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Local termination criteria for Swarm Intelligence: a comparison between local Stochastic Diffusion Search and ant nest-site selection[⋆]

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Abstract. Stochastic diffusion search (SDS) is a global Swarm Intelligence optimisation technique based on the behaviour of ants, rooted in the partial evaluation of an objective function and direct communication between agents. Although population based decision mechanisms employed by many Swarm Intelligence methods can suffer poor convergence resulting in ill-defined halting criteria and loss of the best solution, as a result of its resource allocation mechanism, the solutions found by Stochastic Diffusion Search enjoy excellent stability.
Previous implementations of SDS have deployed stopping criteria derived from global properties of the agent population; this paper examines new *local* SDS halting criteria and compares their performance with 'quorum sensing' (a termination criterion naturally deployed by some species of tandem-running ants). In this paper we discuss two experiments investigating the robustness and efficiency of the new *local* termination criteria; our results demonstrate these to be (a) effectively as robust as the classical SDS termination criteria and (b) almost three times faster.

Keywords: Collective Decision Making; Ant Nest Selection; Stochastic Diffusion Search; Swarm Intelligence; Global Search.

1 Introduction

In recent years there has been growing interest in Swarm Intelligence (SI), a distributed mode of computation utilising interaction between simple agents [37].

[⋆] This paper offers extended discussion of results first presented at ICCCI 2016 (Halkidiki) and published in the conference proceedings [11].

Such systems have often been inspired by observing interactions between social insects: ants, bees, termites (cf. Ant Algorithms and Particle Swarm Optimisers) see Bonabeau [12] for a comprehensive review. SI algorithms also include methods inspired by natural evolution such as Genetic Algorithms [29] [33] or indeed Evolutionary Algorithms [5]. The problem solving ability of SI methods emerges from positive feedback reinforcing potentially good solutions and the spatial/temporal characteristics of their agent interactions.

Independently of these algorithms, Stochastic Diffusion Search (SDS), historically positioned as the first Swarm Intelligence meta-heuristic, was initially described in 1989 as a population-based, pattern-matching algorithm [8] [7]. Unlike stigmergic communication employed in Ant Algorithms, which is based on modification of the physical properties of a simulated environment, SDS uses a form of direct communication between the agents similar to the tandem running mechanism employed by some species of ants (e.g. *Temnothorax* species, [23]).

SDS is an efficient probabilistic multi-agent global search, optimisation and decision making technique [42] that has been applied to diverse problems such as site selection for wireless networks [**?**], mobile robot self-localisation [6], object recognition [9] and text search [7]. Additionally, a hybrid SDS and n-tuple RAM [1] technique has been used to track facial features in video sequences [9] [31]. Previous analysis of SDS has investigated its global convergence [51], linear time complexity [52] and resource allocation [50] under a variety of search conditions. For a recent review of the theoretical foundations, and applications of SDS see Al-Rifaie and Bishop [2].

In arriving at a 'decision' - *halting* - standard implementations of SDS examine the *stability* of the agent population as a whole; in this manner halting is defined as a *global* property of the agent population. However such global mechanisms are both less biologically/naturally plausible and more complex to implement on parallel computational systems, than local decision making mechanisms.

The organisation of this paper is as follows. Firstly we outline *Swarm Intelligence* meta-heuristic and against the background of communication in the social insects. Next we describe how collective decision making occurs in nature by analysing the behaviour of house-hunting ants. We subsequently introduce the Stochastic Diffusion Search.

SDS has subsequently been thoroughly mathematically explored; it is not appropriate to include its full analysis herein (see [9], [51], [52], [50], [55] for detail), however a simplified description (under-pinning that suggested in [31]) and based on the 'practical' characterisation[4] from Myatt et al. [48] is included. Together these analyses make SDS one of the best characterised of all the SI meta-heuristics.

Finally this paper examines the local quorum sensing behaviour observed in some natural (ant) systems and uses this as the inspiration for two new local

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[4] The simplifying assumption is that, by considering only the mean transition of agents between different clusters of agents, rather than the full probability distribution (as investigated in [51]), a *sufficiently* accurate model of SDS may be obtained.

termination mechanisms - one mechanism, ‘independent termination’, seeks to implement a protocol in SDS that is as close as possible to the quorum sensing method used by real ants; a second method - confirmation termination - aims to implement a mechanism closely related to quorum sensing on a more conventional SDS architecture; both halting criteria are algorithmically outlined and their performance experimentally evaluated.

2 Swarm Intelligence

Natural examples of swarm intelligence systems that exhibit such forms of collective interactions and decision-making are: fish schooling, bird flocking, bacterial growth, animal herding, nesting and foraging in the social insects etc. and in recent years, abstractions of such natural behaviour have suggested several new meta-heuristics for use in modelling collective intelligence. The simple and often successful deployment of these new meta-heuristics on traditionally difficult optimisation problems has in turn generated increasing interest in the nascent field of swarm intelligence algorithms: nature-inspired algorithms instantiating distributed computation via the interaction of simple agents and their environment (e.g. ant algorithms [20] [21] and particle swarm optimisation [38] etc).

In this paper we will illustrate Stochastic Diffusion Search - in which interactions between agents cause a population of agents to evolve towards potential solution states - and show that it shares many of the characteristics and behaviours of classical swarm intelligence algorithms; furthermore, we show that core stochastic diffusion processes are illustrated in the behaviours of some social insects (e.g. bees in identifying potential new food sources and ants in choosing a new nest site location) in the following sections of the paper (Sections 2.1 and 2.2); we explore SDS in this context.

2.1 Communication in Social Insects

In the study of interaction in social insects, two key elements are the individuals and the environment, which results in two modes of interaction: the first defines the way in which individuals interact with each other and the second defines the interaction of individuals with the environment [13]. Interaction between individual agents is typically carried out via agent recruitment processes and it has been demonstrated that various recruitment strategies are deployed by ants [15] [32] and honey bees [30] [67]. These recruitment strategies may be used, for example, to attract other members of the population to gather around one or more desired areas in the search space, either for foraging purposes or in order to facilitate a colony relocation to a better nest site.

It has been observed that recruitment strategies in social insects may take the form of: localised or global recruitment; one-to-one or one-to-many recruitment; and may operate stochastically or deterministically. The nature of information exchange also varies in different environments and with different types of social insects. Sometimes the information exchange is quite complex and, for example, might communicate data about the direction, distance and suitability of the target; or sometimes the information sharing is relatively simple, for example, a stimulation forcing a particular triggered action. Nonetheless, what all recruitment and information exchange strategies have in common is an ability to distribute useful information across their community [42].

2.2 Methods of Communication

Chemical communication through pheromones forms the primary method of recruitment in many species of ants; however, in certain species a 'tandem running' mechanism (one-to-one communication) is used [23]. In this process, the scouting ant that finds the resource location attracts a single recruit upon its return to the nest (tandem calling) and physically leads the recruit to the resource (tandem running); by this action the location of the resource is physically publicised [75] to the population.

