to the nature of the objective function: in most situations exact ENBS evaluation is not possible due to the nested expectation and maximisation required in the definition (4) of PEVSI. The problem of  $ENBS(\cdot)$  not being generally available in closed form is usually circumvented by replacing (7) with a suitable approximation, ensuring that the maximum it produces would consistently estimate  $n^*$ . A number of approximation-based solutions dealing with difficulties in PEVSI calculations recently appeared in the statistical literature: see in particular Brennan and Kharroubi (2007a), Oakley (2005) and O'Hagan et al. (2005). The most established and straightforward method (see Gilks et al., 1996, ch. 14), which is also adopted throughout the paper, relies on the empirical MC estimator  $\hat{ENBS}(n)$ . This is obtained from taking sample averages over both the predictive distribution of the data  $x$  (for the outer expectation in (4)) and the posterior distribution of the model inputs  $\mathcal{G}$  conditional on data  $x$  (for the inner expectation).

### 4.1 Designing a single study

A simple research design which would inform the zanamivir vs standard care decision problem consists of a RCT allocating equal number of entrants to its arms (a fixed sample design) and simultaneously informing all model parameters. In this case, the ENBS maximising design  $n^*$  indicates how many patients should be enrolled in the trial for it to provide the highest social value. This initial simplicity of the design space ( $D = \mathbb{N}$ ) allowed full enumeration of a sufficient range of sample sizes to identify the optimal sample size and the maximum ENBS at very high MC resolutions ( $50^6$  iterations).



**Figure 2: PEVSI( $n$ ), C( $n$ ), ENBS( $n$ ) and  $n^*$  (a), ENBS( $n$ ) and  $n^*$  for three cost-effectiveness thresholds (b) and  $n^*$  over a range of cost-effectiveness thresholds (c)**

Figure 2 and the first two rows of Table 2 summarises the results of fully exploring this limited design space when fixed costs  $c_f$  are excluded for simplicity, as they do not influence optimal sample size (or its allocation) and reporting costs  $c_{Std}$  and  $c_{Zan}$  for each arm are assumed equal at £600 per patient. Figure 2a illustrates  $PEVSI(n)$ ,  $C(n)$  and  $ENBS(n)$  for a range of sample sizes when  $\lambda = £50,000$ . As expected  $PEVSI(n)$  increases with  $n$  but at a declining rate, approaching PEVPI in the limit. Therefore, the ENBS reaches a ‘non sharp’ maximum where the ENBS curve is relatively flat, i.e., sample sizes slightly greater or less than optimal have little impact on ENBS. Figure 2b illustrates ENBS curves at three cost-effectiveness thresholds over the same range of sample sizes. The ENBS is highest when  $\lambda = £50,000$  reflecting the fact that the decision is most uncertain (see Figure 1a) and information is more valuable when  $\lambda = £50,000$  rather than  $\lambda = £60,000$  or  $\lambda = £40,000$  (see Figure 1b). Again, the ENBS curve is relatively flat at its maximum. The optimal sample size also changes with the cost-effectiveness threshold. This relationship is explored more fully in Figure 2c which plots the optimal sample size for a range of cost-effectiveness thresholds. At  $\lambda < £30,000$  the optimal sample size is zero and no further research is needed. When  $\lambda > £30,000$  the optimal sample size increases with  $\lambda$  reaching a maximum of 999 at  $\lambda = £61,000$  and declines thereafter because the increased opportunity costs incurred by enrolled patients starts to outweigh the marginally diminishing gains generated from additional sample information. Figure 2b and 2c show that both the value of conducting research and its optimal design (in this case only sample size) depends critically on economic considerations represented by the cost-effectiveness threshold.