Conversely in group recruitment, one ant summons a group of ants, leading them to the resource location. Group recruitment may entail laying a pheromone trail from the resource to the nest; a more complex process in which the recruiting ant is no longer necessarily in physical contact with the recruited ants.

The largest-scale ant recruitment mechanism is called 'mass recruitment' [76]; in this mechanism, worker ants both follow the pheromone trail and incrementally add an amount of pheromone on their journeys to and from the resource location. In such 'mass recruitment', the concentration of pheromone plays an important role in attracting other ants to the resource trail.

Different recruitment and communication algorithms thus induce differing search performances. Ants communicating through group recruitment are faster than tandem running ants, and similarly, ants utilising mass recruitment are more efficient in their performances than the former recruitment strategies [15]. Ant algorithms have been successfully applied to hard optimisation and search problems such as travelling salesman problem and the quadratic assignment problem [22].

However, as mentioned in [17], the success of the ants in reaching the food they have been recruited to obtain, varies from one species to another. In another form of communication, indirect or stigmergic communication, the exchange of information is based on modifying the physical properties of the environment and its success lies in spatial and temporal attributes of mass recruitment and the positive feedback mechanism it employs. In this mode, which is based on using pheromone, short routes are loaded with more pheromone (because of the shorter time it takes the ants to travel back and forth between the source and target [34]).

An ant-like task allocation has been investigated in [35] where robots were used to simulate different non-communication and communication strategies, concluding that ant-inspired techniques of decentralised control, namely tandem running recruitment mechanism [75] shows better results than single robots doing the same task. This technique of information exchange is an instance of a broader type of recruitment strategy utilised in stochastic diffusion search [42], which will be discussed in more detail, later in this paper.

In honeybees, group recruitment is performed by means of waggle dances, in which the angle of the dance relative to gravity shows the direction of the resource, and the duration of the central part of the dance represents the distance to the target area. Each scouting bee can choose one of the dancing bees as a guide to a food source or a new nest.

3 Collective decision-making in house hunting ants

Quorum sensing is widespread throughout biological systems. When a collective decision is required, a quick and effective way of moving from an information-gathering phase to an implementation phase is to use a quorum threshold. A quorum response can be said to occur when an individual's probability of exhibiting a behaviour (e.g. choosing a given option) is a sharply nonlinear function of the number of other individuals already performing this behaviour (or having chosen that option) [72]. Quorum sensing is used by biological systems as diverse as bacteria, insects, fish, and primates - including humans [44], [73], [71], [43]. A model system for collective decision-making and the use of quorum thresholds is provided by the process of house-hunting in social insects, such as honeybees and cavity-nesting ants. Ants that nest in fragile cavities in rocks or twigs have only limited scope to modify their nest-site, and readily relocate their entire colony when the need arises. The processes by which cavity-nesting ant colonies of the genus *Temnothorax* decide whether to emigrate, choose their new nest and implement that choice have been well-explored, both empirically and theoretically, as they provide a key model of animal collective decision-making.

The stages of an emergency emigration by a colony of cavity-nesting ants are summarised in Figure (1). Even when no emigration is needed, scouts survey the surrounding area for potential new nest sites [25] [70]. If the home nest cavity is damaged or degraded, then scouts leave the home nest and both visit the sites they already know and also search for new nest sites. Scouts assess the available nests across a number of metrics using a weighted additive strategy [27]. If a nest is judged as unsuitable, a scout continues searching; if a nest is assessed as suitable by a scouting ant (Scout A), this scout will return towards the home nest and recruit a second scout (Scout B) from in or near the home nest [64]. Scout A will lead Scout B to the new nest by a slow tandem-running process whereby Scout A moves slowly, with Scout B walking behind, making contact with its antennae and learning the route [23]. Scout B will then make an independent assessment of the nest, and will either reject it and keep searching, or accept the nest and spend some time in it, before returning home and recruiting a further scout. By this process of positive feedback, a good quality nest will accumulate ants [61], [63]. Different ants appear to have differing thresholds for accepting a nest and starting to recruit; this means that even low quality nests can attract some ants, but scouts will accumulate more quickly and to a higher level at higher quality nests [70] [62]. This assessment and recruitment process is terminated when a nest reaches a quorum of scouts. When scouts sense that a nest site has reached quorum, they change into a ‘post-quorum’ behavioural state [59]. From this point on they stop leading other scouts by tandem-running, and are no longer willing to be recruited by tandem-running themselves. Instead, they transport brood, queen, nest workers and other scouts to the chosen site. Transported workers do not learn the route between the home nest and the new nest, so are unable to return home, and thus cannot challenge the decision that has been implemented [59]. Transported ants therefore contribute strongly to the quorum by staying in the new nest, so once a few scouts have moved into a ‘post-quorum’ state and started transporting, other scouts will quickly follow suit.

4 Quorum sensing in house hunting ants

The sensing of a quorum threshold is clearly central to the decision-making process, as it marks the transition from information-gathering and assessment, to implementation. Terminating information-gathering promotes cohesion, which is very important for ant colonies that only have one reproductively active queen. For cavity-nesting ants, quorum thresholds appear to be based on direct encounter rate, rather than any indirect cue such as pheromone concentration [61]. Scouts spend 1-2 minutes in a nest assessing the number of workers present via their encounter rate [58]. Quorum threshold as a proportion of colony size is remarkably constant across a range of colony sizes (3.5%) [19], and this is intriguing, because the relationship between colony size and cavity size is not simple positive correlation: although larger colonies do inhabit larger cavities in the wild, in laboratory tests both small and large colonies prefer larger cavities, presumably to allow for growth [60], [14], [39] & [45].

Quorum sensing is a separate process from quality assessment and recruitment. This means that the quorum sensing process in effect detects an average quality assessment across many scouts, and has the potential to smooth out differences in individual nest acceptance thresholds (or indeed, artificially-induced fluctuations in nest quality) [28]. Once quorum is reached and transport has begun, scouts do not re-assess quorum on subsequent visits - they will continue to bring brood even if the nest is artificially emptied of ants [58]. The quorum threshold itself is not modulated depending on the quality of the new nest [57]. If nest quality is artificially manipulated during the assessment phase of an emigration, the ants are able to respond flexibly to the new nest qualities; if quality is manipulated after quorum is reach and implementation has begun, then colonies often become ‘trapped’ in an inferior nest [26], [65]. This indicates that quality is not re-assessed after quorum has been reached in these cavity-dwelling *Temnothorax* species. In contrast, a different ant species, *Diacamma indicum* recruits only by tandem-running with no clear quorum point and no adult transport [3]. Colonies of *Diacamma indicum* are able to respond flexibly to manipulated qualities at any stage of the emigration - but overall colony cohesion is lower, supporting the idea that using a quorum threshold increases cohesion, but at a cost to flexibility.

Although quorum sensing behaviour is not modulated by the quality of the options available, it is influenced by the scout's experience, and by the context of the emigration. Naive scouts use different quorum thresholds to those used by more experienced scouts, but the direction of this difference differs between species [57]. This suggests that there is a learning component to quorum sensing behaviour, but the mechanisms by which appropriate quorum thresholds are learnt is unknown. Figure (1) describes an emergency emigration, but cavity-nesting ants do also sometimes emigrate even when their home nest is undamaged, if a better nest is available in the neighbouring area. This is not due to direct comparison of the quality of the two nests, but due to quality-dependent nest leaving by scouts, and quality-dependent nest acceptance [63], [70]. In these non-emergency migrations, scouts appear to use a quorum threshold around twice as high as in emergency migrations [18], suggesting that colonies prioritise speed over accuracy when conditions are harsher.

5 Stochastic Diffusion Search

SDS is based on distributed computation, in which the operations of simple computational units, or agents are inherently probabilistic. Agents collectively construct the solution by performing independent searches followed by diffusion of information through the population. Positive feedback promotes better solutions by allocating to them more agents for their exploration. Limited resources induce strong competition from which the largest population of agents corresponding to the best-fit solution rapidly emerges.

In many search problems the solution can be thought of as composed of many subparts and in contrast to most Swarm Intelligence methods SDS explicitly utilises such decomposition to increase the search efficiency of individual agents. In what is known as standard, *or vanilla*, SDS each agent poses a hypothesis about the possible solution and evaluates it partially. Successful agents repeatedly test their hypothesis while recruiting unsuccessful agents by direct communication. This creates a positive feedback mechanism ensuring rapid convergence of agents onto promising solutions in the space of all solutions. Regions of the solution space labelled by the presence of agent clusters *with the same hypothesis* can be interpreted as good candidate solutions. A global solution is thus constructed from the interaction of many simple, locally operating agents forming the largest cluster. Such a cluster is dynamic in nature, yet stable, analogous to, "*a forest whose contours do not change but whose individual trees do*", [4] [10] [53] [54]. Below the SDS mechanism is illustrated by analogy in 'The Restaurant Game'.

5.1 The restaurant game analogy

A group of delegates attends a long conference in an unfamiliar town. Each night they have to find somewhere to dine. There is a large choice of restaurants, each

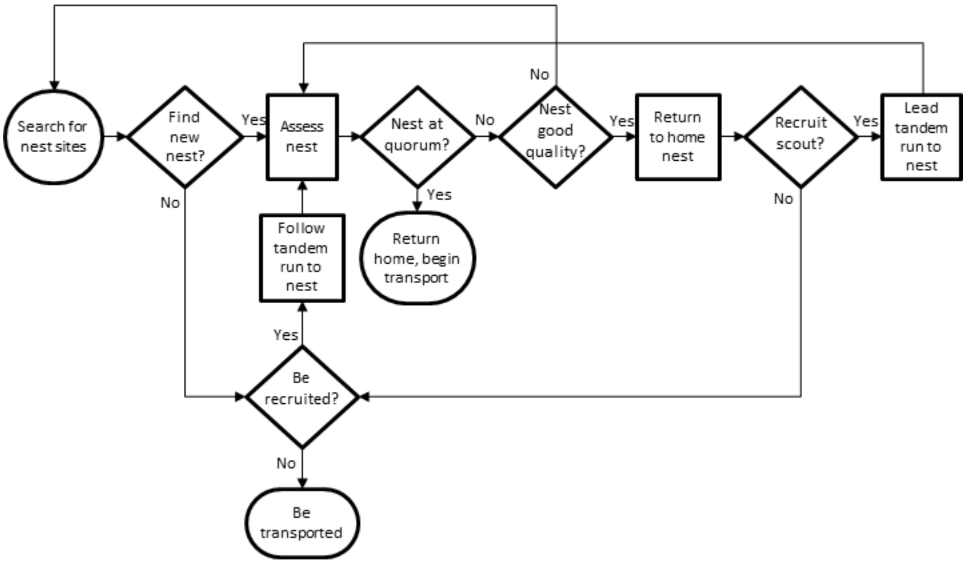

Fig. 1. The process of assessment and recruitment for an individual scouting ant of the genus *Temnothorax*, triggered by damage to the home nest. The scout starts by searching for new nest sites, and may finish by detecting quorum at a site and starting implementation of the decision to choose that site. Alternatively, if it is unable to recruit other scouts or be recruited by other scouts, this indicates that quorum has been reached at another site, and the scout will then be transported to that site and will cease to search. Non-emergency emigrations also occur, but in non-emergency situations scouts may spontaneously cease to search for a nest or attempting to recruit and instead enter a quiescent state. Probability of becoming re-activated is dependent on the home nest quality.

of which offers a large variety of meals. The problem the group faces is to find the best restaurant, that is the restaurant where the maximum number of delegates would enjoy dining. Even a parallel exhaustive search through the restaurant and meal combinations would take too long to accomplish. To solve the problem delegates decide to employ a Stochastic Diffusion Search[5].

Each delegate acts as an agent maintaining a hypothesis identifying the best restaurant in town. Each night each delegate tests his hypothesis by dining there and randomly selecting one of the meals on offer. The next morning at breakfast every delegate who did not enjoy his meal the previous night, asks one randomly selected colleague to share his dinner impressions. If the experience was good, he also adopts this restaurant as his choice. Otherwise he simply selects another restaurant at random from those listed in 'Yellow Pages'.

Using this strategy it is found that very rapidly significant number of delegates congregate around the best restaurant in town.

Abstracting from the above we get the algorithmic process defined in Table 1. By iterating through test and diffusion phases agents stochastically explore the solution space. However, since tests succeed more often on good candidate solutions than in regions with irrelevant information, an individual agent will spend more time examining 'good' regions, at the same time recruiting other agents, which in turn recruit even more agents. Candidate solutions are thus identified by concentrations of a substantial population of agents.

Table 1. Algorithmic description of the restaurant game

```
Initialisation phase
  whereby all agents (delegates) generate an initial hypothesis
  (select a restaurant at random)
loop
  Test phase
    Each agent evaluates evidence for its hypothesis (meal quality).
    Agents are partitioned into active (content) and inactive (disgruntled)
    groups (of diners).
  Diffusion phase
    Inactive agents adopt a new hypothesis by either communication with
    another agent or, if the selected agent is also inactive, there is no
    information flow between the agents; instead the selecting agent must
    adopt a new hypothesis (restaurant) at random.
endloop
```

Central to the power of SDS (see Algorithm 1) is its ability to escape local minima. This is achieved by the probabilistic outcome of the partial hypothesis

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[5] It should be emphasised that this analogy is provided simply to illustrate the communication and feedback mechanisms at the heart of a stochastic diffusion search, and **not** as a heuristic to be employed by a group of hungry conference delegates.