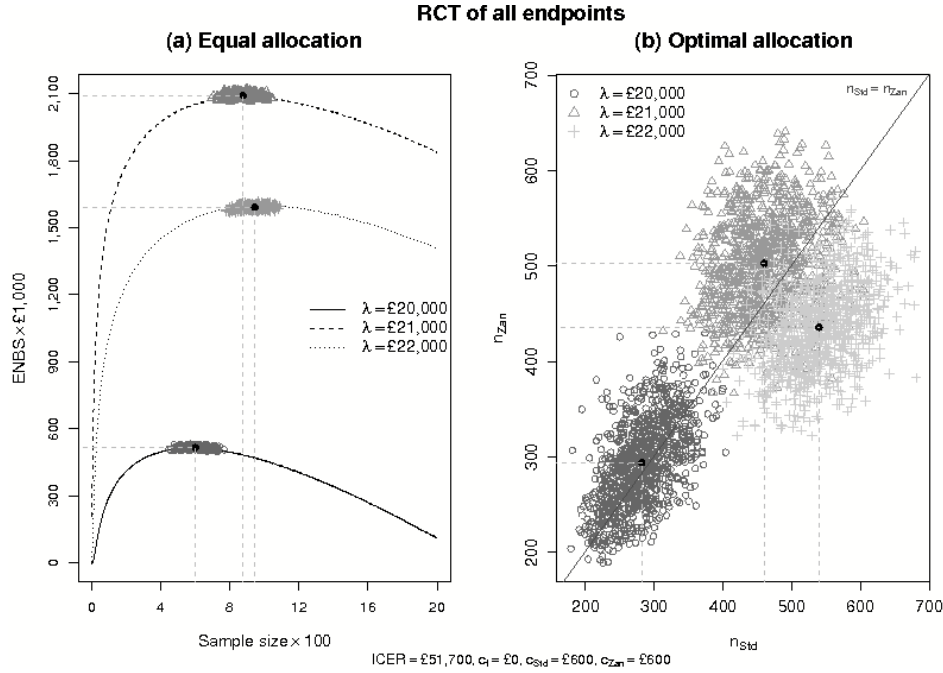
## 4.2 Optimisation algorithms

The limited design space  $D = \mathbb{N}$  considered above makes an exhaustive search for  $n^*$  using estimates of ENBS at very high MC resolutions feasible. However, such an exhaustive search is unlikely to be possible when the other dimensions of the design space described are considered because the computational burden increases geometrically with the dimensions that need to be explored, e.g., considering optimal allocation of each possible sample between two arms of a trial means increasing the dimension of  $D$  from 1 to 2. One way of mitigating this ‘curse of dimensionality’ is to replace the previous exhaustive search with a more efficient search for the maximum ENBS and the associated optimal sample size. A number of more efficient searches or optimisation algorithms are available which when started at a randomly chosen point will evaluate ENBS at candidate sample sizes iterating to the sample size that provides the maximum ENBS. The improved adaptive Nelder-Mead simplex method (see e.g. Nocedal and Wright, 1999, ch. 9) was adopted for its robustness and parsimony in general multi-variate optimisation settings.

This optimisation algorithm searches for the global maximum on the ENBS surface. If the surface is smooth with a single global maximum then a single optimisation routine would be required to identify the maximum ENBS and optimal sample size. However, although  $\hat{ENBS}(n)$  from MC simulation is a consistent estimator of (7) the estimated surface  $ENBS(\cdot)$ , is rough, and its roughness depends on the MC resolution used in its estimation. This rough surface does not preserve the convergence properties underpinning most optimisation algorithms and a single optimisation routine will not necessarily locate  $n^*$ . However, the Sample Average Approximation (Shapiro, 2000) indicates that the estimated optimal design  $\hat{n} = \arg \max_{n \in D} \hat{ENBS}(n)$  converges to the optimal design  $n^* = \arg \max_{n \in D} ENBS(n)$  at an exponential rate with the MC resolution. This suggests that a number ( $L$ ) of optimisation routines are required, each started at random sample sizes. This multiple optimisation provides a sample of estimated optimal sample sizes  $\hat{n}_1, \dots, \hat{n}_L$  and associated  $\hat{ENBS}(\cdot)$  values. This sample of estimates provides credible ranges for  $n^*$  and  $ENBS(n^*)$ , with the average optimal sample size  $(\bar{n} = \sum \hat{n}_i / L)$  and average maximum ENBS

$\left( \overline{ENBS} = \sum \hat{ENBS}_i / L \right)$  providing central estimates. By selecting the number of optimisations and

the MC resolution used to estimate each candidate ENBS, a credible range for optimal design can be established for feasible computing times. A narrower range can be achieved with higher MC resolutions but at the cost of additional computation time. Of course, a sufficient number of optimisations needs to be conducted to ensure the credible range includes the optimum (this will tend to be greater for lower MC resolutions as the surface will be rougher).



**Figure 3: Estimates of maximum ENBS (equal allocation) (a) and optimal sample allocation (b) (filled circles = mean values)**