Algorithm 1 Vanilla SDS

```
procedure step($swarm$, $search\_space$)
    for each $agent$ in $swarm$ do                                ▷ Diffuse Phase
        if not $agent.active$ then
            $polled\_agent = swarm.random\_agent()$
            if $polled\_agent.active$ then
                $agent.hypothesis = polled\_agent.hypothesis$
            else
                $agent.hypothesis = search\_space.random\_hypothesis()$
    for each $agent$ in $swarm$ do                                ▷ Test Phase
        $test\_result = perform\_random\_test(hypothesis)$
        $agent.active = test\_result$
```

evaluation in combination with reallocation of resources (agents) via stochastic recruitment mechanisms. Partial hypothesis evaluation allows an agent to quickly form its opinion on the quality of the investigated solution without exhaustive testing (e.g. it can find the best restaurant in town without having to try all the meals available in each).

6 An approximate characterisation of the Stochastic Diffusion Search

Stochastic Diffusion Search[6] has often been used for pattern matching; such problems can be more generally redefined in terms of an optimisation, by defining the objective function, $F(\mathbf{x})$, for a hypothesis, $\mathbf{x}$, as the similarity between the target pattern and the corresponding region at $\mathbf{x}$ in the search space and finding $\mathbf{x}$, such that $F(\mathbf{x})$ is maximised. In general SDS is most easily applied to optimisation problems where the objective function is decomposable into components that can be evaluated independently:

$$F(\mathbf{x}) = \sum_{i=1}^{n} F_i(\mathbf{x}), \tag{1}$$

where $F_i(\mathbf{x})$ is defined as the i^{th} partial evaluation of $F(\mathbf{x})$.

In a standard SDS a ‘Test Function’ returns boolean value indicating whether a randomly selected partial evaluation of the objective function is suggestive of a ‘good’ hypothesis. In pattern matching the Test Function may return True if the i^{th} sub-feature of the target pattern is present at position $(\mathbf{x}, i)$ in the search space. Thus the *Test Score* for a given hypothesis is the probability that the Test Function will return true, and is hence indicative of the underlying value of the objective function.

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[6] The material in Section (6) was included at the suggestion of one of the reviewers of this manuscript; it forms a much extended and enhanced presentation of the ‘practical’ analysis of SDS behaviour first outlined by Myatt et al in [48].

6.1 Homogeneous background noise

In [50] Nasuto first derived a comprehensive analysis of SDS in which he presented several alternative noise cases and examined the resource allocation of SDS in each using an Ehrenfest Urn model. One of these cases was that of homogeneous noise, where there are a number of distractors with the same Test Score (the probability of an agent becoming active for a given hypothesis). However, practical issues in the selected experimentation domain of text search prevented the exploration of high values of both p_d, (the probability of selecting a distractor from the search space) and p^+ (the overlap of the distractor with the target).

Nonetheless, in many real-world situations there will be high values of p_d, in that any non-optimal hypothesis can be regarded as a significant distractor. In the brief and simplified analysis presented herein we consider the case of 'homogeneous background noise', where $p_d \approx 1$ and the Test Score of each distractor is identical. Moreover, even if the distractors show significant deviation from homogeneity, some search space could be constructed that would produce the same mean response from SDS; hence in many real-world situations the homogeneous background noise model is both practical and useful.

In a typical convergence of SDS in the presence of homogeneous background noise it can be seen that before an optimal solution is found the proportion of active agents remains approximately constant [51]. Let this level of homogeneous background noise be called β, and be equal to the proportion of active agents given that the optimal solution has not yet been selected. Additionally, β is also equivalent to the Test Score of the homogeneous distractors, since if all agents are currently associated with distractors of Test Score β then the proportion of the agents active will also be β. Therefore, the noise parameter β replaces the two parameters p_d and p^+ used in previous analyses by Nasuto. It is noted that in practise the background noise of a search can be estimated simply by iterating SDS for a short time and taking the mean of the number of active agents per iteration.

6.2 Expected cluster size formulation of SDS

In this section the minimum Test Score, $\alpha_{\min}$, for which a stable cluster of agents can form will be derived. A simplifying assumption is that, by considering only the mean transition of agents between different clusters, rather than the full probability distribution (as investigated in [50]), a *sufficiently accurate* characterisation of SDS can be produced. The noise model that will be assumed is that of 'homogeneous background noise', where every non-optimal hypothesis corresponds to a distractor of Test Score β. It is also assumed that there is a single optimal solution with Test Score α that has a negligible probability of being selected. Let $\bar{c}_i$ be the mean number of active agents with the optimal solution as a proportion of the total population.

Consider a function, f, that defines the mean transition between the size of a cluster of active agents with the optimal hypothesis, in consecutive iterations, as a proportion of the total number of agents. It can be seen that f is therefore a function of the current cluster size $\bar{c}_i$, the Test Score of the optimal solution α and the level of homogeneous background noise β.

$$\bar{c}_{i+1} = f\left(\bar{c}_i, \alpha, \beta\right). \tag{2}$$

By calculating f for a given variant of SDS it is possible to extract both the mean optimal cluster size once a stable convergence has occurred and also the minimum value of α for which a stable convergence can occur as a function of β, α_{min}. The stationary state γ occurs when the mean cluster size (the number of *active* agents with the same hypothesis) remains constant between successive iterations i.e.

$$\gamma = f\left(\gamma, \alpha, \beta\right). \tag{3}$$

It is apparent that (3) will have two solutions, one of which will be zero (when $\bar{c}_i = 0$). A cluster will, on average, increase in size if the constraint

$$\bar{c}_{i+1} > f\left(\bar{c}_i, \alpha, \beta\right) \tag{4}$$

holds. α_{min} may therefore be found using the constraint

$$f\left(\frac{1}{k}, \alpha_{\mathrm{min}}, \beta\right) > \frac{1}{k}, \tag{5}$$

which assumes the worst case that only one agent has the correct hypothesis.

For a given value of α_{min} SDS will then stably converge if the constraint

$$\alpha_{\mathrm{min}} < \alpha \leq 1 \tag{6}$$

is satisfied. For a non-negligible p_m, the value of α_{min} derived will be artificially large but will still provide a useful guide.

6.3 Convergence level of SDS

In the following analysis it is assumed that p_m is negligible and hence over a small number of iterations the optimal solution will not be found.