An indication of the performance of this approach to optimisation in terms of computation time, credible intervals and central estimates for  $n^*$  and  $ENBS(n^*)$  is possible by comparison with the results of an exhaustive search at very high MC resolutions (taken to represent ‘true’ values of  $n^*$  and  $ENBS(n^*)$ ). Optimisation was applied to the  $D = \mathbb{N}$  RCT design described above and the results are illustrated in Figure 3a when using  $L=1,000$ , each started at random over the range  $\{1, \dots, 1,000\}$  and with a MC resolution of  $10^5$  for each of the candidate estimates of ENBS. An informal comparison of Figures 2b and 3a shows the optimisation performs well, locating optimal sample size and the maximum ENBS. These results are reported in more detail in Table 2 and confirm that the credible ranges included  $n^*$  and  $ENBS(n^*)$  and that the means of the samples of estimates were close to these ‘true’ values at each of the three cost-effectiveness thresholds. The credible range for  $n^*$  clearly tends to be wider than the relatively narrow ranges for  $ENBS(n^*)$ . This is due to the relative flatness of the ENBS surface around its maximum which, when combined with MC noise, reduces the precision of inferences about  $n^*$  but enables accurate estimation of the relatively stable  $ENBS(n^*)$ . As might be expected, the sampling variability of the estimates appears to be greater at cost-effectiveness thresholds where decision uncertainty is greater (those closer to the ICER). Overall, optimisation appears to provide reliable estimates of  $n^*$  and  $ENBS(n^*)$ ; give an indication of the credible range of possible values; and substantially reduce computation time with a optimisation to exhaustive search ratio of 1/8user computing time.



In the simple  $D = \aleph$  research design considered above, the ENBS can be expected to reach a single global maximum.<sup>2</sup> However, for higher-dimensional design problems there may not necessarily be a single maximum on the ENBS surface, so a single optimisation routine may locate a local rather than global maximum even on a smooth surface. Therefore, using a number of optimisation routines each started at random points in the design space also ensures that the global rather than a local maximum will be located. The presence of a local maximum would be indicated by a bi- or multi-modal distribution of the estimated maximum ENBS. It should be noted that the distribution of the estimates of maximum ENBS were unimodal for all the dimensions of design space explored below, indicating that in this example at least the ENBS surface had a single maximum for all the designs explored.

**Table 2: Summary results for a single trial design**

Rows 1-2: equal allocation (high MC resolution); 3-6: equal allocation (optimisation); rows 7-10: optimal allocation (optimisation).

|                                 | $\lambda = £40,000$   | $\lambda = £50,000$       | $\lambda = £60,000$       |
|---------------------------------|-----------------------|---------------------------|---------------------------|
| $n^*$                           | 578                   | 882                       | 988                       |
| $ENBS(n^*)$                     | £515,000              | £2,086,700                | £1,600,400                |
| $\bar{n}$                       | 600                   | 875                       | 943                       |
| (95% bounds)                    | (500 – 726)           | (762 – 1,008)             | (812 – 1,058)             |
| $\overline{ENBS}$               | £516,600              | £2,092,100                | £1,592,000                |
| (95% bounds)                    | (£505,200 – £527,500) | (£2,074,500 – £2,111,900) | (£1,577,400 – £1,606,500) |
| $\bar{n}_{Std} + \bar{n}_{Zan}$ | 576                   | 963                       | 976                       |
| (95% bounds)                    | (432 – 724)           | (811 – 1119)              | (828 – 1127)              |
| $\overline{ENBS}$               | £529,700              | £2,092,400                | £1,618,400                |
| (95% bounds)                    | (£518,600 – £536,700) | (£2,076,800 – £2,103,500) | (£1,608,300 – £1,626,000) |

### 4.3 Optimal sample allocation

So far the exploration of research design space has been restricted to a single RCT allocating an equal number of entrants to its arms and informing all parameters. The problem is simply to identify the sample size that maximises ENBS, i.e., a single dimension of design. However, evaluation of the alternative designs available requires consideration of both the total number of patients to enrol and how they should be allocated between each of the alternative interventions available. The benefits of allocating the sample to different arms of a trial will tend to differ because the variance of the net benefit of each intervention considered is unlikely to be the same. Even if the benefits are similar, the costs of allocating the sample to different arms will differ because even if the reporting costs are the same (as in this case) the opportunity costs (net benefit forgone) of allocation will differ when  $\lambda \neq ICER$ . Therefore the assumption of equal allocation is unlikely to be justified (Claxton and Thompson, 2001).

In this example, the problem is to establish the total sample size and its allocation between the two arms which provide the maximum ENBS,  $n^* = (n_{Std}^*, n_{Zan}^*)$ . The design space increases from one ( $D = \aleph$ ) to two dimensions ( $D = \aleph^2$ ) and the computational burden makes the type of exhaustive search at high MC resolutions illustrated in Figure 2 infeasible. The optimisation algorithm described above was implemented and the results for three values of  $\lambda$  are illustrated in Figure 3b and reported

<sup>2</sup> This is because PEVSI increases with sample size ultimately at a declining rate and although the costs of sampling also increases at a declining rate it does so more slowly than the PEVSI. This is because two of the three elements of (6) increase at a constant rate and the third (EVSI forgone) necessarily decreases at the same rate as the PEVSI.

in Table 2. Informal inspection of Figure 3b suggests that in this example the optimal allocation does not differ markedly from equal allocation (the rising diagonal) – the joint density of the estimated  $n_{Std}^*$  and  $n_{Zan}^*$  combinations is diffuse and covers equal allocation. This might be expected as the variance of the net benefit of the two arms are similar, and the only difference in sampling costs is the expected net benefit forgone (third term in (6)). However, when  $\lambda > ICER$  the optimal allocation does suggest that more of the sample should be allocated to standard care, i.e., when the prior decision favours zanamivir it is additional information about standard care rather than zanamivir which is more likely to revise this decision<sup>3</sup>. These additional benefits of allocation to standard care outweigh the additional opportunity costs of sampling (the expected net benefit forgone).

Optimal allocation allows designs to be adopted which either provide more benefit for a particular total sample size and/or reduce the costs of sampling. Therefore, the maximum ENBS must be greater with optimal allocation but the total sample size may not necessarily be higher as more efficient use is made of a smaller sample when allocated efficiently. The results reported in Table 2 confirm this. The estimated maximum ENBS is higher at each of the three cost-effectiveness thresholds, although not markedly so, reflecting the similarity between optimal and equal allocations. The optimal total sample size appears higher when  $\lambda = £50,000$  but not necessarily at other values of  $\lambda$ . The credible intervals for the total optimal sample size are wider than with equal allocation, reflecting the greater complexity of the design space. The credible ranges for the maximum ENBS, however, are narrower indicating that the ENBS surface with optimal allocation is ‘flatter’ and therefore estimates are more stable at the maximum. This is because there are more combinations of  $(n_{Std}, n_{Zan})$  which result in ENBS values which are similar and close to their maximum.

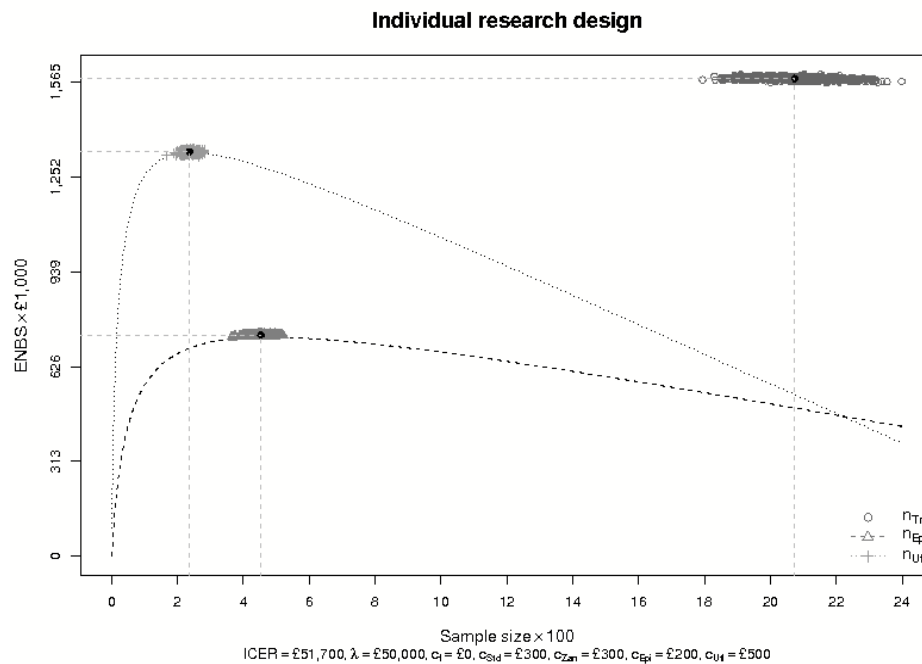
## 5. Designing a portfolio of studies

A single RCT which simultaneously informs all the parameters is not the only type of research which could be conducted. Only some of the parameters (reduction in symptom days and the log-odds ratios for complication and hospitalisation) require experimental design to avoid selection bias. Other parameters associated with significant EVPPI (see Figure 1b) may not require experimental design and could be informed by other designs. For example, a survey of quality of life could be undertaken to estimate the quality of life associated with influenza symptoms. An observational epidemiological study could be designed to estimate the probability that a patient presenting with influenza like symptoms does in fact have influenza, as well as the base line probabilities of complication and hospitalisation. If more than one type of study can be conducted, then optimal sample size and its allocation can be established for each type of study independently. If these are mutually exclusive alternatives then the research decision is simply to choose the optimally designed single study with offers the highest ENBS. However, if they are not mutually exclusive but can be conducted together the ENBS of the combination (portfolio) is not the sum of the ENBS of each but must be estimated simultaneously.