During the diffusion phase, the mean number of inactive agents selecting an agent within the optimal cluster is given by $g(\bar{c}_i, \alpha, \beta)\bar{c}_i$, where g yields the number of inactive agents for a given iteration. From Figure 2 g can be immediately written as

$$g(\bar{c}_i, \alpha, \beta) = \frac{1-\alpha}{\alpha}\bar{c}_i + (1-\beta)\left(1 - \frac{\bar{c}_i}{\alpha}\right). \tag{7}$$

Therefore, the function f that defines the mean 1-step optimal cluster size evolution is

$$\bar{c}_{i+1} = f(\bar{c}_i, \alpha, \beta) = \alpha\left(\bar{c}_i + g(\bar{c}_i, \alpha, \beta)\bar{c}_i\right). \tag{8}$$

and consequently

$$f(\bar{c}_i, \alpha, \beta) = \alpha\left[\bar{c}_i + \left(\frac{1-\alpha}{\alpha}\bar{c}_i + (1-\beta)\left(1 - \frac{\bar{c}_i}{\alpha}\right)\right)\bar{c}_i\right]. \tag{9}$$

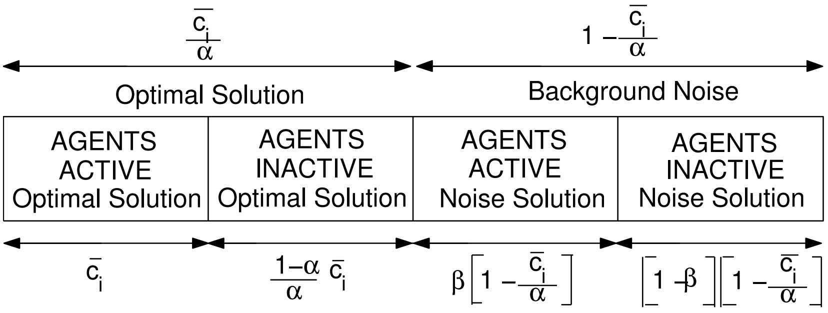

Fig. 2. Illustration of the current state of the agent population in iteration i in terms of $\bar{c}_i$, α and β

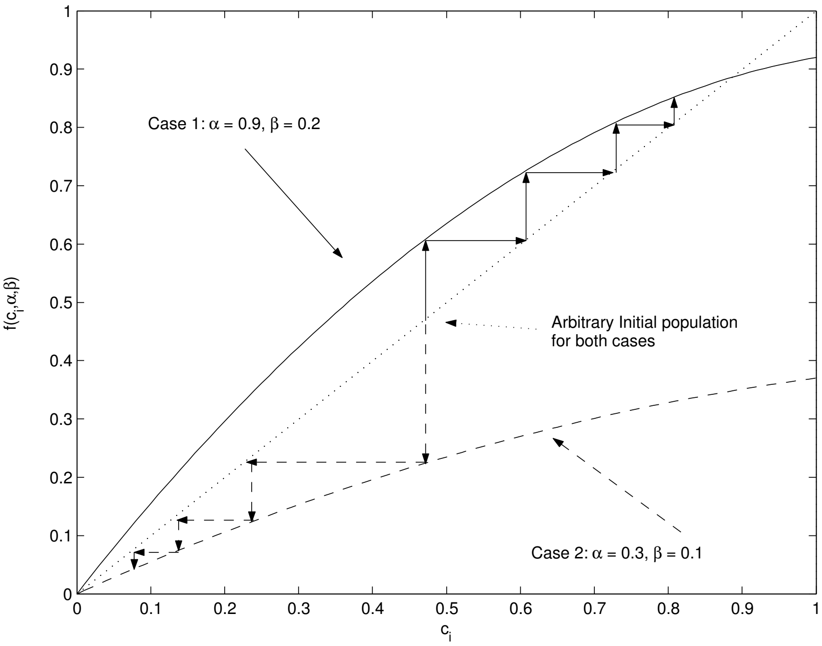

Fig. 3. A 1 dimensional iterated map showing how different values of α and β can either result in a stable optimal cluster or a return to zero.

Substituting this into (3) yields

$$\gamma = \frac{\alpha\left(2-\beta\right)-1}{\alpha-\beta}. \tag{10}$$

Similarly, substituting (9) into (5) gives

$$\alpha_{\min} = \frac{1}{2-\beta}, \tag{11}$$

assuming that k is large. Figure 3 considers (9) as a 1 dimensional iterated map, and graphically it can be seen that for a non-zero attractor to exist the condition

$$\frac{df}{d\bar{c}_i} > 1 \tag{12}$$

must hold for $\bar{c}_i = 0$. Differentiating (9) wrt $\bar{c}_i$ yields

$$\frac{df}{d\bar{c}_i} = \alpha\left(2-\beta\right) - 2\bar{c}_i\left(\alpha-\beta\right) \tag{13}$$

and it follows that the minimum value of α for which the constraint in (12) holds is

$$\alpha_{\min} = \frac{1}{2-\beta}. \tag{14}$$

Hence, for any $\alpha < \alpha_{\min}$ the size of the cluster will tend to zero for *any* initial cluster size and the search will fail. NB. This maximum level of background noise is equal to the convergence level of SDS with $\beta = 0$.

6.4 Multiple testing

In situations where the Test Score of the global optima cannot cause a stable convergence as defined by (5), the corresponding Test Function may be manipulated such that the Test Score of all hypotheses is boosted, thus allowing stable convergence. One simple of achieving this is the utilisation of *multiple testing.* If an agent performs t samples of the Test Function at a given hypothesis (rather than just one) and becomes active *iff* one or more of these tests are passed, then the Test Score α for all hypotheses will be increased, such that

$$\alpha' = 1 - \left(1-\alpha\right)^t \tag{15}$$

Let $e = 1-\alpha$, then

$$\frac{d\alpha'}{de} = -te^{t-1} \tag{16}$$

and

$$\frac{de}{d\alpha} = -1. \tag{17}$$

Therefore

$$\frac{d\alpha'}{d\alpha} = \frac{d\alpha'}{de}.\frac{de}{d\alpha} \tag{18}$$

and thus

$$\frac{d\alpha'}{d\alpha} = t\,(1-\alpha)^{t-1}\,. \tag{19}$$

Hence for $\alpha \in \mathbb{R}, 0 < \alpha < 1$ and $t \in \mathbb{I}, t > 0$,

$$\frac{d\alpha'}{d\alpha} > 0. \tag{20}$$

Therefore, because the increase of α' is monotonic with respect to α, the topology of the Objective Function landscape will be preserved (except as $t \to \infty$), and thus the optima of α' will still correspond to the optima of α. Multiple testing is therefore a generalisation of the standard testing technique in *Vanilla* SDS. Figure 4 shows the transformation applied by tuple testing for varying t (where t is the number of tests evaluated for each agent-hypothesis).

Considered in the domain of nest selection in social insects, multiple testing is analogous to a scout spending time at a potential nest site and evaluating it repeatedly using t *different* metrics, with the nest judged suitable if anyone of the metrics is positive[7].

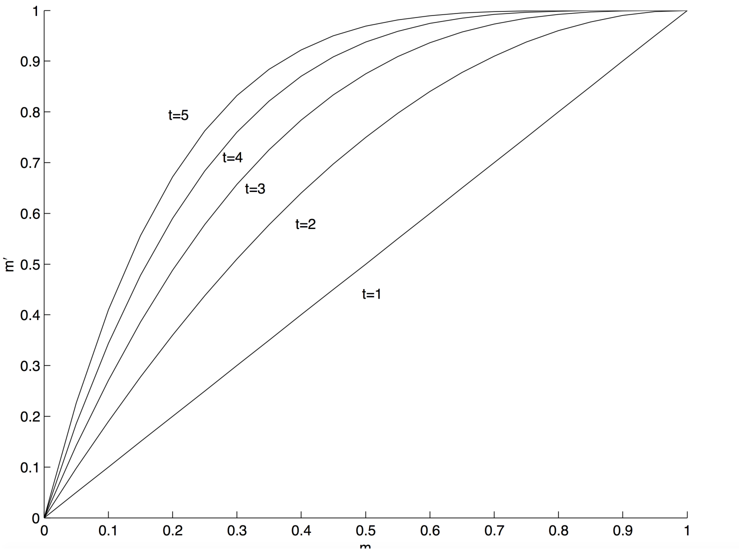

Fig. 4. The relationship between α and α' for varying tuple tests t

Consequently, for sufficiently large t, SDS can be made to converge successfully for any arbitrary optimal match. However, this may pose problems if such

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[7] Thus *multiple testing* is very similar to the *repeated metrics* strategy deployed by real scouts in nest selection, albeit in the latter case scouts assess potential nest sites across a number of metrics using a *weighted additive strategy* [27].

a value of t results in the *background noise*, β, of the search being greater than 50%, because it will be difficult to establish when convergence to the optimal solution has actually taken place (as the search will always be in some state of convergence). In such cases, it may be necessary to evaluate the search a number of times (to a given termination criterion, say, a terminal number of search iterations, this number being selected to ensure a high probability of the search having located the optimal match). In summary we observe that the principle of *multiple testing* is robust and not domain specific (i.e. it may be applied to any problem in which the Test Score of the standard Test Function is not sufficient to ensure stable convergence); an example multiple testing SDS application (locating eyes in images of human faces) was outlined in [9].

7 Halting criteria

The termination of SDS has historically been defined in terms of the *stability* of the population size of a group of active agents. Such methods are termed *global* halting criteria as they are a function of the total number of active agents within the global population of agents.

7.1 Global Halting criteria

Two well documented global methods for determining when SDS should halt are the *Weak Halting Criterion* and *Strong Halting criterion*[51] [50]; the former is a function of the *total* number of active agents and the latter the *total number of active agents sharing the same hypothesis*; the 'Weak' halting criterion is simpler to compute and from Figure 5 can be broadly seen to demonstrate the same halting behaviour as the 'Strong'.

7.2 Local Halting criterion

In order to approximate the behaviour of *Temnothorax* ants in their nest selection behaviour, the halting behaviour of SDS was redesigned such that it would emerge from purely *local* interactions of agents.

By analogy with the behaviour of tandem running *Temnothorax* ants (as outlined in Section (3) and Section (4) herein), in the following we propose two new variants of the process for determining when an agent should switch from the classical SDS *explore-exploit* behaviour to a new, so called, '*terminating*' behaviour which we term the *independent* and *confirmation* halting criteria.

In these variants agents can take on an additional behaviour in which they enter a new state we define 'Terminating', wherein their hypothesis becomes fixed and they subsequently seek to actively remove agents from the dynamic swarm[8] and give them their own (now fixed) termination hypothesis (analogous

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[8] Standard SDS has previously been shown to be a global search algorithm[51] - it will eventually converge to the global best solution in a given search space; by removing agents form the swarm, relative to standard SDS the number of potential agents remaining available for explore-exploit behaviour is reduced.

to the ‘post-quorum’ behavioural state in ants of the genus *Temnothorax*, wherein post-quorum ants literally carry other ants they encounter to the new nest site).

As this decision making process successively removes agents from the swarm we name the removal behaviour *Terminating*, and the whole process *Reducing SDS*. In this way a collective ‘decision’ is made (and the local halting condition met) when all agents are either active and/or have been removed from the swarm.

7.3 ‘Independent’ termination behaviour

In *independent reducing SDS* we relax the assumption that all SDS agents update synchronously in iterative ‘cycles’ (wherein one such cycle corresponds to all agents being updated).

Algorithm 2 independent SDS

```
procedure step($swarm$, $search\_space$)
    $swarm = shuffle(swarm)$
    for each $agent$ in $swarm$ do
        $polled\_agent = swarm.random\_agent()$                ▷ Diffusion behaviour
        if Both agents are inactive then
            Both agents randomise hypothesis
        else if One agent is inactive and other is active but not terminating then
            Inactive agent assumes active agent’s hypothesis
        else if One agent is terminating then
            Other agent is removed from the swarm
        else if Agents share a hypothesis then
            Both agents become terminating
        if not $agent.terminating$ then                      ▷ Testing behaviour
            $test\_result = perform\_random\_test(hypothesis)$
            $agent.active = test\_result$
```

In independent SDS agents update independently[9]; furthermore the diffusion process endeavours to mirror the behaviour of real ants when two ants ’meet’:

- if neither agent is active both reselect new random hypotheses;
- one agent is inactive and other is active but not terminating then the inactive agent assumes active agent’s hypothesis.
- one of the agents is in terminating mode then the other is ‘removed’ from the swarm and give (placed at) this hypothesis.
- if the two agents meet that both have the same hypothesis then both switch to *terminating* mode.

The above process is more fully outlined in **Algorithm 2**.

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[9] To facilitate the use of homogenous performance metrics, we assume that in a population of k agents, k single asynchronous updates corresponds to one standard synchronous iteration cycle.

7.4 ‘Confirmation’ termination behaviour

Since its inception in 1989 [7] a substantial body of algorithmic analysis (describing the theoretical behaviour of SDS), empirical studies and practical applications have been published (for a recent review see [2]). To more readily facilitate the future use of these results in both local termination variants and potentially to extend this work to the analysis of real ants, we also present a further simplification of independent SDS to a second reducing behaviour that more closely aligns with that of standard SDS diffusion; we define this termination mode *confirmation reducing SDS.*

Algorithm 3 Confirmation SDS

```
procedure step(swarm, search_space)
    for each agent in swarm do                                        ▷ Diffuse Phase
        polled_agent = swarm.random_agent()
        if agent.active then
            hyp_1 = agent.hypothesis
            hyp_2 = polled_agent.hypothesis
            if polled_agent.active and hyp_1 == hyp_2 then
                agent.terminating == True
        else
            if polled_agent.active then
                if polled_agent.terminating then
                    swarm.remove(agent)
                else
                    agent.hypothesis = polled_agent.hypothesis
            else
                agent.hypothesis = search_space.random_hypothesis()
    for each agent in swarm do                                        ▷ Test Phase
        test_result = perform_random_test(hypothesis)
        agent.active = test_result
```

In confirmation reduction SDS agents are assumed to update synchronously and the diffusion of information is changed to more closely resemble that of classical dual mode (passive and active) recruitment SDS[49]. In *confirmation* SDS an active agent polls random agents in the diffusion phase. Active agents become *terminating* if their polled agent is also active and both agents share a hypothesis. The agent is then locked into being active, maintaining that hypothesis. If an inactive agent polls a terminating agent, the inactive agent is removed from the population (see **Algorithm 3** for details).