3 The prior distribution of incremental net benefit of zanamivir compared to standard care is positively skewed, but some parameters contribute to this skewness more than others. In this example it is those parameters which require some of the sample to be allocation to zanamivir (e.g., LORc, LORh and rsd) which contribute most to the positive skew. Other parameters which can be updated by allocating sample to standard care alone contribute least. Clearly the distribution of predicted posterior values must also be positively skewed, but its skewness will depend on which parameters are being updated and with ‘how much’ sample information. Therefore, when proportionately more of the sample is allocated to zanamivir the distribution of predicted posterior values tends to be more positively skewed and less so when more sample is allocated to standard care arm. When  $\lambda > ICER$  the mean incremental net benefit is positive, so it is only predicted posterior values on the left of this distribution which could revise the prior decision which is in favour of zanamivir and generate value to research. If this distribution is less positively skewed (more sample allocated to standard care) then such values are more likely and the EVSI will be greater. Conversely when  $\lambda < ICER$  the mean incremental net benefit is negative and only predicted posterior values on the right of this distribution could revise the prior decision which is in favour of standard care. If this distribution is more positively skewed (greater allocation to zanamivir) then such values are more likely and the EVSI is greater. The general point is that the benefits of allocating the sample to different arms of a trial do not just depend on the differences in the variance of net benefit but also on the effect of sample information on its shape.

## 5.1 Individual study designs

Three alternative types of study can be considered: a simple RCT which only provides information about the clinical endpoints (rsd, LORc and LORh) which are vulnerable to selection bias; an epidemiological study updating pip, pcs and phs, and a survey of quality of life providing estimates of utl. The maximum ENBS and optimal sample size can be established for each study independently of the others. Therefore, one search over two dimensions of design space (optimal sample size and allocation) for the clinical trial,  $n_{Tri} = n_{Std} + n_{Zan}$ , and two separate one-dimensional searches for optimal sample size for the epidemiological study  $n_{Epi}$  and survey of quality of life  $n_{Utl}$  is required.



**Figure 4: Estimates of maximum ENBS and optimal sample sizes (filled circles = mean values)**

The optimal design of each of these independent studies is illustrated in Figure 4 when  $\lambda = £50,000$  and details are provided in Table 3 at three cost-effectiveness thresholds. Of these three studies the clinical trial provides the highest ENBS with an optimal sample size much larger than the RCT updating all parameters in Figure 2. This is due to the lower reporting costs of this simpler design (£300 rather than £600) and the fact that the clinical endpoint of reduction in symptom days is associated with the highest value of information.<sup>4 5</sup> It should be noted that the credible range for the optimal sample size of this design is greater than previously, reflecting the relative ‘flatness’ of the ENBS surface.<sup>6</sup> The survey of quality of life generates the next highest ENBS but at the lowest optimal sample size. This is for three reasons: it is associated with high PEVPPI (see Figure 1b); it has a diffuse prior which means that relatively small amounts of sample information are valuable (PEVSI increases rapidly with  $n_{Utl}$ ); and the reporting costs are assumed to be relatively high (£500). Consequently the ENBS curve has a more pronounced maximum and the credible range for the estimates of optimal sample size is relatively narrow. The epidemiological study generates a lower

- 4 Sample size is higher and the ENBS surface is ‘flatter’ for 2 reasons: i) When the marginal sampling costs are lower optimal sample size will always be higher and the ENBS surface will always be flatter at its maximum other things (i.e., EVSI) being equal. ii) This trial design excludes those parameters with diffuse prior distributions where a relatively small amount of information is valuable and PEVSI increases rapidly with  $n$ . The parameter rsd is associated with the highest value of information but its prior distribution already has substantial informational content so the PEVSI increases more slowly with sample size. The combined effect of both leads to a higher optimal sample size on a ‘flatter’ ENBS surface.
- 5 Only the estimated optimal total sample sizes from multiple optimisations rather than the optimal allocation between standard care and zanamivir can be represented in two dimensions in Figure 4. Due to computational burden an exhaustive search at very high MC resolution is only possible and illustrated for the single dimensional designs of an epidemiological study and a survey of quality of life.
- 6 See fn. 5.

ENBS but at a higher sample size. This should be expected given the lower PEVPPI associated with these parameters (see Figure 1b), relatively low reporting costs (£200) and less diffuse prior distributions.