Thus *independent* SDS has two major distinctions from confirmation SDS, firstly no distinction is made between the polling agent and the polled agent, the effect of their diffusion is resolved simultaneously; secondly, rather than alternating diffusion and test phases, agents are randomly selected to **individually** perform a *hypothesis diffusion* behaviour immediately followed by a *hypothesis test* behaviour. These two features are intended to more closely model the actual termination behaviour of real ants.

8 Experiments

A series of experiments was performed to investigate the diffusion behaviour of the two new halting criteria over a variety of search parameters to establish (a) if the algorithms' gross behaviour remains characteristic of SDS and (b) to evaluate their robustness over a variety of search parameters (which effectively characterise the quality of the putative best solution, α $(0 \leq \alpha < 1)$, relative to β, $(0 \leq \beta < 1)$, the quality of the distractor solution[10]); in the 'ant migration' problem, α is analogous to a measure of the quality of the potential new nest site and β effectively a measure of the quality of the original nest.

In all experiments the population is initialised with one agent maintaining the hypothesis representing the potential best solution and the probability of an agent randomly selecting the hypothesis of the potential best solution is set to zero; this ensures that only the *diffusion* behaviour of the algorithm is explored[11].

In the first experiment each of the four termination functions (weak, strong, independent and confirmation) was modelled in a population of 10000 agents, one of which was active and at the solution hypothesis at time zero, with all other agents set inactive pointing to the 'noise' hypothesis. The algorithm was then evaluated 25 times from these conditions against a range of possible values of α and β (from 0 to 0.875 with a step of 0.125). The number of times the algorithm successfully halted within 250 iterations was recorded as was the mean average number of iterations before halting in these cases.

In the case of weak and strong halting SDS, halting was considered successful if the halting criterion was reached. All four algorithms would also halt if all agents were active at the solution hypothesis, as this is analogous to a successful migration of agents to an optimal state. Two further halting conditions were included, when the algorithm had run for more than a specified number of iterations and when all agents held the noise hypothesis. Any experiment that halted for these reasons was considered unsuccessful.

In a second experiment the four algorithms were run against fixed values of α and β which the first experiment had shown would be likely to successfully halt. The state of all agents was recorded at every iteration and number of agents (as a proportion of the total population) in various states was graphed over time to visualise the characteristic behaviour of the halting criteria (see Figure 6).

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[10] β defines a "uniform random noise" hypothesis; an aggregate of all the possible hypotheses an agent could have other than the putative solution hypothesis.

[11] These parameters define a problem analogous to the search space being infinitely large, wherein the only way an agent can adopt the 'best' solution is to receive it via diffusion from an active agent.

8.1 Experiment 1

The graphs in Figure 5 and Figure 6 plot the cluster size of a population of 10,000 SDS agents over time using the four different halting criteria - weak, strong, independent and confirmation - with search parameters (solution hypothesis ($\alpha = 0.750$, noise hypothesis ($\beta = 0.375$); it is noted that the shapes of cluster sizes using all four termination criteria broadly follow the characteristic SDS S-shaped convergence curve.

8.2 Experiment 2

Table 2 lists i the average number of iterations before halting and c the number of times that the algorithm successfully halted for a SDS experiment for all four algorithms using a population of 10,000 agents across a variety of parameter values of the noise hypothesis (β) and solution hypothesis (α).

NB. Pairs of values for α and β for which *all four* algorithms failed to converge 25 times out of 25 are not listed.

Examining the results presented in **Table 2**, the following comparative observations can be made:

weak versus strong halting the convergence time for weak and strong halting are almost identical whilst their robustness is similar (strong halting is more robust in 8 cases and less robust in 8);

strong halting versus independent reduction on average the convergence time is 3.3 times faster for independent whilst its robustness is similar (strong halting is more robust in 11 cases, less robust in 6);

strong halting versus confirmation reduction on average the convergence time is around 2.8 times faster for confirmation whilst its robustness is similar (strong halting is more robust in 8 cases, less robust in 6);

9 Comparison of ant quorum-sensing and SDS local halting criteria

The termination criterion of an SDS algorithm can be considered analogous to the ant quorum threshold, i.e. the point at which ants cease to search, and instead implement their chosen decision (Figure 1). Once an ant has sensed that quorum is met, it does not re-visit that decision and is essentially committed to that hypothesis. This process can be compared to theoretical SDS local halting criteria that are able to terminate a decision-process rapidly. Specifically, the *independent SDS* termination criterion can be considered analogous to a process whereby an ant's decision to enter a post-quorum state is influenced by a one-to-one reinforcement of its own view. That is, Scout A will enter a post-quorum state (=terminating) for Nest A if Scout A has accepted nest A and returned to the home nest to recruit, but is instead itself then recruited by scout B back to nest A. This recruitment event would serve to reinforce Scout

Table 2. Mean average iterations before termination for three different halting criteria (*strong, independent and confirmation*) over varying quality of solutions

		strong		independent		confirmation	
β	α	i	c	i	c	i	c
0.000	0.625	151.2	17	42.0	6	46.2	14
0.000	0.750	126.8	20	25.8	16	27.8	18
0.000	0.875	118.2	21	20.0	22	21.1	21
0.125	0.625	195.0	7	52.2	12	58.4	7
0.125	0.750	130.4	11	29.6	15	34.1	16
0.125	0.875	122.0	23	22.9	21	25.8	20
0.250	0.625	216.0	1	77.1	7	88.6	5
0.250	0.750	138.7	17	35.5	16	42.2	15
0.250	0.875	125.5	22	26.0	21	31.4	22
0.375	0.625	100.0	1	232.0	1	244.0	1
0.375	0.750	165.6	14	48.5	12	56.7	16
0.375	0.875	131.5	21	30.0	19	38.5	20
0.500	0.750	212.0	5	74.2	12	87.9	7
0.500	0.875	140.5	18	38.5	13	51.5	16
0.625	0.750	100.0	4	150.0	3	238.0	1
0.625	0.875	161.9	18	50.6	14	73.5	20
0.750	0.875	211.0	7	92.4	14	142.5	13
0.875	0.875	100.0	1	–	0	–	0