**Table 3: Summary results for different types of studies**

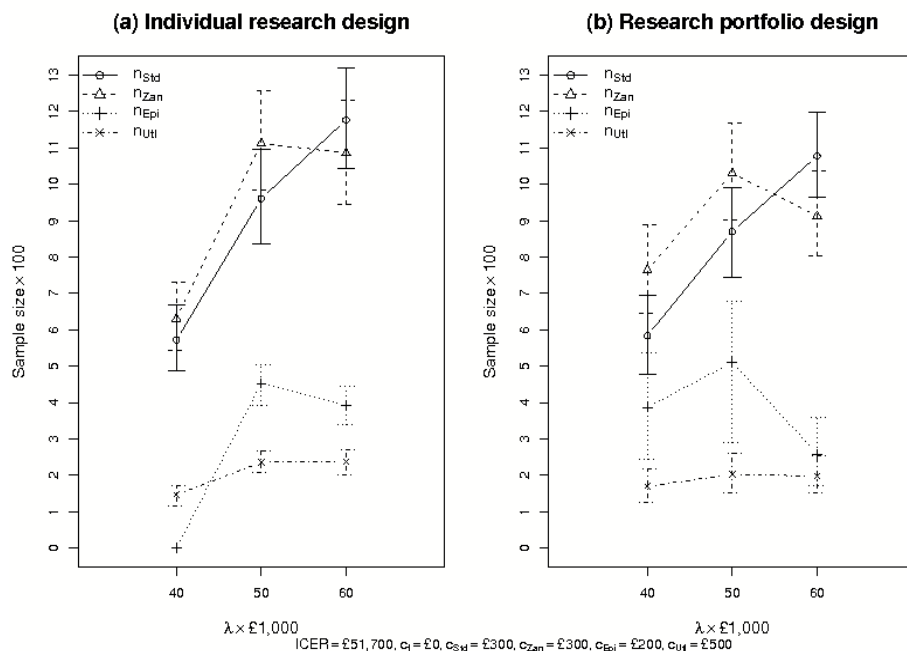
Rows 1-4: clinical trial (optimal allocation); 5-8: epidemiological study; rows 9-12: utility survey; rows 13-16: independent research portfolio.

|   | $\lambda = £40,000$   | $\lambda = £50,000$       | $\lambda = £60,000$       |
|---|-----------------------|---------------------------|---------------------------|
| $\bar{n}_{Std} + \bar{n}_{Zan}$                                 | 1,203                 | 2,074                     | 2,263                     |
| (95% bounds)  | (1,055 – 1,366)       | (1,896 – 2,260)           | (2,060 – 2,458)           |
| $\overline{ENBS}(n_{Std}, n_{Zan})$                             | £69,600               | £1,577,800                | £1,207,700                |
| (95% bounds)  | (£65,500 – £63,200)   | (£1,569,200 – £1,585,900) | (£1,200,100 – £1,216,000) |
| $\bar{n}_{Epi}$   | 0                     | 453                       | 392                       |
| (95% bounds)  | N.A.                  | (393 – 504)               | (339 – 444)               |
| $\overline{ENBS}(n_{Epi})$                                      | 0                     | £731,100                  | £199,900                  |
| (95% bounds)  | N.A.                  | (£726,200 – £735,900)     | (£196,500 – £202,800)     |
| $\bar{n}_{Utl}$   | 147                   | 236                       | 239                       |
| (95% bounds)  | (115 – 171)           | (208 – 267)               | (203 – 270)               |
| $\overline{ENBS}(n_{Utl})$                                      | £121,000              | £1,335,200                | £838,000                  |
| (95% bounds)  | (£117,500 – £123,700) | (£1,328,600 – £1,341,800) | (£831,800 – £844,000)     |
| $\bar{n}_{Std} + \bar{n}_{Zan} + \bar{n}_{Epi} + \bar{n}_{Utl}$ | 1,350                 | 2,763                     | 2,894                     |
| (95% bounds)  | (1,170 – 1,537)       | (2,497 – 3,031)           | (2,602 – 3,172)           |
| $\overline{ENBS}$   | £433,800              | £2,129,700                | £1,692,100                |
| (95% bounds)  | (£432,300 – £435,400) | (£2,126,700 – £2,132,500) | (£1,689,900 – £1,694,600) |

If all the four studies considered and reported in Tables 2 and 3 are regarded as mutually exclusive then the research decision is simply to choose the optimally designed single study which offers the highest ENBS. In these circumstances it would be the RCT which simultaneously informed all the parameters (see Table 2). This optimally designed study provides an estimated maximum ENBS which is greater than the ENBS of any one of the other three studies considered individually (see Table 3) at each cost-effectiveness threshold. However, the three studies reported in Table 3 are not necessarily mutually exclusive and could be conducted simultaneously. The ENBS for this combination is not the sum of the individual ENBS but the ENBS for the combined optimal samples sizes for each study and is reported in the last row of Table 3. The ENBS for the combination of studies when designed independently is greater than the ENBS for the single RCT informing all parameters when  $\lambda = £50,000$  or  $\lambda = £60,000$ . However, at a lower cost-effectiveness threshold of  $\lambda = £40,000$  the single RCT would be better than the combination of the three independently designed studies.

## 5.1 Optimal portfolio design

While relatively straightforward to implement, the strategy of designing each study independently before combining them to construct a portfolio of research will generally fail to identify the optimal combination which maximises ENBS. Identifying individual research designs independently is sub-optimal because it neglects the relationships between studies designed to investigate different but related aspects of the same decision problem, i.e., it assumes the contribution provided by each study (the parameters they investigate) to the value of information is additive. Therefore the optimal design of a portfolio of research can be regarded as finding the total sample size which when allocated optimally between different studies and/or arms of studies will maximise ENBS. In this example it requires simultaneously searching for  $n^* = (n_{Std}^*, n_{Zan}^*, n_{Epi}^*, n_{Utl}^*)$ . The computational scale of this problem and the need for optimisation should be apparent – the dimensions of the design space is now 4 ( $D = \aleph^4$ ). Despite the scale of this problem the optimisation procedures outlined above can be used to provide central estimates of total sample size, its allocation, maximum ENBS and the associated credible intervals. Computational burden can be made feasible by the selection of sufficient number of optimisation routines at a manageable MC resolution. Therefore the feasibility of exploring the design space is not really a question of whether it is feasible or not but more a question of what precision is affordable.



**Figure 5: Optimal sample sizes and allocations from individual (a) and optimal portfolio (b) research designs**

The results of optimisation over these 4 dimensions of design space are illustrated in Figure 5 and reported in more detail in Table 4. Figures 5a and 5b illustrate the optimal sample allocations with their credible intervals when studies are respectively designed independently and simultaneously. An informal comparison confirms that that a portfolio of research made up of combining independently designed studies will not necessarily be the same as an optimal portfolio which simultaneously allocates sample between different studies and the arms of studies. There are similarities: for example, the relative size of total sample allocated to the clinical trial, epidemiological study and the survey of quality of life are similar. Also within the clinical trial optimal allocation to the two arms is similar to previous results (see Table 2) where a greater proportion is allocated to standard care when  $\lambda < ICER$  but more to zanamivir when  $\lambda > ICER$ <sup>7</sup>. However these differences in allocation are smaller with independent design rather than simultaneous optimisation across these studies. One noticeable difference, at  $\lambda = \pounds 40,000$ , is that optimal sample size for the epidemiological study is

7 See fn. 5.

zero, indicating it should be excluded from the portfolio of research when designed independently of the others. However, when designed simultaneously the sample size is positive and it should be included in a portfolio of research.

Simultaneous allocation within and between studies allows optimal portfolios to be identified which either provide more benefit for a particular total sample size and/or reduce the costs of sampling.

**Table 4: Summary results for an optimal portfolio**

Rows 1-2: clinical trial (optimal allocation); 3-4: epidemiological study; rows 5-6: utility survey; rows 7-10: optimal research portfolio.

|                                 | $\lambda = £40,000$ | $\lambda = £50,000$ | $\lambda = £60,000$ |
|---------------------------------|---------------------|---------------------|---------------------|
| $\bar{n}_{Std} + \bar{n}_{Zan}$ | 1,349               | 1,901               | 1,991               |
| (95% bounds)                    | (1,179 – 1,529)     | (1,709 – 2,096)     | (1,823 – 2,167)     |
| $\bar{n}_{Epi}$                 | 387                 | 511                 | 259                 |
| (95% bounds)                    | (243 – 538)         | (291 – 677)         | (173 – 360)         |
| $\bar{n}_{Utl}$                 | 170                 | 203                 | 198                 |
| (95% bounds)                    | (127 – 219)         | (151 – 260)         | (153 – 254)         |
| $\bar{n}$                       | 1,906               | 2,614               | 2,449               |
| (95% bounds)                    | (1,687 – 2,134)     | (2,376 – 2,850)     | (2,251 – 2,655)     |
| $\overline{ENBS}$               | £524,200            | £2,180,400          | £1,756,700          |

Therefore, the maximum ENBS must be greater with optimal design but the total sample size may not necessarily be higher as more efficient use is made of a smaller sample when allocated optimally. In this case the overall total sample size of the optimal portfolio is lower than the combination of individually designed studies at  $\lambda = £50,000$  or  $\lambda = £60,000$  (i.e., it makes more efficient use of a smaller sample) but is greater when  $\lambda = £40,000$ . The ENBS for the optimal portfolio reported in Table 4 confirms that the ENBS for an optimally designed portfolio always exceeds the ENBS for the combination of individually designed studies reported in Table 3 at each of the cost-effectiveness thresholds.