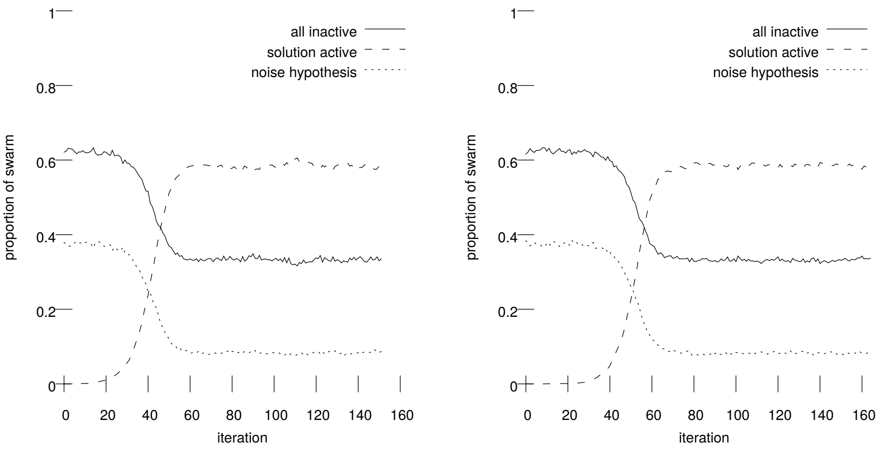

Fig. 5. Cluster size evolution over time for SDS using the *weak halting criterion* (left) and the *strong halting criterion* (right). The x-axis counts iterations, the y-axis shows cluster size as a proportion of the entire population. The behaviours are practically identical, the slightly later convergence of the strong halting SDS is a result of the inherent randomness. The positive feedback effect can be seen in the sharp S-curve of the solution cluster size.

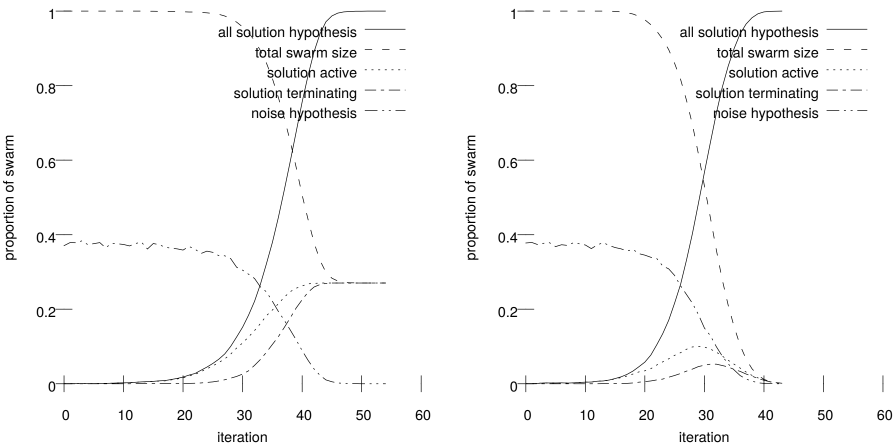

Fig. 6. Cluster size evolution over time for SDS using *confirmation SDS* (left) and *independent SDS* (right). The x-axis counts iterations, the y-axis shows cluster size as a proportion of the entire population. Both show an accelerating growth in the number of agents at the solution hypothesis followed by a similar growth of terminating agents at the solution hypothesis until the entire swarm is either active (in the case of confirmation SDS) or removed from the swarm (in the case of independent SDS).

A's initial judgement. *Temnothorax* ants are indeed sometimes recruited back to nests they have already visited (Figure 1), so there is potential for this 'reinforcement recruitment' process to play a role for ant colonies. For example, 'reinforcement recruitment' could cause ants to enter a post-quorum state at a lowered encounter rate. This would help extra rapid acceptance of a nest if there were only one new nest site available. This idea could be tested empirically, ideally in a complex arena that would promote tandem-running behaviour, allowing communication of preference.

The main limitation on the use of the *independent SDS* termination criterion by the ants is that the criterion relies on ants being able to communicate their current preference, which they are able to do only by the slow and fairly infrequent process of tandem-running. Empirical observations have shown that scouting ants can judge quorum to have been reached (through encounter rate) without having followed a tandem run, so clearly they are not relying solely on *independent*-like rules. An analogous - Swarm Intelligence - process has however been observed in house-hunting honeybees.

Honeybees, in contrast to ants, are able to directly 'poll' another scouting bee for its preference, without having to actually visit that bee's preferred site. This is because honeybees communicate the location of their preferred site using a waggle-dance performed back at the main swarm and honeybees are able to determine whether a waggle-dance indicates their chosen site or not, without flying to the location indicated by the dance ([69]). This gives the bees more power to use *independent*-like rules in their decision-making - and honeybees do indeed use the unanimity of the waggle-dances in their assessment of when to terminate the search and relocate to a new site ([68]). While in this way, honeybee decision-behaviour is closer to *confirmation SDS* than *Temnothorax* house-hunting is, on the other hand we note that bees use a one-to-many broadcasting communication when they waggle-dance, rather than pair-wise communication as used by tandem running ants and SDS agents.

10 Conclusion

This paper has looked at cooperative decision making in house-hunting ants and in the Stochastic Diffusion Search algorithm. Decision making in standard SDS is typically based on the use of one of two well established 'halting' functions - the Strong and Weak Halting criteria - both of which entail *global* access to the activity of the SDS population as a whole. Conversely in this paper, inspired by the quorum sensing mechanism deployed by some species of ants in nest moving, we have successfully demonstrated two new *local* termination criteria for SDS which have broadly been demonstrated to have similar behaviour to the standard SDS meta-heuristic using strong and weak halting criteria (in terms of their robustness to noise). Furthermore, it is observed that the use of a local halting mechanism substantially speeds up the collective decision making time; both independent and confirmation terminate are around three times faster than via the use of the strong halting criterion.

Although the independent and confirmation termination processes described in this paper found inspiration from the nest hunting behaviour of *Temnothorax* ants, we do not claim that the nest selection behaviour of these ants is isomorphic to SDS :- one critical difference between the two systems is that SDS relies on its agents being easily able to communicate their current hypothesis to each other, whereas *Temnothorax* ants are only able to do this by the slow [and relatively infrequent] process of tandem-running. Empirical observations have shown that scouting ants can judge quorum to have been reached (through encounter rate) without having followed a tandem run, so clearly *Temnothorax* ants do not solely rely on *independent*-SDS like termination rules. In this context, future research will investigate the degree to which appropriately modified SDS characterisations can be used to describe the behaviour of *Temnothorax* ants.

In conclusion we suggest that the successful deployment of local halting criteria (a) significantly simplifies the implementation of SDS on parallel computational hardware and (b) potentially open up analogical study of ant and honeybee decision-making behaviour through the transformational lens of SDS theory, where there is a significant extant body of proven mathematical results; for example outlining: speed of convergence, robustness, time complexity, stability of solutions etc.[2]; further mathematical study aims to better understand the empirical performance of the local termination criteria described herein, using extant mathematical models of SDS behaviour (ibid).

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