## 6. Discussion

A Bayesian decision theoretic approach to both adoption and research decisions means that a rational and consistent approach to both is possible. The decision to adopt a technology can be based on expected net benefits (whilst explicitly taking account of any sunk costs and other irreversibilities) rather than traditional rules of inference. Similarly research design issues cannot be resolved using simple universal rules of precedent either, e.g., the traditional power calculation is based on established benchmarks of power and statistical significance and can be seen as essentially arbitrary. Efficient research design is an empirical and partly economic question determined by the characteristics of the decision problem, the prior information available and the monetary valuation of health outcome which will be applied once the research is completed and an adoption decision must be made. In other words, the efficient design of research cannot be separated from the objectives and constraints on service provision.

To date most applications of Bayesian decision theory and value of information analysis have been restricted to establishing the expected value of perfect information, so can only provide a necessary condition for conducting further research and cannot address the question of efficient research design. The demonstration and application of an analysis of the expected value of sample information

has been more limited and restricted to considering the value and optimal sample size of a single study to inform a decision to adopt or reject a technology. The analysis reported above demonstrates that Bayesian decision theory can be used to more fully inform the research decision by exploring more dimensions of research design space when considering the question of whether more evidence is needed to support a decision to adopt or reject a health technology. The research decision is not simply a binary question of whether to conduct further research or not, nor is it restricted to the more complex question of the optimal size of a single study. The research decision requires an examination of much wider dimensions of design space, including how patients should be allocated between the arms of a single study as well as the range of possible combinations of different types of studies that could be conducted to inform particular uncertain parameters.

In this particular example, even when considering a single study with fixed sample design, the optimal allocation of patients between the arms of the proposed trial is important even when the direct costs of allocation between different arms are the same. In this sense optimal allocation is a more fundamental question than the issues which arise in sequential trial designs. As well as an efficiently designed single trial informing all parameters (with optimal sample size and allocation) the research decision space also includes: i) conducting a simpler (and cheaper) clinical trial providing evidence only on clinical endpoints; ii) an epidemiological study of natural history of disease; and iii) a survey of quality of life. The research decision not only requires consideration of each of these independent alternatives but also their possible combinations. The example demonstrates that where a number of different types of study informing different parameters in the decision problem could be conducted, the simultaneous estimation of ENBS across all dimensions of the design space is required to identify the optimal sample sizes and allocations within such a research portfolio.

The computational challenges of exploring these wider dimensions of design space are considerable. However, the use of more efficient algorithms to search the ENBS surface for its maximum rather than exhaustively search over such large dimensions makes this task feasible and substantially reduces computation time. In this example, multiple optimisation routines were used which appear to provide reliable estimates of optimal sample size, allocation and maximum societal payoff to research. Usefully they also provide a credible range for optimal samples sizes and social payoff. The computational burden of exploring design space can be made feasible by the selection of sufficient number of optimisation routines at a manageable MC resolution. Indeed, it is not really a question of whether it is feasible to explore these additional dimensions of design space, but rather a question of what precision is affordable.

There are other dimensions of possible design space that have not been explored in this example. Most importantly the question of sequential design has not been considered. Sequential issues arise in two respects. Firstly, it has been assumed that the proposed trials are a fixed sample rather than sequential design, where the results from earlier trial entrants are available and able to inform the allocation of subsequent patients. A substantial body of literature exists, which examines the optimal allocation of entrants in sequential clinical trials (see e.g. Zelen, 1969; Bather, 1981; Armitage, 1985; and Whitehead, 1997) but traditional approaches to this problem, which favour patient allocation into the most effective arm in the light of its interim results, tend to neglect the dynamics of the marginal benefits and costs of sample information and hence fail to address the issue of socially efficient design. Secondly, in constructing the portfolio it was assumed that all studies would be conducted concurrently so the results from one could not inform the design of another. It is possible to consider the order in which different studies could be conducted (Griffin et al., 2008). For example, it might be better to conduct a survey of quality of life before designing and conducting a clinical trial. However, it should be apparent that the dimensions of this design space are considerable, requiring estimates of social payoff for every possible subsequent sequence and possible design conditional on every predicted posterior for each initial sample size. This becomes even more burdensome when there are sequential issues within and between the studies under consideration. There are ways to reduce the scale of this type of problem using dynamic programming. However, using Bayesian decision theory and value of information analysis to fully explore the design space which arises from these types of sequential problems is an area for future investigation.

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